Improving Multiset Canonical Correlation Analysis in High Dimensional Sample Deficient Settings

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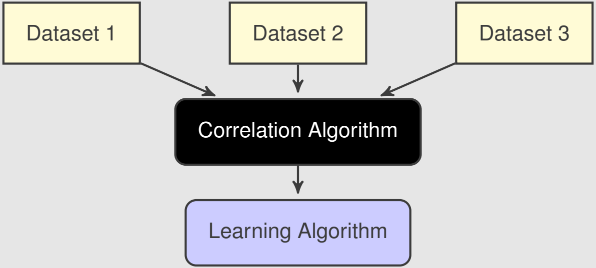
Motivation



Goal

Develop theoretically justified, robust correlation algorithms for multi-dataset fusion

A Myriad of Applications

Multiple Datasets

- Audio-Video
- Audio-Audio

Machine Learning

- emotion identification
- shopping predictions
- music genre classification

Medical Signal Processing

- MRI, fMRi, EEG, MEG, etc.

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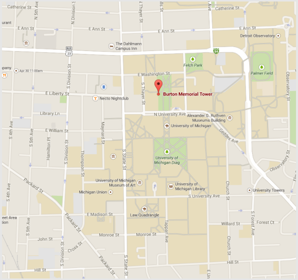

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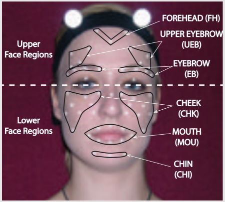

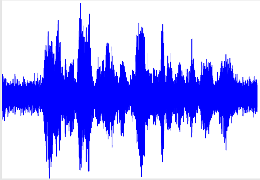

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- disco influences
- danceable grooves
- repetitive melodic phrasing
- extensive vamping
- minor key tonality

MCCA - Multiset Canonical Correlation Analysis

Notation

- data: $y_1 \in \mathbb{C}^{d_1 \times 1}, \ldots, y_m \in \mathbb{C}^{d_m \times 1}$
- covariance matrices: $R_{ij} = \mathbb{E}\left[y_i y_j^H\right]$
- canonical vectors: $x_1, \ldots, x_m$
- canonical variates: $w_1, \ldots, w_m$, $w_i = x_i^H y_i$

$$\Phi(x) = E[ww^H] = \begin{bmatrix} x_1^H R_{11} x_1 & \ldots & x_1^H R_{1m} x_m \\ \vdots & \ddots & \vdots \\ x_m^H R_{m1} x_1 & \ldots & x_m^H R_{mm} x_m \end{bmatrix}$$

Optimization Problem

$$\begin{array}{ll} \underset{x}{\text{optimize}} & J(\Phi(x)) \\ \text{subject to} & h(x, R) \end{array}$$

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MAXVAR Optimization Problem

$$\begin{array}{ll} \underset{x_1,\ldots,x_m}{\text{argmax}} & \rho_{\text{mcca}} = \lambda_1\left(\Phi(x)\right) \\ \text{subject to} & x_i^H R_{ii} x_i = 1 \end{array}$$

MCCA Solution and Empirical Solution

Notation

- $R = [R_{ij}]_{i,j=1}^{m}$
- $R_D = \textbf{blkdiag}(R_{11}, \ldots, R_{mm})$
- $C_{\text{mcca}} = R_D^{-/12} R R_D^{-1/2}$

MCCA Solution

$$\boxed{\rho_{\text{mcca}}^{(j)} = \lambda_j\left(C_{\text{mcca}}\right)}$$

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MCCA Solution

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Empirical MAXVAR

- Covariance matrices R_{ij} are typically unknown in practice
- Training data $Y_j = \left[y_1^{(j)}, \ldots, y_n^{(j)}\right]$, $j = 1, \ldots, m$
- Sample covariance matrices $\widehat{R}_{ij} = \frac{1}{n} Y_i Y_j^H$
- Form $\widehat{R}$ and $\widehat{R}_D$
- $\widehat{C}_{\text{mcca}} = \widehat{R}_D^{-/12} \widehat{R} \widehat{R}_D^{-1/2}$

Proposed correlation statistic

$$\widehat{\rho}_{\text{mcca}}^{(j)} = \lambda_j \left(\widehat{C}_{\text{mcca}} - I\right)$$

