# Machine Learning

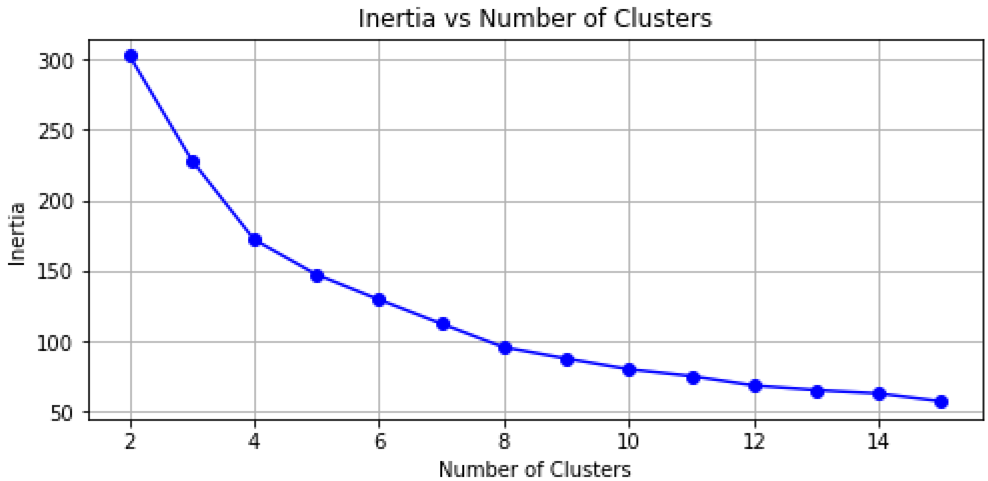
I used various Python libraries to process/scale the data. I then ran the K-means algorithm on the scaled dataset to find centroid (and moreso ensure that K-Means was working as needed). Afterwards, I applied PCA to our dataset to reduce the dimensionality of the data to 2. This will lead to an obvious loss in some data information, but the resulting two prinicpal components still explain almost 50% of the variance in the data and lead to what appears to be reasonable clusterings of data. I plotted the clustering results also.

In [1]:
```
# We import everything we need
import numpy as np # Data manipulation
import pandas as pd # Data Entry
from sklearn.cluster import KMeans # K means algorithm
import matplotlib.pyplot as plt # Data plotting/visualization
from sklearn.decomposition import PCA # Data dimension reduction
from sklearn.preprocessing import StandardScaler # Data scaling
from amplpy import AMPL # LP Stuff
```

```
/Users/alexsemyonov/opt/anaconda3/lib/python3.9/site-packages/scipy/__init__.p
y:146: UserWarning: A NumPy version >=1.16.5 and <1.23.0 is required for this
version of SciPy (detected version 1.26.2
  warnings.warn(f"A NumPy version >={np_minversion} and <{np_maxversion}"
```

In [2]:
```
# We first import our data as a Pandas dataframe
neighborhood_df = pd.read_csv("LP Neighborhood Data Revised.csv") # Note: I've
# We now drop the non-numeric columns and convert this data to a numpy array
neighborhood_temp = neighborhood_df.drop(columns = ["Neighborhood"])
data_matrix = neighborhood_temp.to_numpy()
# Let's standardize our data
scaler = StandardScaler()
scaled_data_matrix = scaler.fit_transform(data_matrix) # We fit (to find mean a
```

In [3]:
```
# Let's try to find the optimal number of clusters without any sort of dimensio
# The code below generates clusters using k-means for {2,...,16} clusters
kmeans_per_k = [KMeans(n_clusters=k).fit(scaled_data_matrix)for k in range(2, 1
# We create an array that stores the inertias for each clustering above
inertias = [model.inertia_ for model in kmeans_per_k]
# We plot Interia vs Number of Clusters
plt.figure(figsize=(8, 3.5))
plt.plot(range(2, 16), inertias, "bo-")
plt.xlabel("Number of Clusters")
plt.ylabel("Inertia")
plt.title("Inertia vs Number of Clusters")
plt.grid()
plt.show()
# It looks like 5 could be an "elbow" point (but trying higher values may be wo
```



