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Project B - Arjun Sekhar (Student ID: 18653508)

1)

Question: Consider the following construction. Start with a horizontal line of length 100 units. Fix two angles $\alpha = 62$, $\beta = 60$ and length l=40. At the end of the line draw another horizontal line of length l. Then start to zigzag by drawing a sequence of lines 100, l, 100, 1, 100,..., the anticlockwise angles these lines make with the horizontal being α, β, 2α, 2β, 3α, 3β... Write a program to repeat this construction 180 times. Produce the graph.

```
(*Working*)
```

Answer: So we start off denoting the angles and the length of the line. We must note that for the length of the line, given that l is added onto the original horizontal line, we make the length equal to 100+l

```
Graphics[Line[{{0, 0}, {100, 0}}]]
```

```
In[281]:= α = 62
Out[281]= 62

In[282]:= β = 60
Out[282]= 60

In[283]:= l = 40
Out[283]= 40
```

Here we show how the line of length 100+l looks, just to show that we have taken into account l=40. We will remember this point for the end, as we attempt to create our construction with 180 repeated lines.

```
Graphics[Line[{{0, 0}, {100, 0}, {100 + l, 0}}]]
```

To start off, we find the first five lines and their respective changes in direction with respect to α and β. We combine α and β by employing the 'Riffle' function, which is a step we follow in the next step. 'Riffle' function enables the alternation of α, β, 2α, 2β, 3α, 3β etc.

```
Table[{100, i α °}, {i, 90}]
```

```
{{100, 62 °}, {100, 124 °}, {100, 186 °}, {100, 248 °}, {100, 310 °}, {100, 372 °},
 {100, 434 °}, {100, 496 °}, {100, 558 °}, {100, 620 °}, {100, 682 °}, {100, 744 °},
 {100, 806 °}, {100, 868 °}, {100, 930 °}, {100, 992 °}, {100, 1054 °}, {100, 1116 °},
 {100, 1178 °}, {100, 1240 °}, {100, 1302 °}, {100, 1364 °}, {100, 1426 °}, {100, 1488 °},
 {100, 1550 °}, {100, 1612 °}, {100, 1674 °}, {100, 1736 °}, {100, 1798 °}, {100, 1860 °},
 {100, 1922 °}, {100, 1984 °}, {100, 2046 °}, {100, 2108 °}, {100, 2170 °}, {100, 2232 °},
 {100, 2294 °}, {100, 2356 °}, {100, 2418 °}, {100, 2480 °}, {100, 2542 °}, {100, 2604 °},
 {100, 2666 °}, {100, 2728 °}, {100, 2790 °}, {100, 2852 °}, {100, 2914 °},
 {100, 2976 °}, {100, 3038 °}, {100, 3100 °}, {100, 3162 °}, {100, 3224 °},
 {100, 3286 °}, {100, 3348 °}, {100, 3410 °}, {100, 3472 °}, {100, 3534 °},
 {100, 3596 °}, {100, 3658 °}, {100, 3720 °}, {100, 3782 °}, {100, 3844 °},
 {100, 3906 °}, {100, 3968 °}, {100, 4030 °}, {100, 4092 °}, {100, 4154 °},
 {100, 4216 °}, {100, 4278 °}, {100, 4340 °}, {100, 4402 °}, {100, 4464 °}, {100, 4526 °},
 {100, 4588 °}, {100, 4650 °}, {100, 4712 °}, {100, 4774 °}, {100, 4836 °}, {100, 4898 °},
 {100, 4960 °}, {100, 5022 °}, {100, 5084 °}, {100, 5146 °}, {100, 5208 °}, {100, 5270 °},
 {100, 5332 °}, {100, 5394 °}, {100, 5456 °}, {100, 5518 °}, {100, 5580 °}}
```

```
Table[{l, i β °}, {i, 90}]
```

```
{{40, 60 °}, {40, 120 °}, {40, 180 °}, {40, 240 °}, {40, 300 °}, {40, 360 °},
 {40, 420 °}, {40, 480 °}, {40, 540 °}, {40, 600 °}, {40, 660 °}, {40, 720 °},
 {40, 780 °}, {40, 840 °}, {40, 900 °}, {40, 960 °}, {40, 1020 °}, {40, 1080 °},
 {40, 1140 °}, {40, 1200 °}, {40, 1260 °}, {40, 1320 °}, {40, 1380 °}, {40, 1440 °},
 {40, 1500 °}, {40, 1560 °}, {40, 1620 °}, {40, 1680 °}, {40, 1740 °}, {40, 1800 °},
 {40, 1860 °}, {40, 1920 °}, {40, 1980 °}, {40, 2040 °}, {40, 2100 °}, {40, 2160 °},
 {40, 2220 °}, {40, 2280 °}, {40, 2340 °}, {40, 2400 °}, {40, 2460 °}, {40, 2520 °},
 {40, 2580 °}, {40, 2640 °}, {40, 2700 °}, {40, 2760 °}, {40, 2820 °}, {40, 2880 °},
 {40, 2940 °}, {40, 3000 °}, {40, 3060 °}, {40, 3120 °}, {40, 3180 °}, {40, 3240 °},
 {40, 3300 °}, {40, 3360 °}, {40, 3420 °}, {40, 3480 °}, {40, 3540 °}, {40, 3600 °},
 {40, 3660 °}, {40, 3720 °}, {40, 3780 °}, {40, 3840 °}, {40, 3900 °}, {40, 3960 °},
 {40, 4020 °}, {40, 4080 °}, {40, 4140 °}, {40, 4200 °}, {40, 4260 °}, {40, 4320 °},
 {40, 4380 °}, {40, 4440 °}, {40, 4500 °}, {40, 4560 °}, {40, 4620 °}, {40, 4680 °},
 {40, 4740 °}, {40, 4800 °}, {40, 4860 °}, {40, 4920 °}, {40, 4980 °}, {40, 5040 °},
 {40, 5100 °}, {40, 5160 °}, {40, 5220 °}, {40, 5280 °}, {40, 5340 °}, {40, 5400 °}}
```

It is here that we use the 'Riffle' function, and thus we can spot the incremental alternation. This means that for every length of 100 units, the angle increments by 62°, and for every length 40 units, the angle increments by 60°. Essentially we are combining the results from above and enabling the function to alternate. This pattern alternates between these respective measurements, thus generating the following numerical output below:

```
Riffle[Table[{100, i α °}, {i, 90}], Table[{l, i β °}, {i, 90}]]
```

```
{{100, 62 °}, {40, 60 °}, {100, 124 °}, {40, 120 °}, {100, 186 °}, {40, 180 °},
 {100, 248 °}, {40, 240 °}, {100, 310 °}, {40, 300 °}, {100, 372 °}, {40, 360 °},
 {100, 434 °}, {40, 420 °}, {100, 496 °}, {40, 480 °}, {100, 558 °}, {40, 540 °},
 {100, 620 °}, {40, 600 °}, {100, 682 °}, {40, 660 °}, {100, 744 °}, {40, 720 °},
 {100, 806 °}, {40, 780 °}, {100, 868 °}, {40, 840 °}, {100, 930 °}, {40, 900 °},
 {100, 992 °}, {40, 960 °}, {100, 1054 °}, {40, 1020 °}, {100, 1116 °}, {40, 1080 °},
 {100, 1178 °}, {40, 1140 °}, {100, 1240 °}, {40, 1200 °}, {100, 1302 °}, {40, 1260 °},
 {100, 1364 °}, {40, 1320 °}, {100, 1426 °}, {40, 1380 °}, {100, 1488 °}, {40, 1440 °},
 {100, 1550 °}, {40, 1500 °}, {100, 1612 °}, {40, 1560 °}, {100, 1674 °}, {40, 1620 °},
 {100, 1736 °}, {40, 1680 °}, {100, 1798 °}, {40, 1740 °}, {100, 1860 °}, {40, 1800 °},
 {100, 1922 °}, {40, 1860 °}, {100, 1984 °}, {40, 1920 °}, {100, 2046 °}, {40, 1980 °},
 {100, 2108 °}, {40, 2040 °}, {100, 2170 °}, {40, 2100 °}, {100, 2232 °}, {40, 2160 °},
 {100, 2294 °}, {40, 2220 °}, {100, 2356 °}, {40, 2280 °}, {100, 2418 °}, {40, 2340 °},
 {100, 2480 °}, {40, 2400 °}, {100, 2542 °}, {40, 2460 °}, {100, 2604 °}, {40, 2520 °},
 {100, 2666 °}, {40, 2580 °}, {100, 2728 °}, {40, 2640 °}, {100, 2790 °}, {40, 2700 °},
 {100, 2852 °}, {40, 2760 °}, {100, 2914 °}, {40, 2820 °}, {100, 2976 °}, {40, 2880 °},
 {100, 3038 °}, {40, 2940 °}, {100, 3100 °}, {40, 3000 °}, {100, 3162 °}, {40, 3060 °},
 {100, 3224 °}, {40, 3120 °}, {100, 3286 °}, {40, 3180 °}, {100, 3348 °}, {40, 3240 °},
 {100, 3410 °}, {40, 3300 °}, {100, 3472 °}, {40, 3360 °}, {100, 3534 °}, {40, 3420 °},
 {100, 3596 °}, {40, 3480 °}, {100, 3658 °}, {40, 3540 °}, {100, 3720 °}, {40, 3600 °},
 {100, 3782 °}, {40, 3660 °}, {100, 3844 °}, {40, 3720 °}, {100, 3906 °}, {40, 3780 °},
 {100, 3968 °}, {40, 3840 °}, {100, 4030 °}, {40, 3900 °}, {100, 4092 °}, {40, 3960 °},
 {100, 4154 °}, {40, 4020 °}, {100, 4216 °}, {40, 4080 °}, {100, 4278 °}, {40, 4140 °},
 {100, 4340 °}, {40, 4200 °}, {100, 4402 °}, {40, 4260 °}, {100, 4464 °}, {40, 4320 °},
 {100, 4526 °}, {40, 4380 °}, {100, 4588 °}, {40, 4440 °}, {100, 4650 °}, {40, 4500 °},
 {100, 4712 °}, {40, 4560 °}, {100, 4774 °}, {40, 4620 °}, {100, 4836 °}, {40, 4680 °},
 {100, 4898 °}, {40, 4740 °}, {100, 4960 °}, {40, 4800 °}, {100, 5022 °}, {40, 4860 °},
 {100, 5084 °}, {40, 4920 °}, {100, 5146 °}, {40, 4980 °}, {100, 5208 °}, {40, 5040 °},
 {100, 5270 °}, {40, 5100 °}, {100, 5332 °}, {40, 5160 °}, {100, 5394 °}, {40, 5220 °},
 {100, 5456 °}, {40, 5280 °}, {100, 5518 °}, {40, 5340 °}, {100, 5580 °}, {40, 5400 °}}
```

We use the ‘FoldList’ function to establish the start length and angle measurement. Given that we start at 100 + 40 = 140, we state this in our code, where l = 40, and our start length is 100 units, thus making it (100 + l) = 100 + 40 = 140 units. We employ the *Riffle* function and the results are displayed below, labelled as *RoozbehHazrat* (the best lecturer in UWS).

In[284]:=
```
RoozbehHazrat = FoldList[AngleVector, {100 + l, 0},
  Riffle[Table[{100, i α °}, {i, 90}], Table[{l, i β °}, {i, 90}]]]
```

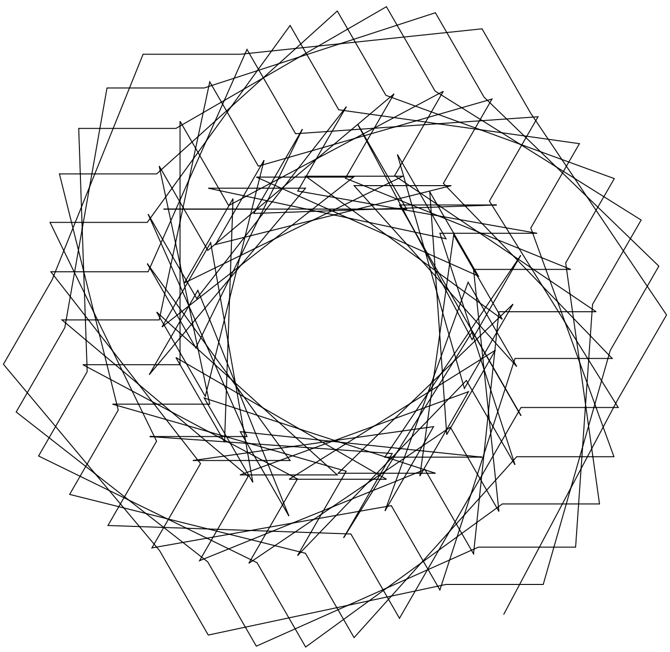
Out[284]=
```
{{140, 0}, {140 + 100 Sin[28 °], 100 Cos[28 °]}, ⋯ 177 ⋯, {⋯ 1 ⋯, ⋯ 1 ⋯},
 {40 + 25 (-1 - √5) + 25 (1 - √5) + 25 (-1 + √5) + 25 (1 + √5),
  -200 + 100 √3 + 50 (-1 - √5) + 50 (1 - √5) + 200 √(5/8 - √5/8) + 200 √(5/8 + √5/8) -
   200 Cos[2 °] + 200 Cos[4 °] + 200 Cos[6 °] + 200 Cos[8 °] - 200 Cos[10 °] -
   200 Cos[12 °] - 200 Cos[14 °] + 200 Cos[16 °] + 200 Cos[20 °] - 200 Cos[22 °] -
   200 Cos[24 °] - 200 Cos[26 °] + 200 Cos[28 °] + ⋯ 14 ⋯ + 200 Sin[8 °] +
   200 Sin[10 °] + 200 Sin[12 °] - 200 Sin[14 °] - 200 Sin[16 °] + 200 Sin[20 °] +
   200 Sin[22 °] + 200 Sin[24 °] - 200 Sin[26 °] - 200 Sin[28 °] + 200 Sin[32 °] +
   200 Sin[34 °] - 200 Sin[38 °] - 200 Sin[40 °] - 200 Sin[42 °] + 200 Sin[44 °]}}
```

large output | show less | show more | show all | set size limit...

Given that we call our list as *RoozbehHazrat*, we attempt graphing this using the *Graphics* function, and get the following output:

In[286]:=
```
Graphics[Line[RoozbehHazrat]]
```

Out[286]=



Another aspect we must note is the start point. We start at {0,0} and the graph above gives the start at{140,0}. Hence this is where the base function comes of its own. We start at {0,0} and move 100 units horizontally, and then a further l = 40 units (as what the question asks), which makes the points flow from {0,0} to {100,0}, and then to {100 + l,0}. We join them together with *Join*, with our graph *RoozbehHazrat* using *Graphics* and then produce the following construction:

In[285]:=
```
base = {{0, 0}, {100, 0}, {100 + l, 0}}
```
Out[285]=
```
{{0, 0}, {100, 0}, {140, 0}}
```

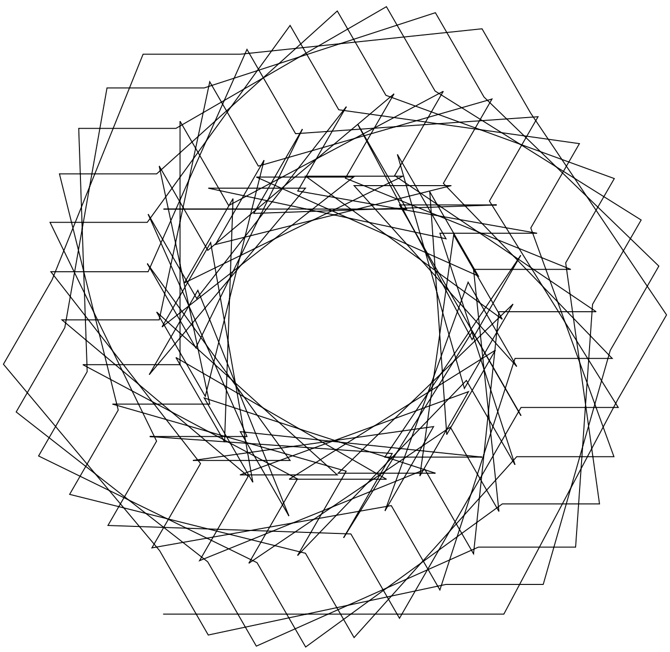
In[287]:=
```
Join[base, RoozbehHazrat]
```

Out[287]=

$$\{\{0,0\},\{100,0\}, \cdots 180 \cdots, \{25(-1-\sqrt{5})+25(1-\cdots 1\cdots)+\cdots 1\cdots+25(1+\sqrt{5}), \cdots 1\cdots\}, \{40+25(-1-\sqrt{5})+25(1-\sqrt{5})+25(-1+\sqrt{5})+25(1+\sqrt{5}),$$
$$-200+100\sqrt{3}+50(-1-\sqrt{5})+50(1-\sqrt{5})+200\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}+200\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}-$$
$$200\,\mathrm{Cos}[2°]+200\,\mathrm{Cos}[4°]+200\,\mathrm{Cos}[6°]+200\,\mathrm{Cos}[8°]-200\,\mathrm{Cos}[10°]-$$
$$200\,\mathrm{Cos}[12°]-200\,\mathrm{Cos}[14°]+200\,\mathrm{Cos}[16°]+200\,\mathrm{Cos}[20°]-200\,\mathrm{Cos}[22°]-$$
$$200\,\mathrm{Cos}[24°]-200\,\mathrm{Cos}[26°]+200\,\mathrm{Cos}[28°]+\cdots 14\cdots+200\,\mathrm{Sin}[8°]+$$
$$200\,\mathrm{Sin}[10°]+200\,\mathrm{Sin}[12°]-200\,\mathrm{Sin}[14°]-200\,\mathrm{Sin}[16°]+200\,\mathrm{Sin}[20°]+$$
$$200\,\mathrm{Sin}[22°]+200\,\mathrm{Sin}[24°]-200\,\mathrm{Sin}[26°]-200\,\mathrm{Sin}[28°]+200\,\mathrm{Sin}[32°]+$$
$$200\,\mathrm{Sin}[34°]-200\,\mathrm{Sin}[38°]-200\,\mathrm{Sin}[40°]-200\,\mathrm{Sin}[42°]+200\,\mathrm{Sin}[44°]\}\}$$

large output | show less | show more | show all | set size limit...

```
Graphics[Line[Join[base, RoozbehHazrat]]]
```



3)

Question: Write a function copymatrix[] such that if A and B are matrices then copymatrix[A,B,col,row] makes a copy of B inside A by replacing A[[i+col,j+row]] by B[[i,j]]. You may assume that the copy of B fits inside A. Hence write a function bigmatrix[] such that if M1, M2, M3 and M4 are (n x n) matrices, then bigmatrix[M1, M2, M3, M4] is the 2n x 2n matrix formed by arranging the entries in these four matrices thus:

$$\begin{pmatrix} M1 & M2 \\ M3 & M4 \end{pmatrix}.$$

Define a sequence of matrices A_n of size (2^n x 2^n) by defining

$$A_1 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

and if n ≥ 1

$$A_{n+1} = \begin{pmatrix} A_n & -A_n \\ A_n & A_n \end{pmatrix}$$

Find the determinant of A_n for n = 1, 2, 3, 4, 5.

Let

$$J = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } o = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Check the following identity for a 2×2 matrix A,

$$\begin{pmatrix} A & o \\ o & A^{-1} \end{pmatrix} = \begin{pmatrix} J & A \\ o & J \end{pmatrix}\begin{pmatrix} J & o \\ -A^{-1} & J \end{pmatrix}\begin{pmatrix} J & A \\ o & J \end{pmatrix}\begin{pmatrix} o & -J \\ J & 0 \end{pmatrix}$$

```
(*Working*)
```

Answer: Here we start off by using the array function to create our general formula for a our matrix size (which we aim to make it 10 by 10). We make the dimensions of our matrix A as ‘m’ and ‘n’ and then substitute m=10 and n=10 to create our (10×10) matrix.

In[270]:=
```
A[m_, n_] := Array[a_{#1,#2} &, {m, n}]
```

In[271]:=
```
A[10, 10] // MatrixForm
```

Out[271]//MatrixForm=

$$\begin{pmatrix}
a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9} & a_{1,10} \\
a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} \\
a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} \\
a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} & a_{4,8} & a_{4,9} & a_{4,10} \\
a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6} & a_{5,7} & a_{5,8} & a_{5,9} & a_{5,10} \\
a_{6,1} & a_{6,2} & a_{6,3} & a_{6,4} & a_{6,5} & a_{6,6} & a_{6,7} & a_{6,8} & a_{6,9} & a_{6,10} \\
a_{7,1} & a_{7,2} & a_{7,3} & a_{7,4} & a_{7,5} & a_{7,6} & a_{7,7} & a_{7,8} & a_{7,9} & a_{7,10} \\
a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & a_{8,8} & a_{8,9} & a_{8,10} \\
a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & a_{9,9} & a_{9,10} \\
a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & a_{10,10}
\end{pmatrix}$$

We apply the same structure for B, except naming the rows and columns as ‘p’ and ‘q’ respectively, enabling us to create our general formula for B. Here, we aim to make a 3 by 4 matrix, thereby making p=3 and q=4.

```
In[272]:= B[p_, q_] := Array[b#1,#2 &, {p, q}]

In[273]:= B[3, 4] // MatrixForm
```

Out[273]//MatrixForm=

$$\begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \\ b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} \end{pmatrix}$$

We split our matrix into P1, P2, P3 and P4, and leave space for B in the middle (it can really be wherever you want, except for convenience sake I have chosen the middle). The variables ‘i’ and ‘j’ defines the location in which matrix B is placed in matrix A (which means for ‘i’, it is placed at row 5, and for ‘j’ it is placed in column 4). We use ‘i’ and ‘j’ to define the sizes of our smaller matrices, and so they get used to . The variables ‘m’ and ‘n’ defines the structure of matrix A (which is a 10 by 10 matrix) and lastly ‘p’ and ‘q’ are the dimensions of B (which is a 3 by 4 matrix). So the first one below is P1, which has rows spanning from 1 to 10 and columns spanning from 1 to 3 It is a 10 by 3 matrix, and lies to the left of matrix B:

```
In[274]:= P1[i_, j_, m_, n_, p_, q_] := Take[A[m, n], {1, m}, {1, j - 1}]

In[275]:= P1[5, 4, 10, 10, 3, 4] // MatrixForm
```

Out[275]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \\ a_{4,1} & a_{4,2} & a_{4,3} \\ a_{5,1} & a_{5,2} & a_{5,3} \\ a_{6,1} & a_{6,2} & a_{6,3} \\ a_{7,1} & a_{7,2} & a_{7,3} \\ a_{8,1} & a_{8,2} & a_{8,3} \\ a_{9,1} & a_{9,2} & a_{9,3} \\ a_{10,1} & a_{10,2} & a_{10,3} \end{pmatrix}$$

The next matrix defined below is P2. Here the matrix rows span from 1 to 4, and the columns span from 4 to 7. It is a 4 by 4 matrix, and lies on top of matrix B:

```
In[276]:= P2[i_, j_, m_, n_, p_, q_] := Take[A[m, n], {1, i - 1}, {j, j + q - 1}]

P2[5, 4, 10, 10, 3, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} \\ a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} \\ a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} \\ a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} \end{pmatrix}$$

The matrix defined below is P3. Here the matrix rows span from 8 to 10, and the columns span from 4 to 7. It is a 3 by 5 matrix and lies underneath matrix B:

```
In[277]:= P3[i_, j_, m_, n_, p_, q_] := Take[A[m, n], {p + i, m}, {j, j + q - 1}]

          P3[5, 4, 10, 10, 3, 4] // MatrixForm
```

$$\begin{pmatrix} a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} \\ a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} \\ a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} \end{pmatrix}$$

The matrix defined below is P4. Here the matrix rows span from 1 to 10, and the columns span from 8 to 10. It is a 10 by 3 matrix and lies to the right of matrix B.

```
In[278]:= P4[i_, j_, m_, n_, p_, q_] := Take[A[m, n], {1, m}, {j + q, n}]

          P4[5, 4, 10, 10, 3, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,8} & a_{1,9} & a_{1,10} \\ a_{2,8} & a_{2,9} & a_{2,10} \\ a_{3,8} & a_{3,9} & a_{3,10} \\ a_{4,8} & a_{4,9} & a_{4,10} \\ a_{5,8} & a_{5,9} & a_{5,10} \\ a_{6,8} & a_{6,9} & a_{6,10} \\ a_{7,8} & a_{7,9} & a_{7,10} \\ a_{8,8} & a_{8,9} & a_{8,10} \\ a_{9,8} & a_{9,9} & a_{9,10} \\ a_{10,8} & a_{10,9} & a_{10,10} \end{pmatrix}$$

Now that we have split the matrices (with space left in the centre of the overall matrix), we now bring matrix B into the picture by placing it between P2 and P3. We employ ‘Join’ to merge the matrices and ‘ArrayFlatten’ to create it into a whole matrix:

```
ArrayFlatten[Join[{{P2[5, 4, 10, 10, 3, 4]}, {B[3, 4]}, {P3[5, 4, 10, 10, 3, 4]}}, 2]] //
 MatrixForm
```

$$\begin{pmatrix} a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} \\ a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} \\ a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} \\ a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} \\ b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} \\ b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} \\ a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} \\ a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} \\ a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} \end{pmatrix}$$

We bring the notion of CopyMatrix[] into the picture, with space for our variables ‘i’, ‘j’, ‘m’, ‘n’, ‘p’ and ‘q’, which enables us to keep a matrix B within any size of A (provided matrix A is always longer and wider than matrix B). Moreover we have used our ‘ArrayFlatten’ and ‘Join’ functions from above and merged it further with ‘Join’ to combine P1, P2, B, P3 and P4. Hence as we desired, we have an overall formula to place matrix B any matrix of A, provided matrix A is always bigger and longer than matrix B.

```
In[279]:= CopyMatrix[i_, j_, m_, n_, p_, q_] := Join[Join[P1[i, j, m, n, p, q],
     ArrayFlatten[Join[{{P2[i, j, m, n, p, q]}, {B[p, q]}, {P3[i, j, m, n, p, q]}}, 2]],
     P4[i, j, m, n, p, q], 2]] // MatrixForm

CopyMatrix[5, 4, 10, 10, 3, 4]
```

there is more elegant way to write this…. 13/15

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9} & a_{1,10} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} & a_{3,6} & a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & a_{4,6} & a_{4,7} & a_{4,8} & a_{4,9} & a_{4,10} \\ a_{5,1} & a_{5,2} & a_{5,3} & b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} & a_{5,8} & a_{5,9} & a_{5,10} \\ a_{6,1} & a_{6,2} & a_{6,3} & b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} & a_{6,8} & a_{6,9} & a_{6,10} \\ a_{7,1} & a_{7,2} & a_{7,3} & b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} & a_{7,8} & a_{7,9} & a_{7,10} \\ a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & a_{8,8} & a_{8,9} & a_{8,10} \\ a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & a_{9,9} & a_{9,10} \\ a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & a_{10,10} \end{pmatrix}$$

We further experiment with a matrix B of a 2 by 5 size and place it between the 3rd - 4th rows and 5th - 9th columns on a 11 by 12 matrix, effectively solving this portion of our problem.

```
CopyMatrix[3, 5, 11, 12, 2, 5]
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} & a_{1,7} & a_{1,8} & a_{1,9} & a_{1,10} & a_{1,11} & a_{1,12} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} & a_{2,11} & a_{2,12} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} & b_{1,5} & a_{3,10} & a_{3,11} & a_{3,12} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} & b_{2,5} & a_{4,10} & a_{4,11} & a_{4,12} \\ a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6} & a_{5,7} & a_{5,8} & a_{5,9} & a_{5,10} & a_{5,11} & a_{5,12} \\ a_{6,1} & a_{6,2} & a_{6,3} & a_{6,4} & a_{6,5} & a_{6,6} & a_{6,7} & a_{6,8} & a_{6,9} & a_{6,10} & a_{6,11} & a_{6,12} \\ a_{7,1} & a_{7,2} & a_{7,3} & a_{7,4} & a_{7,5} & a_{7,6} & a_{7,7} & a_{7,8} & a_{7,9} & a_{7,10} & a_{7,11} & a_{7,12} \\ a_{8,1} & a_{8,2} & a_{8,3} & a_{8,4} & a_{8,5} & a_{8,6} & a_{8,7} & a_{8,8} & a_{8,9} & a_{8,10} & a_{8,11} & a_{8,12} \\ a_{9,1} & a_{9,2} & a_{9,3} & a_{9,4} & a_{9,5} & a_{9,6} & a_{9,7} & a_{9,8} & a_{9,9} & a_{9,10} & a_{9,11} & a_{9,12} \\ a_{10,1} & a_{10,2} & a_{10,3} & a_{10,4} & a_{10,5} & a_{10,6} & a_{10,7} & a_{10,8} & a_{10,9} & a_{10,10} & a_{10,11} & a_{10,12} \\ a_{11,1} & a_{11,2} & a_{11,3} & a_{11,4} & a_{11,5} & a_{11,6} & a_{11,7} & a_{11,8} & a_{11,9} & a_{11,10} & a_{11,11} & a_{11,12} \end{pmatrix}$$

Over to the BigMatrix[], we ensure the matrices M1, M2, M3 and M4 were of size (n×n), and create an overall equation that allow us to create our square-sized matrices. We use n=4 as an example.

```
In[280]:= A[n_, n_] := Array[a#1,#2 &, {n, n}]

M1[n_, n_] := Array[a#1,#2 &, {n, n}]

M1[4, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

```
M2[n_, n_] := Array[a#1,#2 &, {n, n}]

M2[4, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

```
M3[n_, n_] := Array[a_{#1,#2} &, {n, n}]
```

```
M3[4, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

```
M4[n_, n_] := Array[a_{#1,#2} &, {n, n}]
```

```
M4[4, 4] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

Following the concept from CopyMatrix[], we combine the matrices using *Join* and *ArrayFlatten* and merge M1 and M3 together.

```
ArrayFlatten[Join[{{M1[4, 4]}, {M3[4, 4]}}, 2]] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \\ a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

Similarly, we do the same operation of *Join* and *ArrayFlatten*, except this time it is to merge M2 and M4 together.

```
ArrayFlatten[Join[{{M2[4, 4]}, {M4[4, 4]}}, 2]] // MatrixForm
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \\ a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

Now that that has been established, we combine our merged matrices to create our overall formula for BigMatrix[], which essentially combines M1, M2, M3 and M4.

```
BigMatrix[n_, n_] := Join[Join[ArrayFlatten[Join[{{M1[n, n]}, {M3[n, n]}}, 2]],
    ArrayFlatten[Join[{{M2[n, n]}, {M4[n, n]}}, 2]], 2]] // MatrixForm
```

```
BigMatrix[4, 4]
```

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \\ a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

Now that we completed the BigMatrix[] problem, we solve the remainder of the problem, firstly by defining a sequence of matrices A_n of size $2^n \times 2^n$.

In[267]:=
```
A[n_] := ArrayFlatten[{{A[n - 1], -A[n - 1]}, {A[n - 1], A[n - 1]}}]
```

The matrix and determinant when n = 1:

In[268]:=
```
A[1] := ArrayFlatten[{{1, -1}, {1, 1}}]
```

In[269]:=
```
A[1] // MatrixForm
```

Out[269]//MatrixForm=

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

In[231]:=
```
Det[A[1]]
```

Out[231]= 2

The matrix and determinant when n = 2:

In[232]:=
```
A[2] // MatrixForm
```

Out[232]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

In[233]:=
```
Det[A[2]]
```

Out[233]= 16

The matrix and determinant when n = 3:

In[234]:=
```
A[3] // MatrixForm
```

Out[234]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

In[235]:=
```
Det[A[3]]
```

Out[235]= 4096

The matrix and determinant when n = 4:

In[236]:= **A[4] // MatrixForm**

Out[236]//MatrixForm=

$$\begin{pmatrix}
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

In[237]:= **Det[A[4]]**

Out[237]= 4 294 967 296

The matrix and determinant when n = 5:

A[5] // MatrixForm

$$\begin{pmatrix}
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}$$

In[238]:= `Det[A[5]]`

Out[238]= 1 208 925 819 614 629 174 706 176

Over to the final stretch of this question, we check the identity for a 2×2 matrix A. Here we firstly define our A, A^{-1}, J and o, as part of our components of the matrix using *ArrayFlatten* and *MatrixForm*. This helps us determine the matrices of the following:

```
A[n_] := ArrayFlatten[{{A[n - 1], -A[n - 1]}, {A[n - 1], A[n - 1]}}]
```

In[243]:= `A[1] // MatrixForm`

Out[243]//MatrixForm=

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

In[245]:= `Inverse[A[1]] // MatrixForm`

Out[245]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

In[250]:= `J = ArrayFlatten[{{1, 0}, {0, 1}}] // MatrixForm`

Out[250]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[252]:= `o = ArrayFlatten[{{0, 0}, {0, 0}}] // MatrixForm`

Out[252]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Our penultimate step is to check whether the matrices are true using *MatrixQ:*

In[266]:= $\texttt{MatrixQ}\left[\begin{pmatrix} \texttt{A[1]} & \texttt{o} \\ \texttt{o} & \texttt{Inverse[A[1]]} \end{pmatrix}\right] ==$

$\texttt{MatrixQ}\left[\begin{pmatrix} \texttt{J} & \texttt{A[1]} \\ \texttt{o} & \texttt{J} \end{pmatrix}.\begin{pmatrix} \texttt{J} & \texttt{o} \\ \texttt{-Inverse[A[1]]} & \texttt{J} \end{pmatrix}.\begin{pmatrix} \texttt{J} & \texttt{A[1]} \\ \texttt{o} & \texttt{J} \end{pmatrix}.\begin{pmatrix} \texttt{o} & \texttt{-J} \\ \texttt{J} & \texttt{0} \end{pmatrix}\right]$

... Dot: Nonrectangular tensor encountered.

... Dot: Nonrectangular tensor encountered.

... Dot: Nonrectangular tensor encountered.

... General: Further output of Dot::rect will be suppressed during this calculation.

Out[266]= True

As shown above, it is, which successfully solves this project!