Fourier Trigo Form → Complex Exponential Form

$$x(t) = A_0 + \sum_{k=1}^{\infty} \left(A_k \cos k\Omega_0 t + B_k \sin k\Omega_0 t \right)$$

A_0: DC term; $k\Omega_0$: k^{th} harmonic. * Trigo Form

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\frac{1}{j} = \frac{1}{j}\cdot\frac{j}{j} = \frac{j}{j^2} = -j \; *$$

$$\therefore \cos\theta = \frac{e^{+j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{+j\theta} - e^{-j\theta}}{2j}$$

$$\therefore A_k \cos k\Omega_0 t = A_k \left[\frac{e^{+jk\Omega_0 t} + e^{-jk\Omega_0 t}}{2} \right]$$

$$B_k \sin k\Omega_0 t = B_k \left[\frac{e^{+jk\Omega_0 t} - e^{-jk\Omega_0 t}}{2j} \right]$$

Collect $e^{+jk\Omega_0 t}$: $\frac{A_k}{2} e^{+jk\Omega_0 t} + \frac{B_k}{2j} e^{+jk\Omega_0 t}$

$e^{-jk\Omega_0 t}$: $\frac{A_k}{2} e^{-jk\Omega_0 t} - \frac{B_k}{2j} e^{-jk\Omega_0 t}$

$$\therefore e^{+jk\Omega_0 t}\left(\frac{A_k}{2} + \frac{B_k}{2j} \right) \Big| \; e^{-jk\Omega_0 t}\left(\frac{A_k}{2} - \frac{B_k}{2j} \right)$$
$$= e^{+jk\Omega_0 t}\left(\frac{A_k}{2} - \frac{jB_k}{2} \right) \Big| = e^{+jk\Omega_0 t}\left(\frac{A_k}{2} + \frac{jB_k}{2} \right)$$

Compare to $\sum_{k=-\infty}^{\infty} C_k e^{+jk\Omega_0 t}$. } complex exponential form.

$$\therefore \; C_0 = A_0.$$
$$C_k = \frac{A_k}{2} - j\frac{B_k}{2}.$$
$$C_{-k} = \frac{A_k}{2} + j\frac{B_k}{2}$$

Complex Exponential Coeff to Trigo Coeff relationships.

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