

Initial report on analysis of the Anganwadi 2010 dataset

Avishek Sen Gupta
ThoughtWorks

November 2, 2011

Contents

1	Abstract	5
2	Methodology	6
2.1	CRISP-DM	6
2.2	Relevance of CRISP-DM to this report	6
3	Data Preparation	7
3.1	Nature of the source data	8
3.2	Data representation	9
3.2.1	Data store	9
3.2.2	Schema	10
3.3	Data migration: Identifying invalid data	12
4	Bias	13
4.1	Sampling bias	14
5	Shape of the Data	15
5.1	Univariate distributions	16
5.1.1	Observations/Notes	16
5.2	Outlier analysis	19
5.3	Bivariate distribution	20
5.3.1	Observations/Notes	20
5.4	Summary plots	22
5.4.1	Observations/Notes	22
5.5	Rank Order Charts: Geoclusters	23
5.5.1	Observations/Notes	24
6	Data Analysis and Exploration	25
6.1	Parallel Coordinates	26
6.2	Covariance plot	27
6.2.1	Observations/Notes	29
6.3	Geographical distribution	30
7	Models and Statistical Tests	31
7.1	Tests for conformance to distributions	31
7.1.1	Jarque-Bera test	31
7.1.2	Quantile-Quantile plots	32
7.1.3	Observations/Notes	35
7.2	The Central Limit Theorem	36
7.3	Answer distribution	40
7.3.1	Observations/Notes	40
7.4	Effectiveness of intervention	42
7.4.1	Intervention effect on individual responses: McNemar's Test	42
7.5	Modeling responses as Bernoulli trials	43
7.5.1	The Binomial Distribution	44

7.5.2	Typical questions	45
7.6	Test for variable independence	45
7.6.1	Chi-square test	46
8	Prediction and Classification	47
8.1	Decision Trees	48
8.2	Bayes classifier	49
8.3	Density estimators	50
8.3.1	Naive Bayes density	50
8.3.2	Kernel density estimation	50
8.3.3	Results	50
9	Dimension reduction/Factor analysis	51
9.1	Principal Component Analysis	53
10	Technical notes	54

List of Figures

1	Probability distributions of pre-, post-intervention, and improvement	17
2	Bivariate probability distribution of pre- vs. post-intervention scores	21
3	Box plots of pre- and post-intervention scores, broken down by language	22
4	Parallel Coordinates showing language, gender, school, pre- and post-intervention scores	26
5	Covariance plot of pre-intervention responses	27
6	Covariance plot of post-intervention responses	28
7	Geocoded populations by cluster	30
8	Improvement data overlaid with theoretical Gaussian ($\sigma^2=245.47$, $\bar{x}=7.22$)	33
9	Translated log of data overlaid with theoretical Cauchy (scale=0.14, $\bar{x}=4.13$)	34
10	Quantile-Quantile plot of score improvement vs. theoretical Gaussian	35
11	Curve-fitted theoretical exponential for reflected post-intervention probability distribution	37
12	Quantile-Quantile plot of post-intervention score (reflected) vs. theoretical exponential	38
13	Central Limit Theorem illustration on the pre-intervention score	39
14	Central Limit Theorem illustration on the post-intervention score	39
15	Central Limit Theorem illustration on the improvement metric	40
16	Answer distribution vs. question number	41
17	Bayes posterior distribution of language from score improvement	50

18	Bayes posterior distribution of gender from score improvement .	51
19	Bayes posterior distribution of geocluster from score improvement	52

1 Abstract

This report summarises the results of exploration of the Anganwadi dataset provided by the Akshara Foundation. The analysis aims to characterise the structure of the data, and reveal trends (which would otherwise be obscured by the format of the source data) which may inform strategy through subsequent prediction and/or classification procedures.

2 Methodology

2.1 CRISP-DM

CRISP-DM is a process model distilled from the most common approaches used in data mining procedures. It stands for Cross Industry Standard Process for Data Mining. Not so much a prescription as a collection of 'good practices' followed by data mining professionals, **CRISP-DM** has the following characteristics.

- Domain-neutral
- Tool-neutral
- Provides a structural approach to the data mining process

CRISP-DM segregates data mining endeavours into the following phases.

- Business Understanding
- Data Understanding
- Data Preparation
- Modeling
- Evaluation
- Deployment

2.2 Relevance of CRISP-DM to this report

As far as this report is concerned, the relevant or most significant phases we focus on are:

- Data Understanding
- Data Preparation
- Modeling

Work on the Evaluation step is still preliminary, and will probably be the subject of another report. In a full-fledged project, the rest of the activities upstream and downstream to the above list will assume more importance, and require corresponding investment.

3 Data Preparation

3.1 Nature of the source data

The dataset comes from the education domain. The source data is a file, with each line corresponding to a single student evaluation record. Roughly, there are 29000 records, prior to any data sanitisation. Each line is pipe()-delimited into multiple fields. The fields salient to this analysis are listed below:

- Location of the student's school
- Language of the student
- Student's score before intervention
- Student's score after intervention

The score is not a single number, it is a set of 56 responses marked as 0/1. Generally, a 1 may be treated as a favourable answer, therefore, adding them up to get a single aggregate score has natural ordering: a sense of who did better. We reproduce two such records below, with the original formatting.

[illegible]

Looking at the second row, we see that the location of the Anganwadi is BADAMAKAAN I, the student is female and speaks Urdu. The first contiguous set of 0s and 1s is the pre-intervention score, and the next set is the post-intervention one.

3.2 Data representation

3.2.1 Data store

Before any sort of sanitisation or analysis may be performed, it is important to ensure that the source data is stored in a format/datastore which makes querying and modifying the data relatively painless. This decision is largely driven by technological considerations, like:

- Scale of data (centralised/distributed store?)
- Sophistication of queries (OLAP/OLTP?)
- Structure of data, or lack thereof (SQL/NoSQL?)

We were dealing with only about 29000 records, and most of the analysis would probably be performed outside the database. Thus, we opted to use MySQL as our datastore.

3.2.2 Schema

The decisions when creating the database schema affect the ease of querying for relevant information. Apart from the attributes of interest, we wanted to store the individual binary responses as well. One way is to create one column for each response, giving us a total of 112 columns for storing these responses (56 for pre-intervention, 56 for post-intervention). The other way, and that is the one that we chose was to store this information as a 64-bit integer (bigint for MySQL). When required, we could unpack the individual response bits from this number.

A `desc responses;` command on the table reveals the schema we ended up with.

Field	Type	Null	Key	Default	Extra
student_id	int(11)	YES		NULL	
area	char(50)	YES		NULL	
pre_performance	bigint(20)	YES		NULL	
post_performance	bigint(20)	YES		NULL	
language	char(50)	YES		NULL	
gender	char(20)	YES		NULL	
pre_total	int(11)	YES		NULL	
post_total	int(11)	YES		NULL	
id	int(11)	NO	PRI	NULL	auto_increment
school_id	int(11)	YES		NULL	
year	int(11)	YES		NULL	

The most important use of the reference data is to locate the schools geographically. Given that geocoding the school from its name, we used the cluster to locate schools in 2D space. We deal with geographical analysis in a later section. The schema of the master data mostly mirrors the CSV master data file format, with the addition of latitude and longitude, like so:

Field	Type	Null	Key	Default	Extra
district	char(50)	YES		NULL	
block	char(50)	YES		NULL	
cluster	char(50)	YES		NULL	
school_id	int(11)	YES		NULL	
school_code	char(20)	YES		NULL	
school_name	char(50)	YES		NULL	
id	int(11)	NO	PRI	NULL	auto_increment
latitude	decimal(20,10)	YES		NULL	
longitude	decimal(20,10)	YES		NULL	

+-----+-----+-----+-----+-----+-----+

To identify latitude and longitude, we used Google's Map API to geocode the cluster information. It is to be noted that there may be some clusters which weren't located by the Map API, and some more work is needed to cross-validate the coordinate information taken from the Map API.

3.3 Data migration: Identifying invalid data

It is natural to expect missing or corrupted data. The most crucial attribute are the score data, as any misinterpretation of that data may adversely bias the quality of our analysis. Thus, specific checks were put in place to ensure that none of the binary responses was null or some string other than 0 or 1.

Using this check, we found 1067 responses which violated it. All of them had either empty pre- or post-intervention scores. We did not migrate these response records, though it may be possible to do Monte Carlo simulations to predict the missing data.

As a result, out of a total of 28535 records in the original source, 27468 were migrated to the database.

We also found a large fraction of records which did not have a LANGUAGE attribute, i.e., that field was empty. Nevertheless, they were included in the migration.

4 Bias

Analysis is most susceptible to bias in the data collection stage. Sampling is one such activity. If, for a statistical study, participating individuals are not equally likely to have been selected, it may be difficult to distinguish between the actual phenomenon and this biased sampling. This sort of bias is called sampling bias.

4.1 Sampling bias

To find evidence for bias, we looked at a few parameters. Here is the breakdown of the population by language, with the biggest language bucket highlighted.

Unspecified=869
URDU=3564
KANNADA=18685
TELUGU=1688
TAMIL=2051
MARATHI=91
OTHER=239
HINDI=243
KONKANI=18
GUJARATHI=12
NOT KNOWN=3
ORIYA=2
MULTI LNG=1
BENGALI=1
NEPALI=1

There is an overwhelming proportion of students who speak Kannada as their mother tongue (leading by an order of magnitude), a fact that is very likely to bias any sort of analysis where language is involved. We must remain cognizant of such biases, and interpret the results accordingly.

Here is the breakdown of the population by gender.

Girl=14822
Boy=12646

There is not a huge disparity between the two sexes, which indicates that any analysis/prediction based on gender may be less biased.

5 Shape of the Data

Before embarking on any deep analysis of data, it behooves us to look at the shape of the raw data. There are a few reasons why we want to do this.

- **Evident trends/outliers:** Visualisation of the raw data set is always a quick way to spot trends without doing too much analysis. Of course, visualisation is best suited for 1,2 and 3-dimensional data: data of higher dimensionality must usually be either sliced prior to visualisation, or have its dimensionality reduced, before projecting it onto the 2D plane. Having said that, there are other ways of visualising the data without sacrificing any dimensions at all, such as Parallel Coordinates, though it is more suited to data exploration.
- **Evidence of conformance to well-known distributions:** There exist many probability distributions, some of whose properties are well-studied and well-known, like the Normal distribution. If the data approximates one of these distributions, there are several mature statistical methods which may be applied to test different hypotheses and properties of the data. Indeed, many of the classical statistical analyses make the assumption that the underlying data is (approximately) normally distributed.

