Hands-on: Molecular Dynamics

Content

MD Visualization

- Analyse MD data
 - Property 1
 - Property 2
 - o Property 3

Run:

vmd trajectory.xyz

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Configuration:

Graphics/Representations.../Drawing Method/VDW

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vmd trajectory.xyz

Configuration:

Graphics/Representations.../Drawing Method/VDW

Edit:

Sphere scale

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vmd trajectory.xyz

Configuration:

Graphics/Representations.../Drawing Method/VDW

Edit:

Sphere scale

Movie:



- Analyse MD data
 - Property 1
 - o Property 2
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- Analyse MD data
 - Property 1
 - Property 2
 - Property 3

Questions

• Is the material a solid, liquid, or gas?

Questions

Is the material a solid, liquid, or gas?

 Given the positions of every atom at each time step, what analyses would you perform to corroborate it?

MSD

$$MSD(t) = < (x(0) - x(t))^2 >$$

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Diffusion Coefficient

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Problem: PBC

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Diffusion Coefficient

$$6 D = d MSD(t) / dt$$

Problem: PBC

Steps: 1) "unwrap" or "unfold" trajectories, 2) Compute MSD(t) of unfolded system

[0,0] jumps_cumulated [0,0] current_jumps

[1,0] jumps_cumulated [1,0] current_jumps

[1,0] jumps_cumulated [1,0] current_jumps

[1, -1] jumps_cumulated [0, -1] current_jumps

[1,0] jumps_cumulated [1,0] current_jumps



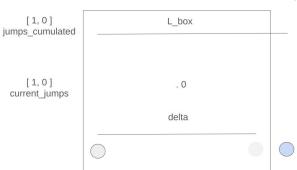
For all atoms!

[1,0] L_box jumps_cumulated [1,0] . 0 current_jumps delta

Unwrap trajectories

```
def unwrap trajectory(trajectory wrapped):
    Unwrap the trajectory!
    unwrapped = np.zeros like(trajectory wrapped)
    unwrapped[0] = trajectory wrapped[0]
    jumps cumulated = np.zeros((N atoms, 3)) # counts boundary crossings
    for t in range(N frames - 1):
        delta =
                                              # delta in real units
        current jumps = np.rint(delta / L box)
        # Update jumps cumulated
        # Reconstruct next unwrapped position
        unwrapped[t + 1] =
    return unwrapped
```

For all atoms!



Unwrap trajectories: Solution

```
def unwrap trajectory(trajectory wrapped):
   Unwrap the trajectory!
    111
    unwrapped = np.zeros like(trajectory wrapped)
    unwrapped[0] = trajectory wrapped[0]
    jumps cumulated = np.zeros((N atoms, 3)) # counts boundary crossings
    for t in range(N frames - 1):
        delta = trajectory wrapped[t] - trajectory wrapped[t+1] # delta in real units
        current jumps = np.rint(delta / L box)
        # Update jumps cumulated
        jumps cumulated += current jumps
        # Reconstruct next unwrapped position
        unwrapped[t + 1] = trajectory wrapped[t+1] + jumps cumulated * L box
    return unwrapped
```

[1, 0] jumps_cumulated



Compute mean square displacement

MSD

$$MSD(t) = < (x(0) - x(t))^2 >$$

For a given t:

$$(x_1(0) - x_1(t))^2$$

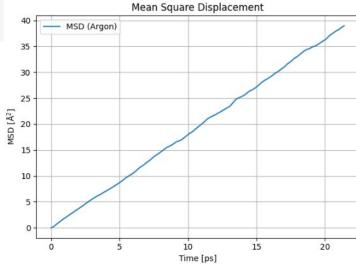
 $(x_2(0) - x_2(t))^2$
 $(x_3(0) - x_3(t))^2$
 $(x_4(0) - x_4(t))^2$

Average over particles

```
def compute_msd(unwrapped):
    Compute the mean squared displacement!
    r0 = unwrapped[0] # initial positions
    displacements = unwrapped - r0 # displacement from initial position
    return msd
```

