

# Hands-on: Molecular Dynamics

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# Content

- MD Visualization
- Analyse MD data
  - Property 1
  - Property 2
  - Property 3
- Produce MD data

# MD Visualization

Run:

```
vmd trajectory.xyz
```

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Configuration:

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Graphics/Representations.../Drawing Method/VDW
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Movie:



- MD Visualization

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- MD Visualization



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# Questions

- Is the material a solid, liquid, or gas?

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- Is the material a solid, liquid, or gas?
- Given the positions of every atom at each time step, what analyses would you perform to corroborate it?

# MD Properties

## Exercise 0: Loading and Parameters

### Loading Data

```
: Trajectory = read_xyz_file("trajectory.xyz") # Position of the atoms in angstrom  
Trajectory.shape
```

### Simulation parameters

```
: N_atoms = Trajectory.shape[1]  
dt_sampling = 10  
dt_reduced = 0.005  
rho_reduced = 0.84  
N_box = 6
```

### Physical units parameters

```
: sigma_angstrom = 3.4  
sigma_m = sigma_angstrom * 1e-10  
eps_joule = 120 * 1.380649e-23  
mass_kg = 39.948 * 1.66054e-27  
tau = sigma_m * (mass_kg / eps_joule) ** 0.5 # in seconds  
tau_ps = tau * 1e12 # convert to picoseconds  
  
dt_ps = dt_reduced * tau_ps * dt_sampling # in picoseconds
```

### Removing equilibration

```
: N_frames = 200  
Trajectory = Trajectory[-N_frames:]  
Trajectory.shape
```

### Time array and L\_box

```
: time = np.arange(N_frames) * dt_ps # time array in ps  
L_box = ((4/rho_reduced)**(1./3.)) * sigma_angstrom * N_box # L_box in angstroms
```

# Mean square displacement & Diffusion

- MSD

$$\text{MSD}(t) = \langle (x(0) - x(t))^2 \rangle$$

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- Diffusion Coefficient

$$6D = d \text{MSD}(t) / dt$$

- Problem: PBC

- Steps: 1) “unwrap” or “unfold” trajectories, 2) Compute MSD(t) of unfolded system



[ 0, 0 ]  
jumps\_cumulated

[ 0, 0 ]  
current\_jumps

. 0



[ 1, 0 ]  
jumps\_cumulated

[ 1, 0 ]  
current\_jumps

. 0



[ 1, 0 ]  
jumps\_cumulated

[ 1, 0 ]  
current\_jumps

. 0



[ 1, -1 ]  
jumps\_cumulated

[ 0, -1 ]  
current\_jumps

. 0



[ 1, 0 ]  
jumps\_cumulated

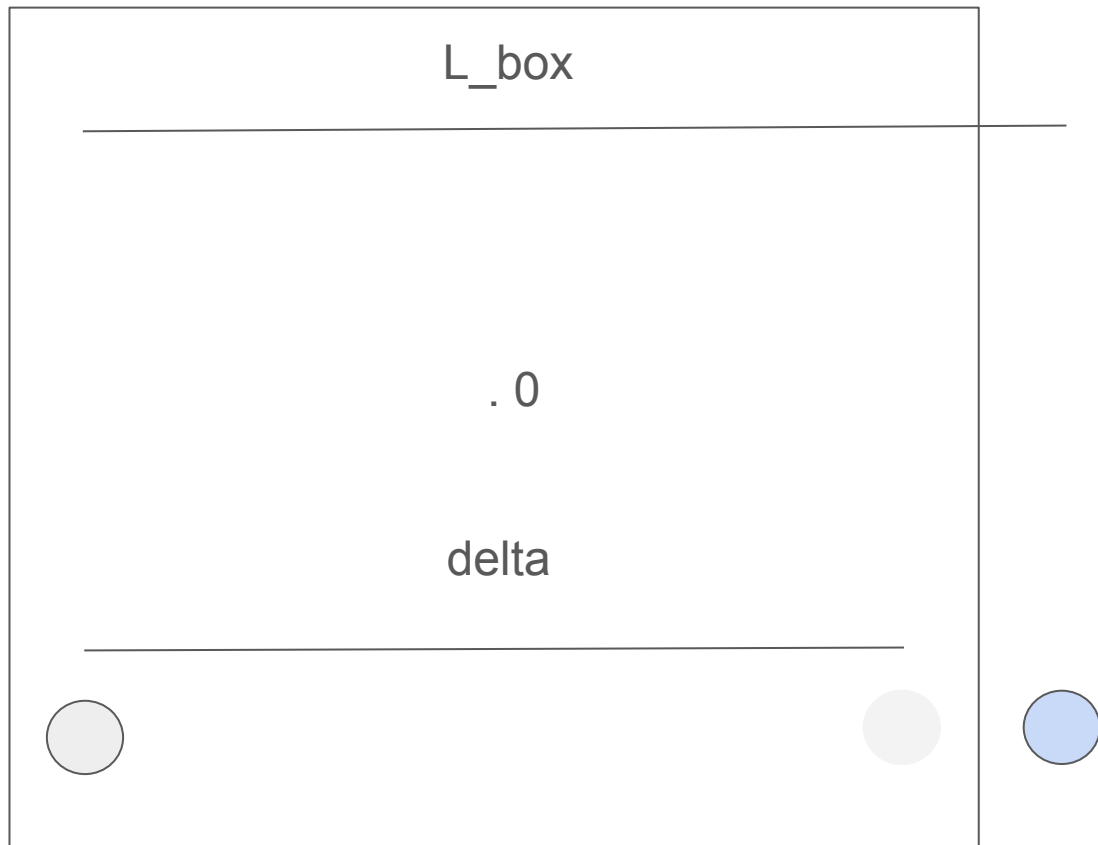
[ 1, 0 ]  
current\_jumps

. 0



[ 1, 0 ]  
jumps\_cumulated

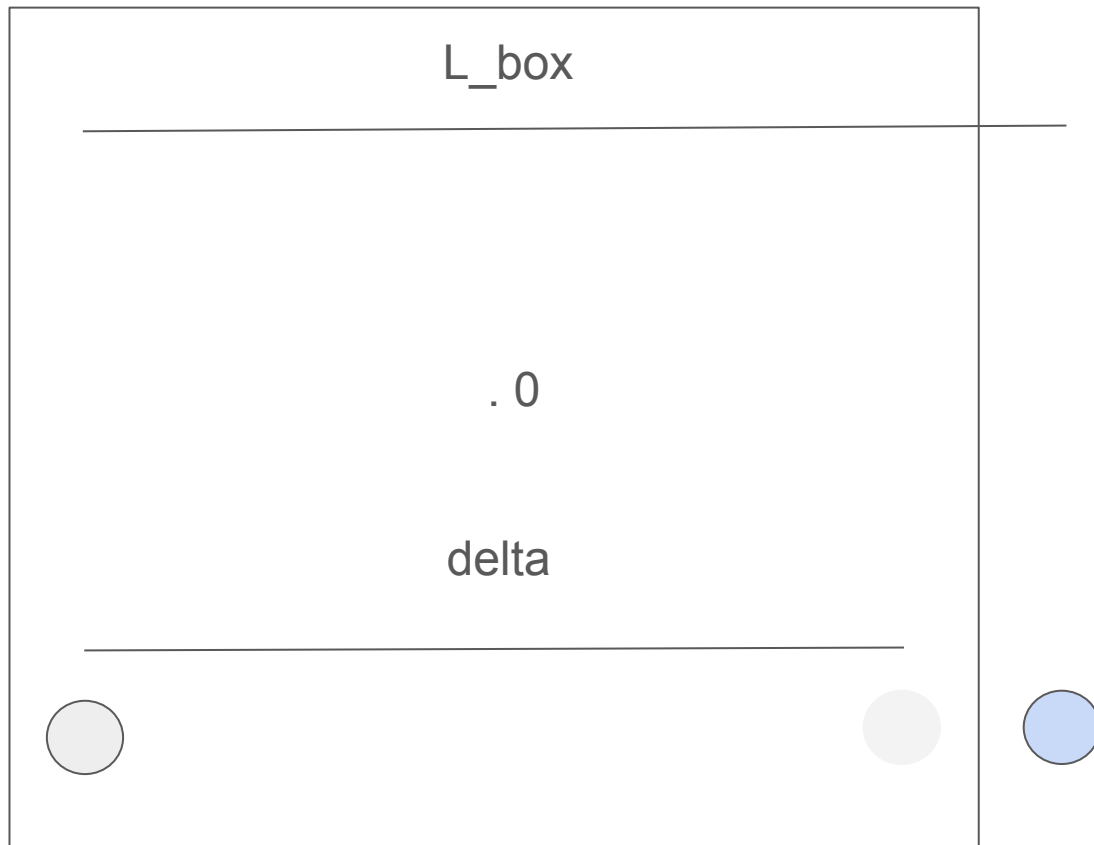
[ 1, 0 ]  
current\_jumps



For all atoms!

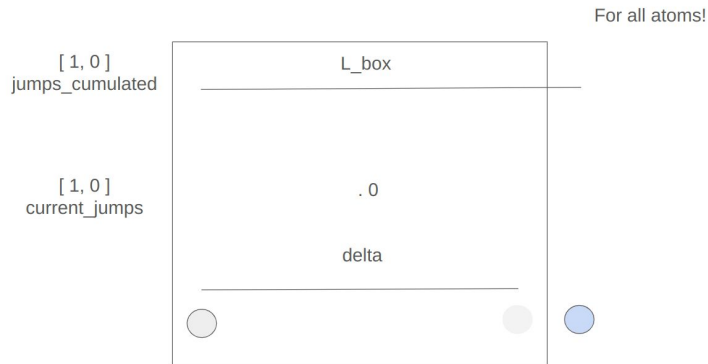
[ 1, 0 ]  
jumps\_cumulated

[ 1, 0 ]  
current\_jumps



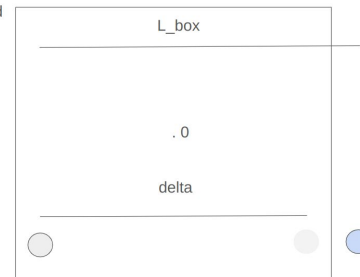
# Unwrap trajectories

```
def unwrap_trajectory(trajectory_wrapped):  
    """  
    Unwrap the trajectory!  
    """  
  
    unwrapped = np.zeros_like(trajectory_wrapped)  
    unwrapped[0] = trajectory_wrapped[0]  
  
    jumps_cumulated = np.zeros((N_atoms, 3)) # counts boundary crossings  
  
    for t in range(N_frames - 1):  
  
        delta = # delta in real units  
  
        current_jumps = np rint(delta / L_box)  
  
        # Update jumps_cumulated  
  
        # Reconstruct next unwrapped position  
        unwrapped[t + 1] =  
  
    return unwrapped
```





[1, 0]  
jumps\_cumulated



# Unwrap trajectories: Solution

```
def unwrap_trajectory(trajectory_wrapped):  
    '''  
    Unwrap the trajectory!  
    '''  
  
    unwrapped = np.zeros_like(trajectory_wrapped)  
    unwrapped[0] = trajectory_wrapped[0]  
  
    jumps_cumulated = np.zeros((N_atoms, 3)) # counts boundary crossings  
  
    for t in range(N_frames - 1):  
  
        delta = trajectory_wrapped[t] - trajectory_wrapped[t+1] # delta in real units  
  
        current_jumps = np rint(delta / L_box)  
  
        # Update jumps_cumulated  
        jumps_cumulated += current_jumps  
  
        # Reconstruct next unwrapped position  
        unwrapped[t + 1] = trajectory_wrapped[t+1] + jumps_cumulated * L_box  
  
    return unwrapped
```

# Compute mean square displacement

- MSD

$$\text{MSD}(t) = \langle (x(0) - x(t))^2 \rangle$$

- For a given t:

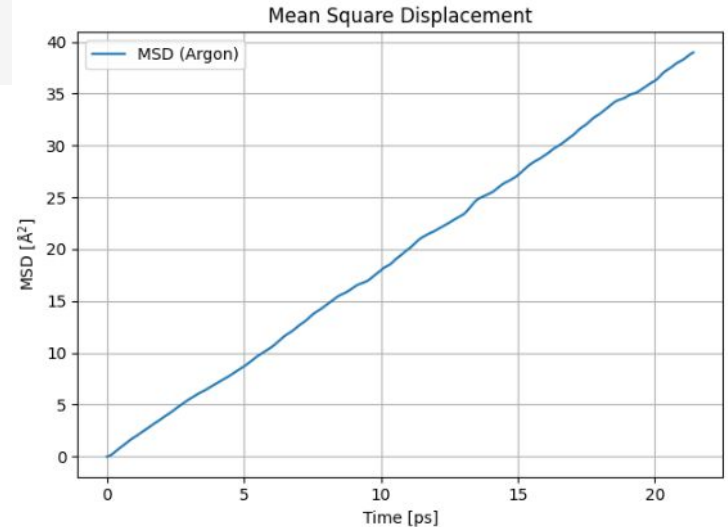
$$\left. \begin{array}{l} (x_1(0) - x_1(t))^2 \\ (x_2(0) - x_2(t))^2 \\ (x_3(0) - x_3(t))^2 \\ (x_4(0) - x_4(t))^2 \end{array} \right\}$$

Average over particles

```
def compute_msd(unwrapped):  
    ...  
    Compute the mean squared displacement!  
    ...  
    r0 = unwrapped[0] # initial positions  
  
    displacements = unwrapped - r0 # displacement from initial position  
  
    return msd
```

# Compute mean square displacement: Solution

```
def compute_msd(unwrapped):  
  
    r0 = unwrapped[0] # initial positions  
  
    displacements = unwrapped - r0 # displacement from initial position  
  
    squared_displacements = np.sum(displacements**2, axis=2) # shape: (n_frames, n_atoms)  
  
    msd = np.mean(squared_displacements, axis=1) # average over atoms  
  
    return msd
```



- MD Visualization



- Analyse MD data

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- MD Visualization



- Analyse MD data
  - Property 1: MSD & Diffusion
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- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
- Property 2
- Property 3



- Produce MD data

# Question

- What if you only have the positions at a single time frame?

# Radial Distribution Function

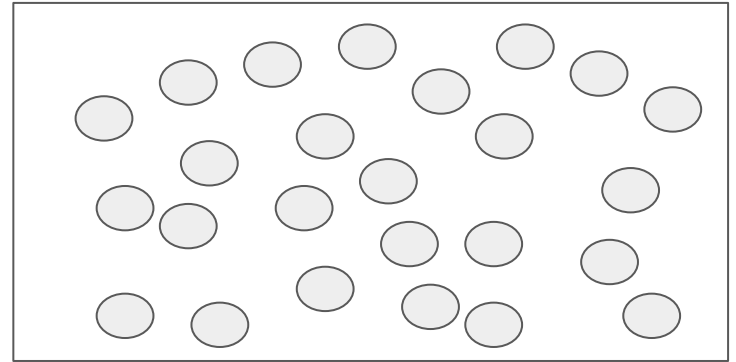
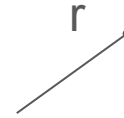
- Definition  $\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_i^N \sum_{j \neq i}^N \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$



# Radial Distribution Function

- Definition

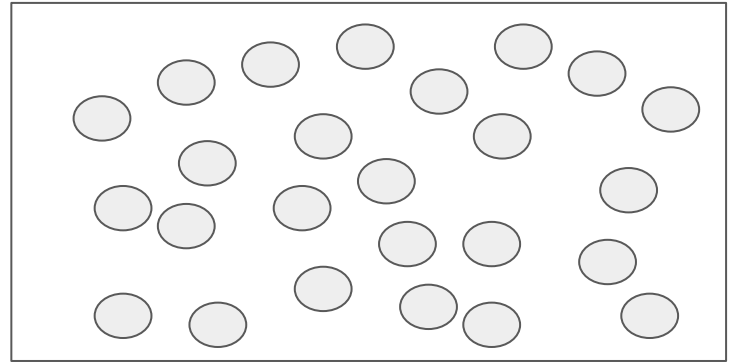
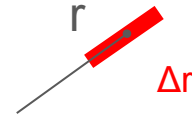
$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_i^N \sum_{j \neq i}^N \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$



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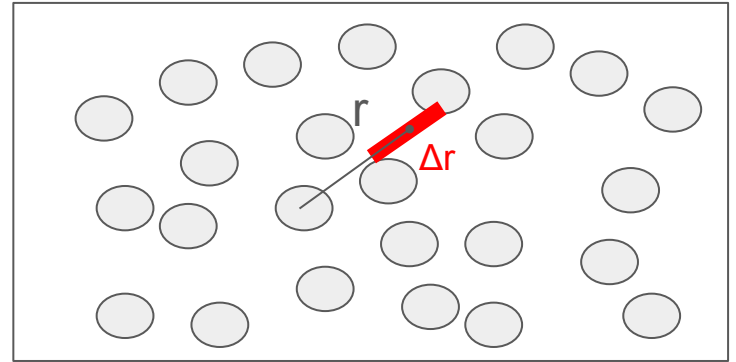
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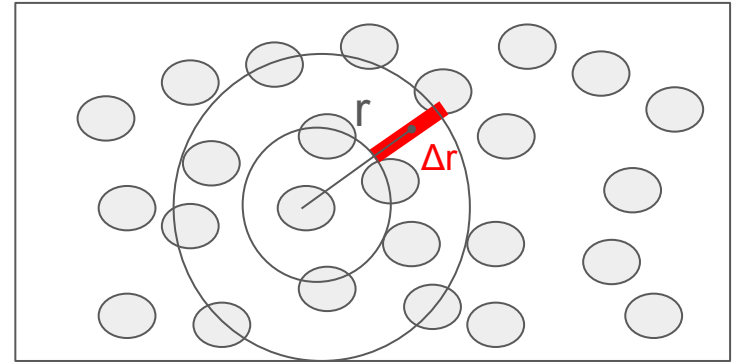
$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_i^N \sum_{j \neq i}^N \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$



# Radial Distribution Function

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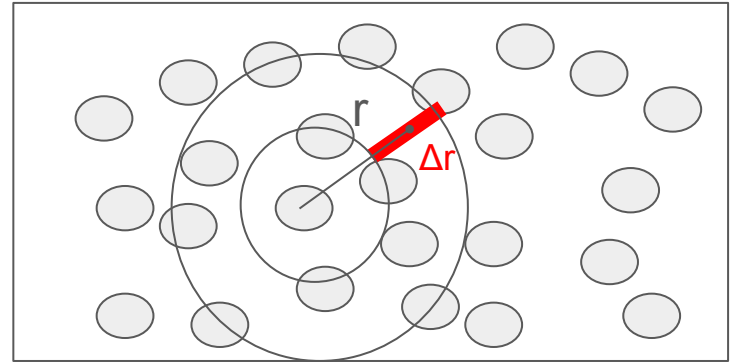
# Radial Distribution Function

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- Steps

1. Counting
2. Normalization



# Radial Distribution Function

- Definition

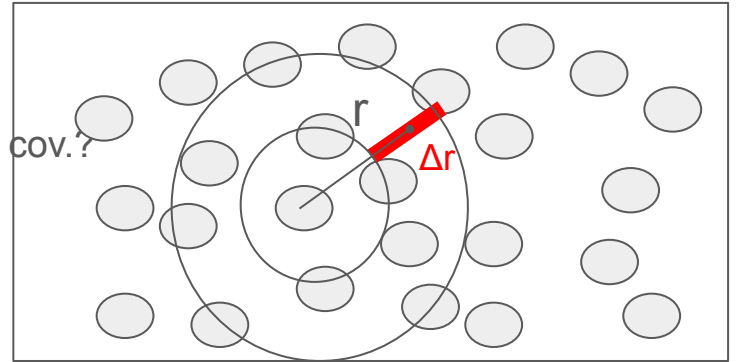
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- Steps

1. Counting

- a. How to consider the min. image cov.?

2. Normalization



# Radial Distribution Function

- Definition

$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_i^N \sum_{j \neq i}^N \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$

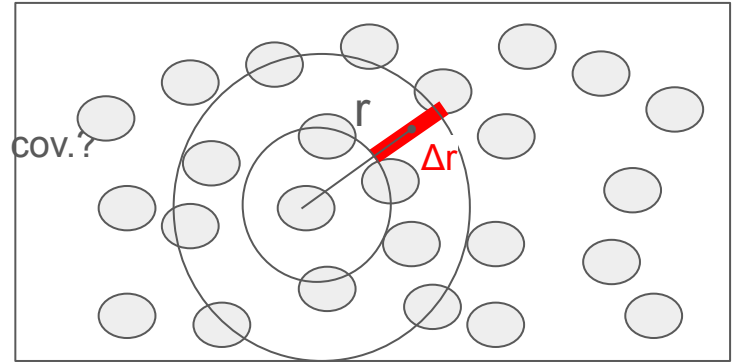
- Steps

1. Counting

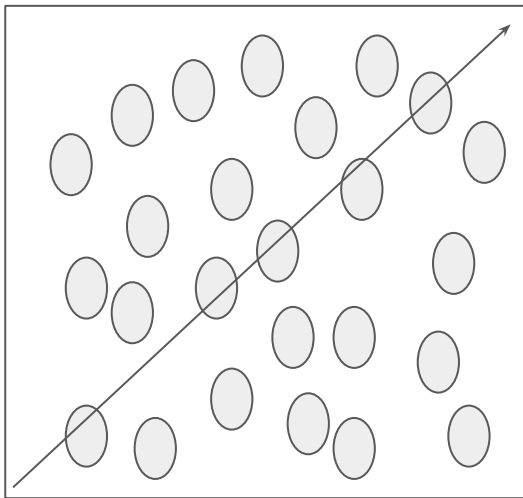
- a. How to consider the min. image cov.?

2. Normalization

- a. How to consider  $\Delta r$  ?

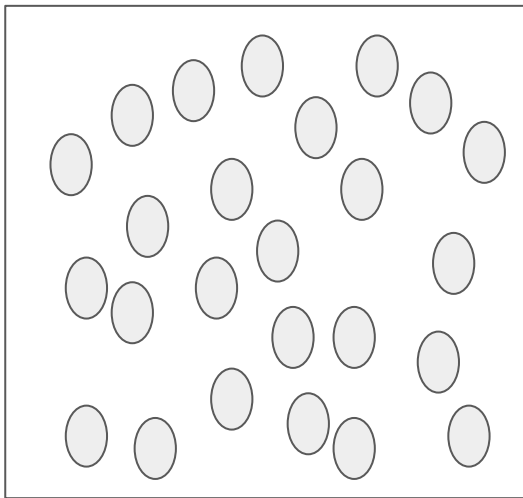


# Counting





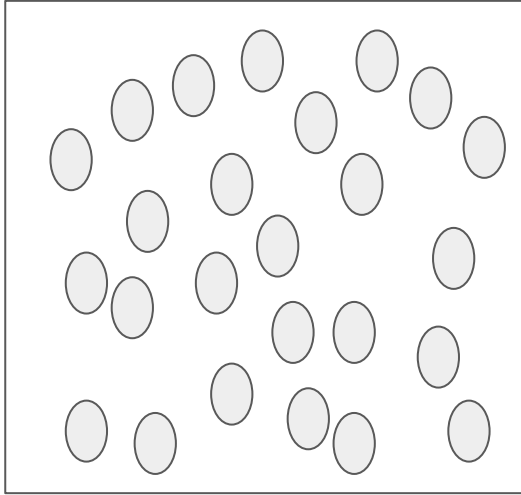
# Counting



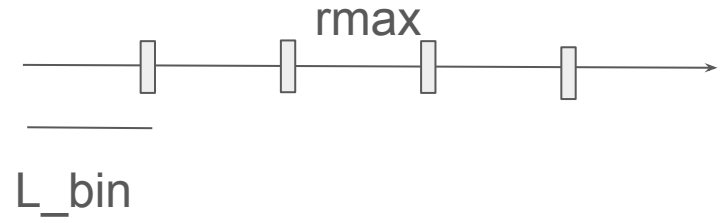
rmax



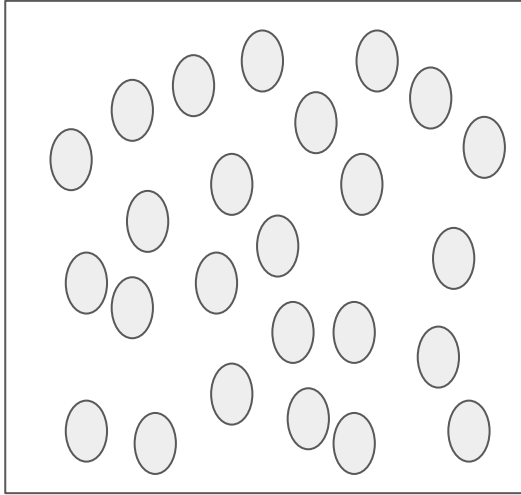
# Counting



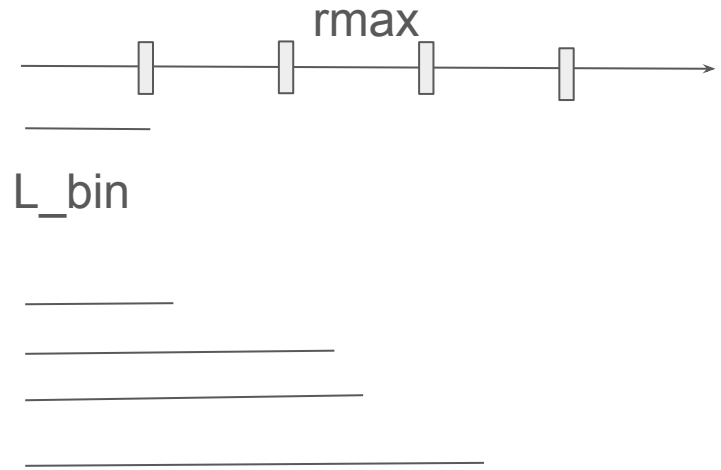
$N_{\text{bins}} = 5$



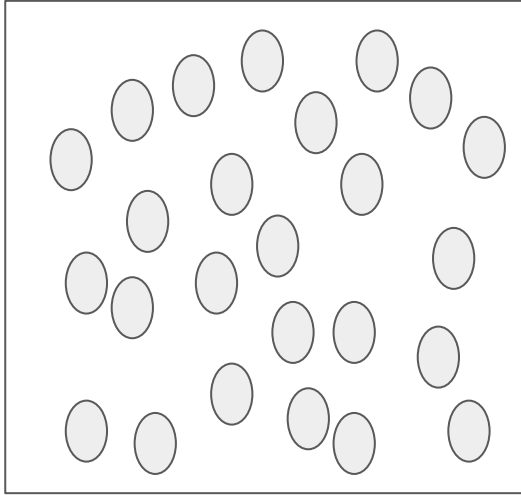
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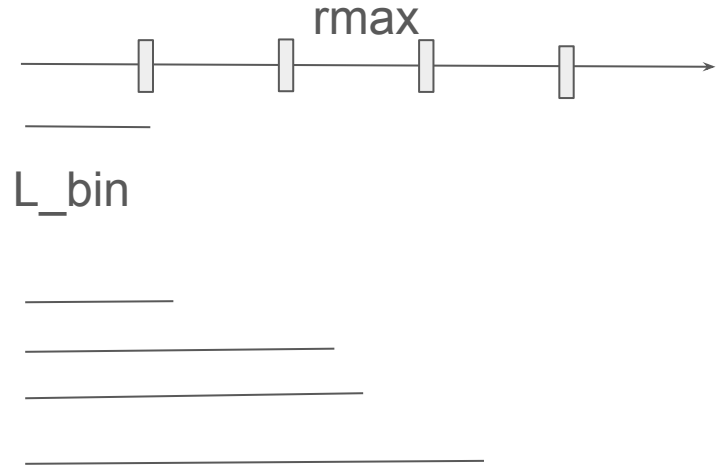
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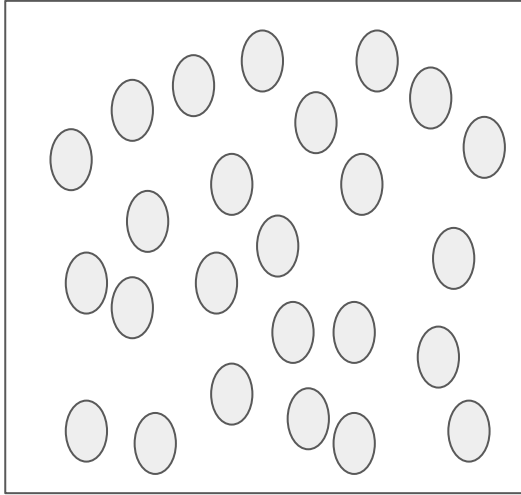


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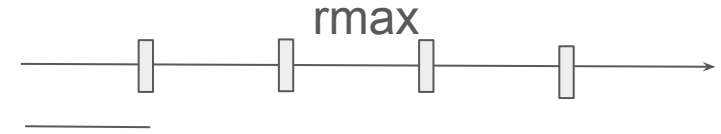


$g_{\text{count}} = [ 0, \quad 1, \quad 2, \quad 1, \quad 0 ]$

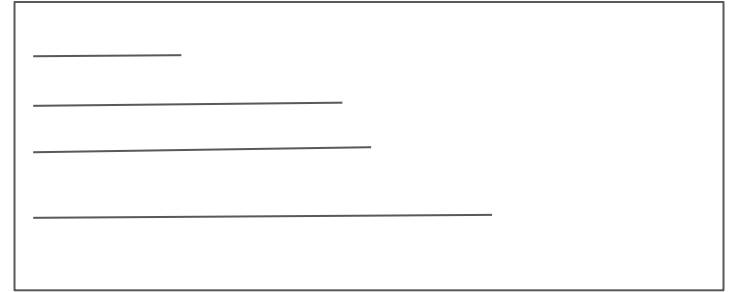
# Counting



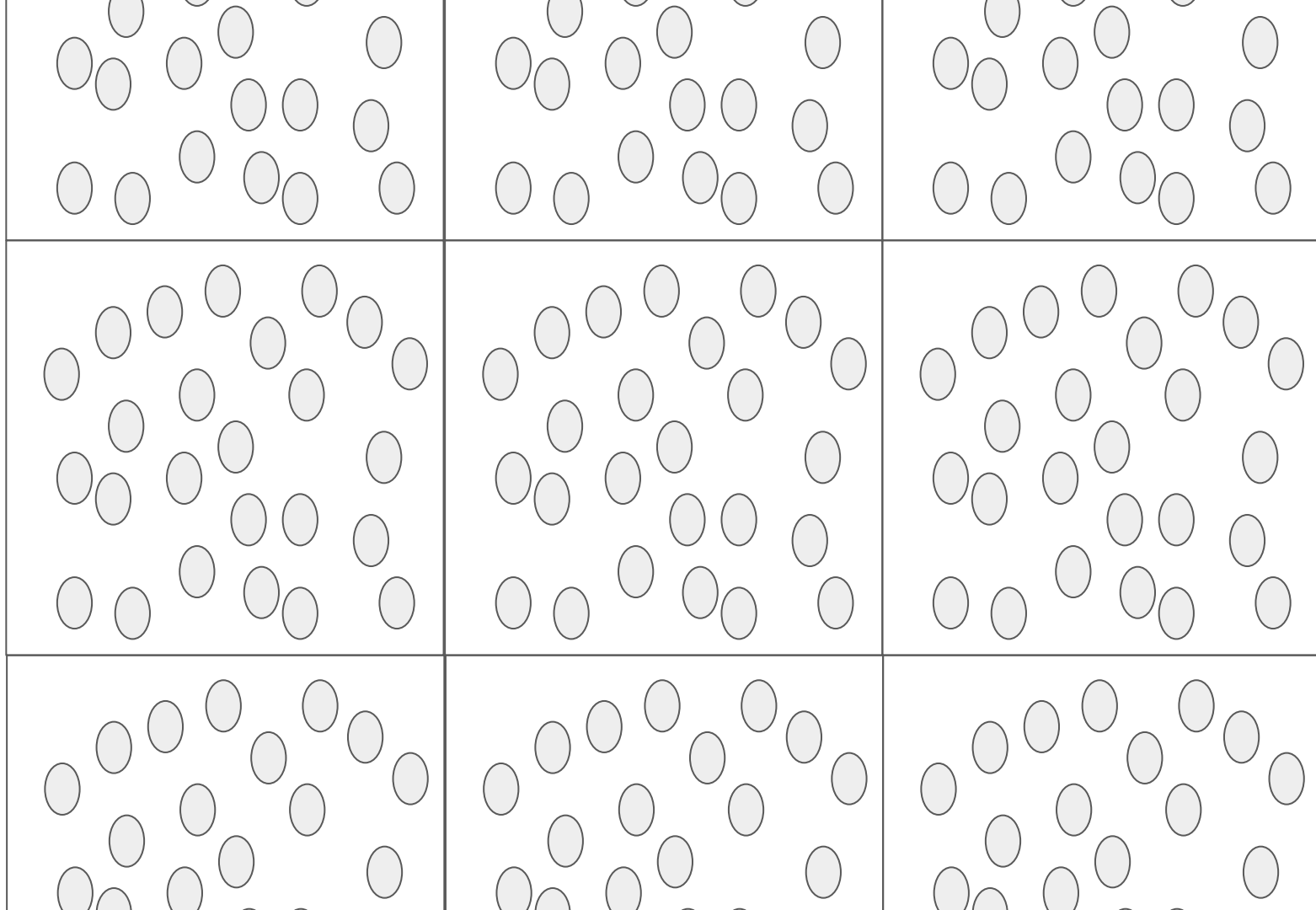
$N_{\text{bins}} = 5$

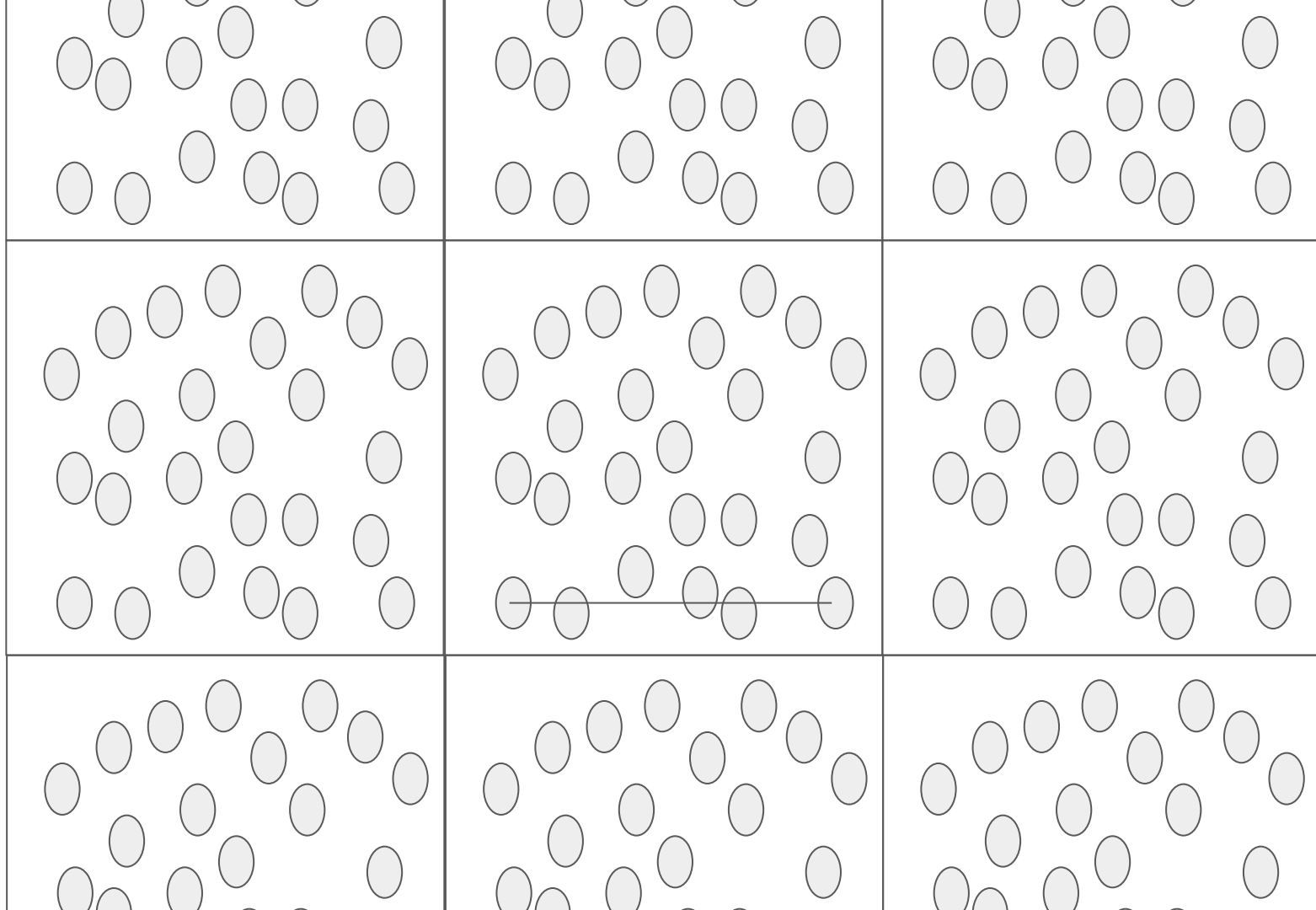


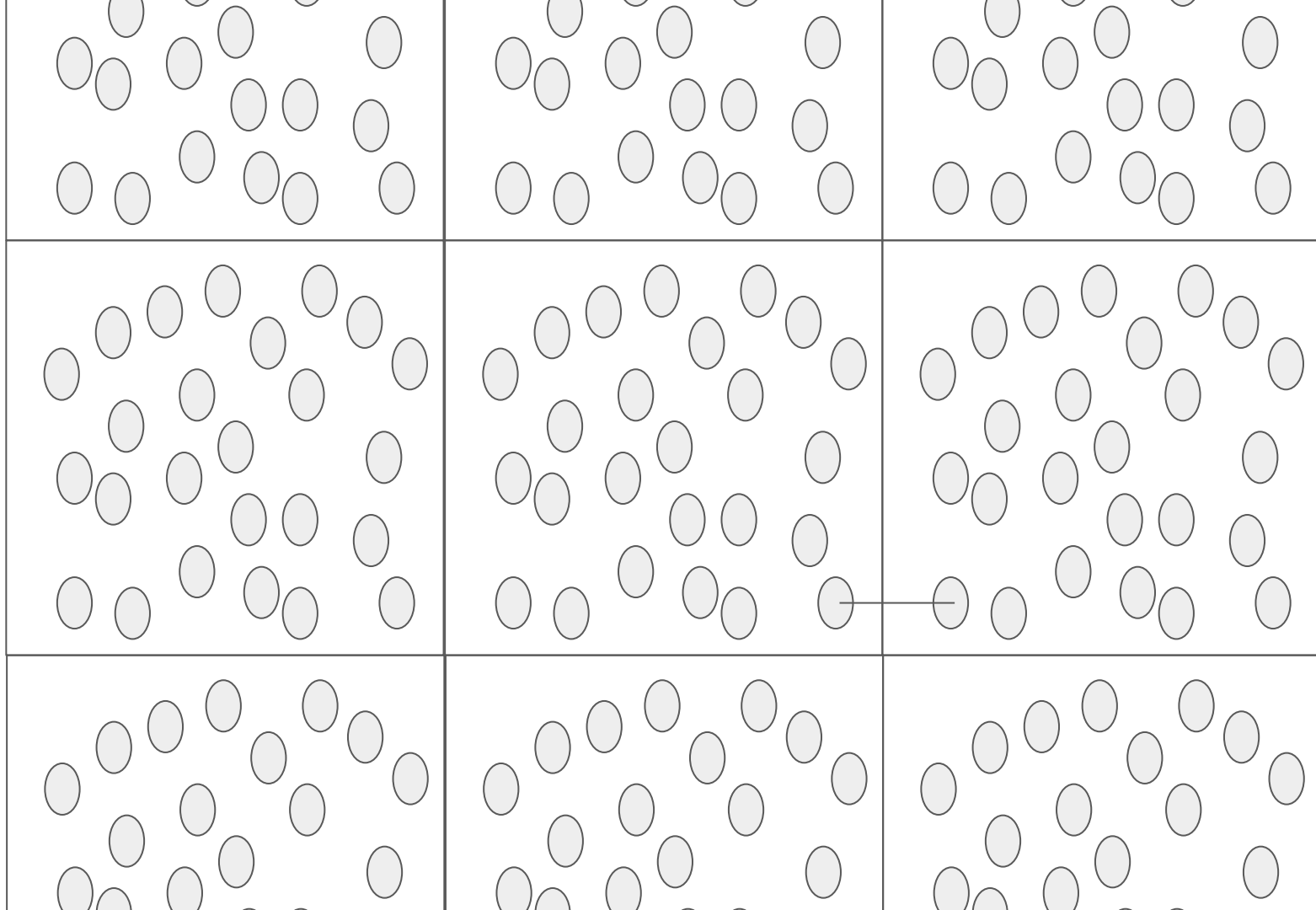
$L_{\text{bin}}$



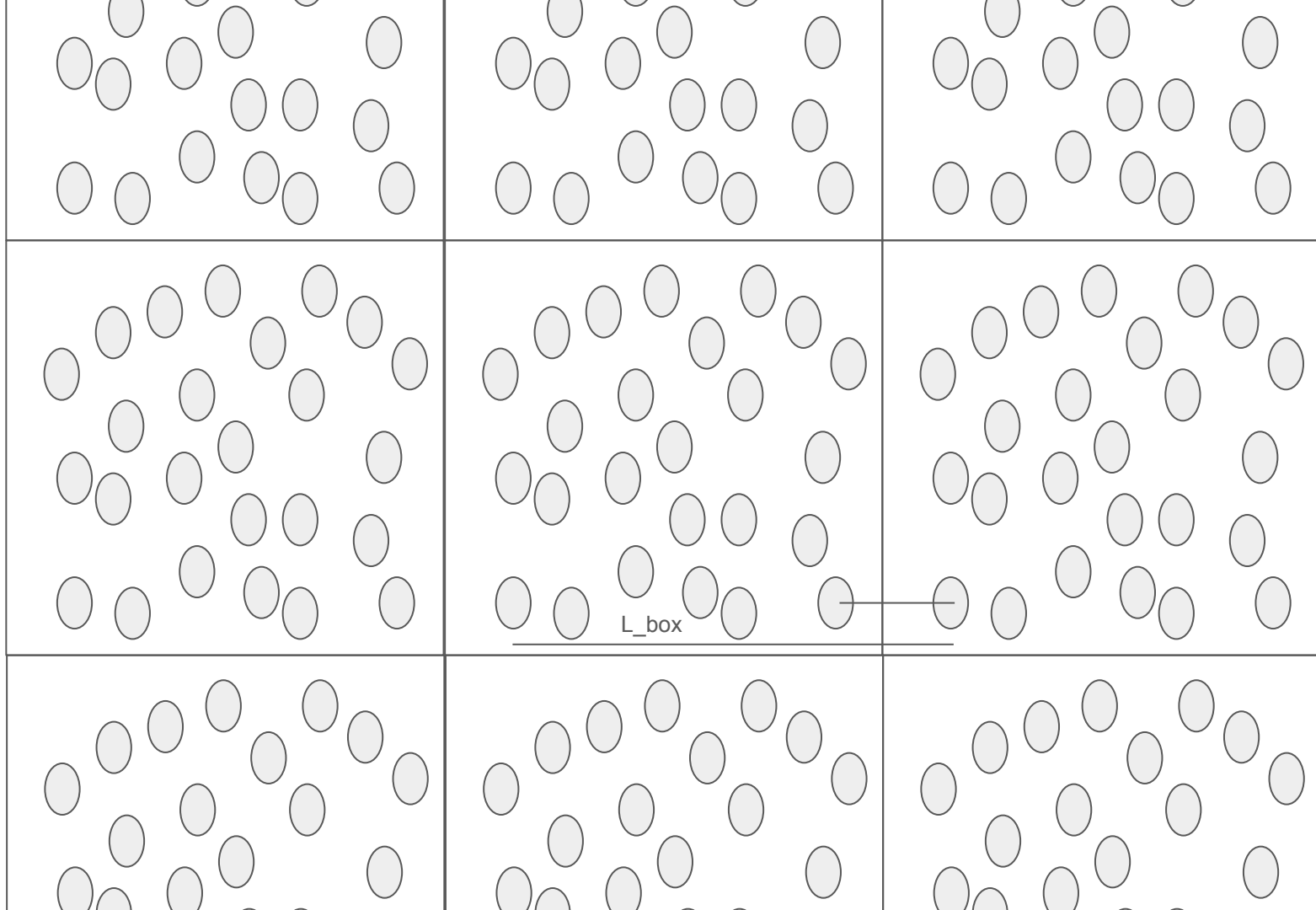
$g_{\text{count}} = [ 0, \quad 1, \quad 2, \quad 1, \quad 0 ]$

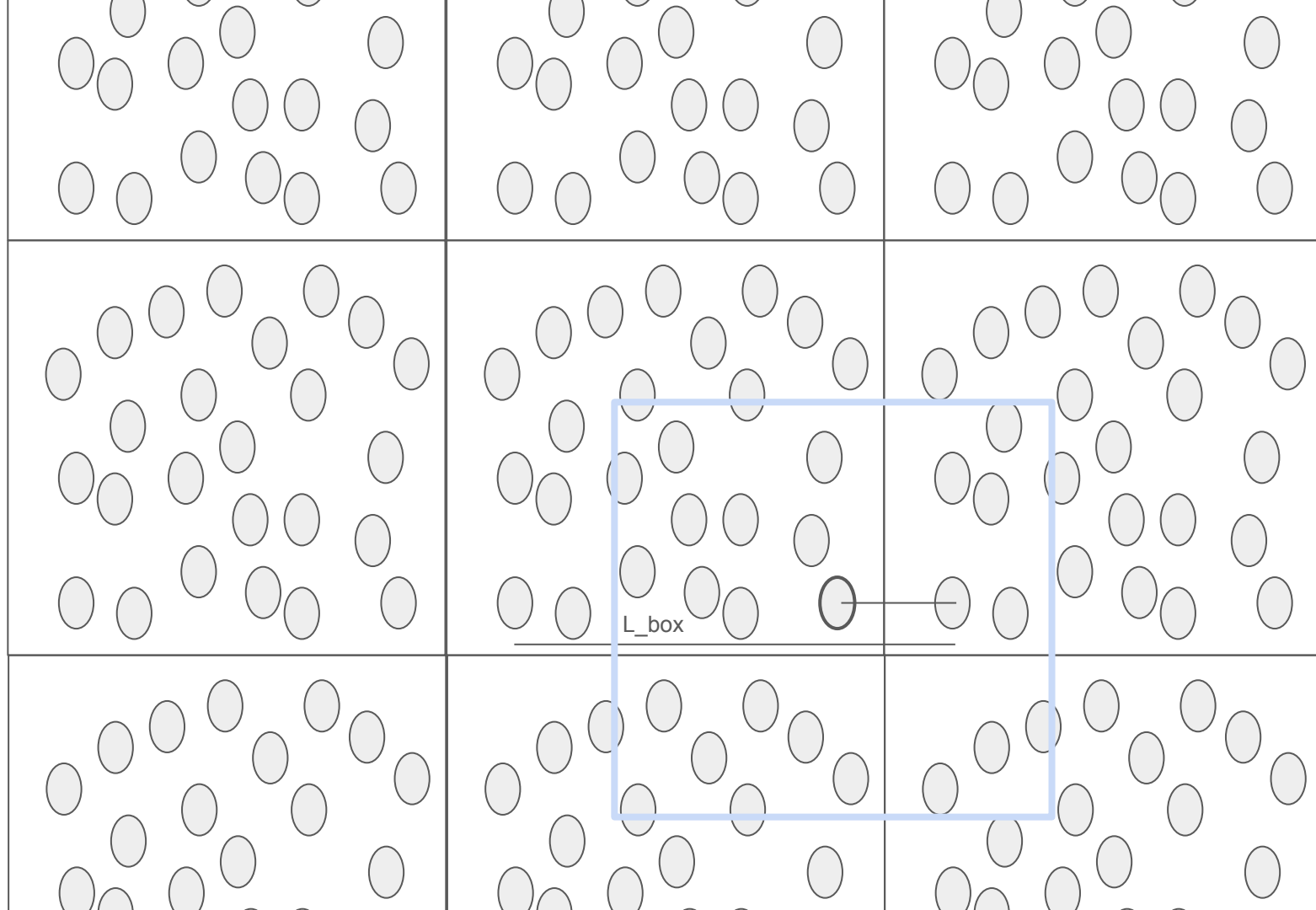




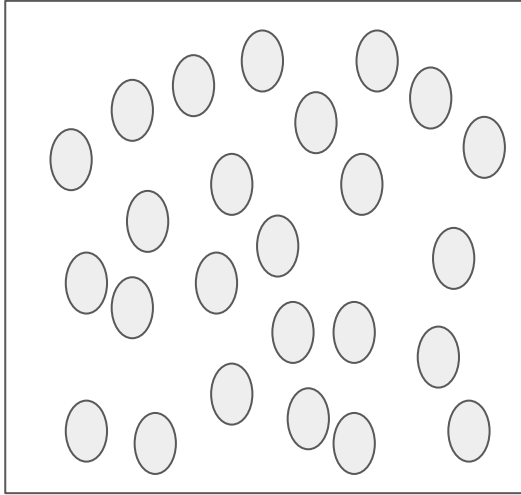




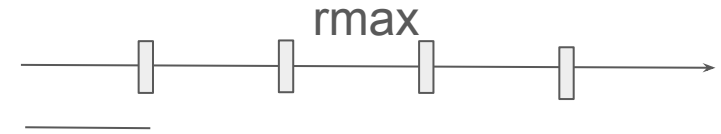




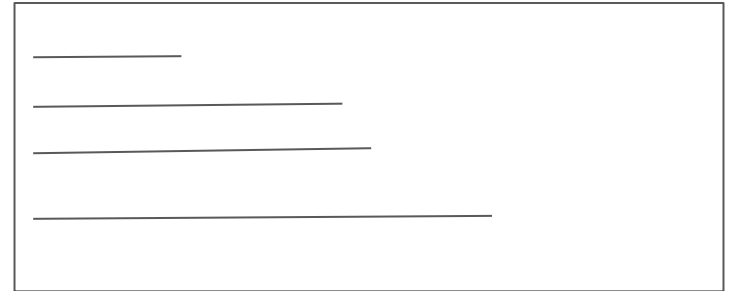
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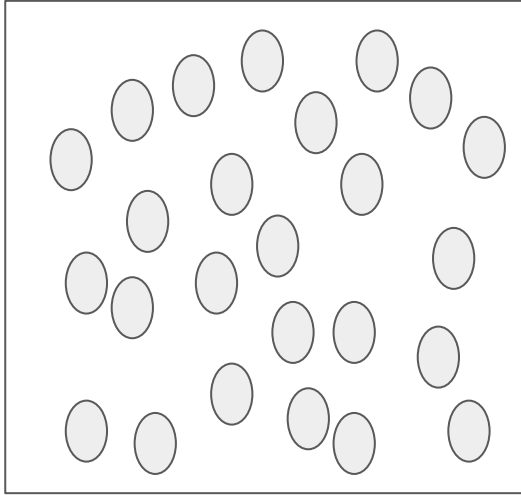


$L_{\text{bin}}$

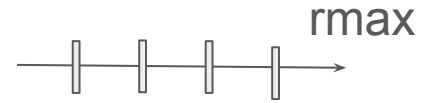


$g_{\text{count}} = [ 0, \quad 1, \quad 2, \quad 1, \quad 0 ]$

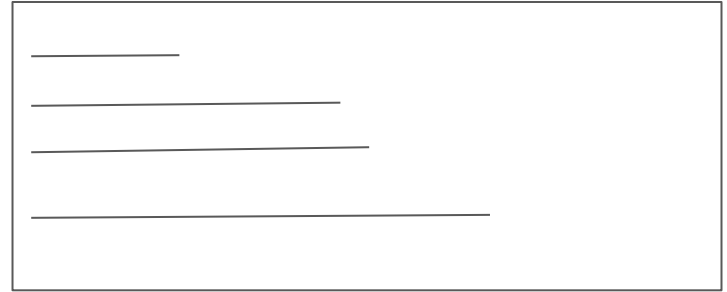
# Counting



$N_{\text{bins}} = 5$

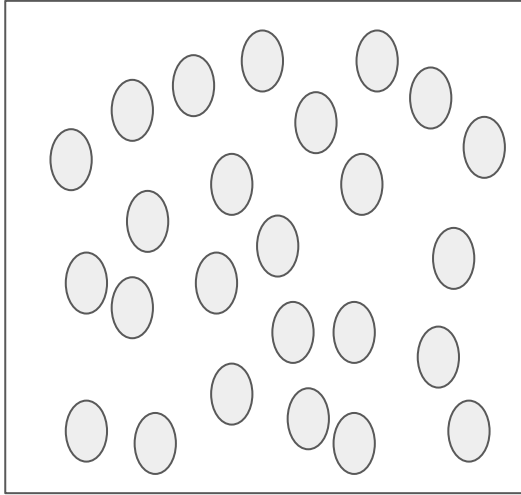


$L_{\text{bin}}$

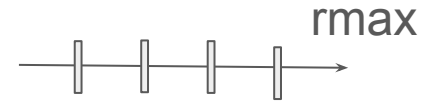


$g_{\text{count}} = [ 0, \quad 1, \quad 2, \quad 1, \quad 0 ]$

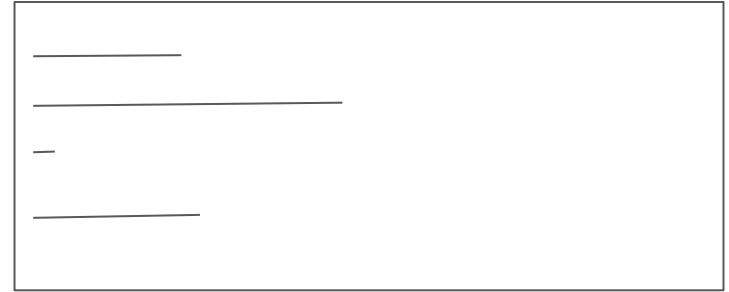
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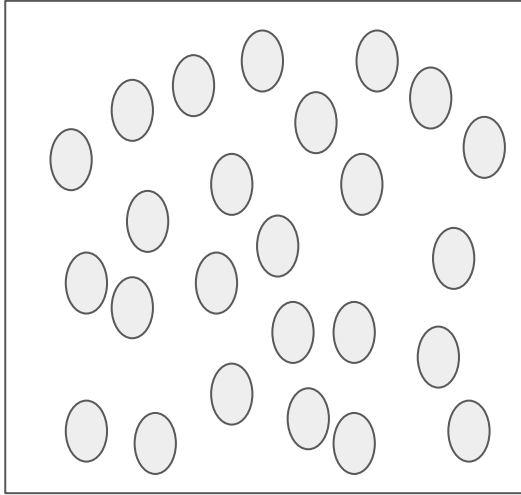


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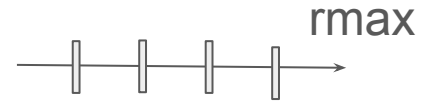


$g_{\text{count}} = [ 0, \quad 1, \quad 2, \quad 1, \quad 0 ]$

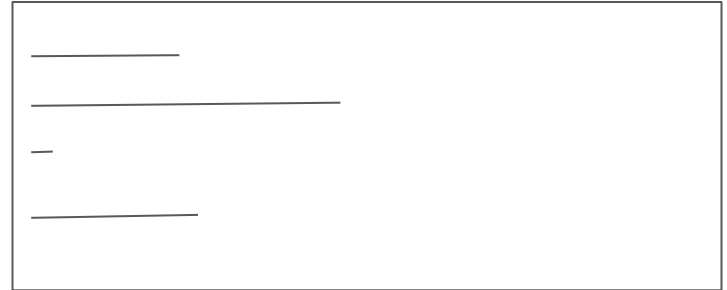
# Counting



$N_{\text{bins}} = 5$

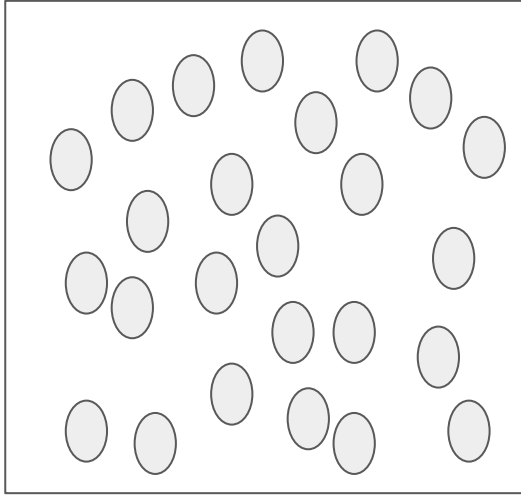


$L_{\text{bin}}$

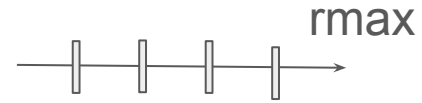


$g_{\text{count}} = [1, 0, 2, 0, 1]$

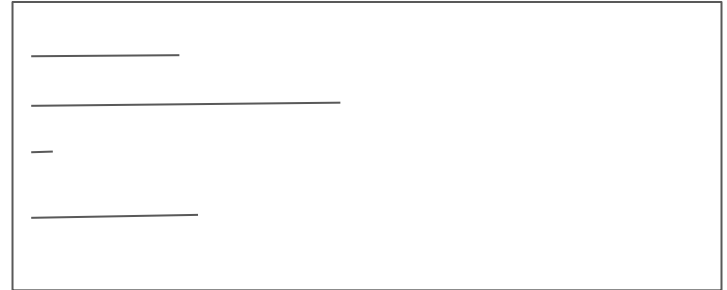
# Counting



$N_{\text{bins}} = 5$



$L_{\text{bin}}$



$g_{\text{count}} = [1, 0, 2, 0, 1]$

# Counting

```
N_bins = 512
rmax = np.sqrt(3*L_box**2)/2
L_bin=rmax/N_bins
g_counter=np.zeros(N_bins)
```

```
def counting_distances_frame(i):
    """
    Add the distances to g_counter corresponding to the i-th frame
    """
    rx = Trajectory[i][:,0];
    ry = Trajectory[i][:,1];
    rz = Trajectory[i][:,2];

    for k in range(N_atoms-1):
        j=k+1

        # Distances to atoms with superior index (not normalized)
        dx = (rx[k]-rx[j:N_atoms])
        dy = (ry[k]-ry[j:N_atoms])
        dz = (rz[k]-rz[j:N_atoms])

        # Apply minimum image convention to dx, dy and dz

        # dx, dy and dz already with the minimum image convention in real units.
        r2 = dx*dx + dy*dy + dz*dz
        r = np.sqrt(r2)

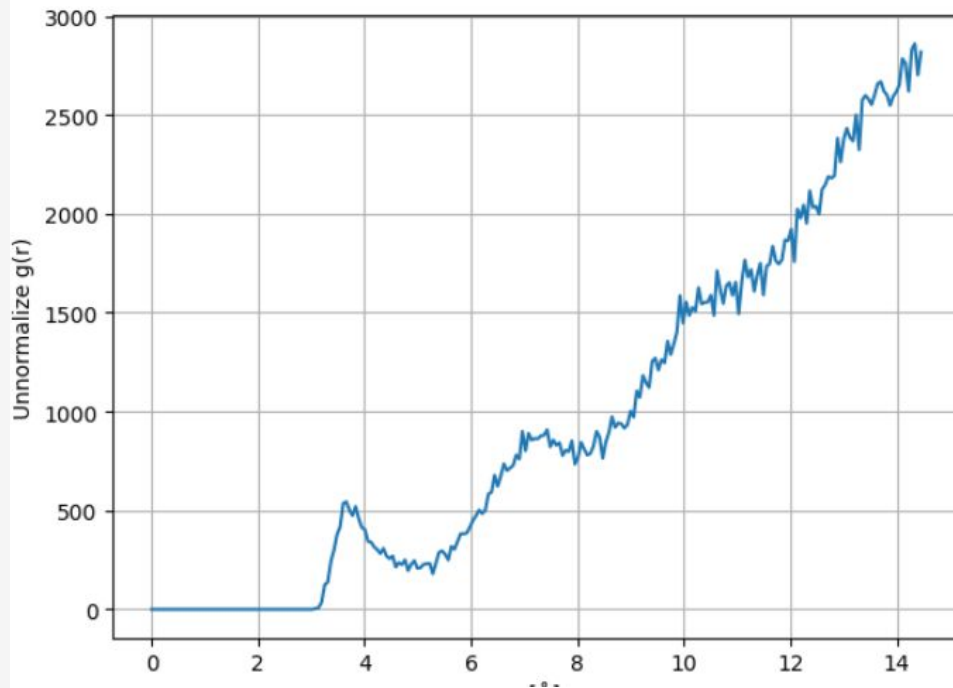
        for corrected_distance in r:
            g_counter[?] += 2    # Find the expression for ?
```

Hint: If  $dx$  is normalized,  
 $dx = dx - \text{np rint}(dx)$   
applies the minimum image  
convention  
in normalized units (why?)



# Counting: Solution

```
def counting_distances_frame(i):  
    """  
    Add the distances to g_counter corresponding to the i-th frame  
    """  
    rx = Trajectory[i][:,0];  
    ry = Trajectory[i][:,1];  
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    for k in range(N_atoms-1):  
        j=k+1  
  
        # Distances to atoms with superior index (not normalized)  
        dx = (rx[k]-rx[j:N_atoms])  
        dy = (ry[k]-ry[j:N_atoms])  
        dz = (rz[k]-rz[j:N_atoms])  
  
        # Apply minimum image convention to dx, dy and dz  
        dx = dx/L_box  
        dy = dy/L_box  
        dz = dz/L_box  
  
        dx = dx - np rint(dx)  
        dy = dy - np rint(dy)  
        dz = dz - np rint(dz)  
  
        dx = dx * L_box  
        dy = dy * L_box  
        dz = dz * L_box  
  
        # dx, dy and dz already with the minimum image convention in real units.  
        r2 = dx*dx + dy*dy + dz*dz  
        r = np.sqrt(r2)  
  
        for corrected_distance in r:  
            g_counter[int(corrected_distance/L_bin)] += 2    # Find the expression for ?
```



# Normalization

- 2 Types

# Normalization

- 2 Types

1.  $r$  &  $\Delta r$  independent

2.  $r$  &  $\Delta r$  dependent

# Normalization

- 2 Types

1.  $r$  &  $\Delta r$  independent

- Averages  $\rightarrow N_{\text{atoms}} * N_{\text{frames}}$

- Definition  $\rightarrow (N_{\text{atoms}} / L_{\text{box}}^3)$

Density

$$\text{norm\_factor} = N_{\text{atoms}} * N_{\text{frames}} * (N_{\text{atoms}} / L_{\text{box}}^3)$$

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# Normalization

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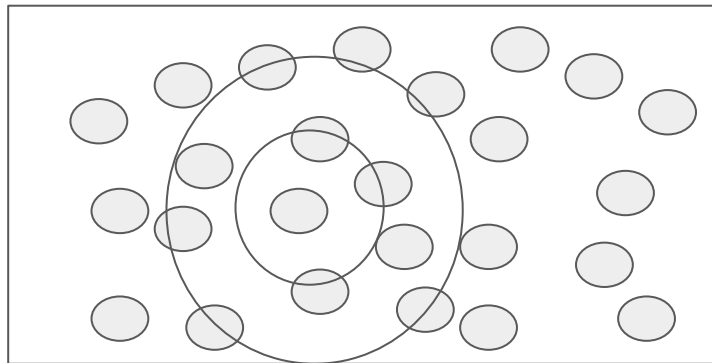
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$g\_count = [ 10, 65 ]$

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Density

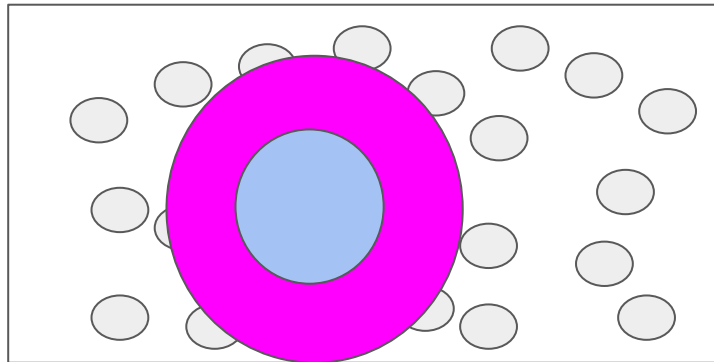
$$\text{norm\_factor} = N_{\text{atoms}} * N_{\text{frames}} * (N_{\text{atoms}} / L_{\text{box}}^3)$$

2.  $r$  &  $\Delta r$  dependent

$g_{\text{count}} = [10, 65]$

$g_{\text{normalization}} = [\text{vol}, \text{vol}] * \text{norm\_factor}$

- Volumen of the shells



# Normalization

$$g_r = g\_count / g\_normalization$$

- 2 Types

1.  $r$  &  $\Delta r$  independent

- Averages  $\rightarrow N\_atoms * N\_frames$

- Definition  $\rightarrow (N\_atoms / L\_box^{**3})$

Density

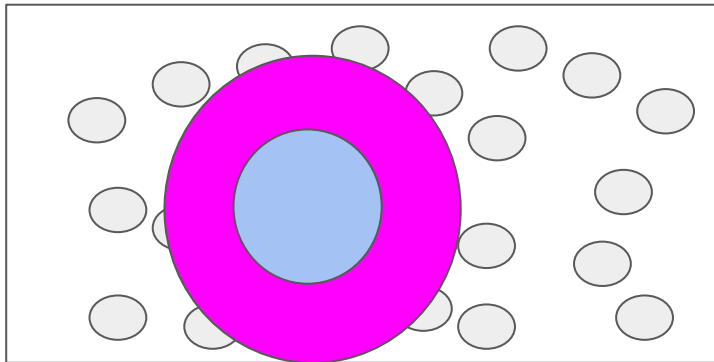
$$norm\_factor = N\_atoms * N\_frames * (N\_atoms / L\_box^{**3})$$

2.  $r$  &  $\Delta r$  dependent

$g\_count = [ 10, 65 ]$

$g\_normalization = [vol, vol] * norm\_factor$

- Volumen of the shells





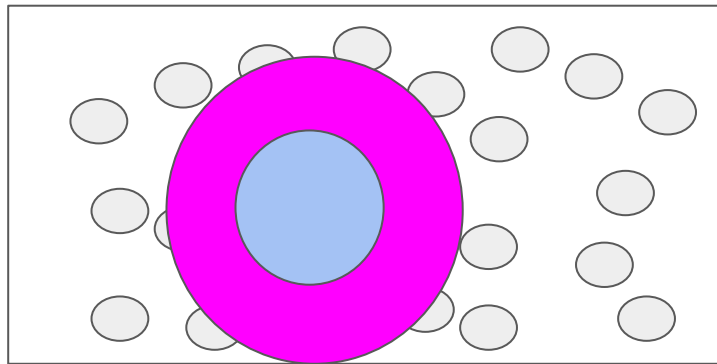
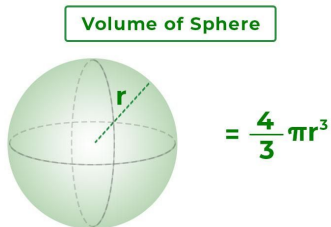
# Normalization

```
g_normalization = np.zeros(N_bins)
for i in range(N_bins):
    g_normalization = # Compute the normalization such that g(r) = g_counter/g_normalization
```

$\text{norm\_factor} = N_{\text{atoms}} * N_{\text{frames}} * (N_{\text{atoms}} / L_{\text{box}}^{**3})$

$g_{\text{normalization}} = [\text{vol}, \text{vol}] * \text{norm\_factor}$

Hint: The volume of a sphere is

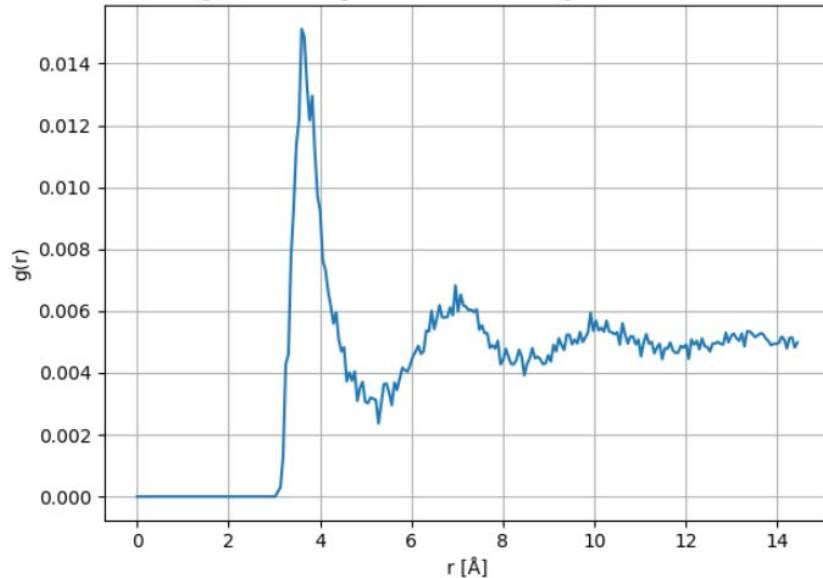


Hint: the radius can be written as  
 $L_{\text{bin}} * \text{integer}$  and contiguous radii are  
 $L_{\text{bin}} * \text{integer}$  &  $L_{\text{bin}} * (\text{integer} + 1)$

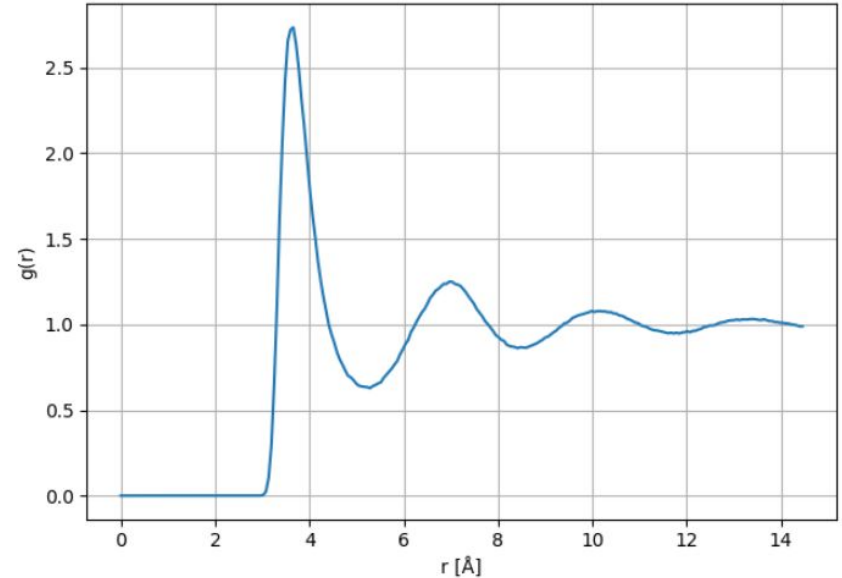
# Normalization: Solution

```
g_normalization = np.zeros(N_bins)
for i in range(N_bins):
    g_normalization[i] = (4/3)*np.pi*((L_bin*(i+1))**3-(L_bin*i)**3)*N_atoms*N_frames*(N_atoms/L_box**3)
```

$g(r)$  for a single frame with wrong normalization



Radial Distribution Function



- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
- Property 2
- Property 3



- Produce MD data

- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
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- Produce MD data

# Velocity autocorrelation function

- Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

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- For a given  $\tau$ :

$t=0$

$v_1(\tau) \cdot v_1(0)$

$v_2(\tau) \cdot v_2(0)$

$v_3(\tau) \cdot v_3(0)$

$v_4(\tau) \cdot v_4(0)$

# Velocity autocorrelation function

- Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

- Average over time and over particles!

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$$\left. \begin{array}{l} \text{t=0} \\ v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} \right\} \begin{array}{l} \text{Particle} \\ \text{av. t=0} \end{array}$$



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$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

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$$\begin{array}{ccc}
 \begin{array}{c} t=0 \\ v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} & \left\{ \begin{array}{c} \text{Particle} \\ \text{av. } t=0 \end{array} \right\} & \begin{array}{c} t=1 \\ v_1(\tau+1) \cdot v_1(1) \\ v_2(\tau+1) \cdot v_2(1) \\ v_3(\tau+1) \cdot v_3(1) \\ v_4(\tau+1) \cdot v_4(1) \end{array} \\
 & & \left\{ \begin{array}{c} \text{Particle} \\ \text{av. } t=1 \end{array} \right\} & \begin{array}{c} t=2 \\ v_1(\tau+2) \cdot v_1(2) \\ v_2(\tau+2) \cdot v_2(2) \\ v_3(\tau+2) \cdot v_3(2) \\ v_4(\tau+2) \cdot v_4(2) \end{array} \\
 & & & \left\{ \begin{array}{c} \text{Particle} \\ \text{av. } t=2 \end{array} \right\}
 \end{array}$$

# Velocity autocorrelation function

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- Average over time and over particles!

Particle & time av.

- For a given  $\tau$ :

$t=0$ $\left. \begin{array}{l} v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} \right\}$	$t=1$ $\left. \begin{array}{l} v_1(\tau+1) \cdot v_1(1) \\ v_2(\tau+1) \cdot v_2(1) \\ v_3(\tau+1) \cdot v_3(1) \\ v_4(\tau+1) \cdot v_4(1) \end{array} \right\}$	$t=2$ $\left. \begin{array}{l} v_1(\tau+2) \cdot v_1(2) \\ v_2(\tau+2) \cdot v_2(2) \\ v_3(\tau+2) \cdot v_3(2) \\ v_4(\tau+2) \cdot v_4(2) \end{array} \right\}$
Particle av. $t=0$	Particle av. $t=1$	Particle av. $t=2$

# Velocity autocorrelation function

```
def compute_vacf(vels, max_lag=N_frames):  
    '''  
    Compute the velocity autocorrelation function  
    '''  
  
    vacf = np.zeros(max_lag)  
    for lag in range(max_lag):  
        dot_sum = 0.0  
        count = 0  
        for t in range(N_frames - lag):  
            v0 = vels[t]  
            vlag = vels[t + lag]  
            dot_sum += v0.dot(vlag)  # Complete the function!  
            count += N_atoms  
        vacf[lag] = dot_sum / count  
    return vacf
```

Particle & time av.

$$\left. \begin{array}{l} \text{t=0} \\ v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} \right\} \begin{array}{l} \text{Particle} \\ \text{av. t=0} \end{array} \quad \left. \begin{array}{l} \text{t=1} \\ v_1(\tau+1) \cdot v_1(1) \\ v_2(\tau+1) \cdot v_2(1) \\ v_3(\tau+1) \cdot v_3(1) \\ v_4(\tau+1) \cdot v_4(1) \end{array} \right\} \begin{array}{l} \text{Particle} \\ \text{av. t=1} \end{array} \quad \left. \begin{array}{l} \text{t=2} \\ v_1(\tau+2) \cdot v_1(2) \\ v_2(\tau+2) \cdot v_2(2) \\ v_3(\tau+2) \cdot v_3(2) \\ v_4(\tau+2) \cdot v_4(2) \end{array} \right\} \begin{array}{l} \text{Particle} \\ \text{av. t=2} \end{array}$$

# Velocity autocorrelation function

```
def compute_vacf(vels, max_lag=N_frames):
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    """
    vacf = np.zeros(max_lag)
    for lag in range(max_lag):
        dot_sum = 0.0
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        for t in range(N_frames - lag):
            v0 = vels[t]
            vlag = vels[t + lag]
            dot_sum +=          # Complete the function!
            count += N_atoms
        vacf[lag] = dot_sum / count
    return vacf
```

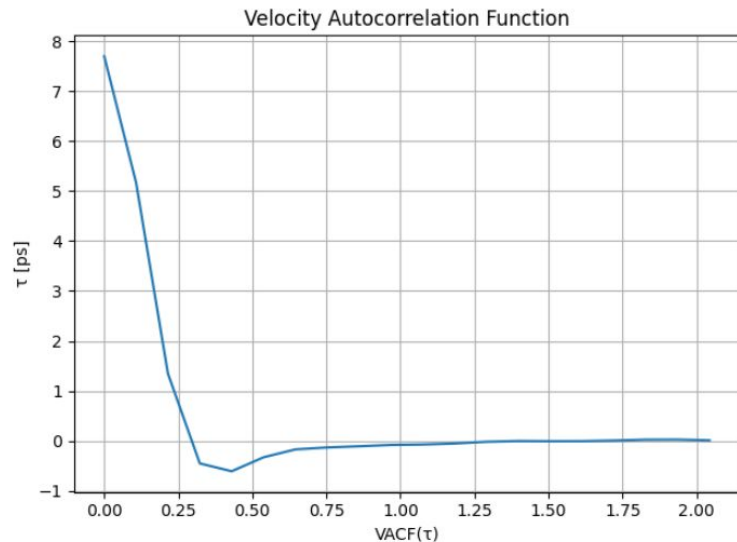
Hint: The missing line includes only one multiplication and two nested np.sum()'s

Particle & time av.

$$\begin{array}{ccc}
 \begin{array}{c} t=0 \\ \left. \begin{array}{l} v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} \right\} \\ \text{Particle} \\ \text{av. } t=0 \end{array} & 
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 \end{array}$$

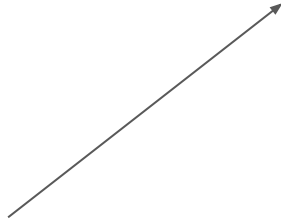
# Velocity autocorrelation function: Solution

```
def compute_vacf(vels, max_lag=N_frames):  
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    vacf = np.zeros(max_lag)  
    for lag in range(max_lag):  
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        count = 0  
        for t in range(N_frames - lag):  
            v0 = vels[t]  
            v1 = vels[t + lag]  
            dot_sum += np.sum(np.sum(v0 * v1, axis=1)) # Complete the function!  
            count += N_atoms  
        vacf[lag] = dot_sum / count  
    return vacf
```



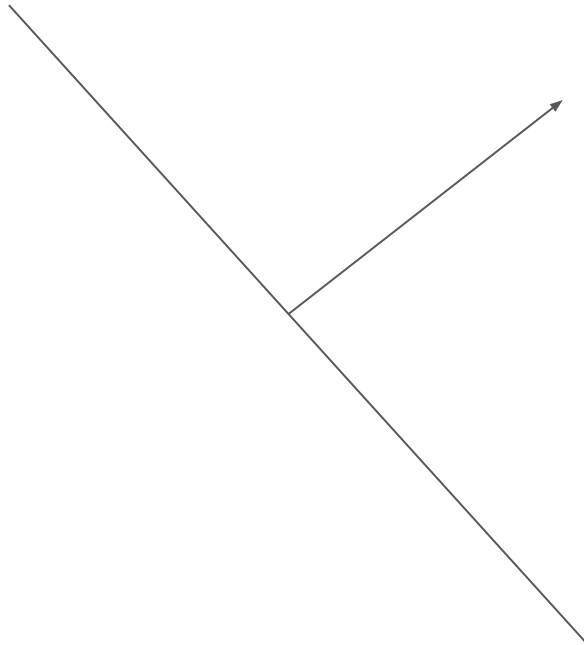
# Velocity autocorrelation function

- When the velocity autocorrelation function is negative?

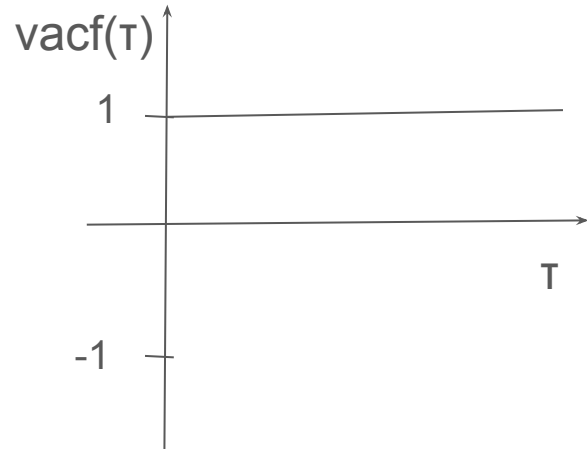


# Velocity autocorrelation function

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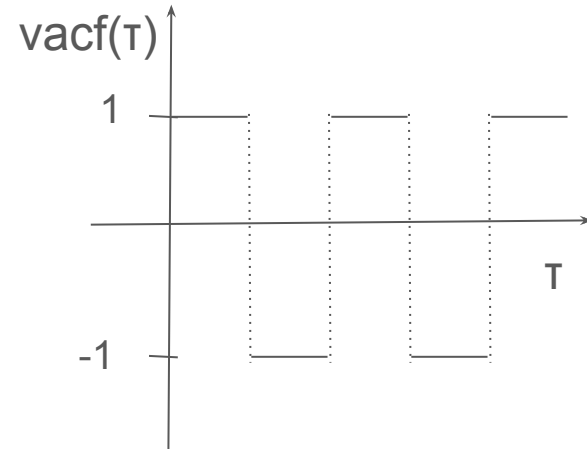
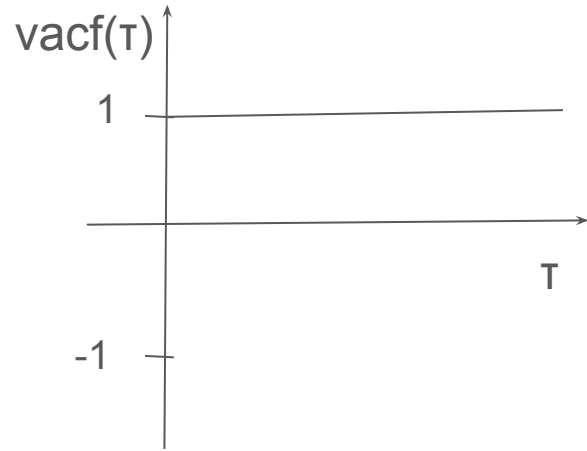


# Velocity autocorrelation function





# Velocity autocorrelation function



# Area below VAFC

Estimate the integral of the VACF

```
np.trapezoid(vacf, time)/3
```

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This is an estimation of a value that you obtained before, which one?

- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
- Property 2: Radial distribution function
- Property 3



- Produce MD data

- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
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- Property 3: Velocity Autocorrelation function & Diffusion



- Produce MD data

# Results comparison

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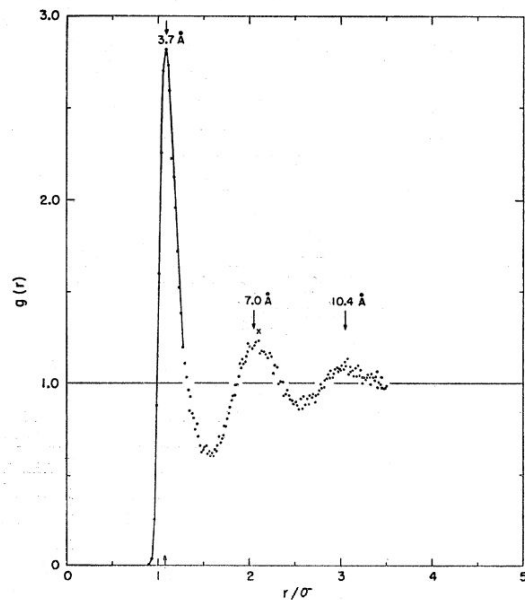


FIG. 2. Pair-correlation function obtained in this calculation at 94.4°K and  $1.374 \text{ g cm}^{-3}$ . The Fourier transform of this function has peaks at  $\kappa\sigma = 6.8, 12.5, 18.5, 24.8$ .

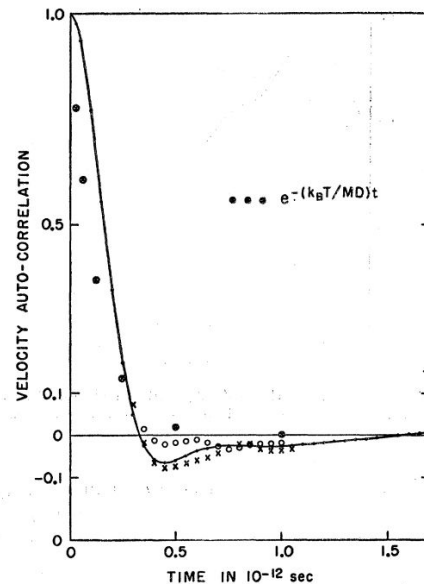


FIG. 4. The velocity autocorrelation function. The Langevin-type exponential function is also shown. The continuous curve, the circles, and the crosses correspond to the curves shown in Fig. 3.

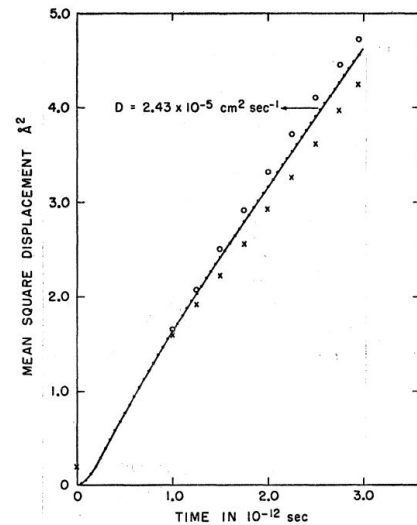


FIG. 3. Mean-square displacement of particles. The continuous curve is the mean of a set of 64 curves; the two members of the set which have *maximum* departures from the mean are shown as circles and as crosses. The asymptotic form of the continuous curve is  $6Dt + C$ , with  $D$  as shown on the figure and  $C = 0.2 \text{ Å}^2$ .

# Results comparison

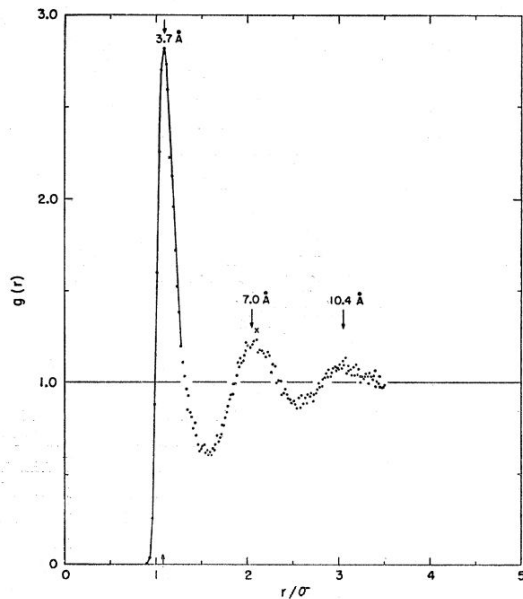


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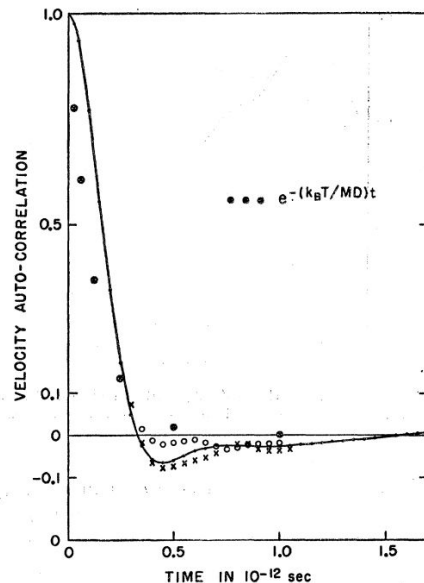


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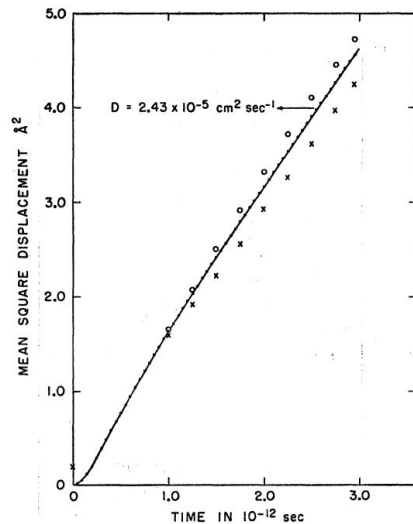


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Question: The temperature of our system is higher or lower than the system in the paper? How do you know it?



# About the paper

## JNCASR - CECAM Conference MD@60

February 25, 2024 - February 29, 2024  
Registration deadline: November 5, 2023  
Location: Bangalore, India

**Visa requirements:** Registered participants holding a passport  
Conference Visa (more information below).  
Hosting node: CECAM-HQ

Description

Participants

### Organisers

- Sara Bonella (CECAM HQ)
- Andrea Cavalli (CECAM HQ)
- Michael Klein (Temple University)
- Balasubramanian Sundaram (Jawaharlal Nehru Centre for Advanced Scientific Research)
- Umesh Waghmare (Jawaharlal Nehru Centre for Advanced Scientific Research)



EDITORIAL | MAY 05 2025

### Aneesur Rahman: Pioneer of molecular simulation

Special Collection: Molecular Dynamics, Methods and Applications 60 Years after Rahman

Srikanth Sastry

Check for updates

+ Author & Article Information

J. Chem. Phys. 162, 170401 (2025)

<https://doi.org/10.1063/5.0273655>

Article history

Split-Screen

PDF

Share

Tools

Aneesur Rahman's paper, "Correlations in the Motion of Atoms in Liquid Argon,"<sup>1,2</sup> published in 1964, is a landmark contribution that launched the application of molecular dynamics to studying properties of a diversity of substances. The present special issue of the *Journal of Chemical Physics* celebrates the 60th anniversary of the publication of this seminal paper, which cemented Rahman's place as among the founding fathers of molecular dynamics and

- MD Visualization



- Analyse MD data

- Property 1: MSD & Diffusion
- Property 2: Radial distribution function
- Property 3: Velocity Autocorrelation function & Diffusion



- Produce MD data

- MD Visualization



- Analyse MD data



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- Discuss your plots with your nearest neighbors.