In the following sections, we shall explore the shape of the Anganwadi 2010 dataset, and record our observations on it.

5.1 Univariate distributions

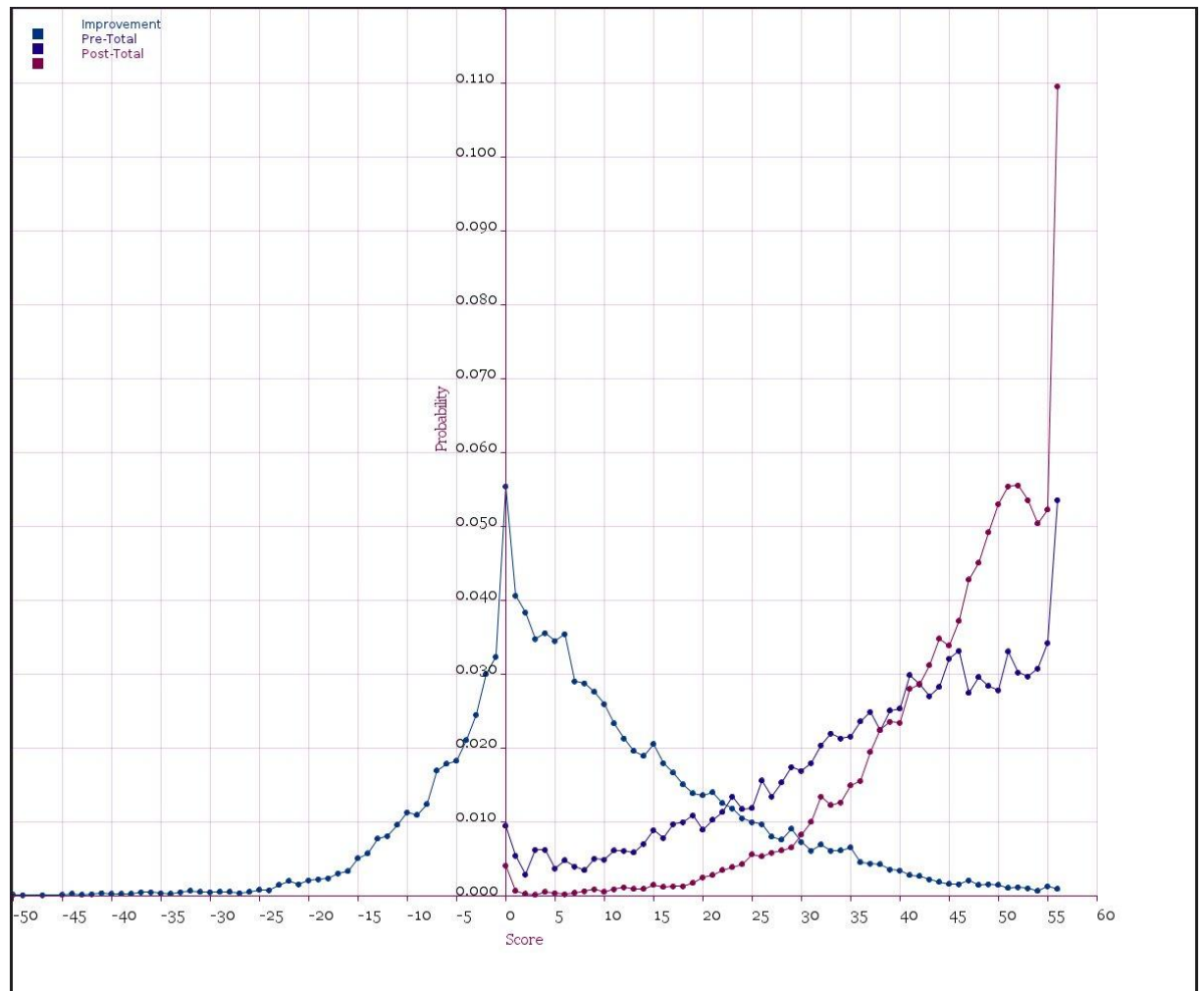
Figure 1 shows 3 distributions: the pre-intervention scores, the post-intervention scores, and the improvements.

5.1.1 Observations/Notes

- The pre-intervention score with the highest number of students is 56, which is the highest score possible. This implies that, even prior to intervention, a sizeable fraction of the students have scored very high on the test.
- The post-intervention score also follows the same trend, albeit with a steeper curve, which implies that many students have scored better in the post-test than in the pre-test.
- The improvement distribution is peaked, the peak being near zero. This makes sense, because if a large fraction of students answered all 56 questions as 1 in the pre-intervention score, there really is no way for them to improve. This is assuming that their performance did not worsen in the post-test. In fact, the calculated mean is around 7.
- There is a significant fraction of students whose performance has worsened in the post-test. This number is 7081. Out of those, we noticed that the worsening was dramatic for a small set. We have listed down the records which had a regression of 40 or more, below.

student_id	area	pre_total	post_total	language
1506619	VABASANDRA	52	0	KANNADA
1444355	KITTAGANA COLONY	56	0	KANNADA
1426910	PRIYA DARSHINI	53	0	KANNADA
1445387	KYALASANAHALLI	51	1	TELUGU
1445382	KOTHANUR	44	0	TAMIL
1445383	KOTHANUR	41	0	KANNADA
1442911	BETTANA PALYA	55	12	KANNADA
1445160	KODIGEHALI	52	0	KANNADA
1457095	KAVERI NAGARA	41	0	KANNADA
1457090	KAVERI NAGARA	44	0	TELUGU
1457092	KAVERI NAGARA	41	0	TELUGU
1507686	REHMATH NAGAR	44	0	URDU
1448385	MALSANDRA	50	0	KANNADA
1455798	KANTEERAVA COLONY	55	15	KANNADA
1444466	PRIYA DARSHINI	52	0	KANNADA
1444467	PRIYA DARSHINI	44	0	KANNADA
1445337	RACHENAHALLI	52	0	KANNADA

Figure 1: Probability distributions of pre-, post-intervention, and improvement



1425269	VINYAKNAGAR	41	0	KANNADA
552534	KYALASANAHALLI	52	1	TELUGU
1448976	KRISHNA SAGARA COLONY	54	14	KANNADA
1507157	BELTHURU	52	7	KANNADA
1444897	MUNESHWARA NAGAR	45	0	TAMIL
1444890	MUNESHWARA NAGAR	40	0	KANNADA
1542861	VERABADRA NAGAR 1	56	1	KANNADA
1358415	KOTHANUR	48	8	
1445437	THRIVENINAGARA	40	0	TELUGU
1442907	BETTANA PALYA	55	8	KANNADA
1457106	KAVERI NAGARA	40	0	TELUGU
1445444	THRIVENINAGARA	41	0	TELUGU
1444474	PRIYA DARSHINI	54	0	KANNADA
1445159	KODIGEHALLI	41	0	KANNADA
511998	KODIGEHALLI	55	0	KANNADA
1356274	KAVERI NAGARA	56	0	
1358429	KOTHANUR	42	0	
1497510	JALAHALLI 1	42	0	KANNADA
1443914	BIDARAHALLI	56	12	KANNADA
1366955	PRIYA DARSHINI	53	0	KANNADA
1366956	PRIYA DARSHINI	53	0	KANNADA
1447019	MADAPPANA HALLI	42	0	TELUGU
1355112	BYRATHI BANDE	55	14	
1504259	MAYASANDRA A	56	0	KANNADA
1457120	KAVERI NAGARA	43	0	TAMIL
1457089	KAVERI NAGARA	43	0	TAMIL
1445171	KODIGEHALLI	44	0	KANNADA
1457203	SARAIPALYA	51	0	URDU
1457179	BYRATHI BANDE	55	14	KANNADA
1504538	KEMPEGOWDA NAGAR	51	7	KANNADA
1444486	PRIYA DARSHINI	49	0	KANNADA
1444488	PRIYA DARSHINI	42	0	KANNADA
1451831	AMBED NAGAR	50	9	TAMIL

- The post-intervention score distribution seems to follow a power law. We shall consider modeling this attribute further on.

5.2 Outlier analysis

5.3 Bivariate distribution

So far, we've been looking at single variables in isolation. Figure 2 shows a bivariate histogram of pre- vs. post-intervention scores. The lighter a cell, the more the number of records in that 'bucket'.

$$y = 0.224.x + 37.134$$

5.3.1 Observations/Notes

- Many of the scores seem to be clustered near the top right. To further highlight this, we have draw a linear regression line as a rough indicator of a trend (To model the trend in more detail, we could use LOESS). What is somewhat puzzling is that there are not a few students whose performance has dropped after the intervention. This is evident even without doing a linear regression.
- The immediate outliers which are visible are the ones on the extreme left (pre 0, post 56) and at the origin (pre=0, post=0). The latter outlier(s) may be an artifact of corrupted data collection; we cannot say.

