Initial report on analysis of the Anganwadi 2010 dataset

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1   Abstract

This report summarises the results of exploration of the Anganwadi dataset provided by the Akshara Foundation. The analysis aims to characterise the structure of the data, and reveal trends (which would otherwise be obscured by the format of the source data) which may inform strategy through subsequent prediction and/or classification procedures.

2 Methodology

2.1 CRISP-DM

CRISP-DM is a process model distilled from the most common approaches used in data mining procedures. It stands for Cross Industry Standard Process for Data Mining. Not so much a prescription as a collection of 'good practices' followed by data mining professionals, **CRISP-DM** has the following characteristics.

- Domain-neutral

- Tool-neutral

- Provides a structural approach to the data mining process

CRISP-DM segregates data mining endeavours into the following phases.

- Business Understanding

- Data Understanding

- Data Preparation

- Modeling

- Evaluation

- Deployment

2.2 Relevance of CRISP-DM to this report

As far as this report is concerned, the relevant or most significant phases we focus on are:

- Data Understanding

- Data Preparation

- Modeling

Work on the Evaluation step is still preliminary, and will probably be the subject of another report. In a full-fledged project, the rest of the activities upstream and downstream to the above list will assume more importance, and require corresponding investment.

3 Data Preparation

3.1 Nature of the source data

The dataset comes from the education domain. The source data is a file, with each line corresponding to a single student evaluation record. Roughly, there are 29000 records, prior to any data sanitisation. Each line is pipe(|)-delimited into multiple fields. The fields salient to this analysis are listed below:

- Location of the student's school

- Language of the student

- Student's score before intervention

- Student's score after intervention

The score is not a single number, it is a set of 56 responses marked as 0/1. Generally, a 1 may be treated as a favourable answer, therefore, adding them up to get a single aggregate score has natural ordering: a sense of who did better. We reproduce two such records below, with the original formatting.

30915|YALLAMMANADODDI|KANNADA|1231468|Girl||1NoSection|Aug 2010 Assessment |1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1 |Anganwadi Post Test|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|1|

29607|BADAMAKAAN I|KANNADA|1445172|Girl|URDU|1NoSection|Aug 2010 Assessment |1|0|1|1|0|1|0|1|1|1|1|1|1|1|0|1|1|1|1|1|0|1|1|1|1|1|0|1|1|1|0|1|1|1|1|1|0|1|1|0|1|1|1|1|1|1|1|0|1|1|0|1|1|1|1|1|1|1|1|1|1|1|1|0 |Anganwadi Post Test|1|0|1|1|0|1|1|0|1|1|0|1|1|1|0|1|1|0|0|1|1|0|1|0|0|1|1|1|0|1|1|0|0|1|1|0|0|1|1|1|1|1|1|1|1|1|1|1|1|1| ...

Looking at the second row, we see that the location of the Anganwadi is BADAMAKAAN I, the student is female and speaks Urdu. The first contiguous set of 0s and 1s is the pre-intervention score, and the next set is the post-intervention one.

3.2 Data representation

3.2.1 Data store

Before any sort of sanitisation or analysis may be performed, it is important to ensure that the source data is stored in a format/datastore which makes querying and modifying the data relatively painless. This decision is largely driven by technological considerations, like:

- Scale of data (centralised/distributed store?)

- Sophistication of queries (OLAP/OLTP?)

- Structure of data, or lack thereof (SQL/NoSQL?)

We were dealing with only about 29000 records, and most of the analysis would probably be performed outside the database. Thus, we opted to use MySQL as our datastore.

3.2.2 Schema

The decisions when creating the database schema affect the ease of querying for relevant information. Apart from the attributes of interest, we wanted to store the individual binary responses as well. One way is to create one column for each response, giving us a total of 112 columns for storing these responses (56 for pre-intervention, 56 for post-intervention). The other way, and that is the one that we chose was to store this information as a 64-bit integer (bigint for MySQL). When required, we could unpack the individual response bits from this number.
A `desc responses;` command on the table reveals the schema we ended up with.

```
+------------------+------------+------+-----+---------+----------------+
| Field            | Type       | Null | Key | Default | Extra          |
+------------------+------------+------+-----+---------+----------------+
| student_id       | int(11)    | YES  |     | NULL    |                |
| area             | char(50)   | YES  |     | NULL    |                |
| pre_performance  | bigint(20) | YES  |     | NULL    |                |
| post_performance | bigint(20) | YES  |     | NULL    |                |
| language         | char(50)   | YES  |     | NULL    |                |
| gender           | char(20)   | YES  |     | NULL    |                |
| pre_total        | int(11)    | YES  |     | NULL    |                |
| post_total       | int(11)    | YES  |     | NULL    |                |
| id               | int(11)    | NO   | PRI | NULL    | auto_increment |
| school_id        | int(11)    | YES  |     | NULL    |                |
| year             | int(11)    | YES  |     | NULL    |                |
+------------------+------------+------+-----+---------+----------------+
```

The most important use of the reference data is to locate the schools geo-graphically. Given that geocoding the school from its name, we used the cluster to locate schools in 2D space. We deal with geographical analysis in a later section. The schema of the master data mostly mirrors the CSV master data file format, with the addition of latitude and longitude, like so:

```
+-------------+----------------+------+-----+---------+----------------+
| Field       | Type           | Null | Key | Default | Extra          |
+-------------+----------------+------+-----+---------+----------------+
| district    | char(50)       | YES  |     | NULL    |                |
| block       | char(50)       | YES  |     | NULL    |                |
| cluster     | char(50)       | YES  |     | NULL    |                |
| school_id   | int(11)        | YES  |     | NULL    |                |
| school_code | char(20)       | YES  |     | NULL    |                |
| school_name | char(50)       | YES  |     | NULL    |                |
| id          | int(11)        | NO   | PRI | NULL    | auto_increment |
| latitude    | decimal(20,10) | YES  |     | NULL    |                |
| longitude   | decimal(20,10) | YES  |     | NULL    |                |
+-------------+----------------+------+-----+---------+----------------+
```

To identify latitude and longitude, we used Google's Map API to geocode the cluster information. It is to be noted that there may be some clusters which weren't located by the Map API, and some more work is needed to cross-validate the coordinate information taken from the Map API.

3.3 Data migration: Identifying invalid data

It is natural to expect missing or corrupted data. The most crucial attribute are the score data, as any misinterpretation of that data may adversely bias the quality of our analysis. Thus, specific checks were put in place to ensure that none of the binary responses was null or some string other than 0 or 1.

Using this check, we found 1067 responses which violated it. All of them had either empty pre- or post-intervention scores. We did not migrate these response records, though it may be possible to do Monte Carlo simulations to predict the missing data.
As a result, out of a total of 28535 records in the original source, 27468 were migrated to the database.
We also found a large fraction of records which did not have a LANGUAGE attribute, i.e., that field was empty. Nevertheless, they were included in the migration.

4 Bias

Analysis is most suscpetible to bias in the data collection stage. Sampling is one such activity. If, for a statistical study, participating individuals are not equally likely to have been selected, it may be difficult to distinguish between the actual phenomenon and this biased sampling. This sort of bias is called sampling bias.

4.1 Sampling bias

To find evidence for bias, we looked at a few parameters. Here is the breakdown of the population by language, with the biggest language bucket highlighted.

Unspecified=869
URDU=3564
KANNADA=18685
TELUGU=1688
TAMIL=2051
MARATHI=91
OTHER=239
HINDI=243
KONKANI=18
GUJARATHI=12
NOT KNOWN=3
ORIYA=2
MULTI LNG=1
BENGALI=1
NEPALI=1

There is an overwhelming proportion of students who speak Kannada as their mother tongue (leading by an order of magnitude), a fact that is very likely to bias any sort of analysis where language is involved. We must remain cognizant of such biases, and interpret the results accordingly.

Here is the breakdown of the population by gender.

Girl=14822
Boy=12646

There is not a huge disparity between the two sexes, which indicates that any analysis/prediction based on gender may be less biased.

5 Shape of the Data

Before embarking on any deep analysis of data, it behooves us to look at the shape of the raw data. There are a few reasons why we want to do this.

- **Evident trends/outliers**: Visualisation of the raw data set is always a quick way to spot trends without doing too much analysis. Of course, visualisation is best suited for 1,2 and 3-dimensional data: data of higher dimensionality must usually be either sliced prior to visualisation, or have its dimensionality reduced, before projecting it onto the 2D plane.
  Having said that, there are other ways of visualising the data without sacrificing any dimensions at all, such as Parallel Coordinates, though it is more suited to data exploration.

- **Evidence of conformance to well-known distributions**: There exist many probability distributions, some of whose properties are well-studied and well-known, like the Normal distribution. If the data approximates one of these distributions, there are several mature statistical methods which may be applied to test different hypotheses and properties of the data.
  Indeed, many of the classical statistical analyses make the assumption that the underlying data is (approximately) normally distributed.

In the following sections, we shall explore the shape of the Anganwadi 2010 dataset, and record our observations on it.

5.1 Univariate distributions

Figure 1 shows 3 distributions: the pre-intervention scores, the post-intervention scores, and the improvements.

5.1.1 Observations/Notes

- The pre-intervention score with the highest number of students is 56, which is the highest score possible. This implies that, even prior to intervention, a sizeable fraction of the students have scored very high on the test.

- The post-intervention score also follows the same trend, albeit with a steeper curve, which implies that many students have scored better in the post-test than in the pre-test.

- The improvement distribution is peaked, the peak being near zero. This makes sense, because if a large fraction of students answered all 56 questions as 1 in the pre-intervention score, there really is no way for them to improve. This is assuming that their performance did not worsen in the post-test. In fact, the calculated mean is around 7.