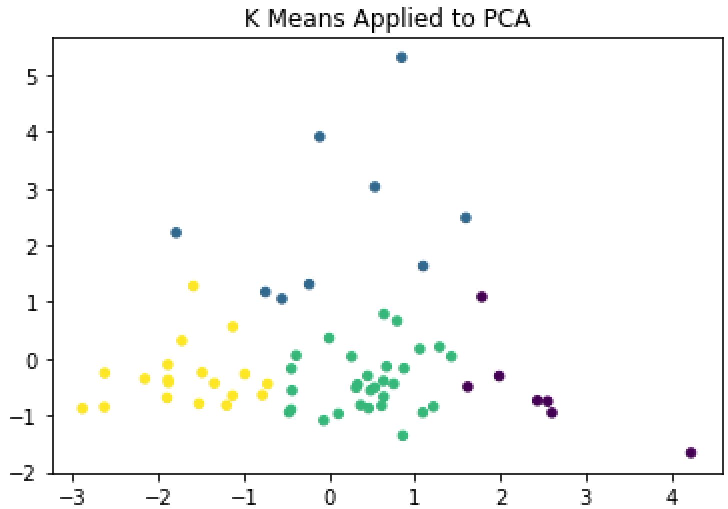

In [4]:
```
# Let's cluster the data using five clusters
kmeans = KMeans(n_clusters = 4)
cluster = kmeans.fit_predict(scaled_data_matrix)
# Let's look at our cluster centers
print(kmeans.cluster_centers_)
# Note: we would want to run multiple times and take best one (best: minimizes
# and cluster centers)
```

```
[[ 0.57505858 -0.2305793   0.60317449 -0.6814973  -0.28639515 -0.13429442]
 [-0.74814058 -0.45992607 -0.70025787  0.925666   -0.39772041 -0.13485835]
 [ 0.17741169  2.89953559 -0.68293189 -0.49440449 -1.31502269  7.9728813 ]
 [-0.14046682  1.3060724  -0.23725493  0.11419715  1.72210195 -0.07662458]]
```

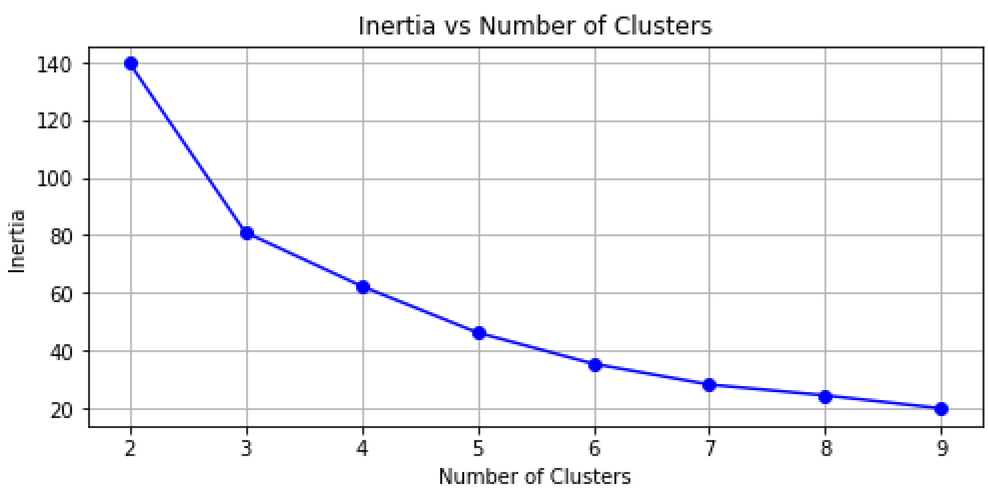
In [5]:
```
# To visualize our data,  need to reduce the dimension of our data from 6 to 2
# We can use principal component analysis (PCA) to do this.
# Note: reducing our dimension to two does lead to inherent data loss.
# We want to find two prinicpal components:
pca = PCA(n_components = 2)
pca_data = pca.fit_transform(scaled_data_matrix)
# Let's look at our variance ratio
print("Variance Ratio: ", pca.explained_variance_ratio_)
print("Total variance explained by principal components are: ", np.sum(pca.exp
# Our reduction still explains almost 50% of the variance in the data
# We fit K-means on the scaled and reduced data.
k_means_PCA = kmeans.fit_predict(pca_data)
plt.scatter(pca_data[:, 0], pca_data[:, 1], c= k_means_PCA, s=20)
plt.title("K Means Applied to PCA")
```

```
Variance Ratio:  [0.33197491 0.24919664]
Total variance explained by principal components are:  0.5811715494723237
```