Figure 2: Bivariate probability distribution of pre- vs. post-intervention scores

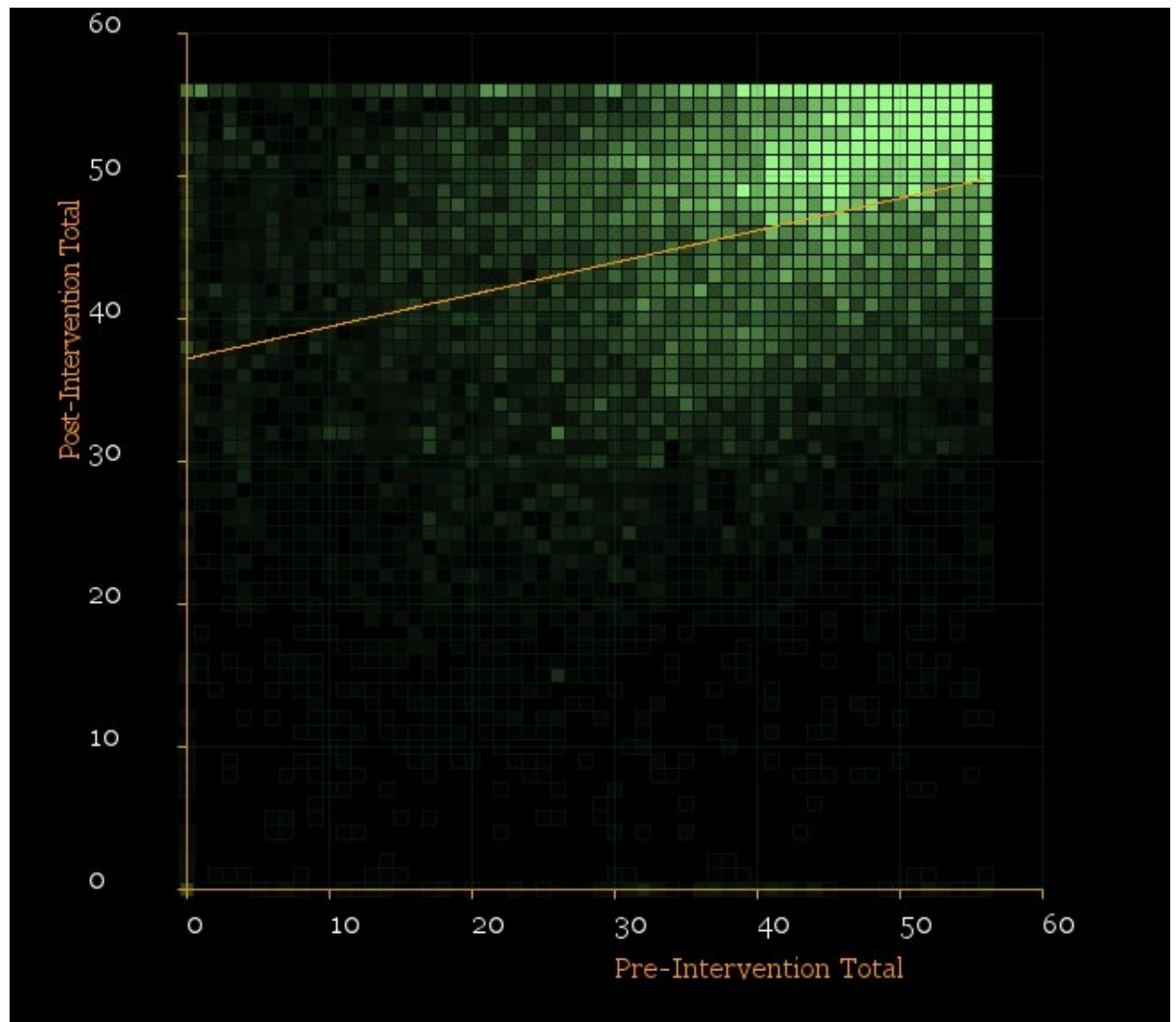
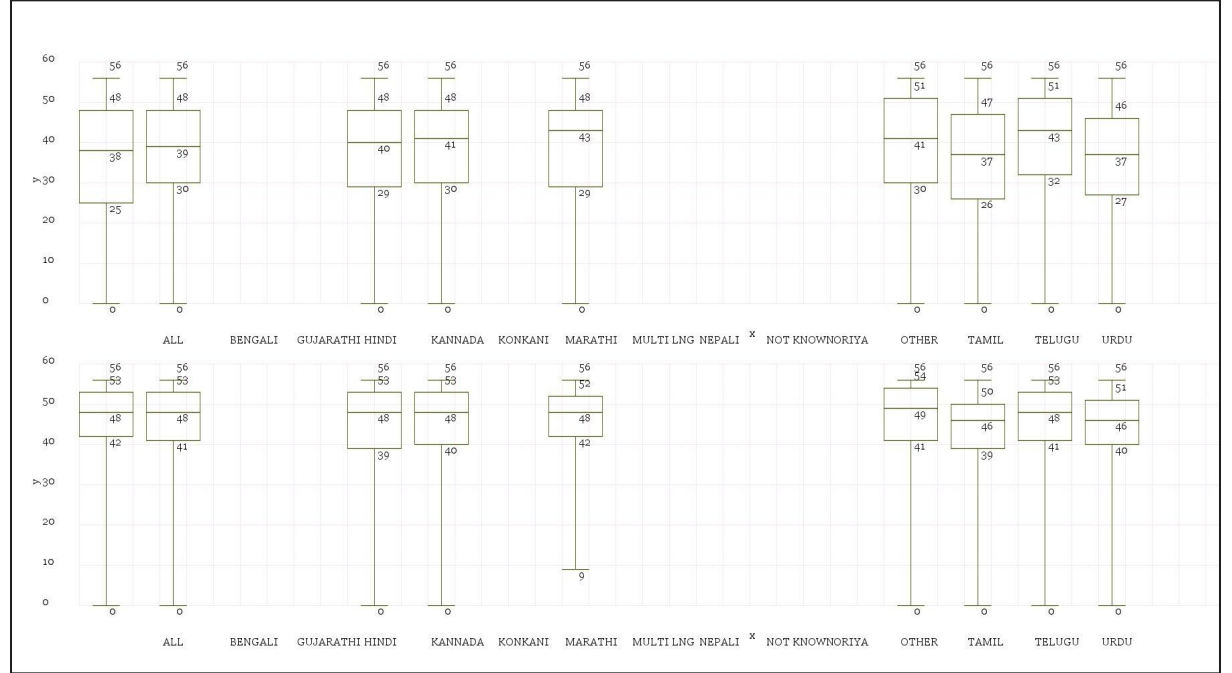


Figure 3: Box plots of pre- and post-intervention scores, broken down by language



5.4 Summary plots

Summary plots are so called for their ability to summarise up a data set as a set of numbers, which can be easily interpreted. We used Box Plots to summarise the data, broken down by language. Figure 3 shows that breakdown. The top row represents the box plots for the pre-intervention assessment, the bottom one for the post-intervention assessment.

5.4.1 Observations/Notes

- Out of all the languages, we were not able to create box plots because the corresponding samples were too few in number to differentiate between the different quartiles. These languages are Bengali, MultiLng, Not Known and Oriya.
- There are no dramatic differences between the plots in each set. The medians, 1st and the 2nd quartiles are rather close to each other.

5.5 Rank Order Charts: Geoclusters

One of the types of data that is pretty common is categorical nominal data. Categorical data is that which is non-numeric. Nominal data is that which does not lend itself to any intrinsic ordering. Names of places (if we ignore their coordinate system representation) are categorical data that is nominal.

The best way to summarise categorical data, especially if it is the Independent Variable, is to order records based on an ordinal Dependent Variable. This sort of a representation is called a Rank Order chart. We have rank ordered the geographical clusters based on the student population in each cluster.

Index	Cluster	Probability	Cumulative Distribution
1.	Sir M Vishweshwariah 1st Circle	0.030690257754477937	0.030690257754477937
2.	Sultanpalya	0.030289791757681667	0.0609800495121596
3.	konnana kunte	0.02668559778651522	0.08766564729867482
4.	kadugodi	0.02584825979321392	0.11351390709188874
5.	Laggere 4th circle	0.025156545798747633	0.13867045289063637
6.	K.R.Puram	0.024756079801951363	0.16342653269258772
7.	Murphy Town	0.024064365807485073	0.1874908985000728
8.	Netaji 2nd Circle	0.02340905781272754	0.21089995631280034
9.	Abbigere	0.02326343381389253	0.23416339012669288
10.	Begure	0.023190621814475027	0.2573540119411679
11.	Thavare kere	0.022462501820299987	0.2798165137614679
12.	Kodihalli	0.022244065822047472	0.30206057958351534
13.	Dodda Banaswadi	0.02202562982379496	0.3240862094073103
14.	Jigani	0.021952817824377458	0.3460390272316878
15.	Koramangala	0.021370321829037427	0.3674093490607252
16.	Byrathi	0.020387359836901122	0.3877967088976263
17.	Rajanakunte	0.020059705839522355	0.4078564147371487
18.	Varthur	0.01944080384447357	0.42729721858162223
19.	Hebbagodi	0.01907674384738605	0.4463739624290083
20.	Hreohalli	0.018639871850881024	0.4650138342798893
21.	Yalahanka	0.01802096985583224	0.48303480413572153
22.	Banashankari	0.017438473860492208	0.5004732779962138
23.	K Gollahalli	0.0172928498616572	0.517766127857871
24.	Sarjapura	0.017183631862530944	0.5349497597204019
25.	Mallasandra	0.01711081986311344	0.5520605795835154
26.	Kodigenahalli	0.01689238386486093	0.5689529634483763
27.	kasaba	0.01667394786660842	0.5856269113149848
28.	Haragadde	0.016564729867482163	0.6021916411824669
29.	Makali	0.016455511868355904	0.6186471530508229
30.	Doddakannelli	0.016419105868647154	0.63506625891947
31.	Mahantalingapura	0.016200669870394643	0.6512669287898646
32.	Baglur	0.015836609873307123	0.6671035386631717
33.	chandapura	0.015836609873307123	0.6829401485364789

34.	Bettahalasuru	0.015690985874472114	0.698631134410951
35.	Hesaragatta	0.015690985874472114	0.7143221202854232
36.	Kaggalipura	0.0154725498762196	0.7297946701616428
37.	Sondekoppa	0.015217707878258336	0.7450123780399012
38.	Wilson Garden	0.01510848987913208	0.7601208679190332
39.	Dommasanddra	0.01510848987913208	0.7752293577981653
40.	Kithuru Rani Channamma 3rd Circle	0.01474442988204456	0.7899737876802099
41.	Triveni 5th Circle	0.014489587884083296	0.8044633755642931
42.	Palace Guttalli Circle	0.014453181884374545	0.8189165574486676
43.	Kengeri circle	0.014343963885248289	0.8332605213339159
44.	Yashwanthpura	0.014052715887578273	0.8473132372214942
45.	Kuvempunagar 4th Circle	0.013543031891655745	0.8608562691131499
46.	Chikkajala	0.012960535896315713	0.8738168050094657
47.	Kadugondanahalli	0.012596475899228193	0.8864132809086939
48.	Attibele	0.012523663899810689	0.8989369448085045
49.	Indlavadi	0.012341633901266929	0.9112785787097715
50.	Sonnenahali	0.012305227901558177	0.9235838066113297
51.	Uttrahalli	0.012232415902140673	0.9358162225134703
52.	Machohalli	0.012159603902723169	0.9479758264161935
53.	Gavipura Guttahalli	0.009210717926314256	0.9571865443425077
54.	Marasur	0.009137905926896752	0.9663244502694045
55.	Netravathi	0.00906509392747925	0.9753895441968837
56.	Gopalapura	0.008955875928352992	0.9843454201252367
57.	Hennagara	0.00873743993010048	0.9930828600553372
58.	PANTHARAPALYA	0.002621231979030144	0.9957040920343674
59.	YARUB NAGAR	0.001783893985728848	0.9974879860200963
60.	VERABADRA NAGAR	0.001310615989515072	0.9987986020096113
61.	Yalahanka upnagar	0.00043687199650502403	0.9992354740061163
62.	DODDA BANASAWADI	0.000400465996796272	0.9996359400029126
63.	ASHRAYA NAGAR	0.00036405999708752004	1.0