Figure 1: Probability distributions of pre-, post-intervention, and improvement
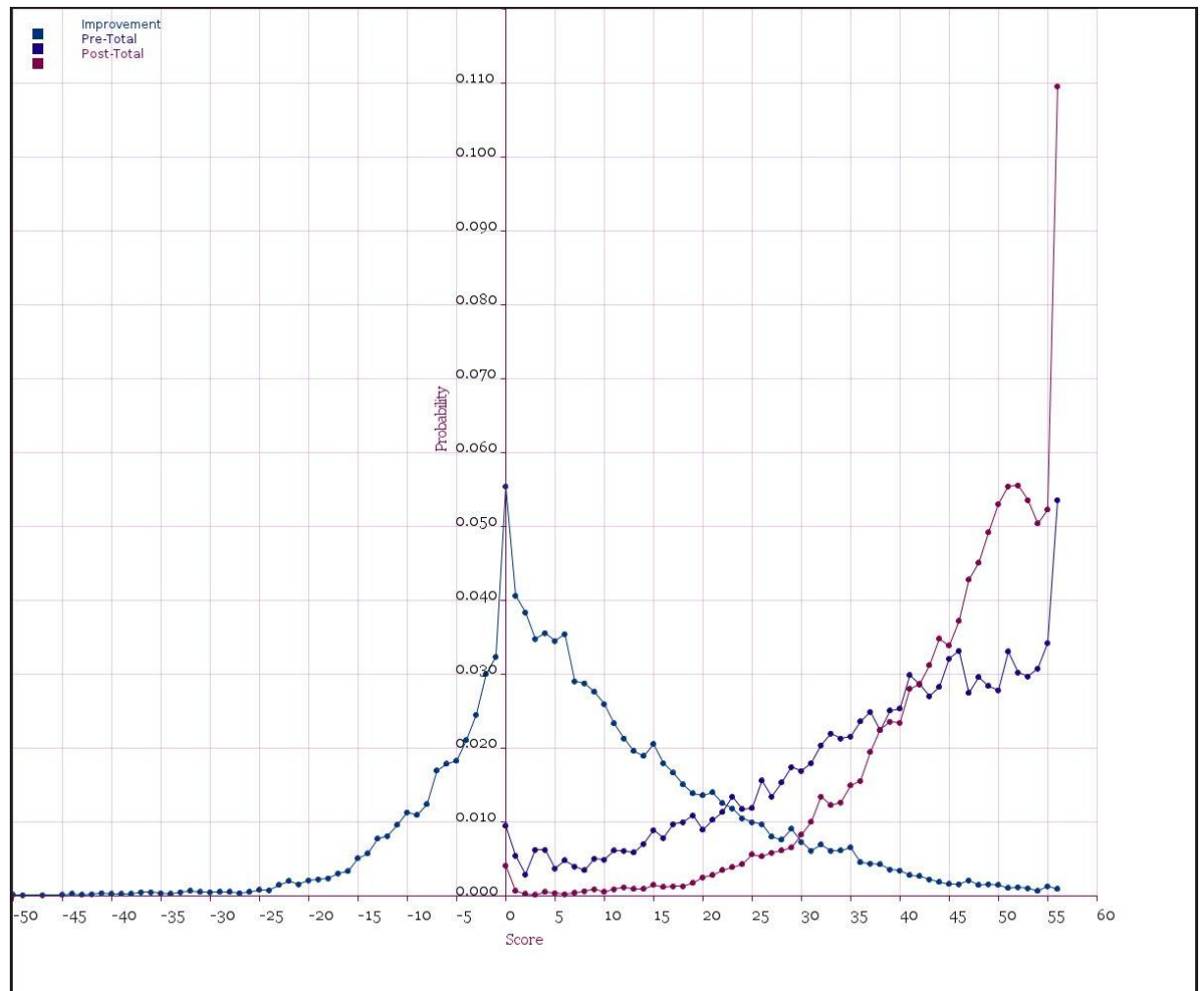
- There is a significant fraction of students whose performance has worsened in the post-test. This number is 7081. Out of those, we noticed that the worsening was dramatic for a small set. We have listed down the records which had a regression of 40 or more, below.

```
+------------+-----------------------+-----------+------------+----------+
| student_id | area                  | pre_total | post_total | language |
+------------+-----------------------+-----------+------------+----------+
|    1506619 | VABASANDRA            |        52 |          0 | KANNADA  |
|    1444355 | KITTAGANA COLONY      |        56 |          0 | KANNADA  |
|    1426910 | PRIYA DARSHINI        |        53 |          0 | KANNADA  |
|    1445387 | KYALASANAHALLI        |        51 |          1 | TELUGU   |
|    1445382 | KOTHANUR              |        44 |          0 | TAMIL    |
|    1445383 | KOTHANUR              |        41 |          0 | KANNADA  |
|    1442911 | BETTANA PALYA         |        55 |         12 | KANNADA  |
|    1445160 | KODIGEHALLI           |        52 |          0 | KANNADA  |
|    1457095 | KAVERI NAGARA         |        41 |          0 | KANNADA  |
|    1457090 | KAVERI NAGARA         |        44 |          0 | TELUGU   |
|    1457092 | KAVERI NAGARA         |        41 |          0 | TELUGU   |
|    1507686 | REHMATH NAGAR         |        44 |          0 | URDU     |
|    1448385 | MALSANDRA             |        50 |          0 | KANNADA  |
|    1455798 | KANTEERAVA COLONY     |        55 |         15 | KANNADA  |
|    1444466 | PRIYA DARSHINI        |        52 |          0 | KANNADA  |
|    1444467 | PRIYA DARSHINI        |        44 |          0 | KANNADA  |
|    1445337 | RACHENAHALLI          |        52 |          0 | KANNADA  |
|    1425269 | VINYAKNAGAR           |        41 |          0 | KANNADA  |
|     552534 | KYALASANAHALLI        |        52 |          1 | TELUGU   |
|    1448976 | KRISHNA SAGARA COLONY |        54 |         14 | KANNADA  |
|    1507157 | BELTHURU              |        52 |          7 | KANNADA  |
|    1444897 | MUNESHWARA NAGAR      |        45 |          0 | TAMIL    |
|    1444890 | MUNESHWARA NAGAR      |        40 |          0 | KANNADA  |
|    1542861 | VERABADRA NAGAR 1     |        56 |          1 | KANNADA  |
|    1358415 | KOTHANUR              |        48 |          8 |          |
|    1445437 | THRIVENINAGARA        |        40 |          0 | TELUGU   |
|    1442907 | BETTANA PALYA         |        55 |          8 | KANNADA  |
|    1457106 | KAVERI NAGARA         |        40 |          0 | TELUGU   |
|    1445444 | THRIVENINAGARA        |        41 |          0 | TELUGU   |
|    1444474 | PRIYA DARSHINI        |        54 |          0 | KANNADA  |
|    1445159 | KODIGEHALLI           |        41 |          0 | KANNADA  |
|     511998 | KODIGEHALLI           |        55 |          0 | KANNADA  |
|    1356274 | KAVERI NAGARA         |        56 |          0 |          |
|    1358429 | KOTHANUR              |        42 |          0 |          |
|    1497510 | JALAHALLI 1           |        42 |          0 | KANNADA  |
|    1443914 | BIDARAHALLI           |        56 |         12 | KANNADA  |
|    1366955 | PRIYA DARSHINI        |        53 |          0 | KANNADA  |
|    1366956 | PRIYA DARSHINI        |        53 |          0 | KANNADA  |
```