Out[5]:
```
Text(0.5, 1.0, 'K Means Applied to PCA')
```



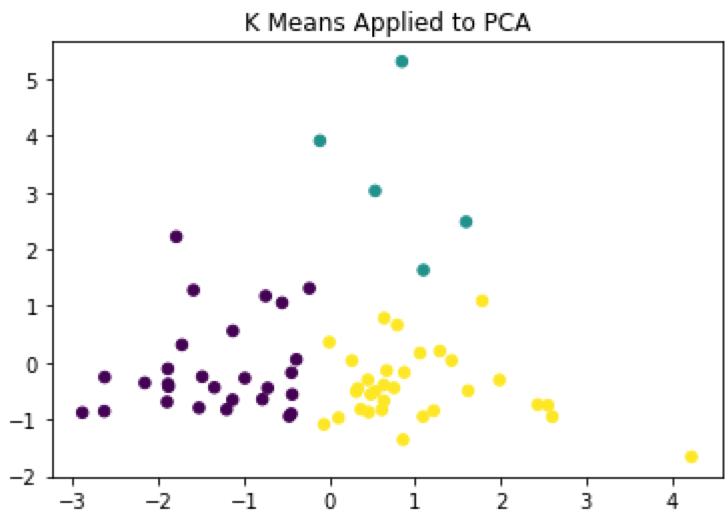

In [6]:
```
# Let's try to find the optimal number of clusters for our PCA data
# The code below generates clusters using k-means for {2,...,10} clusters
kmeans_per_k = [KMeans(n_clusters=k).fit(pca_data)for k in range(2, 10)]
# We create an array that stores the inertias for each clustering above
inertias = [model.inertia_ for model in kmeans_per_k]
# We plot Interia vs Number of Clusters
plt.figure(figsize=(8, 3.5))
plt.plot(range(2, 10), inertias, "bo-")
plt.xlabel("Number of Clusters")
plt.ylabel("Inertia")
plt.title("Inertia vs Number of Clusters")
plt.grid()
plt.show()
# It looks like 3 is a possible "elbow" points
```




In [7]:
```
# Let's cluster the data using three clusters
kmeans_pca = KMeans(n_clusters = 3)
# Let's look at our variance ratio
print("Variance Ratio: ", pca.explained_variance_ratio_)
print("Total variance explained by principal components are: ", np.sum(pca.exp
# The high value of the first entry indicates that there are features which "do
k_means_PCA = kmeans_pca.fit_predict(pca_data)
plt.scatter(pca_data[:, 0], pca_data[:, 1], c= k_means_PCA, s=30)
plt.title("K Means Applied to PCA") # It looks like three clusters are indeed 
```

```
Variance Ratio:  [0.33197491 0.24919664]
Total variance explained by principal components are:  0.5811715494723237
```

Out[7]:
```
Text(0.5, 1.0, 'K Means Applied to PCA')
```




# Linear Programming

We now try and formulate a linear program to solve the constrained version of this problem. Assuming that we do not initially know the cluster centers, this is a non-linear mixed integer program (which is quite difficult to solve). If we do know our cluster centers, this is merely a classic supply and demand LP once we restrict the number of neighborhoods allowed in each cluster (i.e. constrain our problem). One potential approach to solving this problem consists of starting with some initialization of cluster centers, clustering our data (since we have our cluster centers this will be a linear program), moving the cluster centers closer to the bulk of the data in that cluster and then reclustering. We repeat this process until the cluster centers stop moving. Let's also suppose that we have 5 clusters each requiring 13 neighborhoods in each cluster (as this ensures that our data can be evenly divided amongst the clusters). Our task then becomes to find the optimal clustering of points given these cluster centers. We first define our parameters. Initially, let $x_i$ denote the feature vector associated with neighborhood $i$. We can then find the cost associated with cluster $j$ as: $c_{i,j} = \|x_i - \mu_{\mathbf{j}}\|_2$ (note that the $\mu_j$'s are known). We can formulate the following linear program to solve this problem:

$$\text{Minimize} \sum_{j=1}^{5} \sum_{i=1}^{65} c_{i,j} Y_{i,j}$$

$$\text{Subject to: } \sum_{i=1}^{65} Y_{i,j} = 13 \ \forall j = 1,2,3,4,5$$

$$\sum_{j=1}^{5} Y_{i,j} = 1 \ \forall i = 1,2,\ldots,65$$

$$Y_{i,j} \in \{0, 1\}$$

Additionally, we can try relaxing the integrality constraints in the event the problem becomes computationally infeasible. This relaxed problem is given below:

$$\text{Minimize} \sum_{j=1}^{5} \sum_{i=1}^{65} c_{i,j} Y_{i,j}$$

$$\text{Subject to:} \sum_{i=1}^{65} Y_{i,j} = 13 \ \forall j = 1, 2, 3, 4, 5$$

$$\sum_{j=1}^{5} Y_{i,j} = 1 \ \forall i = 1, 2, \ldots, 65$$

$$Y_{i,j} \in [0, 1]$$

We start by building our data:

In [8]:

```
# We redo our K-Means algorithm with 5 clusters (this is to ensure everything 
# and use the generated cluster centers to initialize our cluster centers. Note
# six-dimensional neighborhood data, not the PCA components.
kmeans_LP_init = KMeans(n_clusters = 5)
LP_init_k_means = kmeans_LP_init.fit_predict(scaled_data_matrix)
LP_cluster_centers_init = kmeans_LP_init.cluster_centers_
```

In [9]:

```
# We now construct our cost matrix
cost = np.zeros((65,5))
for i in range(65):
    for j in range(5):
        cost[i,j] = np.linalg.norm(scaled_data_matrix[i]-LP_cluster_centers_ini
# We also create origin (neighborhood) vectors and destination (cluster) vecto
orig = neighborhood_df["Neighborhood"].to_numpy()
dest = np.zeros(5)
for j in range(5):
    dest[j] = j
orig_df = pd.DataFrame(orig, columns = ["Neighborhood"]).set_index("Neighborhoo
dest_df = pd.DataFrame(dest, columns = ["Clusters"]).set_index("Clusters")
cost_df = pd.DataFrame(cost, columns = dest_df.index.to_list(),index = orig_df
print(orig_df)
print(dest_df)
print(cost_df)
```

```
Empty DataFrame
Columns: []
Index: [Athmar Park, Baker, Barnum, Barnum West, Bear Valley, Belcaro, Berkele
y, Capitol Hill, Chaffee Park, Cheesman Park, Cherry Creek, City Park, City Pa
rk West, Clayton, Cole, College View - South Platte, Congress Park, Cory - Mer
rill, Country Club, Denver International Airport, East Colfax, Elyria Swansea,
Five Points, Fort Logan, Globeville, Goldsmith, Green Valley Ranch Denver, Hal
e, Hampden, Hampden South, Harvey Park, Harvey Park South, Highland, Hilltop,
Indian Creek, Jefferson Park, Lincoln Park, Lowry Field, Mar Lee, Montbello, M
ontclair, Overland, Platt Park, Regis, Rosedale, Ruby Hill, Skyland, Sloan Lak
e, Southmoor Park, Speer, Stapleton Denver, Sun Valley, Sunnyside, University,
Valverde, Villa Park, Virginia Village, Washington Park, Washington Park West,
Washington Virginia Vale, Wellshire Denver, West Colfax, West Highland, Westwo
od, Windsor]

[65 rows x 0 columns]
Empty DataFrame
Columns: []
Index: [0.0, 1.0, 2.0, 3.0, 4.0]
                       0.0       1.0       2.0       3.0       4.0
Athmar Park       3.484968  0.448793  8.973069  3.037826  1.910049
Baker             2.009234  3.278900  8.582656  2.918343  1.238257
Barnum            5.012073  1.498559  9.551305  4.399288  3.662466
Barnum West       2.468820  2.177955  8.939866  3.563822  1.147236
Bear Valley       2.676537  1.457800  8.998884  2.805726  1.115492
...                    ...       ...       ...       ...       ...
Wellshire Denver  1.558344  2.862649  9.196678  3.970097  1.823234
West Colfax       3.735638  2.050404  9.133764  3.117457  1.805249
West Highland     0.954720  3.421360  9.131714  3.137973  1.626459
Westwood          4.821666  1.488658  9.542135  3.546128  3.428763
Windsor           4.095947  2.474063  9.387348  2.018513  2.502561

[65 rows x 5 columns]
```