Compute mean square displacement: Solution

```
def compute_msd(unwrapped):
    r0 = unwrapped[0] # initial positions
    displacements = unwrapped - r0 # displacement from initial position
    squared_displacements = np.sum(displacements**2, axis=2) # shape: (n_frames, n_atoms)
    msd = np.mean(squared_displacements, axis=1) # average over atoms
    return msd
```





- Analyse MD data
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 - Property 2
 - Property 3



- Analyse MD data
 - o Property 1: MSD & Diffusion
 - Property 2
 - Property 3



- Property 1: MSD & Diffusion
- Property 2
- o Property 3

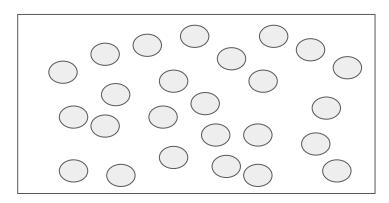
Question

What if you only have the positions at a single time frame?

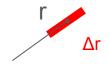
$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_{i}^{N} \sum_{j \neq i}^{N} \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$

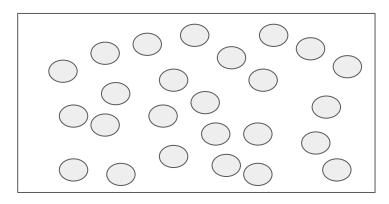
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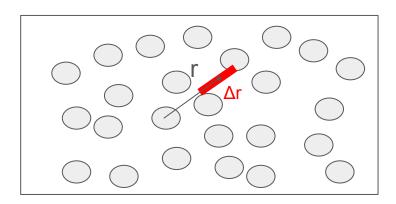


$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_{i}^{N} \sum_{j \neq i}^{N} \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$

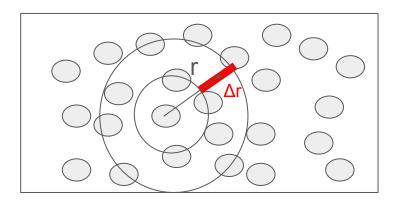




$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_{i}^{N} \sum_{j \neq i}^{N} \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$

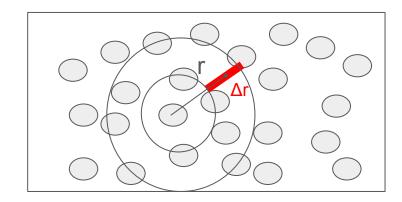


$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_{i}^{N} \sum_{j \neq i}^{N} \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$



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- Steps
 - 1. Counting
 - 2. Normalization

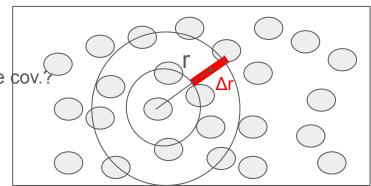


Radial Distribution Function

Definition

$$\rho g(\mathbf{r}) = \frac{1}{N} \left\langle \sum_{i}^{N} \sum_{j \neq i}^{N} \delta[\mathbf{r} - \mathbf{r}_{ij}] \right\rangle$$

- Steps
 - 1. Counting
- a. How to consider the min. image cov.?
- 2. Normalization

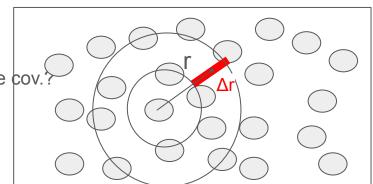


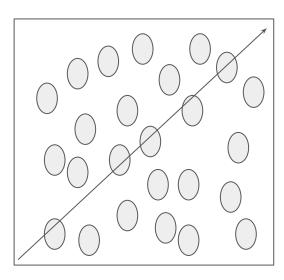
Radial Distribution Function

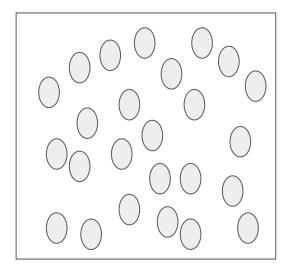
Definition

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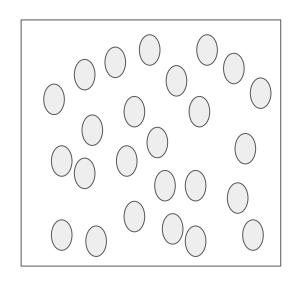
- Steps
 - 1. Counting
- a. How to consider the min. image cov.?
- 2. Normalization
 - a. How to consider Δr ?