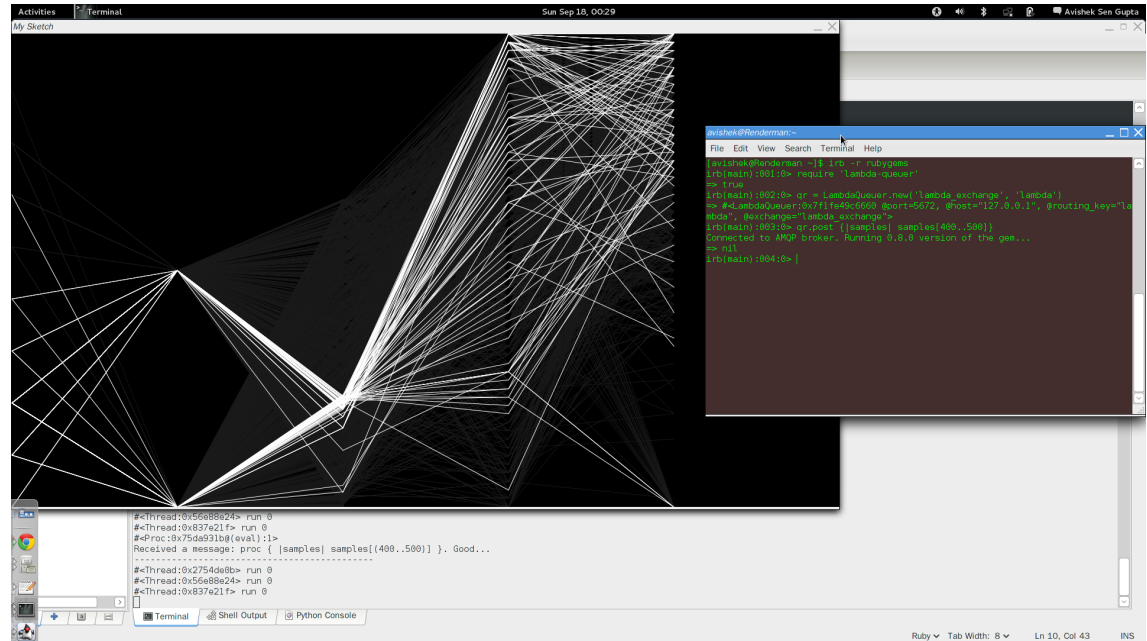
5.5.1 Observations/Notes

- Out of 63 clusters, the top 38 clusters account for 75% of the population.

6 Data Analysis and Exploration

There are multiple ways of exploring datasets when simple visual inspection is tedious and unintuitive. Many of them are standard, but some of them may reveal more about the nature of the data. Often, they are motivated by specific business drivers and questions, and some exploration may be custom to the dataset under analysis. Data exploration is also commonly done through queries to an OLAP database.

Figure 4: Parallel Coordinates showing language, gender, school, pre- and post-intervention scores



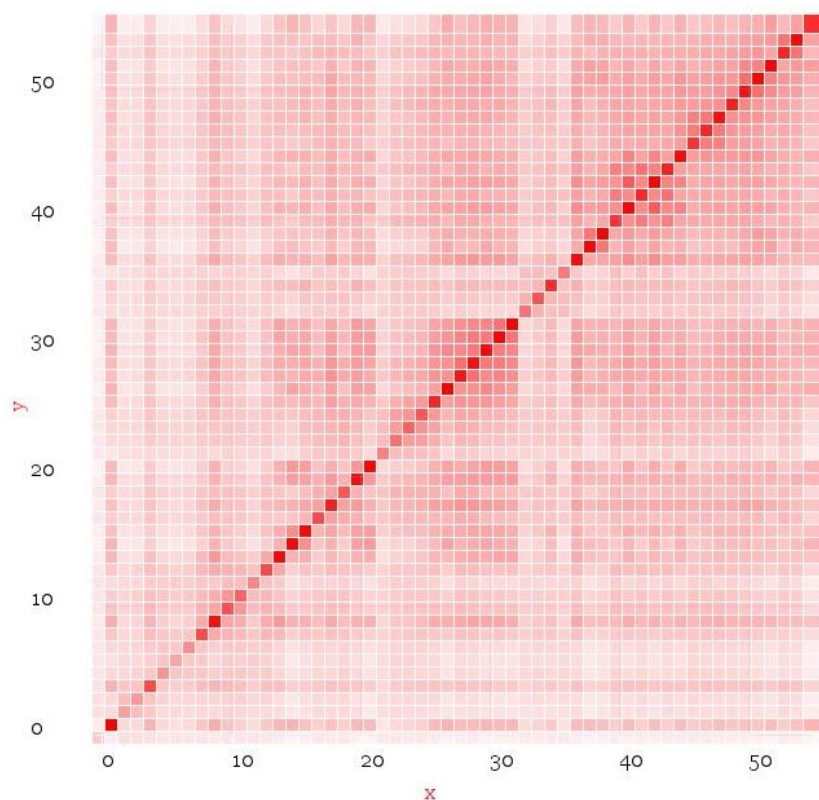
6.1 Parallel Coordinates

Parallel coordinates are an interesting way to explore high-dimensional data without discarding any detail. The only downside is that the elegance of presentation is somewhat sacrificed.

Essentially, instead of aligning axes in orthogonal directions (we can visualise only upto 3), the axes are stacked horizontally, giving the impression of n parallel lines. A single data point in this n -dimensional representation is a set of broken lines spanning the widths between the axes.

The way to explore this visualisation is to highlight the samples of interest, based on some criteria. The highlighted samples can then be inspected visually to discover trends, if any. Figure 4 shows an example of exploration using parallel coordinates on the Anganwadi data.

Figure 5: Covariance plot of pre-intervention responses

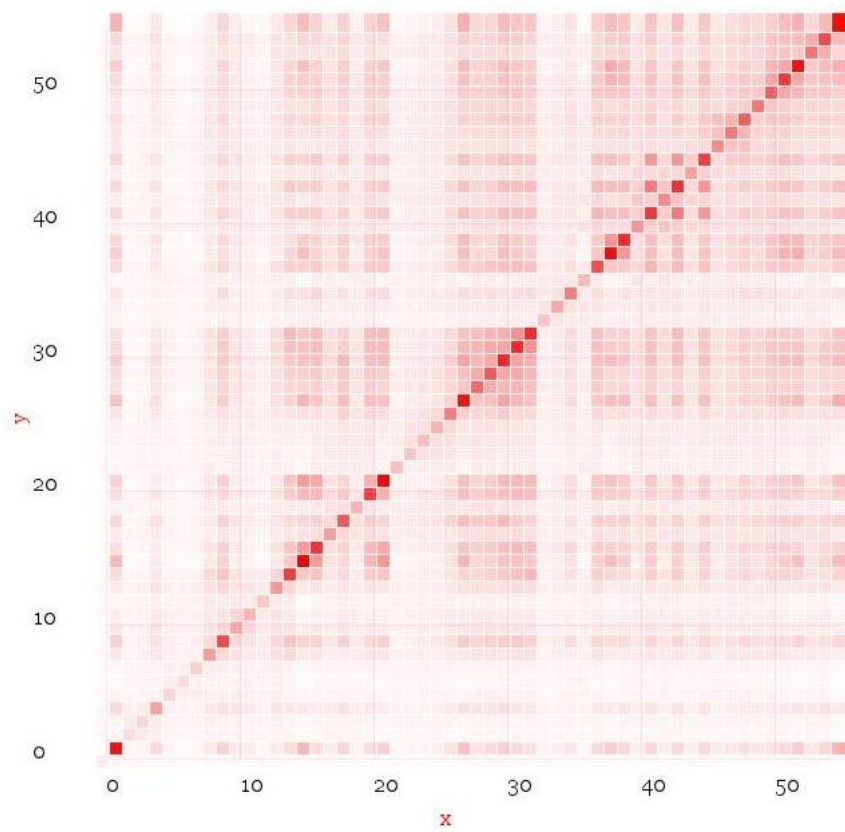


6.2 Covariance plot

The covariance plot is a first step towards looking at the correlations between responses to individual questions. Basically, the kind of question such analysis can answer is, for example, how dependent is a response to question 40 on the response to question 3? Do they change together, i.e., is there a strong dependency between answer to question X and the answer to question Y?

Figure 5 shows the covariance plot for the pre-intervention responses. Figure 6 shows the covariance plot for the post-intervention responses.