Empirical MCCA Insights

Empirical MAXVAR Equivalency

- Let $Y_j = \widehat{U}_j \widehat{\Sigma}_j \widehat{V}_j$ be SVD of dataset j
- Let $\widetilde{U} = \textbf{blkdiag}(\widehat{U}_1, \ldots, \widehat{U}_m)$
- Let $\widetilde{V} = [\widehat{V}_1, \ldots, \widehat{V}_m]$
- $\widehat{C}_{\text{mcca}} = \widetilde{U}\widetilde{V}^H\widetilde{V}\widetilde{U}$
- $\lambda_j\left(\widehat{C}_{\text{mcca}}\right) = \lambda_j\left(\widetilde{V}^H\widetilde{V}\right)$

Insights

- This uses *ALL* right singular vectors
- In many applications, low-rank setting
- In low-SNR, low-sample regime, singular vectors may be inaccurate
- We can use insights from random matrix theory (RMT) to quantify singular vector accuracy to improve MCCA
- Let $\widehat{k}_j$ be RMT estimates of rank of each dataset

Idea - Trim right singular vectors

- $\mathring{V}_i = \widehat{V}_i\left(:, 1 : \widehat{k}_i\right)$
- $\mathring{V} = \left[\mathring{V}_1, \ldots \mathring{V}_m\right]$

New IMCCA Matrix

$$\widehat{C}_{\text{imcca}} = \mathring{V}^H \mathring{V}$$

Proposed correlation statistic

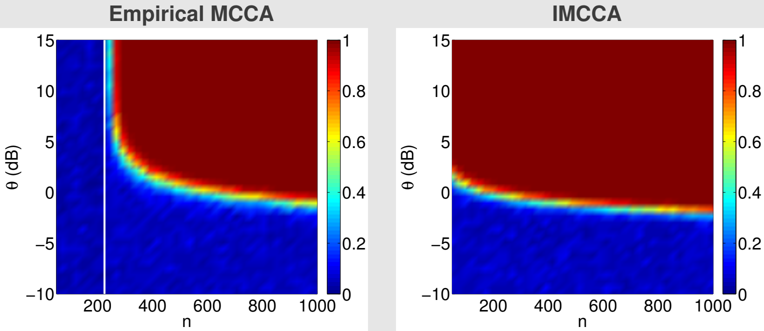
$$\widehat{\rho}^{(j)}_{\text{imcca}} = \lambda_j\left(\widehat{C}_{\text{imcca}} - I\right)$$

Insights

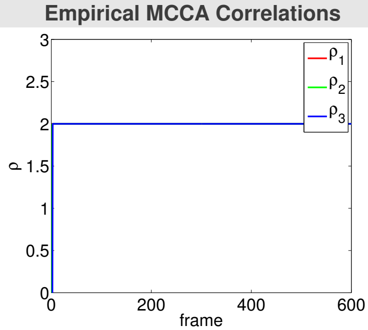
- Eigenvalue detects correlation
- Eigenvector reveals structure

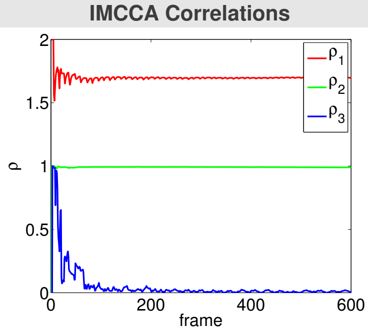
Numerical Simulation

- rank-1 setting, $m = 3$, $d_1 = d_2 = d_3 = 150$
- $y_j^{(i)} = U_j s_j^{(i)} + z_j^{(i)}$
- $z_j \sim \mathcal{CN}(0, I)$, $s_j^{(i)} \sim \mathcal{CN}(0, \theta)$
- $\mathbb{E}\left[s_j s_i\right] = 0.9$
- Plot KS-statistic of $\widehat{\rho}_{\text{mcca}}^{(j)}$ and $\widehat{\rho}_{\text{imcca}}^{(j)}$ for signal-plus-noise vs. noise only



Empirical MCCA and IMCCA Demonstration





Conclusion

Take Home Message

$\boxed{\text{Trim then fuse } \textbf{\textit{NOT}} \text{ fuse then trim}}$

Contributions

- Proposed IMCCA algorithm
- Proposed statistics to detect latent correlations in more than 2 datasets
- Acheived better performance in low-SNR, high dimensional setting