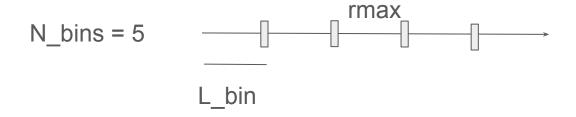


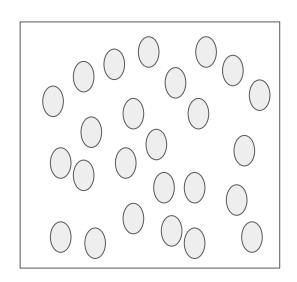




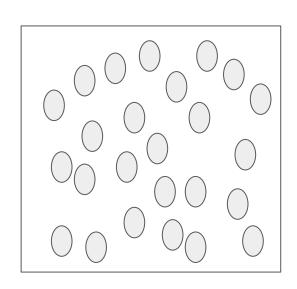
rmax

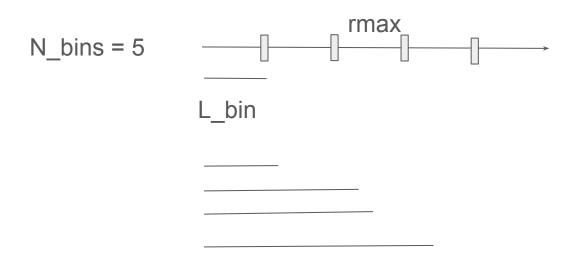




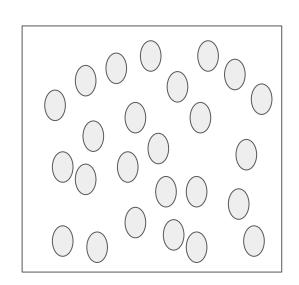