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```
|    1447019 | MADAPPANA HALLI        |       42 |          0 | TELUGU   |
|    1355112 | BYRATHI BANDE          |       55 |         14 |          |
|    1504259 | MAYASANDRA   A         |       56 |          0 | KANNADA  |
|    1457120 | KAVERI NAGARA          |       43 |          0 | TAMIL    |
|    1457089 | KAVERI NAGARA          |       43 |          0 | TAMIL    |
|    1445171 | KODIGEHALLI            |       44 |          0 | KANNADA  |
|    1457203 | SARAIPALYA             |       51 |          0 | URDU     |
|    1457179 | BYRATHI BANDE          |       55 |         14 | KANNADA  |
|    1504538 | KEMPEGOWDA NAGAR       |       51 |          7 | KANNADA  |
|    1444486 | PRIYA DARSHINI         |       49 |          0 | KANNADA  |
|    1444488 | PRIYA DARSHINI         |       42 |          0 | KANNADA  |
|    1451831 | AMBED NAGAR            |       50 |          9 | TAMIL    |
+------------+-----------------------+----------+------------+----------+
```

- The post-intervention score distribution seems to follow a power law. We shall consider modeling this attribute further on.

5.2   Bivariate distribution

So far, we've been looking at single variables in isolation. Figure 2 shows a bivariate histogram of pre- vs. post-intervention scores. The lighter a cell, the more the number of records in that 'bucket'.

5.2.1   Observations/Notes

- Many of the scores seem to be clustered near the top right. To further highlight this, we have draw a linear regression line as a rough indicator of a trend (To model the trend in more detail, we could use LOESS). What is somewhat puzzling is that there are not a few students whose performance has dropped after the intervention. This is evident even without doing a linear regression.

- The immediate outliers which are visible are the ones on the extreme left (pre 0, post 56) and at the origin (pre=0, post=0). The latter outlier(s) may be an artifact of corrupted data collection; we cannot say.

5.3   Summary plots

Summary plots are so called for their ability to summarise up a data set as a set of numbers, which can be easily interpreted. We used Box Plots to summarise the data, broken down by language. Figure 3 shows that breakdown. The top row represents the box plots for the pre-intervention assessment, the bottom one for the post-intervention assessment.

Figure 2: Bivariate probability distribution of pre- vs. post-intervention scores, with linear regression line $y = 0.224.x + 37.134$
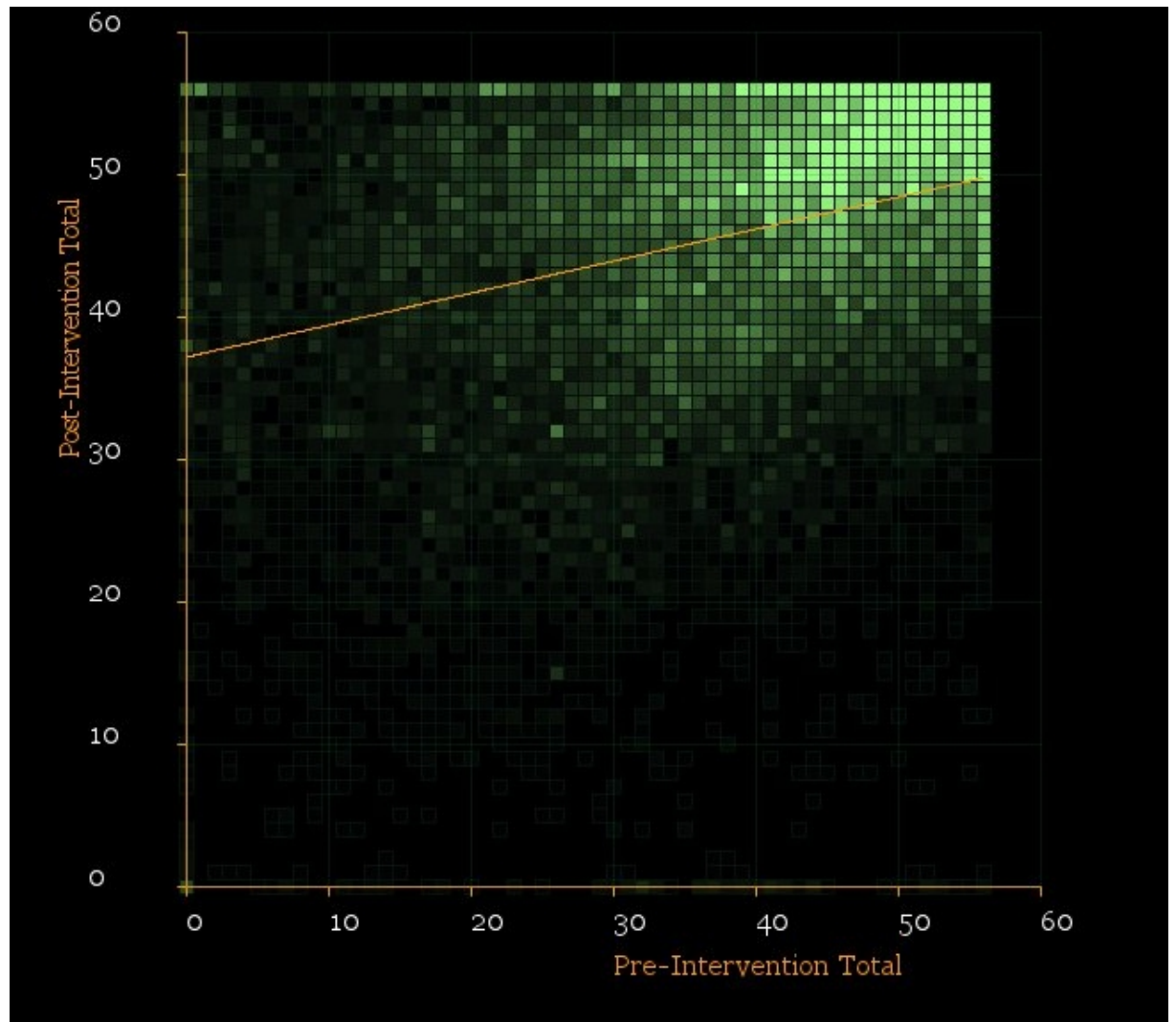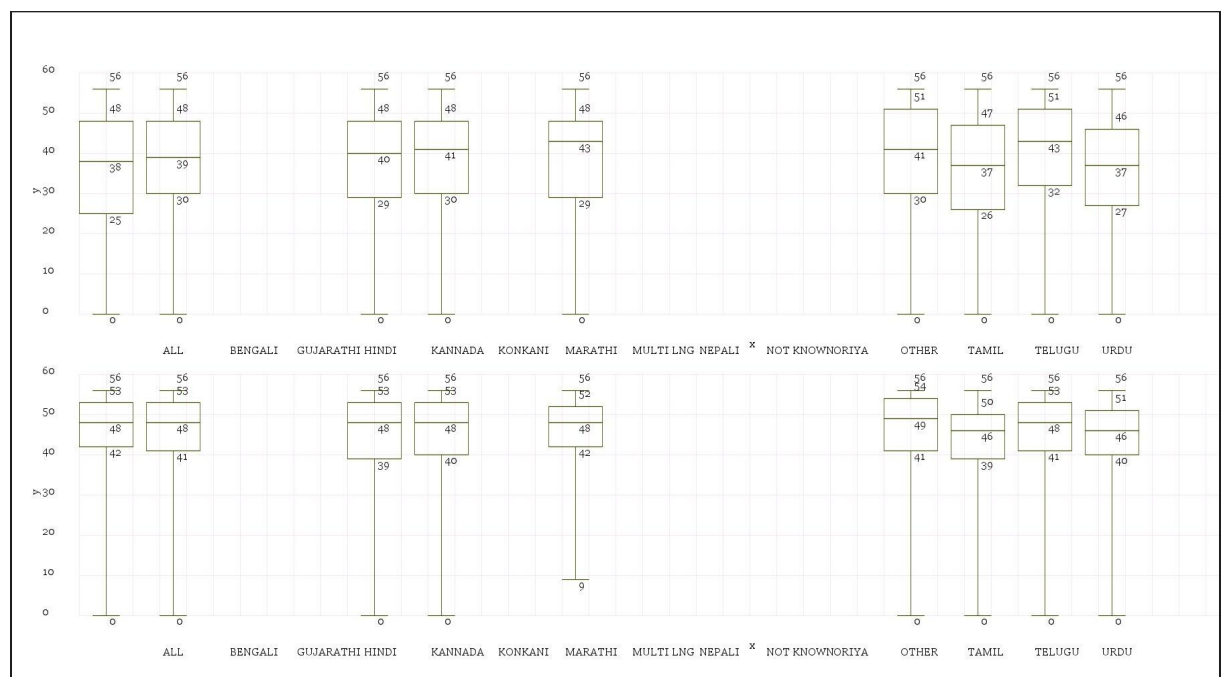
Figure 3: Box plots of pre- and post-intervention scores, broken down by language

5.3.1 Observations/Notes

- Out of all the languages, we were not able to create box plots because the corresponding samples were too few in number to differentiate between the different quartiles. These languages are Bengali, MultiLng, Not Known and Oriya.

- There are no dramatic differences between the plots in each set. The medians, 1st and the 2nd quartiles are rather close to each other.

5.4 Rank Order Charts: Geoclusters

One of the types of data that is pretty common is categorical nominal data. Categorical data is that which is non-numeric. Nominal data is that which does not lend itself to any intrinsic ordering. Names of places (if we ignore their coordinate system representation) are categorical data that is nominal.

The best way to summarise categorical data, especially if it is the Independent Variable, is to order records based on an ordinal Dependent Variable. This sort of a representation is called a Rank Order chart. We have rank ordered the geographical clusters based on the student population in each cluster.

Index	Cluster	Probability	Cumulative Distribution
1.	Sir M Vishweshwariah 1st Circle	0.030690257754477937	0.030690257754477937
2.	Sultanpalya	0.030289791757681667	0.0609800495121596
3.	konnana kunte	0.02668559778651522	0.08766564729867482
4.	kadugodi	0.02584825979321392	0.11351390709188874
5.	Laggere 4th circle	0.025156545798747633	0.13867045289063637
6.	K.R.Puram	0.024756079801951363	0.16342653269258772
7.	Murphy Town	0.024064365807485073	0.1874908985000728
8.	Netaji 2nd Circle	0.02340905781272754	0.21089995631280034
9.	Abbigere	0.02326343381389253	0.23416339012669288
10.	Begure	0.023190621814475027	0.2573540119411679
11.	Thavare kere	0.022462501820299987	0.2798165137614679
12.	Kodihalli	0.022244065822047472	0.3020605795835153
13.	Dodda Banaswadi	0.02202562982379496	0.3240862094073103
14.	Jigani	0.021952817824377458	0.3460390272316878
15.	Koramangala	0.021370321829037427	0.3674093490607252
16.	Byrathi	0.020387359836901122	0.3877967088976263
17.	Rajanakunte	0.020059705839522355	0.4078564147371487
18.	Varthur	0.01944080384447357	0.42729721858162223
19.	Hebbagodi	0.01907674384738605	0.4463739624290083
20.	Hreohalli	0.018639871850881024	0.4650138342798893
21.	Yalahanka	0.01802096985583224	0.48303480413572153
22.	Banashankari	0.017438473860492208	0.5004732779962138
23.	K Gollahalli	0.0172928498616572	0.517766127857871
24.	Sarjapura	0.017183631862530944	0.5349497597204019

25.	Mallasandra	0.01711081986311344	0.5520605795835154
26.	Kodigenahalli	0.01689238386486093	0.5689529634483763
27.	kasaba	0.01667394786660842	0.5856269113149848
28.	Haragadde	0.016564729867482163	0.6021916411824669
29.	Makali	0.016455511868355904	0.6186471530508229