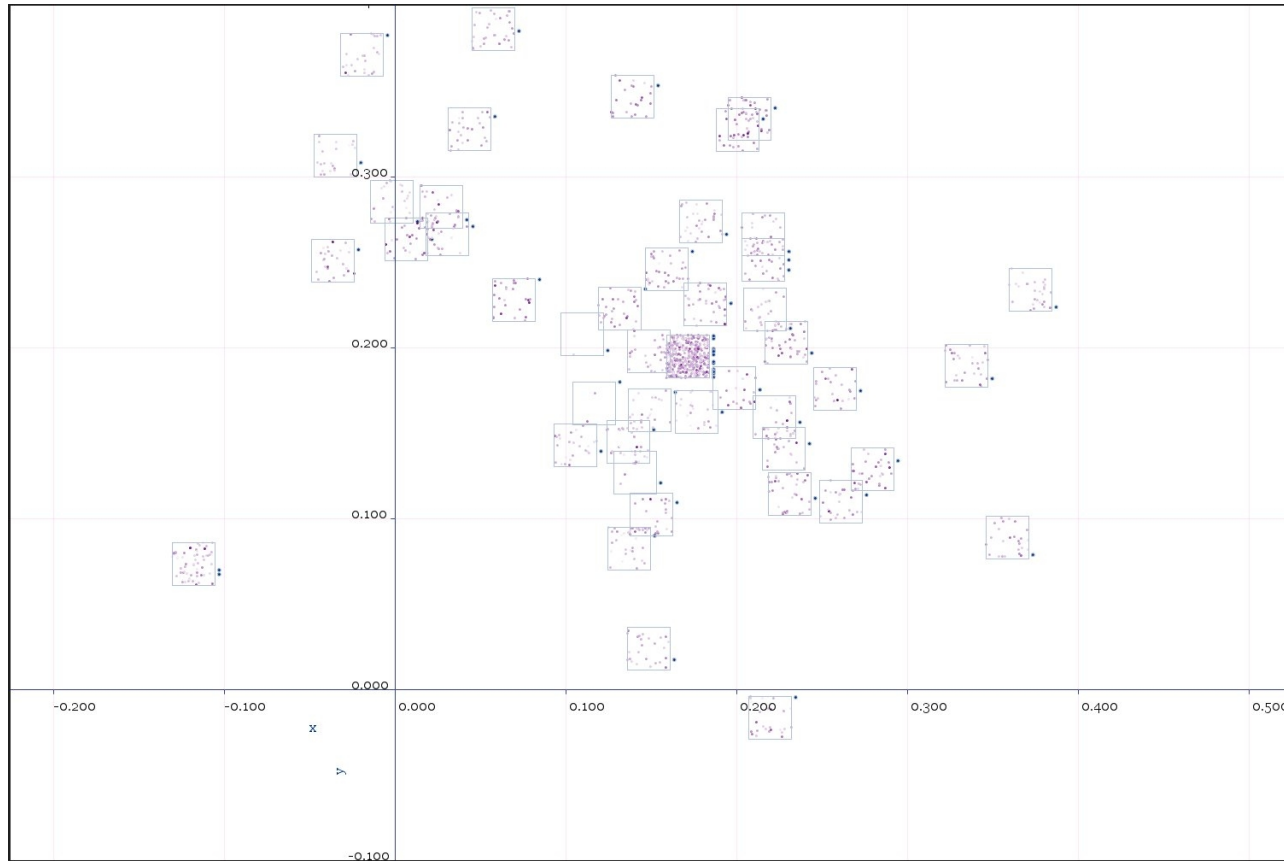
Figure 6: Covariance plot of post-intervention responses



6.2.1 Observations/Notes

- The plots are color coded such that the darker the hue, higher the covariance. The plot is symmetric about the diagonal, and the diagonal is much darker because it is essentially the covariance of a single response with itself. The data has not been normalised yet, hence the diagonal hues vary.
- The covariance between the responses appears weakened in the post-intervention scores, indicating that the degree of dependence between the responses has decreased.
- There appear to be clusters of relatively high covariance, for example, between the questions in the range of (40-43), (26-31), etc. Further investigation may be needed to answer specific questions in this area.
- Correlation does not imply causation. Inferred links between responses may be result of a deeper phenomenon; thus a high covariance doesn't necessarily imply a direct causative link between two responses.

Figure 7: Geocoded populations by cluster



6.3 Geographical distribution

7 Models and Statistical Tests

Models are simplified descriptions of the real-world phenomena, based on simplifying assumptions. Their primary use is to explain variations in observed data, and hopefully make useful predictions about data yet unseen.

Statistical tests, on the other hand, are designed to test certain hypotheses which might be inferred from the data set. These hypotheses may be proved or disproved by these tests, based on the notion of 'statistical significance'.

This section summarises the models and tests we've applied to the Anganwadi dataset.

7.1 Tests for conformance to distributions

Many of the tests in statistics work under the assumption that the underlying dataset (with or without transformation) is normally distributed, i.e., it follows a Gaussian probability distribution, namely:

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}}$$

This verification is important because it dictates, to a great extent, the validity of any tests we run. If the data turns out to be not normal, we will probably want to use nonparametric methods, which do not make assumptions about the shape of the distribution.

7.1.1 Jarque-Bera test

The Jarque-Bera test measures the conformance of a dataset to a Gaussian distribution using its skewness and kurtosis. For reference, the Jarque-Bera statistic is given by:

$$JB = \frac{n}{6} \left(S^2 + \frac{1}{4}(K - 3)^2 \right)$$

where S (skew) and K (kurtosis) are defined as:

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{3/2}}$$
$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2}$$

The null hypothesis that this statistic tests is that the skewness and excess kurtosis are both zero, which is the defining characteristic of a Gaussian distribution. The data below summarises the JB statistics for the pre-, post-intervention scores and the score improvements. Please note that the statistics from the data before the removal of the invalid data. This does not affect the results overmuch, however.

Pre-intervention Score

n = 28535 Skewness = -1.0001504234198

Kurtosis = -1.99959305171352

JB statistic = 34476.3843030411

Conclusion : Pre-intervention scores are not normally distributed.

Post-intervention Score

n = 28535

Skewness = -1.00010106352368

Kurtosis = -1.9997273050813

JB statistic = 34477.5108572449

Conclusion : Post-intervention scores are not normally distributed.

Score Improvement

n = 28535

Skewness = -4.24685767755943

Kurtosis = 37.1765663807246

JB statistic = 1474523.40413686

Conclusion : Score improvements are not normally distributed.

The initial probability distributions provided very clear evidence that the pre- and post-intervention scores were not normal: this is merely validation. However, the test also shows that the improvement is not normally distributed either. If we notice the improvement distribution in Figure 1, we notice a few things:

- Fat tails
- Thin narrow peak (high kurtosis)
- Fair amount of skew (Non-zero excess skew)

We did attempt to assess the visual fit of a Gaussian distribution, but this was not very successful. Figures 8 and 9 show some of our attempts at fitting different theoretical distributions. The Cauchy distribution was a closer fit after a translation and a transform, but there are still outliers.

7.1.2 Quantile-Quantile plots

There is a visually more intuitive way of measuring the goodness of fit of a dataset to any theoretical probability distribution. That method is called a Quantile-Quantile plot. They are points marked off at regular intervals on the cumulative distribution function of a random variable. The Q-Q plot is so named for the two quantile plots that are reflected in a single plot: one for the theoretical distribution, and one for the dataset.

The theoretical distribution's quantile plot comes out as a straight line. If the dataset follows the distribution closely, it will also follow this line, more or less.

Figure 8: Improvement data overlaid with theoretical Gaussian ($\sigma^2=245.47$, $\bar{x}=7.22$)

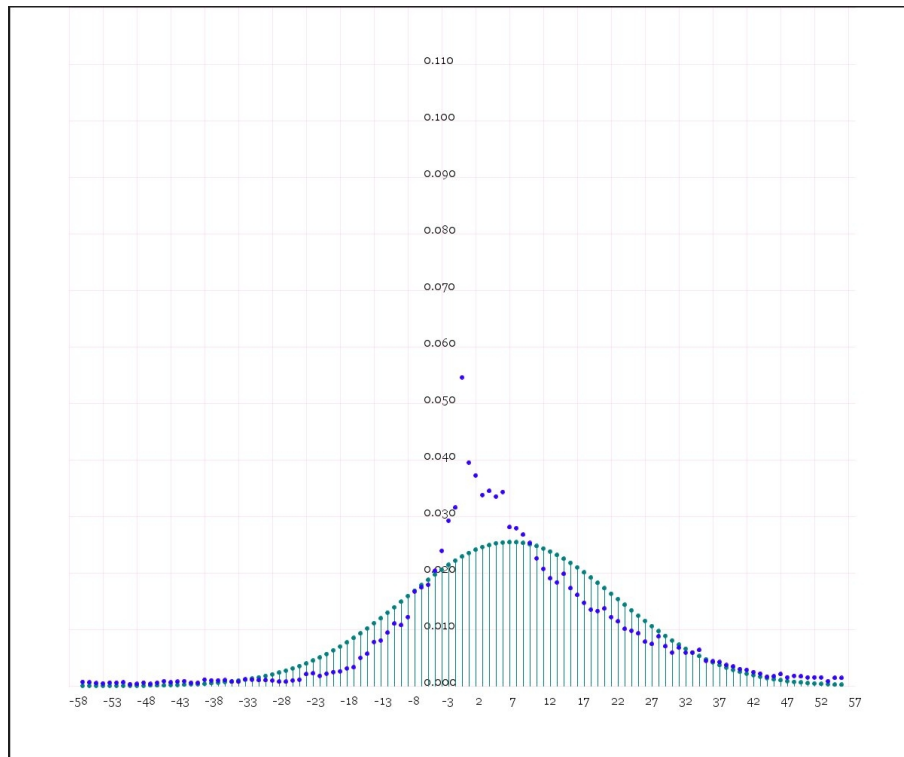


Figure 9: Translated log of data overlaid with theoretical Cauchy (scale=0.14, $\bar{x}=4.13$)