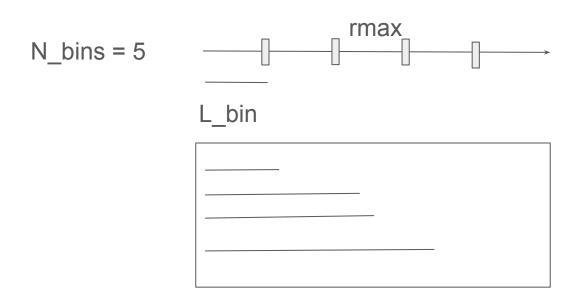
N_bins = 5		rmax —[]—	
	L_bin		



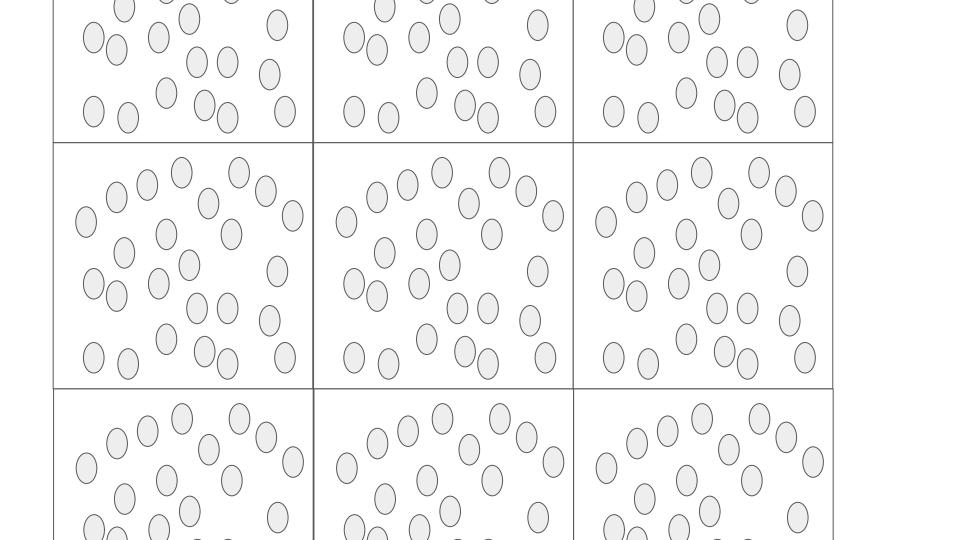


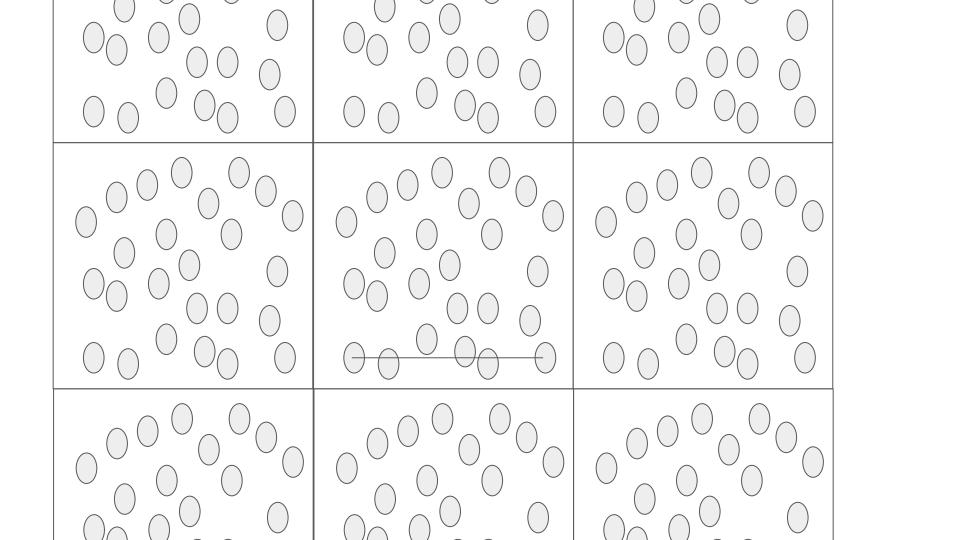
$$g_{\text{count}} = [0, 1, 2, 1, 0]$$

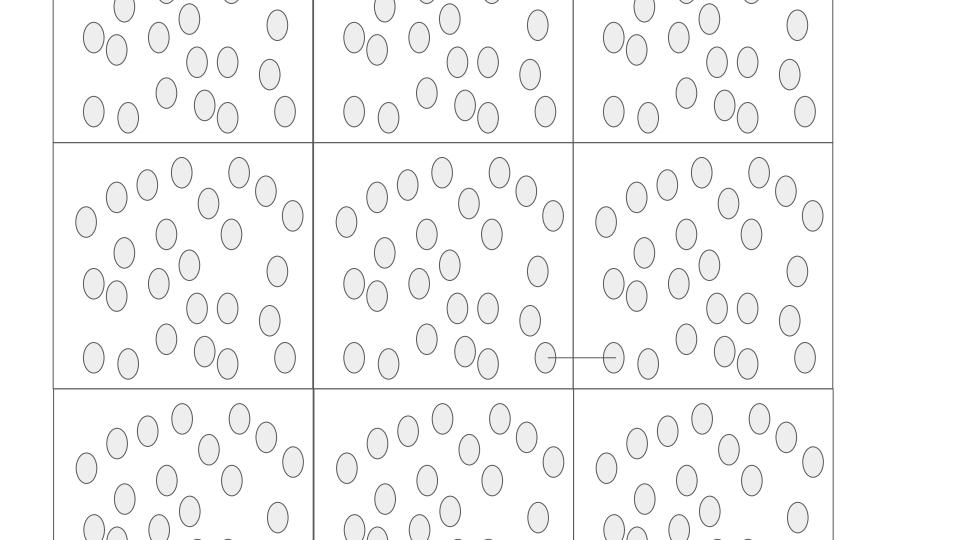


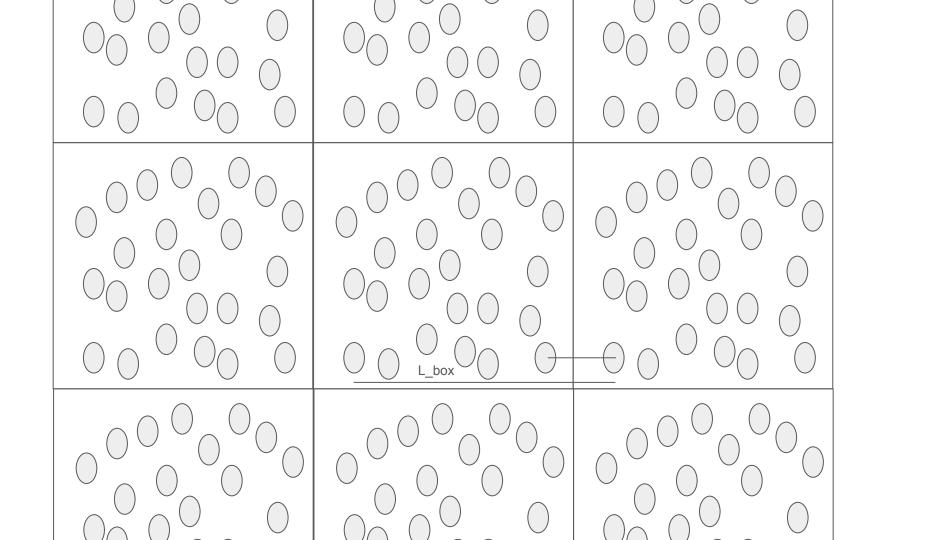


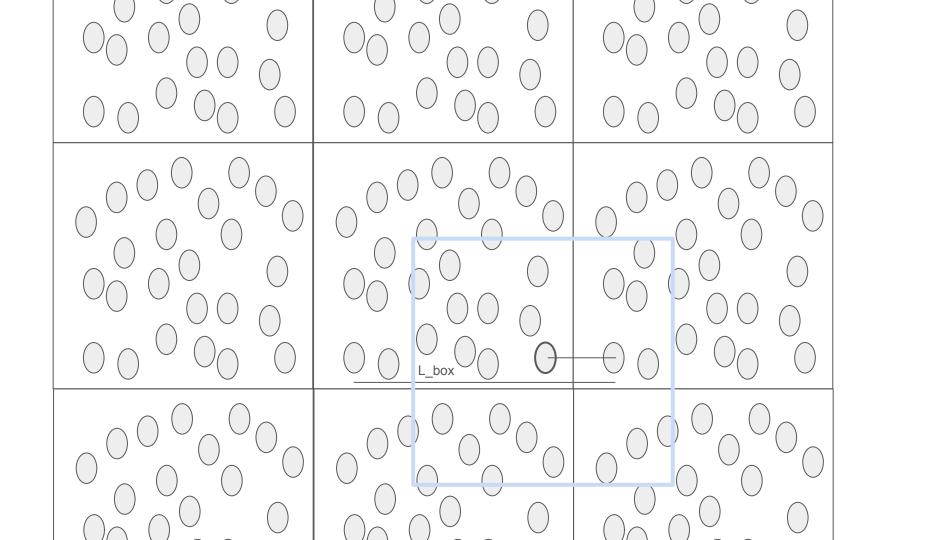
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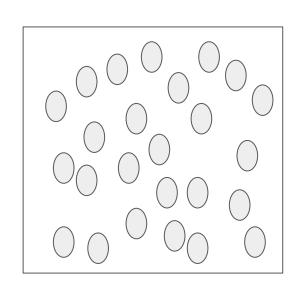