In [10]:

```
# Now that we have our cost matrix, we can formulate an AMPL program to solve 
# supply and demand problem.
ampl = AMPL()
ampl.read("K_means_clustering.mod")
ampl.set_data(orig_df, "Neighborhood")
ampl.set_data(dest_df, "Clusters")
for n in range(10):
    ampl.get_parameter("cost").set_values(cost_df)
    ampl.option["solver"] = "highs"
    ampl.solve()
    # Now, we want to reformat our answer into something more readable.
    clustering = ampl.get_variable("Cluster").get_values().to_pandas()
    clustering_temp = clustering.to_numpy().reshape(65,5) # Reshapes in 65 x 5
    # Code below creates an indicator matrix of neighborhoods and clusters
    clustering_temp2 = pd.DataFrame(clustering_temp, columns = dest_df.index.t
    # Code block below translates indicator variables in to cluster number
    clustering_temp3 = clustering_temp2.idxmax(axis=1)
    # We add a column to be able to select specific clusters
    clustering_final = pd.DataFrame(clustering_temp3, columns = ["Group"],inde
    # We find the neighborhoods in each cluster
    clust_0 = clustering_final.index[clustering_final["Group"] == 0.0].to_list
    clust_1 = clustering_final.index[clustering_final["Group"] == 1.0].to_list
    clust_2 = clustering_final.index[clustering_final["Group"] == 2.0].to_list
    clust_3 = clustering_final.index[clustering_final["Group"] == 3.0].to_list
    clust_4 = clustering_final.index[clustering_final["Group"] == 4.0].to_list
    # We return to our original samples
```

```
    neigh_dat = pd.DataFrame(scaled_data_matrix, index = orig_df.index.to_list
    # We create Numpy arrays of the original samples in each cluster
    orig_clust_0 = neigh_dat.loc[clust_0].to_numpy()
    orig_clust_1 = neigh_dat.loc[clust_1].to_numpy()
    orig_clust_2 = neigh_dat.loc[clust_2].to_numpy()
    orig_clust_3 = neigh_dat.loc[clust_3].to_numpy()
    orig_clust_4 = neigh_dat.loc[clust_4].to_numpy()
    new_clust_centers = np.zeros((5,6))
    new_clust_centers[0] = (1/orig_clust_0.shape[0])*np.sum(orig_clust_0, axis
    new_clust_centers[1] = (1/orig_clust_1.shape[0])*np.sum(orig_clust_1, axis
    new_clust_centers[2] = (1/orig_clust_2.shape[0])*np.sum(orig_clust_2, axis
    new_clust_centers[3] = (1/orig_clust_3.shape[0])*np.sum(orig_clust_3, axis
    new_clust_centers[4] = (1/orig_clust_4.shape[0])*np.sum(orig_clust_4, axis
    # We recompute our cost matrix
    for i in range(65):
        for j in range(5):
            cost[i,j] = np.linalg.norm(scaled_data_matrix[i]-new_clust_centers
    cost_df = pd.DataFrame(cost, columns = [0.0,1.0,2.0,3.0,4.0],index = orig_
print(clustering_final)
print(new_clust_centers)
```

```
HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 178.4889836
78 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 100.2529459
27 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.76794351
21 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.4790646
5 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
6 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
0 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
0 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
0 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
0 simplex iterations
0 barrier iterations

HiGHS 1.5.3:HiGHS 1.5.3: optimal solution; objective 98.06702068
0 simplex iterations
0 barrier iterations

                  Group
Athmar Park         1.0
Baker               2.0
Barnum              1.0
Barnum West         4.0
Bear Valley         4.0
...                 ...
Wellshire Denver    0.0
West Colfax         2.0
West Highland       0.0
Westwood            1.0
Windsor             3.0

[65 rows x 1 columns]
[[ 0.76339103 -0.3547268   1.44987003 -0.8342851  -0.27034594 -0.13705684]
 [-1.33202861 -0.47559548 -0.69190234  1.02598495 -0.43141516 -0.13559128]
 [ 0.63016794  0.33879222 -0.58210626  0.3475316  -0.65589652  0.49286206]
 [-0.1517277   1.08944422 -0.10117147  0.11048163  1.61495506 -0.08637619]
 [ 0.09019734 -0.59791417 -0.07468997 -0.64971309 -0.25729744 -0.13383776]]
```

In [11]:
```
# The code below outputs the entire dataframe as opposed to a truncated display
# We can see what cluster each neighborhood belongs to.
with pd.option_context('display.max_rows', None,
                       'display.max_columns', None,
                       'display.precision', 3,
                       ):
    print(clustering_final)
```