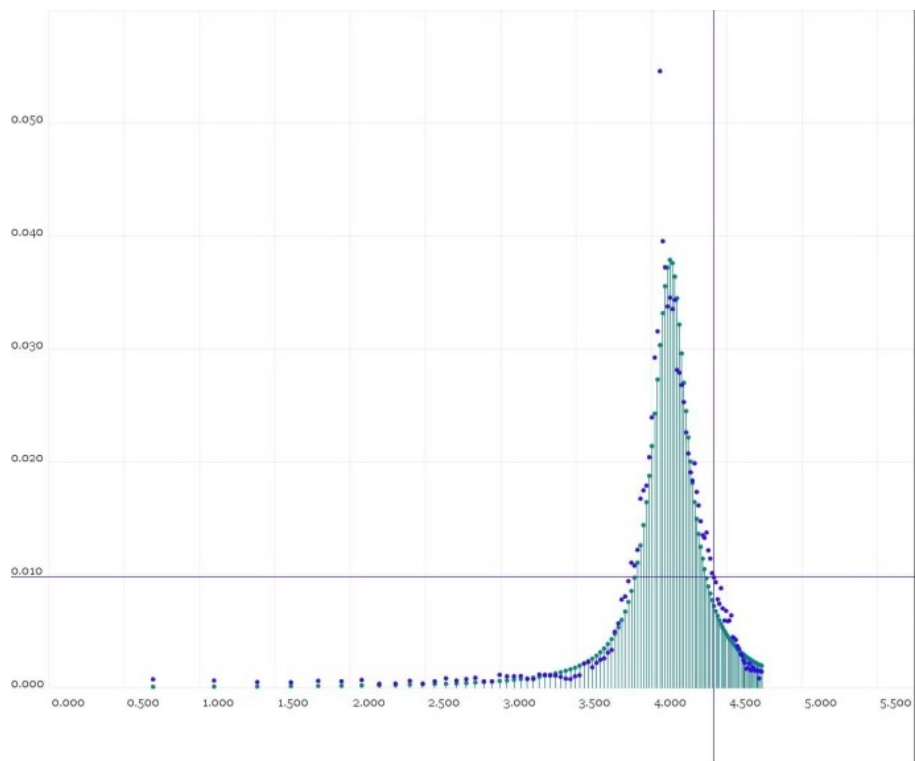
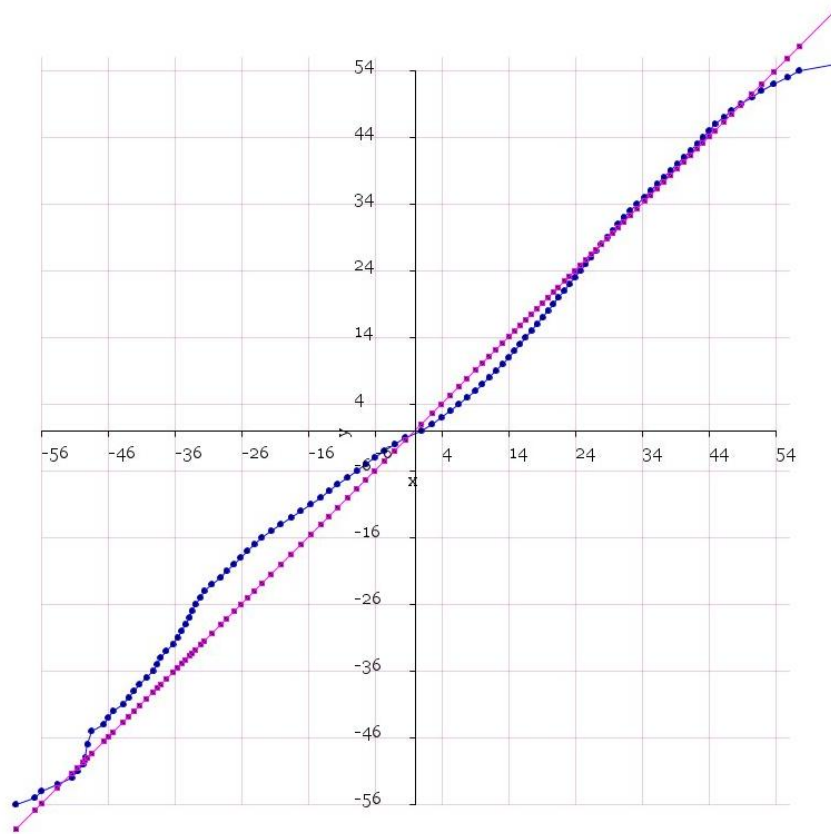


Figure 10: Quantile-Quantile plot of score improvement vs. theoretical Gaussian



Significant deviations indicate violation of fit. Figure 10 shows the Q-Q plot of score improvement and the Gaussian.

7.1.3 Observations/Notes

- The dataset shows a systematic deviation from the Gaussian quantile.
- Conformance to the theoretical distribution is never exactly possible, so the "fat pencil test" is used. Essentially, if the blunt end of a pencil covers both the theoretical and the actual dataset, we can say that the data approximates the model somewhat. In this case, the fat pencil test fails too.

We present another example of curve fitting to derive a probability distribution model, this time, with more success. We attempted to fit the exponential

distribution to the post-intervention data. The exponential distribution is given as:

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Because improvement can be negative as well, we applied the following transform to the data:

$$I'(x) = I(x) + 57$$

As Figure 11 shows, the distribution is a reasonable fit to the data. Further validation comes from the Q-Q plot of the dataset and the theoretical exponential in Figure 12. Note how closely the dataset tracks the theoretical quantile, except near the top right, which is where the 'kink' in the data set lies.

7.2 The Central Limit Theorem

Fortunately, we need not discard classical statistical techniques because the underlying distribution is not normally distributed. This is because even though the data itself is non-normal, its means are very probably normally distributed, regardless of the original distribution of the data.

This statement is known as the Central Limit Theorem, which states that:

The distribution of the sum of a large number of independent, identically distributed variables will be approximately normal, regardless of the underlying distribution.

To illustrate the point, we show the histograms of the means of the pre-intervention scores, post-intervention scores, and the improvements, in Figures 13, 14, and 15, respectively.

The Jarque-Bera statistics for these three cases is well below the threshold. The shape validates that, too.

Why is the Central Limit Theorem important? It is because, even though statistical tests might not be appropriate for the raw, non-normal, data itself, they certainly can be applied to the sampling distribution of the means of the data. We can still proceed with these tests, at the expense of losing the richness of data, and boiling it down to a single metric, namely, the mean.

This report does not cover any further tests on these sampling distributions.

Figure 11: Curve-fitted theoretical exponential for reflected post-intervention probability distribution

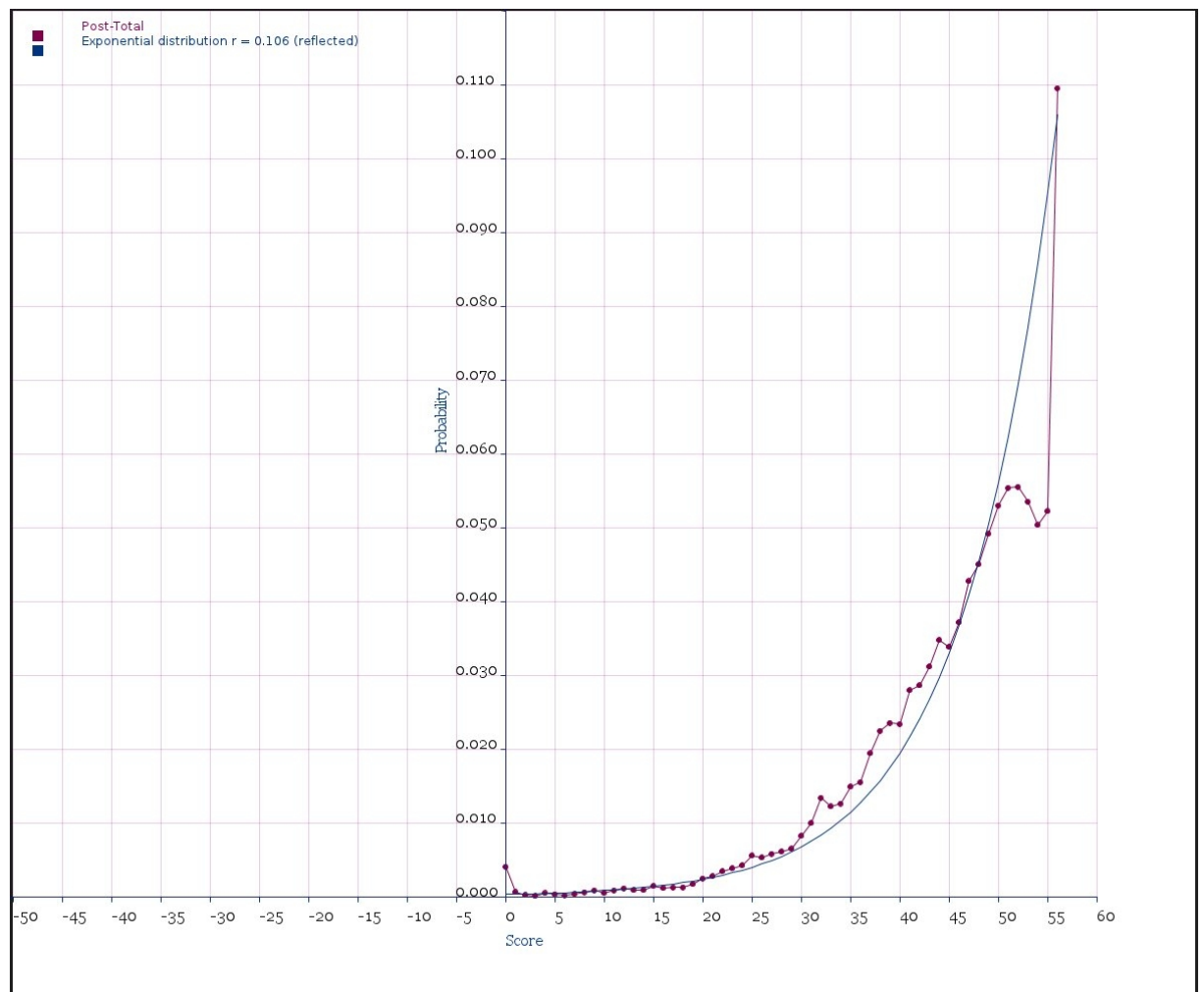


Figure 12: Quantile-Quantile plot of post-intervention score (reflected) vs. theoretical exponential

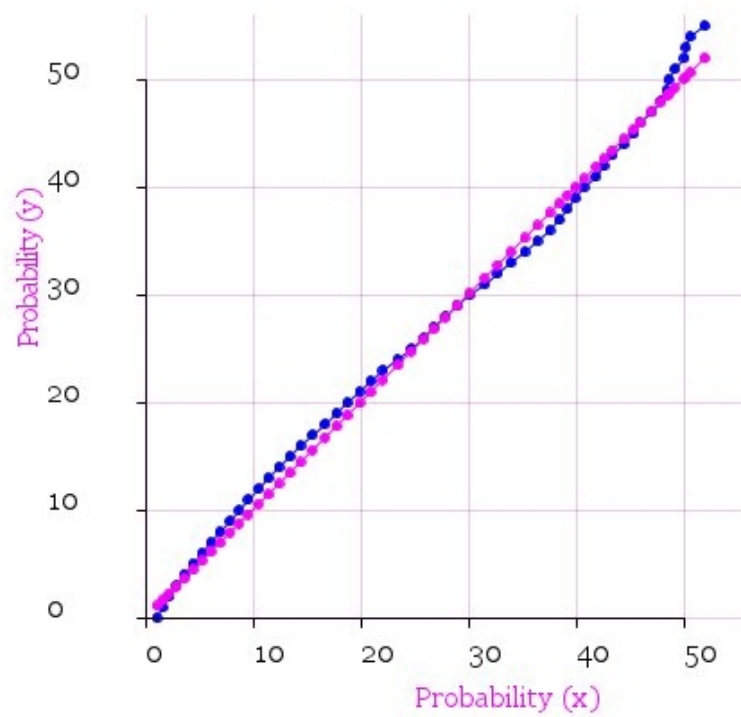
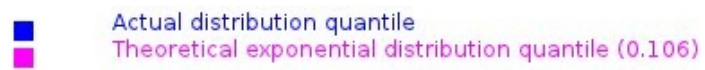


Figure 13: Central Limit Theorem illustration on the pre-intervention score

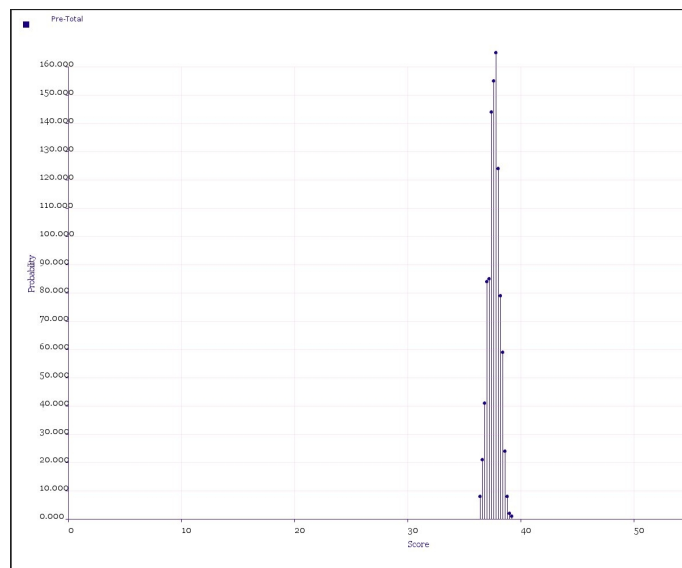


Figure 14: Central Limit Theorem illustration on the post-intervention score

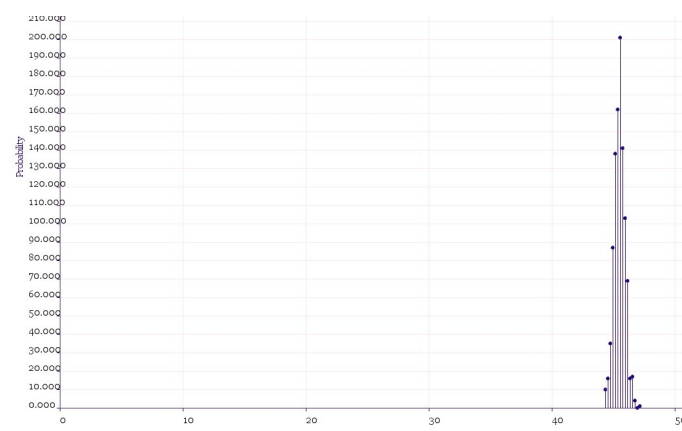
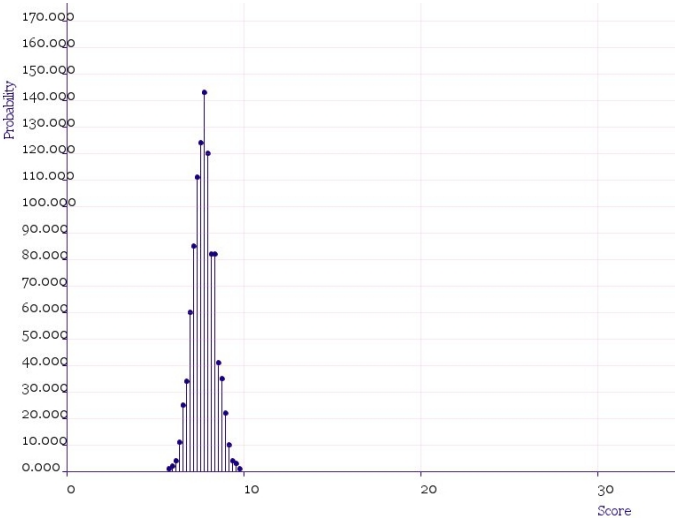


Figure 15: Central Limit Theorem illustration on the improvement metric



7.3 Answer distribution

The answer distribution is a visual map of the pattern of answering in different groups segregated by the pre-intervention score.

Each row in the map corresponds to the population of students in a single performance bracket. Thus, the topmost row corresponds to the group of students who score 56 in the pre-intervention, for example.

Within each row, each cell (left to right) represents the answer to the corresponding question. Thus, the (10,5) represents the response to question number 5 in the bracket of students who scored 10 prior to intervention.

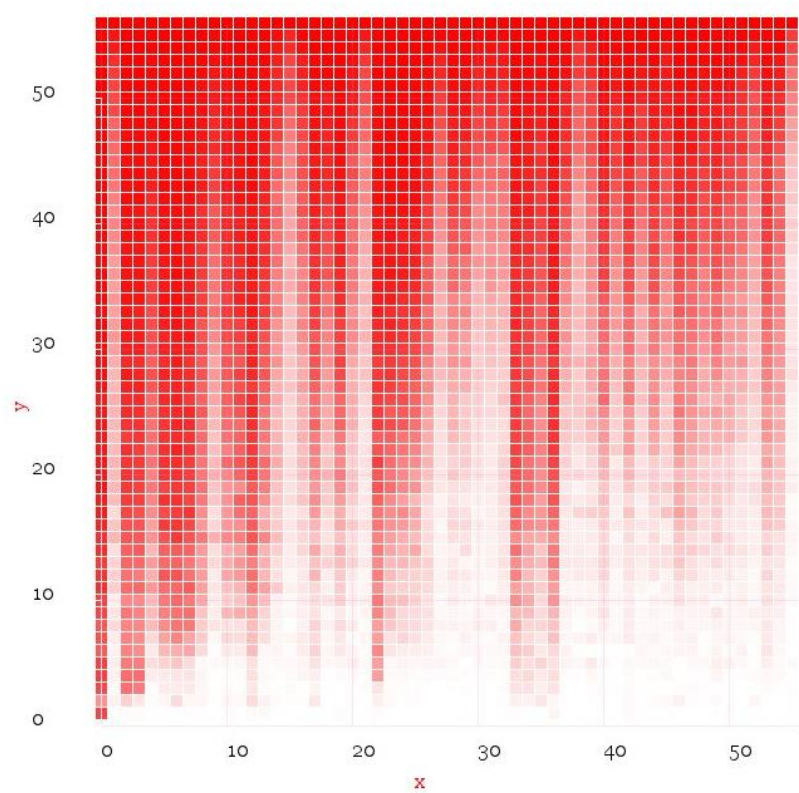
The brightness of the hue in a cell represents the fraction of students in that corresponding bracket who answered 1 to that specific question.

Therefore, starting from the bottom of the map, we get a sense of which questions only began to be answered in the affirmative in the higher brackets. This gives us a sense of the 'difficulty' of each question.

7.3.1 Observations/Notes

- The 'difficulty' metric of questions tends to clump together in bands in the left two-thirds of the map. Answering patterns from around question number 37 and above alternate much more uniformly.

Figure 16: Answer distribution vs. question number



7.4 Effectiveness of intervention

7.4.1 Intervention effect on individual responses: McNemar's Test

McNemar's test for matched pairs is used to measure statistical difference between two tests on matched pairs of data. Examples of matched pairs of data are:

- Test scores of siblings
- Test scores before and after intervention of the same student.

The second point is aligned with the situation that we are dealing with. The two, disjoint, tests essentially are:

- Candidate answered 1 to the question.
- Candidate answered 0 to the question.

Using this, we will be able to deduce whether the intervention had any significant effect on the response to a particular question across the entire population of students. This has the side-effect of allowing us to analyse this data in a fine-grained fashion.

7.5 Modeling responses as Bernoulli trials

7.5.1 The Binomial Distribution

7.5.2 Typical questions

7.6 Test for variable independence

7.6.1 Chi-square test

Null hypothesis: Area and Improvement are NOT related.

For area vs. improvement

Chi-Square statistic = 56499.4692602837

$X^2 = 9652.9739$

Degrees of freedom = 9426

Null hypothesis rejected

^c

Null hypothesis: Area and Pre-Score are NOT related.

For area vs. pre-score

Chi-Square statistic = 58665.7089390644

$X^2 = 8062.2959$

Degrees of freedom = 7855

Null hypothesis rejected

Null hypothesis: Area and Post-Score are NOT related.

For area vs. post-score

Chi-Square statistic = 38567.0016158761

$X^2 = 8062.2959$

Degrees of freedom = 7855

Null hypothesis rejected

Null hypothesis: Language and Post-Score are NOT related.

For language vs. post-score

Chi-Square statistic = 280.234448946825

$X^2 = 96.2166$

Degrees of freedom = 75

Null hypothesis rejected

Null hypothesis: Language and Improvement are NOT related.

For language vs. improvement

Chi-Square statistic = 232.464548410971

$X^2 = 113.1452$

Degrees of freedom = 90

Null hypothesis rejected

Null hypothesis: Language and Pre-Score are NOT related.

For language vs. pre-score

Chi-Square statistic = 277.85501653079

$X^2 = 96.2166$

Degrees of freedom = 75

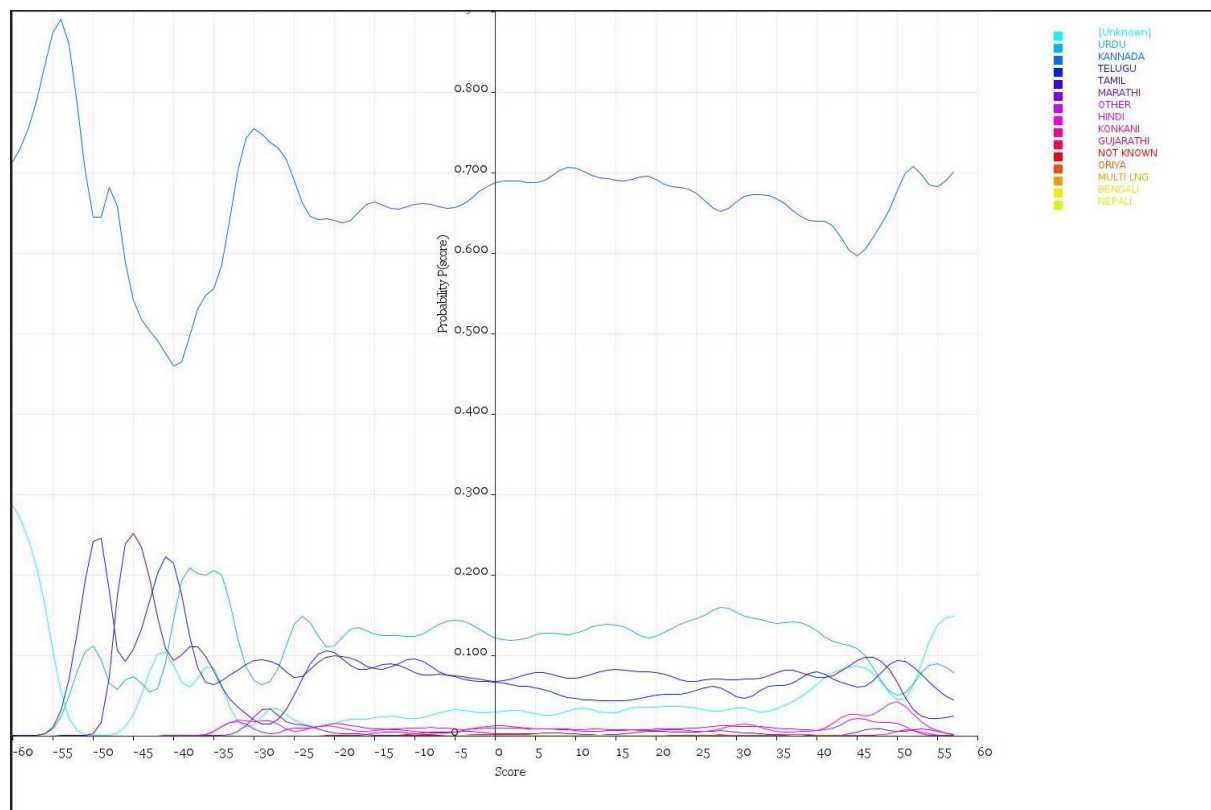
Null hypothesis rejected

8 Prediction and Classification

8.1 Decision Trees

8.2 Bayes classifier

Figure 17: Bayes posterior distribution of language from score improvement



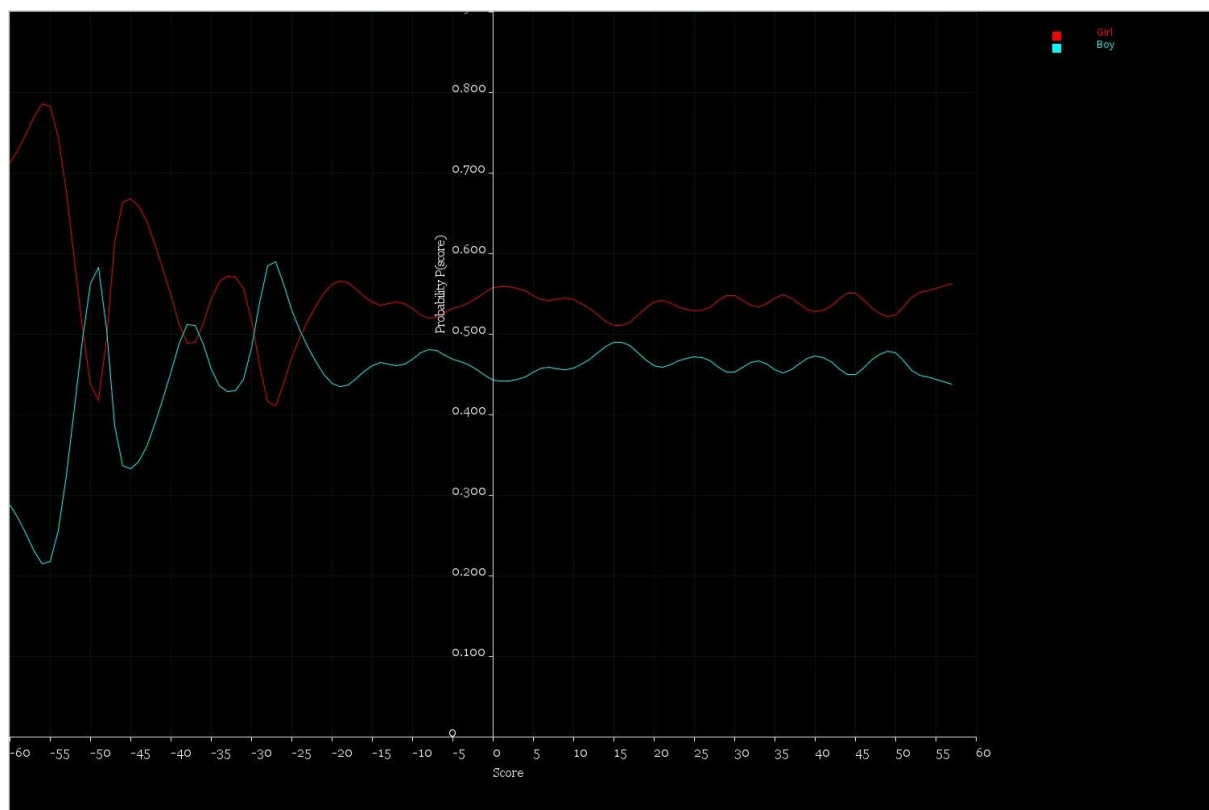
8.3 Density estimators

8.3.1 Naive Bayes density

8.3.2 Kernel density estimation

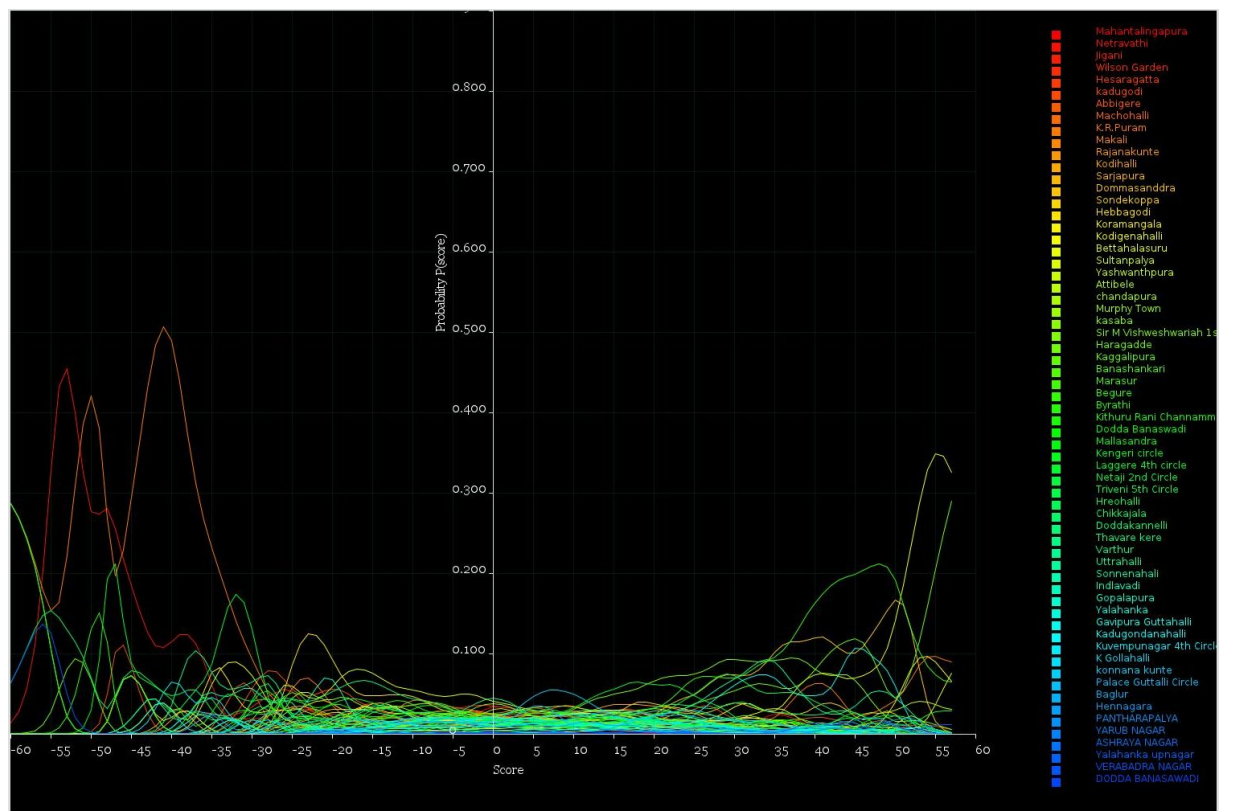
8.3.3 Results

Figure 18: Bayes posterior distribution of gender from score improvement



9 Dimension reduction/Factor analysis

Figure 19: Bayes posterior distribution of geocluster from score improvement



9.1 Principal Component Analysis

10 Technical notes

For base visualisation and interactivity, Processing was used through its Ruby bindings (Ruby-Processing). All plots were done using a coordinate plotting gem called Basis-Processing.

Some calculations like Principal Component Analysis were done using the Stat-sample gem from the SciRuby project.

JRuby 1.6.4 was used for all code needing visualisation; Ruby 1.9.2 was used for everything else.

Data was stored in a MySQL database.