30.	Doddakannelli	0.016419105868647154	0.63506625891947
31.	Mahantalingapura	0.016200669870394643	0.6512669287898646
32.	Baglur	0.015836609873307123	0.6671035386631717
33.	chandapura	0.015836609873307123	0.6829401485364789
34.	Bettahalasuru	0.015690985874472114	0.698631134410951
35.	Hesaragatta	0.015690985874472114	0.7143221202854232
36.	Kaggalipura	0.0154725498762196	0.7297946701616428
37.	Sondekoppa	0.015217707878258336	0.7450123780399012
38.	Wilson Garden	0.01510848987913208	0.7601208679190332
39.	Dommasanddra	0.01510848987913208	0.7752293577981653
40.	Kithuru Rani Channamma 3rd Circle	0.01474442988204456	0.7899737876802099
41.	Triveni 5th Circle	0.014489587884083296	0.8044633755642931
42.	Palace Guttalli Circle	0.014453181884374545	0.8189165574486676
43.	Kengeri circle	0.014343963885248289	0.8332605213339159
44.	Yashwanthpura	0.014052715887578273	0.8473132372214942
45.	Kuvempunagar 4th Circle	0.013543031891655745	0.8608562691131499
46.	Chikkajala	0.012960535896315713	0.8738168050094657
47.	Kadugondanahalli	0.012596475899228193	0.8864132809086939
48.	Attibele	0.012523663899810689	0.8989369448085045
49.	Indlavadi	0.012341633901266929	0.9112785787097715
50.	Sonnenahali	0.012305227901558177	0.9235838066113297
51.	Uttrahalli	0.012232415902140673	0.9358162225134703
52.	Machohalli	0.012159603902723169	0.9479758264161935
53.	Gavipura Guttahalli	0.009210717926314256	0.9571865443425077
54.	Marasur	0.009137905926896752	0.9663244502694045
55.	Netravathi	0.00906509392747925	0.9753895441968837
56.	Gopalapura	0.008955875928352992	0.9843454201252367
57.	Hennagara	0.00873743993010048	0.9930828600553372
58.	PANTHARAPALYA	0.002621231979030144	0.9957040920343674
59.	YARUB NAGAR	0.001783893985728848	0.9974879860200963
60.	VERABADRA NAGAR	0.001310615989515072	0.9987986020096113
61.	Yalahanka upnagar	0.00043687199650502403	0.9992354740061163
62.	DODDA BANASAWADI	0.000400465996796272	0.9996359400029126
63.	ASHRAYA NAGAR	0.00036405999708752004	1.0

5.4.1 Observations/Notes

- Out of 63 clusters, the top 38 clusters account for 75% of the population.

6 Data Analysis and Exploration

There are multiple ways of exploring datasets when simple visual inspection is tedious and unintuitive. Many of them are standard, but some of them may reveal more about the nature of the data. Often, they are motivated by specific business drivers and questions, and some exploration may be custom to the dataset under analysis. Data exploration is also commonly done through queries to an OLAP database.

6.1 Parallel Coordinates

Parallel coordinates are an interesting way to explore high-dimensional data without discarding any detail. The only downside is that the elegance of presentation is somewhat sacrificed.
Essentially, instead of aligning axes in orthogonal directions (we can visualise only upto 3), the axes are stacked horizontally, giving the impression of n parallel lines. A single data point in this n-dimensional representation is a set of broken lines spanning the widths between the axes.

The way to explore this visualisation is to highlight the samples of interest, based on some criteria. The highlighted samples can then be inspected visually to discover trends, if any. Figure 4 shows an example of exploration using parallel coordinates on the Anganwadi data.

6.2 Covariance plot

The covariance plot is a first step towards looking at the correlations between responses to individual questions. Basically, the kind of question such analysis can answer is, for example, how dependent is a response to question 40 on the response to question 3? Do they change together, i.e., is there a strong dependency between answer to question X and the answer to question Y?

Figure 5 shows the covariance plot for the pre-intervention responses.
Figure 6 shows the covariance plot for the post-intervention responses.

6.2.1 Observations/Notes

- The plots are color coded such that the darker the hue, higher the covariance. The plot is symmetric about the diagonal, and the diagonal is much darker because it is essentially the covariance of a single response with itself. The data has not been normalised yet, hence the diagonal hues vary.

- The covariance between the responses appears weakened in the post-intervention scores, indicating that the degree of dependence between the responses has decreased.

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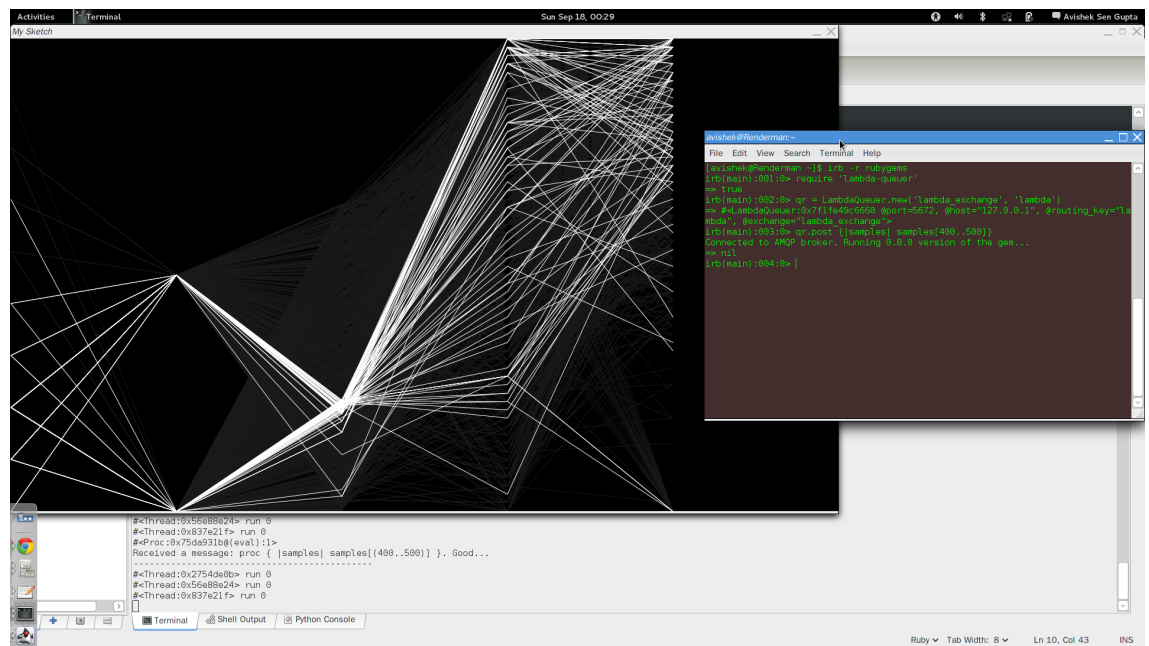Figure 4: Parallel Coordinates showing language, gender, school, pre- and post-intervention scores

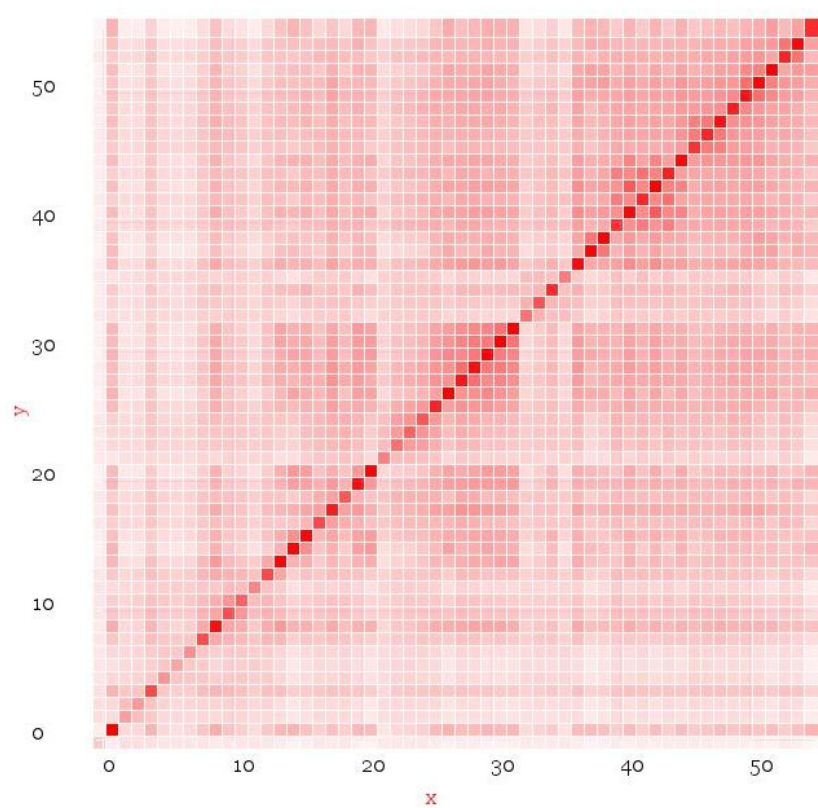Figure 5: Covariance plot of pre-intervention responses

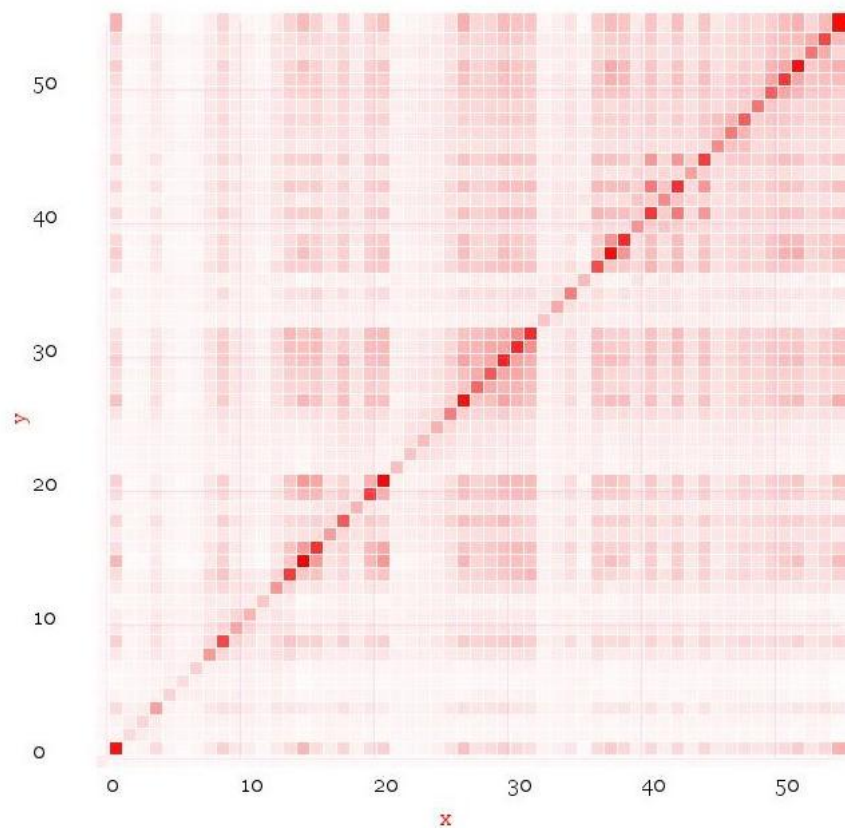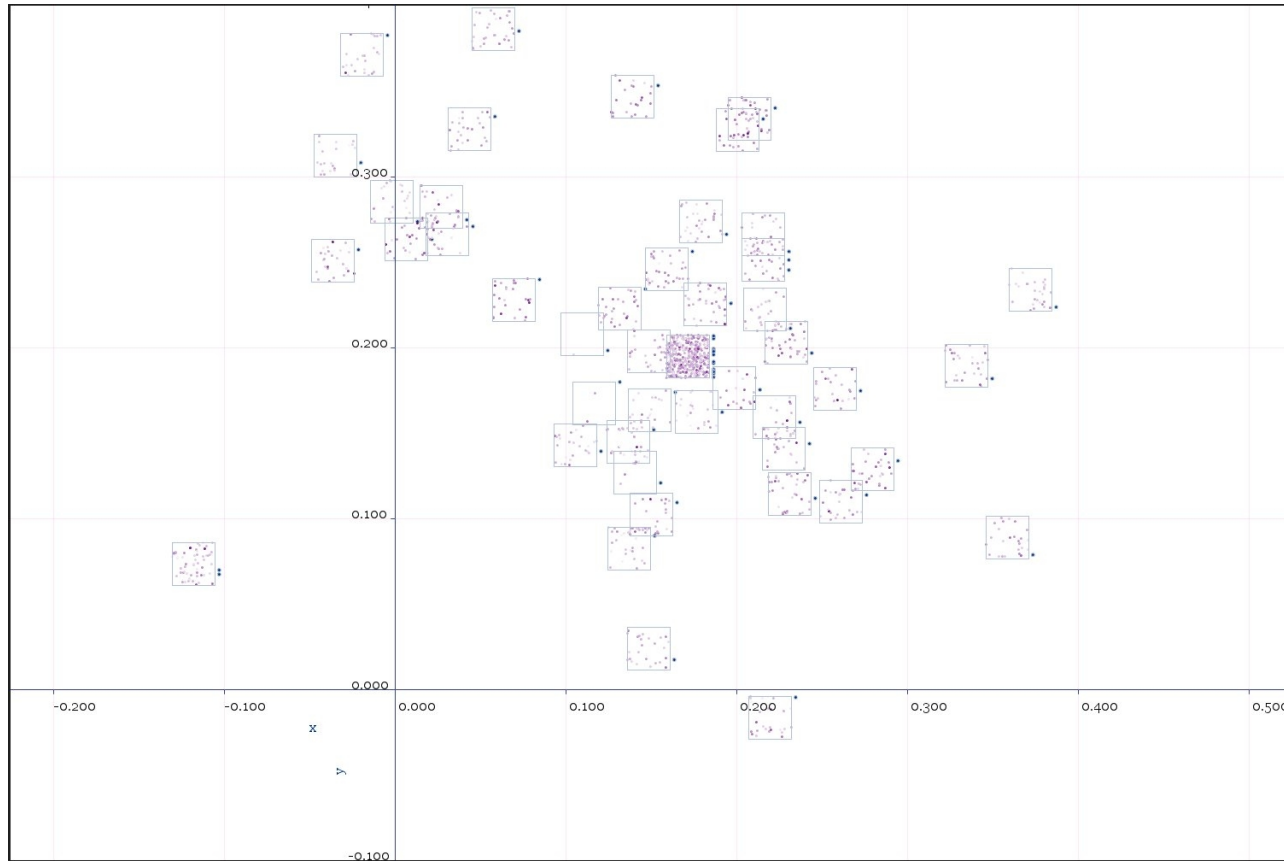Figure 6: Covariance plot of post-intervention responses

Figure 7: Geocoded populations by cluster

- There appear to be clusters of relatively high covariance, for example, between the questions in the range of (40-43), (26-31), etc. Further investigation may be needed to answer specific questions in this area.

- Correlation does not imply causation. Inferred links between responses may be result of a deeper phenomenon; thus a high covariance doesn't necessarily imply a direct causative link between two responses.

6.3 Geographical distribution

7 Models and Statistical Tests

Models are simplified descriptions of the real-world phenomena, based on simplifying assumptions. Their primary use is to explain variations in observed data, and hopefully make useful predictions about data yet unseen.

Statistical tests, on the other hand, are designed to test certain hypotheses which might be inferred from the data set. These hypotheses may be proved or disproved by these tests, based on the notion of 'statistically significance'.

This section summarises the models and tests we've applied to the Anganwadi dataset.

7.1 Tests for conformance to distributions

Many of the tests in statistics work under the assumption that the underlying dataset (with or without transformation) is normally distributed, i.e., it follows a Gaussian probability distribution, namely:

$$f(x) = ae^{\frac{(x-b)^2}{2c^2}}$$

This verification is important because it dictates, to a great extent, the validity of any tests we run. If the data turns out to be not normal, we will probably want to use nonparametric methods, which do not make assumptions about the shape of the distribution.

7.1.1 Jarque-Bera test

The Jarque-Bera test measures the conformance of a dataset to a Gaussian distribution using its skewness and kurtosis. For reference, the Jarque-Bera statistic is given by:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right)$$

where S (skew) and K (kurtosis) are defined as:

$$S = \frac{\hat{\mu}_3}{\hat{\sigma}^3} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{3/2}}$$

$$K = \frac{\hat{\mu}_4}{\hat{\sigma}^4} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4}{\left(\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^2}$$

The null hypothesis that this statistic tests is that the skewness and excess kurtosis are both zero, which is the defining characteristic of a Gaussian distribution. The data below summarises the JB statistics for the pre-, post-intervention scores and the score improvements. Please note that the statistics from the

data before the removal of the invalid data. This does not affect the results overmuch, however.

Pre-intervention Score
n = 28535 Skewness = -1.0001504234198
Kurtosis = -1.99959305171352
JB statistic = 34476.3843030411
Conclusion : Pre-intervention scores are not normally distributed.

Post-intervention Score
n = 28535
Skewness = -1.00010106352368
Kurtosis = -1.9997273050813
JB statistic = 34477.5108572449
Conclusion : Post-intervention scores are not normally distributed.

Score Improvement
n = 28535
Skewness = -4.24685767755943
Kurtosis = 37.1765663807246
JB statistic = 1474523.40413686
Conclusion : Score improvements are not normally distributed.

The initial probability distributions provided very clear evidence that the pre- and post-intervention scores were not normal: this is merely validation. However, the test also shows that the improvement is not normally distributed either. If we notice the improvement distribution in Figure 1, we notice a few things:

- Fat tails

- Thin narrow peak (high kurtosis)

- Fair amount of skew (Non-zero excess skew)

We did attempt to assess the visual fit of a Gaussian distribution, but this was not very successful. Figures 8 and 9 show some of our attempts at fitting different theoretical distributions. The Cauchy distribution was a closer fit after a translation and a transform, but there are still outliers.

7.1.2   Quantile-Quantile plots

There is a visually more intuitive way of measuring the goodness of fit of a dataset to any theoretical probability distribution. That method is called a Quantile-Quantile plot. They are points marked off at regular intervals on the cumulative distribution function of a random variable. The Q-Q plot is so named for the two quantile plots that are reflected in a single plot: one for the theoretical distribution, and one for the dataset.
The theoretical distribution's quantile plot comes out as a straight line. If the

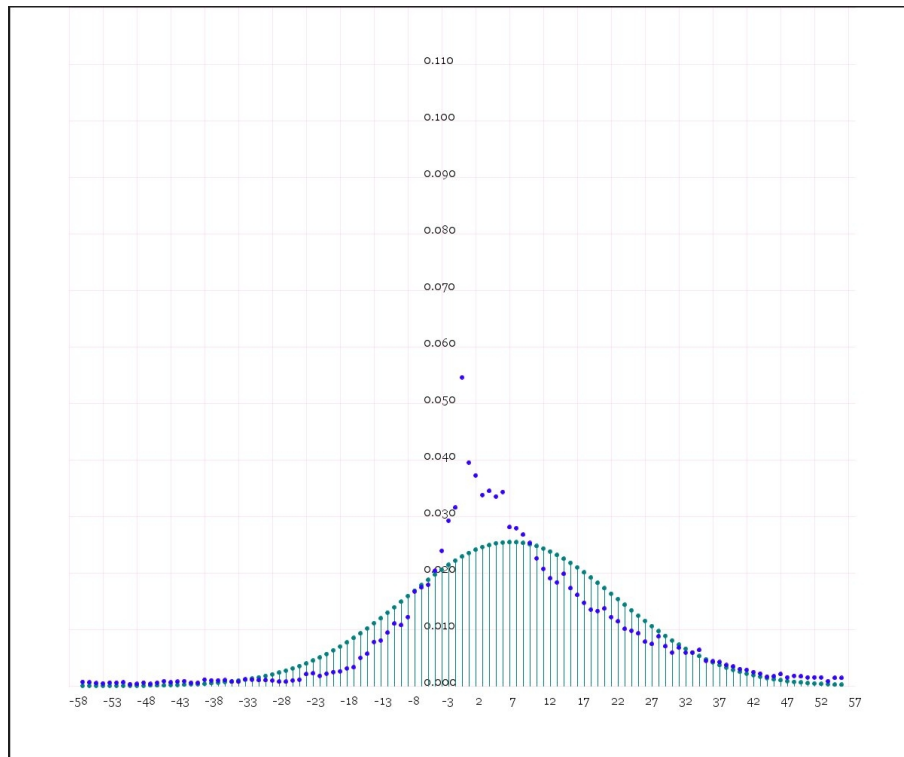Figure 8: Improvement data overlaid with theoretical Gaussian (σ^2=245.47, $\bar{x}$=7.22)

Figure 9: Translated log of data overlaid with theoretical Cauchy (scale=0.14, $\bar{x}$= 4.13)
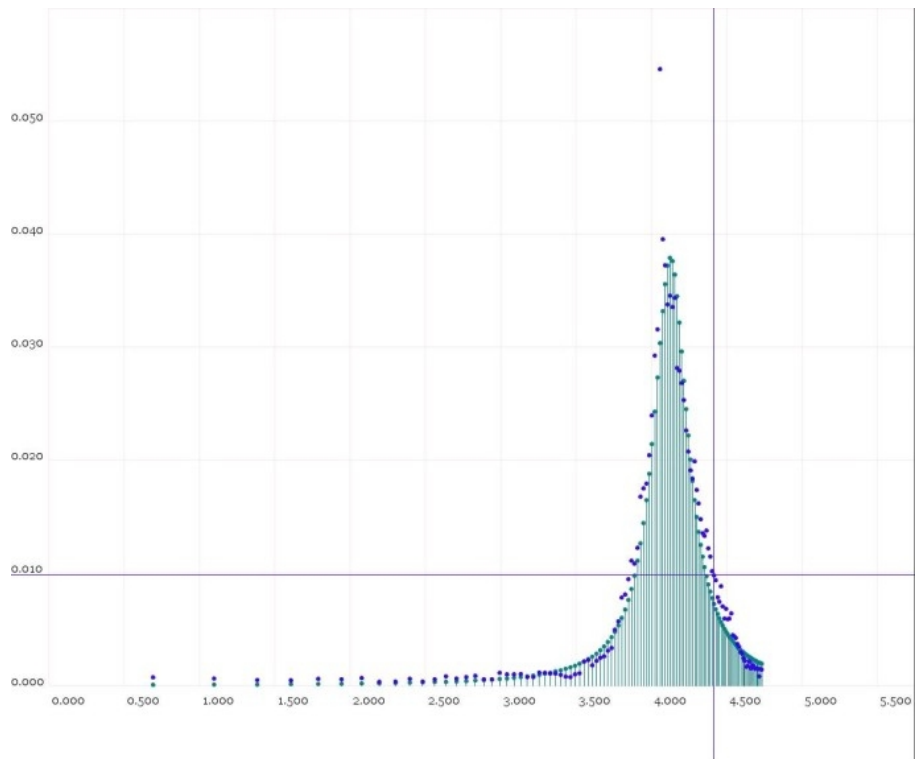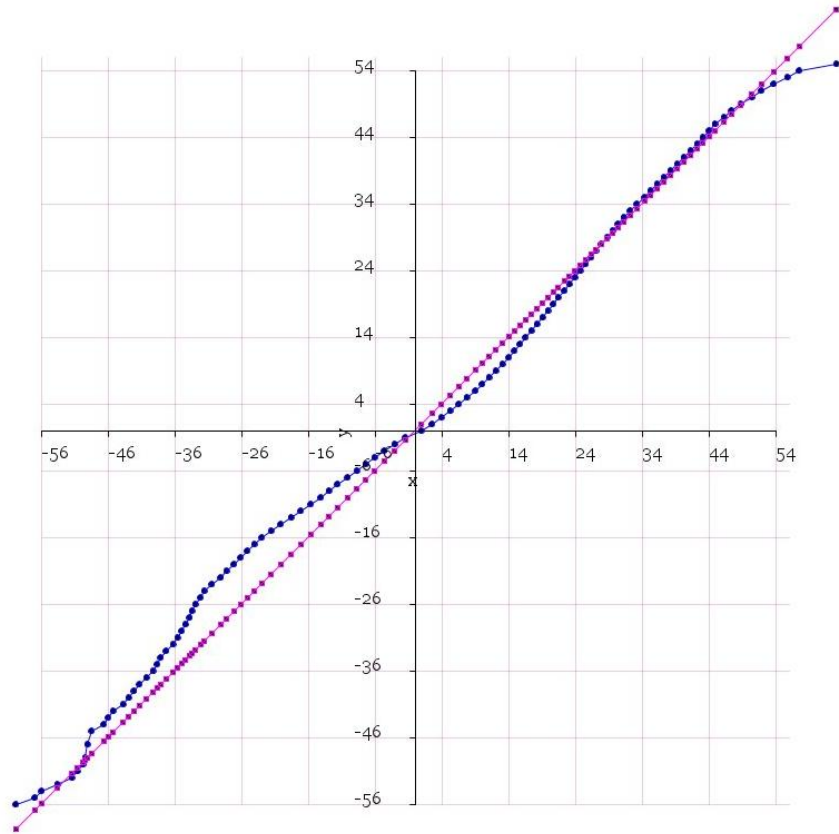
Figure 10: Quantile-Quantile plot of score improvement vs. theoretical Gaussian

dataset follows the distribution closely, it will also follow this line, more or less. Significant deviations indicate violation of fit. Figure 10 shows the Q-Q plot of score improvement and the Gaussian.

7.1.3 Observations/Notes

- The dataset shows a systematic deviation from the Gaussian quantile.

- Conformance to the theoretical distribution is never exactly possible, so the "fat pencil test" is used. Essentially, if the blunt end of a pencil covers both the theoretical and the actual dataset, we can say that the data approximates the model somewhat. In this case, the fat pencil test fails too.

We present another example of curve fitting to derive a probability distribution model, this time, with more success. We attempted to fit the exponential distribution to the post-intervention data. The exponential distribution is given as:

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & if x \geq 0 \\ 0 & if x < 0 \end{cases}$$

Because improvement can be negative is well, we applied the following transform to the data:

$$I'(x) = I(x) + 57$$

As Figure 11 shows, the distribution is a reasonable fit to the data. Further validation comes from the Q-Q plot of the dataset and the theoretical exponential in Figure 12. Note how closely the dataset tracks the theoretical quantile, except near the top right, which is where the 'kink' in the data set lies.

7.2   The Central Limit Theorem

Fortunately, we need not discard classical statistical techniques because the underlying distribution is not normally distributed. This is because even though the data itself is non-normal, its means are very probably normally distributed, regardless of the original distribution of the data.
This statement is known as the Central Limit Theorem, which states that:

The distribution of the sum of a large number of independent, identically distributed variables will be approximately normal, regardless of the underlying distribution.

To illustrate the point, we show the histograms of the means of the pre-intervention scores, post-intervention scores, and the improvements, in Figures 13, 14, and 15, respectively.
The Jarque-Bera statistics for these three cases is well below the threshold. The shape validates that, too.

Why is the Central Limit Theorem important? It is because, even though statistical tests might not be appropriate for the raw, non-normal, data itself, they certainly can be applied to the sampling distribution of the means of the data. We can still proceed with these tests, at the expense of losing the richness of data, and boiling it down to a single metric, namely, the mean.
This report does not cover any further tests on these sampling distributions.

7.3   Answer distribution

The answer distribution is a visual map of the pattern of answering in different groups segregated by the pre-intervention score.
Each row in the map corresponds to the population of students in a single performance bracket. Thus, the topmost row corresponds to the group of students who score 56 in the pre-intervention, for example.

Figure 11: Curve-fitted theoretical exponential for reflected post-intervention probability distribution
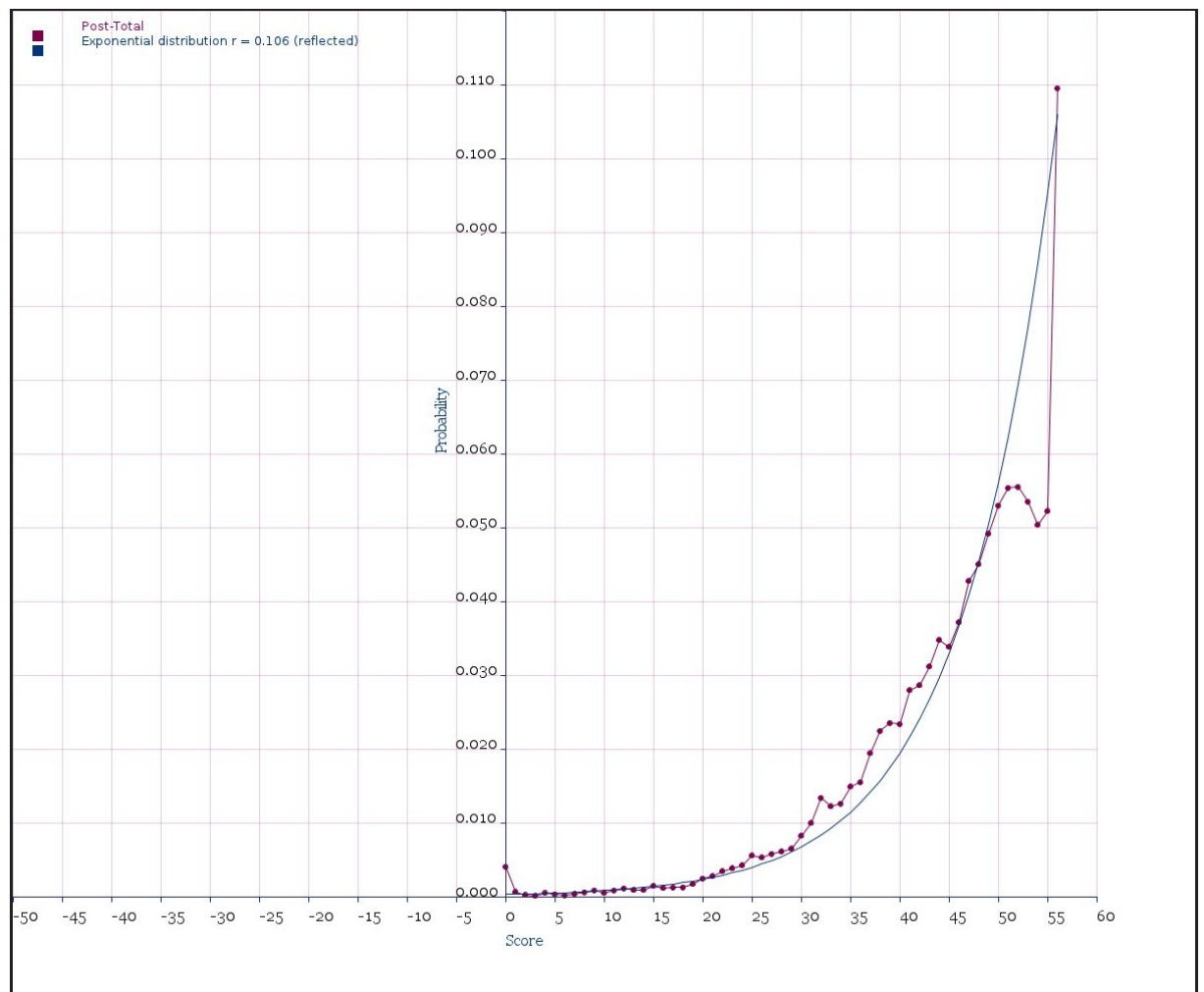
Figure 12: Quantile-Quantile plot of post-intervention score (reflected) vs. theoretical exponential
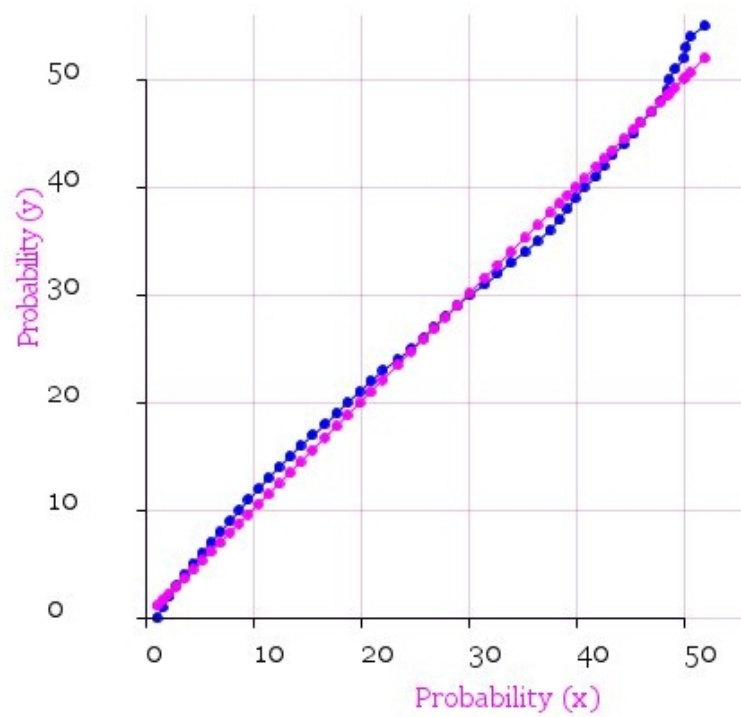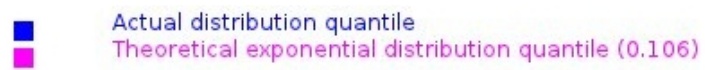
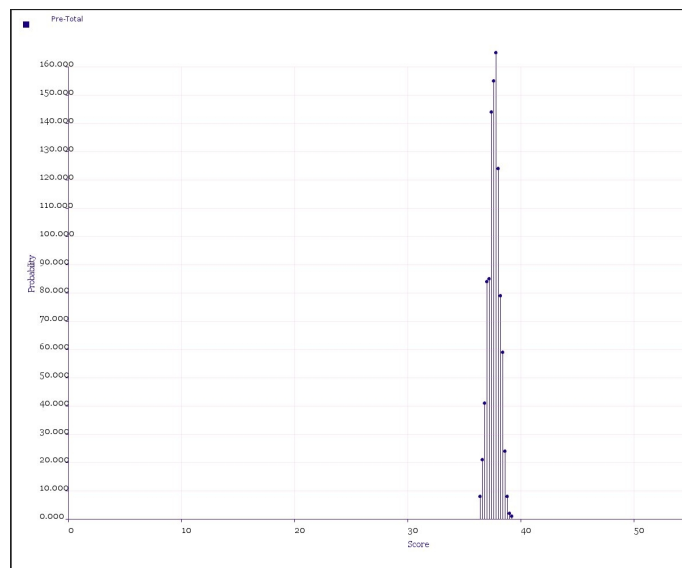Figure 13: Central Limit Theorem illustration on the pre-intervention score
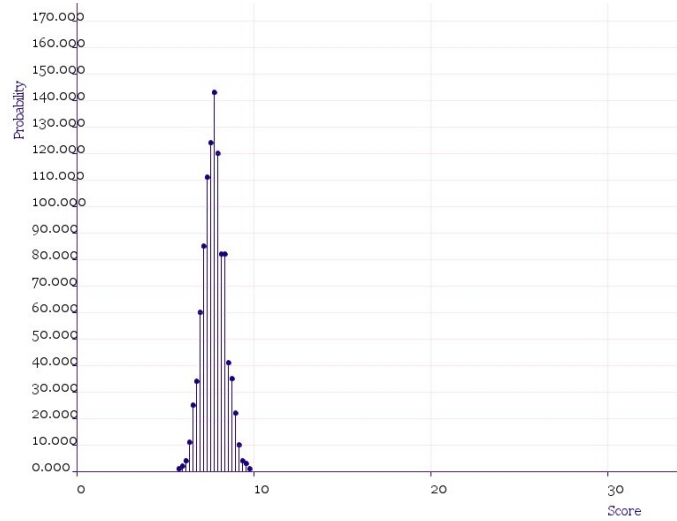

Figure 14: Central Limit Theorem illustration on the post-intervention score

Figure 15: Central Limit Theorem illustration on the improvement metric

Within each row, each cell (left to right) represents the answer to the corresponding question. Thus, the (10,5) represents the response to question number 5 in the bracket of students who scored 10 prior to intervention.

The brightness of the hue in a cell represents the fraction of students in that corresponding bracket who answered 1 to that specific question.

Therefore, starting from the bottom of the map, we get a sense of which questions only began to be answered in the affirmative in the higher brackets. This gives us a sense of the 'difficulty' of each question.

7.3.1 Observations/Notes

- The 'difficulty' metric of questions tends to clump together in bands in the left two-thirds of the map. Answering patterns from around question number 37 and above alternate much more uniformly.

7.4 Effectiveness of intervention

7.4.1 Intervention effect on indivdual responses: McNemar's Test

McNemar's test for matched pairs is used to measure statistical difference between two tests on matched pairs of data. Examples of matched pairs of data are:

- Test scores of siblings

- Test scores before and after intervention of the same student.

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Figure 16: Answer distribution vs. question number
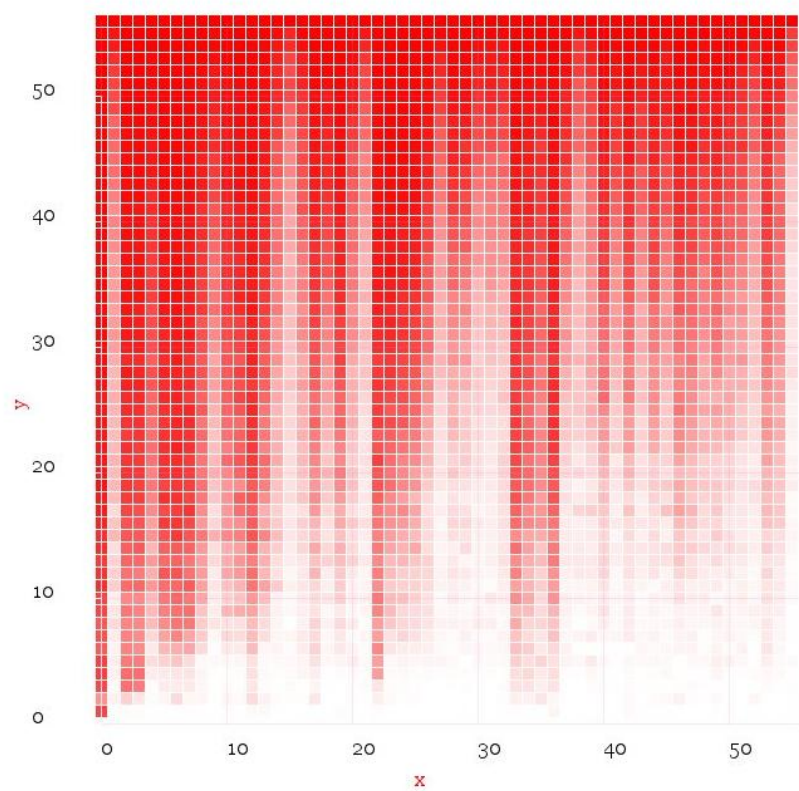
The second point is aligned with the situation that we are dealing with. The two, disjoint, tests essentially are:

- Candidate answered 1 to the question.

- Candidate answered 0 to the question.

Using this, we will be able to deduce whether the intervention had any significant effect on the response to a particular question across the entire population of students. This has the side-effect of allowing us to analyse this data in a fine-grained fashion.
[TODO: Get calculation results]

7.5 Modeling responses as Bernoulli trials

There are n students. All of them have answered either 0 or 1, to, say, question 3. We may restate this phenomenon and say that there were n Bernoulli trials performed on question 3, each outcome being either true or false. The probability of the outcome is, say, p. The definition of 'true' and 'false' in this case is arbitrary, and does not need to specifically correspond to the perceived value of the answer.
Under these conditions, the phenomenon can be modelled as a Binomial Distribution. Formally, the binomial distribution is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, each of which yields success with probability p. The probability p will be specific to each question, and can be estimated from the dataset. Here is an example of the type of question that this modeling will be able to answer:

If I choose 6000 students randomly from the population, what is the probability that at least 100 of them have answered 1 to question 45?

7.5.1 The Binomial Distribution

The binomial distribution for a random variable S (in our case, the response to a particular question), for which n experiments have been conducted is a discrete probability distribution given by:

$$P(S = k) =^n C_r.p^k.(1 - p)^{n-k}$$

where:

$$^nC_r = \frac{n!}{k!(n - k)!}$$

Table 2 summarises the probabilities for the pre- and post-intervention answers.

Index	$P_{pre}(Answer = 1)$	$P_{post}(Answer = 1)$

1	0.9462283384301733	0.982233872142129
2	0.5107397699140819	0.6418377748652978
3	0.8927479248580166	0.963885248288918
4	0.8855759429153924	0.9571137323430902
5	0.7667831658657347	0.8963157128294743
6	0.8795689529634484	0.9554754623561963
7	0.8994466288044269	0.9667613222659094
8	0.8741444590068443	0.9521625163826999
9	0.7712246978302024	0.8880515508955876
10	0.623452745012378	0.7670380078636959
11	0.7710426678316586	0.9049803407601573
12	0.8090505315275958	0.9207077326343381
13	0.8611839231105286	0.9410950924712392
14	0.7652905198776758	0.8864132809086938
15	0.5783457113732343	0.744575506043396
16	0.4260594145915247	0.5671326634629387
17	0.5827144313382846	0.7254259501965924
18	0.7672564438619485	0.8914009028687928
19	0.662807630697539	0.7944881316440949
20	0.7896097276831222	0.9197611766419106
21	0.59188874326489	0.747051114023591
22	0.4380369884957041	0.59247123926023
23	0.8520824231833406	0.9350516965195864
24	0.8316950633464395	0.9384010484927916
25	0.8089413135284695	0.9293723605650211
26	0.7880442696956459	0.9172491626620067
27	0.6848332605213339	0.8484782292121742
28	0.4887869520897044	0.6438401048492791
29	0.6410004368719965	0.8189529634483763
30	0.6163899810688801	0.7939420416484637
31	0.49344692005242463	0.6865807485073541
32	0.5105941459152469	0.7051114023591087
33	0.5081913499344692	0.7071865443425076
34	0.8340978593272171	0.9393840104849279
35	0.780690257754478	0.9070190767438474
36	0.7031454783748362	0.850116499199068
37	0.8453109072375128	0.9269695645842435
38	0.5957477792340178	0.7741735838066113
39	0.44033056647735547	0.6132226590942187
40	0.5306538517547692	0.7051842143585263
41	0.7143221202854231	0.8831731469346148
42	0.5657856414737149	0.7348551041211592
43	0.6976481724188146	0.8690476190476191
44	0.550203873598369	0.7196009902431921
45	0.673074122615407	0.8979903888160768

46	0.5836609873307121	0.7459953400320373
47	0.7170889762632882	0.8698485510412116
48	0.6857798165137615	0.8449104412407165
49	0.6198485510412116	0.8022062035823504
50	0.6704528906363769	0.8373379933012961
51	0.6138779670889762	0.8026066695791466
52	0.5696446774428425	0.7395878840832969
53	0.4630479102956167	0.6245449250036406
54	0.6854885685160914	0.8392675112858599
55	0.5814038153487695	0.7639070918887433
56	0.29219455366244357	0.3939493228484054

Table 2: Known probabilities of pre- and post-intervention scores

7.6 Test for variable independence

Hypothesis testing is one of the practices in classical statistics, where a proposition, or a null hypothesis is either rejected or not. Failure to reject a null hypothesis does not necessarily imply its acceptance. One of the questions we asked is whether any of the variables in the school dataset were independent of each other. To frame this in terms of a hypothesis test, we used the Chi-Square test.

7.6.1 Chi-square test

Null hypothesis: Area and Improvement are NOT related.
For **area vs. improvement**
Chi-Square statistic = 56499.4692602837
X2 = 9652.9739
Degrees of freedom = 9426
Null hypothesis rejected
c
Null hypothesis: Area and Pre-Score are NOT related.
For **area vs. pre-score**
Chi-Square statistic = 58665.7089390644
X2 = 8062.2959
Degrees of freedom = 7855
Null hypothesis rejected

Null hypothesis: Area and Post-Score are NOT related.
For **area vs. post-score**
Chi-Square statistic = 38567.0016158761
X2 = 8062.2959
Degrees of freedom = 7855
Null hypothesis rejected

Null hypothesis: Language and Post-Score are NOT related.
For **language vs. post-score**
Chi-Square statistic = 280.234448946825
X2 = 96.2166
Degrees of freedom = 75
Null hypothesis rejected

Null hypothesis: Language and Improvement are NOT related.
For **language vs. improvement**
Chi-Square statistic = 232.464548410971
X2 = 113.1452
Degrees of freedom = 90
Null hypothesis rejected

Null hypothesis: Language and Pre-Score are NOT related.
For **language vs. pre-score**
Chi-Square statistic = 277.85501653079
X2 = 96.2166
Degrees of freedom = 75
Null hypothesis rejected

8 Prediction and Classification

8.1 Decision Trees

Decision trees are tree-like graphs used to model the possible outcomes reached by taking a certain path through the tree. This path represents a set of decisions or a specific set of predictor attributes. In the domain of classification/prediction, the graph is constructed by splitting the entire dataset recursively based on the predictor attributes.

The order of this recursive splitting is determined by the information gain of an attribute. Intuitively, information gain is the increase in the amount of information gotten when the value of a certain attribute is made known, from the state when this value is not known, i.e., how well a particular attribute is capable of distinguishing between data points. Information gain is also known as Kullback-Leibler divergence.

The decision tree for the dataset, using the predictor attributes of area, language, and pre-intervention score, and the predicted attribute as the score improvement.

The prediction is on the attribute of score improvement, and the categorical distinction for improvement has been made to make it easier to interpret the improvement. Table 8.1 shows a fragment of the decision tree generated for this dataset.

```
area - YALLAMMANADODDI Prediction = NO
area - BADAMAKAAN I
     pre-performance - EXCELLENT Prediction = DECLINE
     pre-performance - GOOD
          gender - Girl Prediction = DECLINE
          gender - Boy
               language - URDU Prediction = NO
     pre-performance - AVERAGE Prediction = NO
area - BINGIPURA HODU
     gender - Girl
          language - KANNADA
               pre-performance - GOOD Prediction = SLIGHT
     gender - Boy
          language - KANNADA
               pre-performance - GOOD Prediction = SLIGHT
          language - TAMIL Prediction = SLIGHT
area - ADUGODI Prediction = DECLINE
area - MUTTAGADAHALLI
     pre-performance - EXCELLENT
          gender - Girl
               language - KANNADA Prediction = DECLINE
          gender - Boy Prediction = DECLINE
     pre-performance - AVERAGE Prediction = GOOD
```

pre-performance - REASONABLE Prediction = GOOD

. . .

8.2   Bayes classifier

The Bayes Theorem is widely used for making statements about the possibility of a particular outcome. In this interpretation, there is no one correct outcome, only probabilities of multiple outcomes. One of the frequently used tools used in Bayesian data analysis is the Bayes classifier.

The Bayes classifier uses a density estimator to make predictions about a particular outcome, given some information. There are many density estimators which can be plugged into the Bayesian classifier, to do this. This accounts for the popularity of the Bayesian classifier. Density estimators may be based on very simple assumptions, like the naive Bayes density estimator, or may assume particular distribution of the data, like the Gaussian density estimator.

8.3   Density estimators

A density estimator is a model of the probability distribution of a random variable. This may be discretely built up (naive density estimator, joint density estimator), or it may be modelled smoothly (kernel density estimator). In our analysis, we have chosen the kernel density estimator as our estimator. In all these scenarios, we have dealt with the bivariate case, but this can be extended to higher dimensions. Kernel density estimation is costlier in higher dimensions, thus a different estimator may be needed depending upon the number of dimensions under analysis.

8.3.1   Results

Figure 17 plots the posterior probabilities of language, predicted from the improvement in score. The kind of question that this plot can answer is:
Given that a student improved by 30 as a result of intervention, what is the probability that the student speaks Marathi? We also note the overwhelming probability that the student speaks Kannada, no matter what the improvement is. This is a direct result of the sampling bias we noted earlier.
Figure 18 show the posterior graphs of gender predicted from improvement.
Figure 19 show the posterior graphs of geographical cluster predicted from improvement.

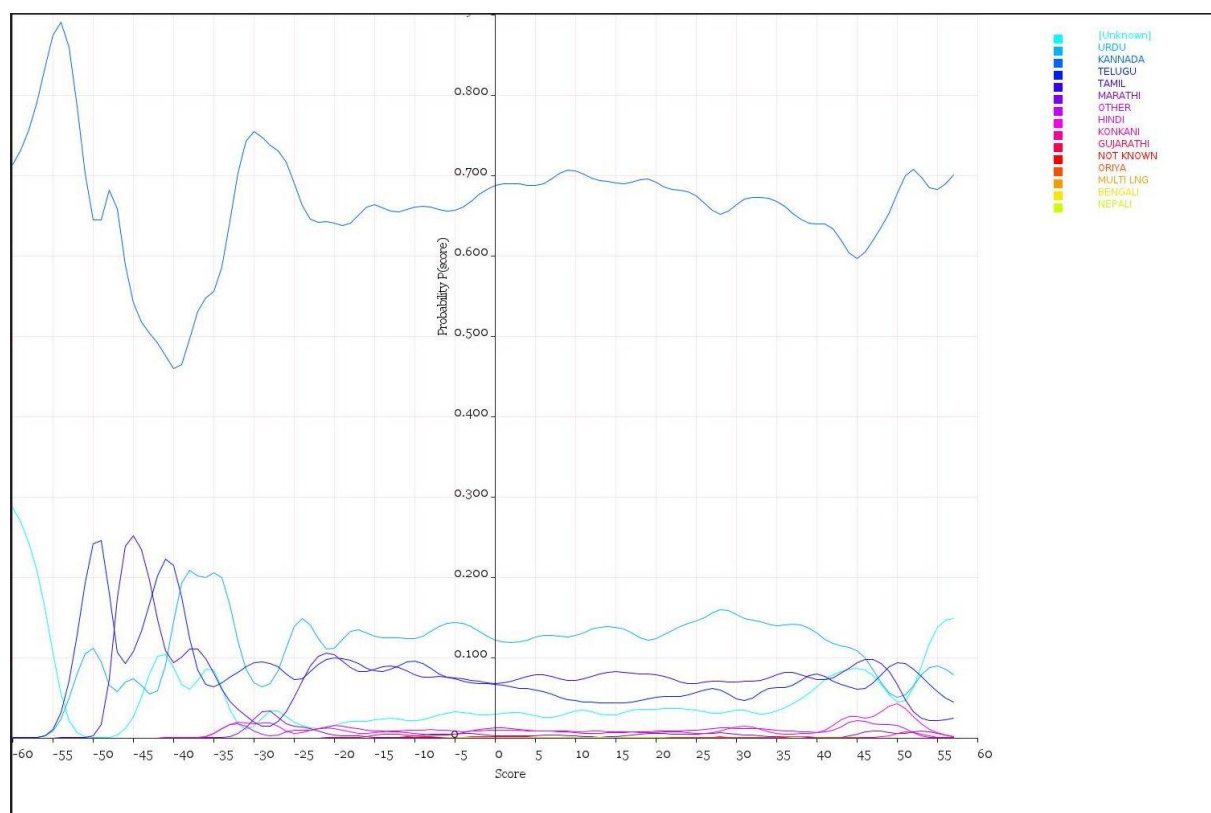Figure 17: Bayes posterior distribution of language from score improvement

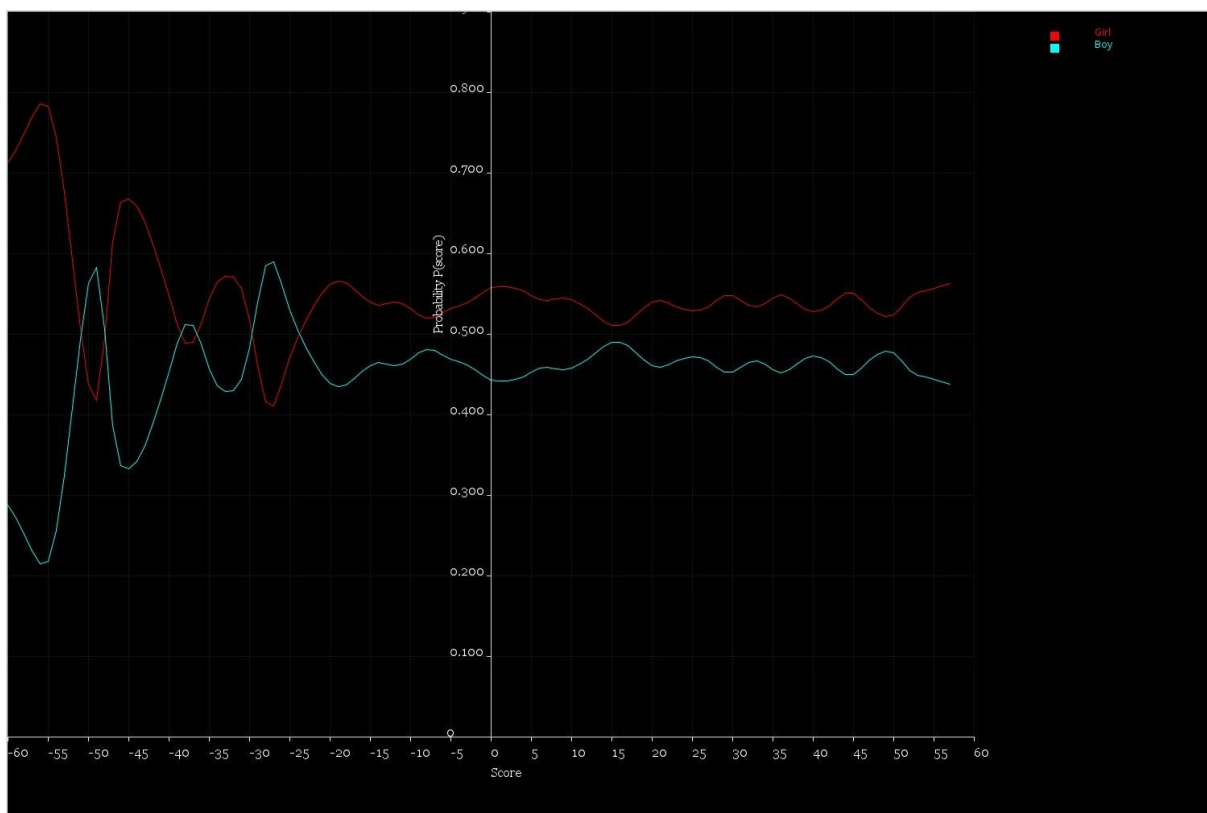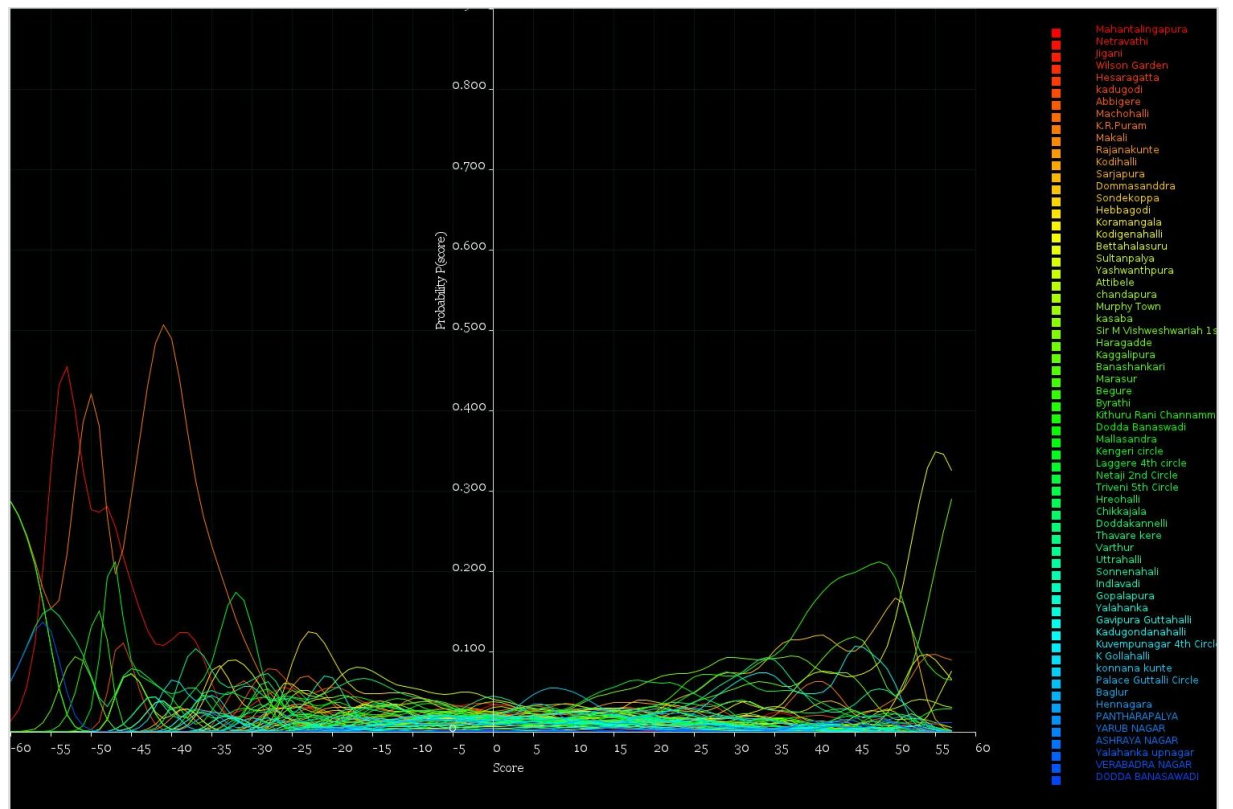Figure 18: Bayes posterior distribution of gender from score improvement

Figure 19: Bayes posterior distribution of geocluster from score improvement

9   Technical notes

For base visualisation and interactivity, Processing was used through its Ruby bindings (Ruby-Processing). All plots were done using a coordinate plotting gem called Basis-Processing.
Some calculations like Principal Component Analysis were done using the Statsample gem from the SciRuby project.
JRuby 1.6.4 was used for all code needing visualisation; Ruby 1.9.2 was used for everything else.
Data was stored in a MySQL database.