| | Group |
|---|---|
| Athmar Park | 1.0 |
| Baker | 2.0 |
| Barnum | 1.0 |
| Barnum West | 4.0 |
| Bear Valley | 4.0 |
| Belcaro | 0.0 |
| Berkeley | 4.0 |
| Capitol Hill | 3.0 |
| Chaffee Park | 1.0 |
| Cheesman Park | 4.0 |
| Cherry Creek | 3.0 |
| City Park | 2.0 |
| City Park West | 2.0 |
| Clayton | 1.0 |
| Cole | 2.0 |
| College View - South Platte | 1.0 |
| Congress Park | 0.0 |
| Cory - Merrill | 0.0 |
| Country Club | 0.0 |
| Denver International Airport | 2.0 |
| East Colfax | 4.0 |
| Elyria Swansea | 2.0 |
| Five Points | 3.0 |
| Fort Logan | 4.0 |
| Globeville | 2.0 |
| Goldsmith | 1.0 |
| Green Valley Ranch Denver | 3.0 |
| Hale | 3.0 |
| Hampden | 3.0 |
| Hampden South | 3.0 |
| Harvey Park | 1.0 |
| Harvey Park South | 1.0 |
| Highland | 0.0 |
| Hilltop | 0.0 |
| Indian Creek | 4.0 |
| Jefferson Park | 2.0 |
| Lincoln Park | 2.0 |
| Lowry Field | 0.0 |
| Mar Lee | 1.0 |
| Montbello | 3.0 |
| Montclair | 4.0 |
| Overland | 4.0 |
| Platt Park | 0.0 |
| Regis | 1.0 |
| Rosedale | 0.0 |
| Ruby Hill | 1.0 |
| Skyland | 4.0 |
| Sloan Lake | 4.0 |
| Southmoor Park | 4.0 |
| Speer | 3.0 |
| Stapleton Denver | 3.0 |
| Sun Valley | 2.0 |
| Sunnyside | 4.0 |
| University | 2.0 |
| Valverde | 1.0 |
| Villa Park | 2.0 |
| Virginia Village | 3.0 |
| Washington Park | 0.0 |
| Washington Park West | 0.0 |

```
Washington Virginia Vale       3.0
Wellshire Denver               0.0
West Colfax                    2.0
West Highland                  0.0
Westwood                       1.0
Windsor                        3.0
```

In [12]:
```
print("The final cluster centers are: ", scaler.inverse_transform(new_clust_cen
```

```
The final cluster centers are:  [[2.24830769e+03 3.02146154e+03 1.14832132e+05
3.78015385e-01
  3.34607692e+03 9.56076923e+00]
 [1.61069231e+03 2.30330769e+03 4.68660054e+04 5.53076923e-01
  2.92638462e+03 1.18861538e+01]
 [2.20776923e+03 7.14207692e+03 5.03502292e+04 4.89230769e-01
  2.34146154e+03 1.00904846e+03]
 [1.96984615e+03 1.16021538e+04 6.56120169e+04 4.66923077e-01
  8.25853846e+03 8.99753846e+01]
 [2.04346154e+03 1.57653846e+03 6.64523700e+04 3.95384615e-01
  3.38007692e+03 1.46684615e+01]]
```

In [13]:
```
# The neighborhoods in each cluster don't seem to change from iteration to ite
print(clust_0)
print(clust_1)
print(clust_2)
print(clust_3)
print(clust_4)
```

```
['Belcaro', 'Congress Park', 'Cory - Merrill', 'Country Club', 'Highland', 'Hi
lltop', 'Lowry Field', 'Platt Park', 'Rosedale', 'Washington Park', 'Washingto
n Park West', 'Wellshire Denver', 'West Highland']
['Athmar Park', 'Barnum', 'Chaffee Park', 'Clayton', 'College View - South Pla
tte', 'Goldsmith', 'Harvey Park', 'Harvey Park South', 'Mar Lee', 'Regis', 'Ru
by Hill', 'Valverde', 'Westwood']
['Baker', 'City Park', 'City Park West', 'Cole', 'Denver International Airpor
t', 'Elyria Swansea', 'Globeville', 'Jefferson Park', 'Lincoln Park', 'Sun Val
ley', 'University', 'Villa Park', 'West Colfax']
['Capitol Hill', 'Cherry Creek', 'Five Points', 'Green Valley Ranch Denver',
'Hale', 'Hampden', 'Hampden South', 'Montbello', 'Speer', 'Stapleton Denver',
'Virginia Village', 'Washington Virginia Vale', 'Windsor']
['Barnum West', 'Bear Valley', 'Berkeley', 'Cheesman Park', 'East Colfax', 'Fo
rt Logan', 'Indian Creek', 'Montclair', 'Overland', 'Skyland', 'Sloan Lake',
'Southmoor Park', 'Sunnyside']
```

In [ ]: