Appendix B: Calculation of SDDR

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To find B_{01}, start with:

$$p(M_0|d) = \int d\Psi \cdot \pi_0(\Psi)\, p(d|\Psi, \omega_*) \,.$$ → Model 0 posterior, integrating out Ψ.

$$p(M_1|d) = \int d\Psi \cdot d\omega \cdot \pi_1(\Psi, \omega) \cdot p(d|\Psi, \omega) \equiv q \,.$$ → Model 1 posterior integrating out Ψ.

Dividing the two,

$$B_{01} = \frac{\int d\Psi \cdot \pi_0(\Psi)\, p(d|\Psi, \omega_*)}{\int d\Psi \cdot d\omega \cdot \pi_1(\Psi, \omega)\, p(d|\Psi, \omega) \equiv q} \equiv \int \frac{d\Psi \cdot \pi_0(\Psi)\, p(d|\Psi, \omega_*)}{q} \,.$$

Now multiply & divide by: $p(\omega_*|d)$.

$$B_{01} = p(\omega_*|d) \cdot \int \frac{d\Psi \cdot \pi_0(\Psi) \cdot p(d|\Psi, \omega_*)}{q \cdot p(\omega_*|d)}$$

→(product rule)

Rewrite using PR: $p(\omega_*|d) \cdot p(\Psi|\omega_*, d) = p(\Psi, \omega_*|d)$

$$\therefore p(\omega_*|d) = \frac{p(\Psi, \omega_*|d)}{p(\Psi|\omega_*, d)} \,.$$

$\therefore$ We obtain: $$B_{01} = p(\omega_*|d) \cdot \int \frac{d\Psi \cdot \pi_0(\Psi) \cdot p(d|\omega_*, \Psi) \cdot p(\Psi|\omega_*, d)}{q \cdot p(\Psi, \omega_*|d)} \,.$$

$$= p(\omega_*|d) \cdot \int \frac{d\Psi \cdot \pi_0(\Psi) \cdot p(\cancel{d|}\omega_*, \Psi)\, p(\Psi|\omega_*, d)}{\pi_1(\Psi, \omega_*) \cdot p(\cancel{d|\Psi}, \omega_*)} \,.$$ $(\rightarrow \omega = \omega_*)$

$$= p(\omega_*|d) \cdot \int \frac{d\Psi \cdot \pi_0(\Psi) \cdot p(\Psi|\omega_*, d)}{\pi_1(\Psi, \omega_*)} \,.$$

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Let's assume separable priors, such that:

$$\pi_1(\psi, \omega_*) = \pi_1(\omega_*)\,\pi_0(\psi),$$

because $\pi_1(\psi|\omega_*) = \underbrace{\pi_0(\psi)}_{\text{Only contains } \psi \text{, no } \omega \text{ variance } (\omega=\omega_*)}$. ✓

$$\therefore B_{01} = p(\omega_*|d) \cdot \int \frac{d\psi \cdot \cancel{\pi_0(\psi)} \cdot p(\psi|\omega_*, d)}{\cancel{\pi_0(\psi)}\,\pi_1(\omega_*)}$$

$$\therefore B_{01} = \frac{p(\omega_*|d)}{\pi_1(\omega_*)} \cdot \underbrace{\int d\psi \cdot p(\psi|\omega_*, d)}_{=1 \text{ (over all } \psi \text{ space)}}.$$

$$\therefore \boxed{B_{01} = \frac{p(\omega_*|d)}{\pi_1(\omega_*)}.}$$ ✓

Eq. (7) reads: $B_{01} = \left.\dfrac{p(\omega|d, M_1)}{\pi(\omega|M_1)}\right|_{\omega=\omega_*}$ ✓

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