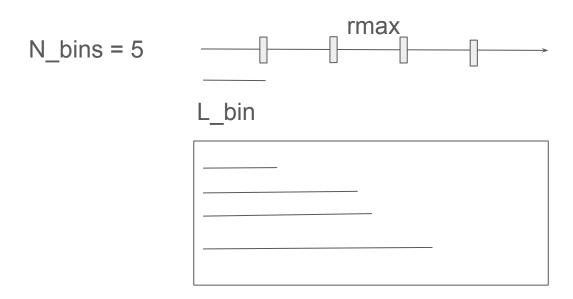




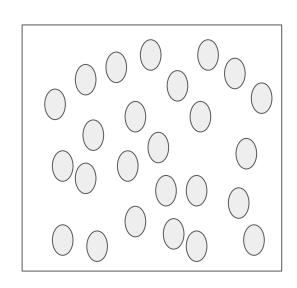


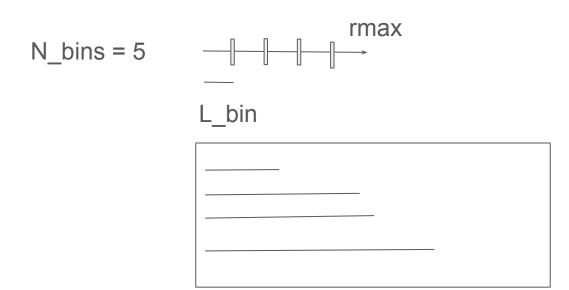


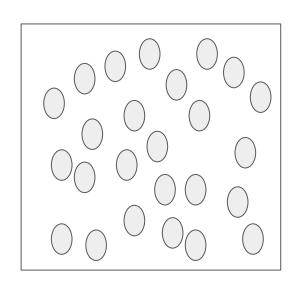


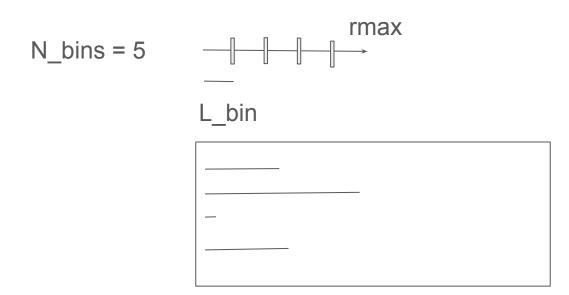


$$g_{\text{count}} = [0, 1, 2, 1, 0]$$

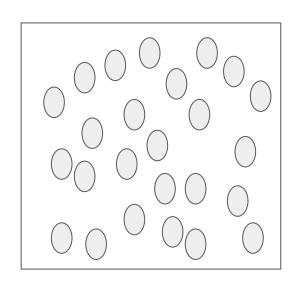




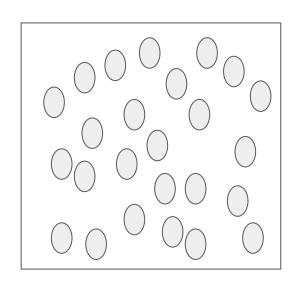




$$g_{\text{count}} = [0, 1, 2, 1, 0]$$



$$g_{ount} = [1, 0, 2, 0, 1]$$



$$g_{ount} = [1, 0, 2, 0, 1]$$

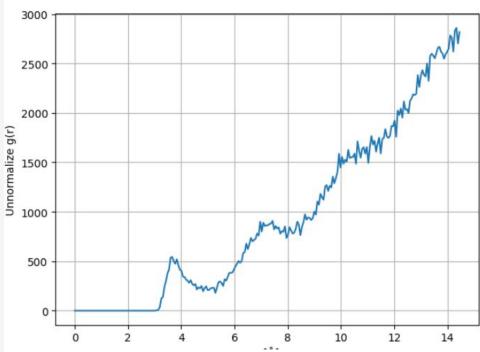
```
N_bins = 512
rmax = np.sqrt(3*L_box**2)/2
L_bin=rmax/N_bins
g_counter=np.zeros(N_bins)
```

```
def counting distances frame(i):
   Add the distances to g counter corresponding to the i-th frame
    rx = Trajectory[i][:,0];
    ry = Trajectory[i][:,1];
    rz = Trajectory[i][:,2];
    for k in range(N atoms-1):
        j=k+1
       # Distances to atoms with superior index (not normalized)
       dx = (rx[k]-rx[j:N atoms])
       dy = (ry[k]-ry[j:N atoms])
       dz = (rz[k]-rz[j:N atoms])
        # Apply minimum image convention to dx, dy and dz
        # dx, dy and dz already with the minimum image convention in real units.
        r2 = dx*dx + dy*dy + dz*dz
        r = np.sqrt(r2)
        for corrected distance in r:
           q counter[?] += 2 # Find the expression for ?
```

Hint: If dx is normalized, dx = dx - np.rint(dx) applies the minimum image convention in normalized units (why?)

Counting: Solution

```
def counting distances frame(i):
    Add the distances to g counter corresponding to the i-th frame
   rx = Trajectory[i][:,0];
    ry = Trajectory[i][:,1];
    rz = Trajectory[i][:,2];
    for k in range(N atoms-1):
        i=k+1
        # Distances to atoms with superior index (not normalized)
        dx = (rx[k]-rx[j:N atoms])
        dy = (ry[k]-ry[j:N atoms])
        dz = (rz[k]-rz[j:N atoms])
        # Apply minimum image convention to dx, dy and dz
        dx = dx/L box
        dy = dy/L box
        dz = dz/L box
        dx = dx - np.rint(dx)
        dy = dy - np.rint(dy)
        dz = dz - np.rint(dz)
        dx = dx * L box
        dy = dy * L box
        dz = dz * L box
        # dx, dy and dz already with the minimum image convention in real units.
        r2 = dx*dx + dy*dy + dz*dz
        r = np.sqrt(r2)
        for corrected distance in r:
            g counter[int(corrected distance/L bin)] += 2 # Find the expression for ?
```



• 2 Types

• 2 Types

1. $r \& \Delta r$ independent

2. r & ∆r dependent

- 2 Types
- 1. $r \& \Delta r$ independent
- Averages -> N_atoms*N_frames
- Definition -> (N_atoms/L_box**3)

Density

norm_factor = N_atoms*N_frames*(N_atoms/L_box**3)

2. $r \& \Delta r$ dependent

- 2 Types
- 1. $r \& \Delta r$ independent
- Averages -> N_atoms*N_frames
- Definition -> (N_atoms/L_box**3)

Density

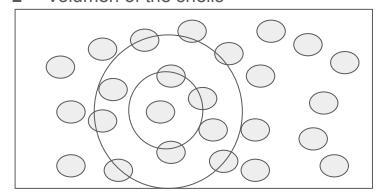
norm_factor = N_atoms*N_frames*(N_atoms/L_box**3)

2. r & Δr dependent

- 2 Types
- 1. $r \& \Delta r$ independent
- Averages -> N_atoms*N_frames
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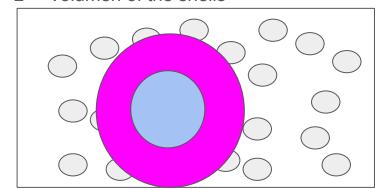
2. r & Δr dependent



- 2 Types
- 1. r & ∆r independent
- Averages -> N_atoms*N_frames
- Definition -> (N_atoms/L_box**3)

Density

2. r & Δr dependent



g_r =g_count/g_normalization

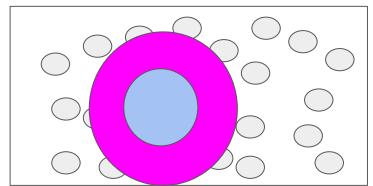
Normalization

- 2 Types
- 1. r & ∆r independent
- Averages -> N atoms*N frames
- Definition -> (N_atoms/L_box**3)

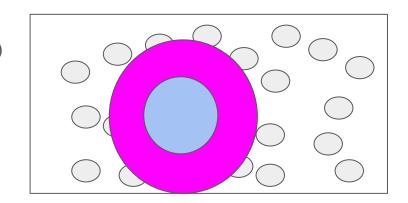
Density

norm_factor = N_atoms*N_frames*(N_atoms/L_box**3)

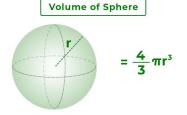
2. r & ∆r dependent



norm_factor = N_atoms*N_frames*(N_atoms/L_box**3)
g_normalization = [vol,vol]*norm_factor



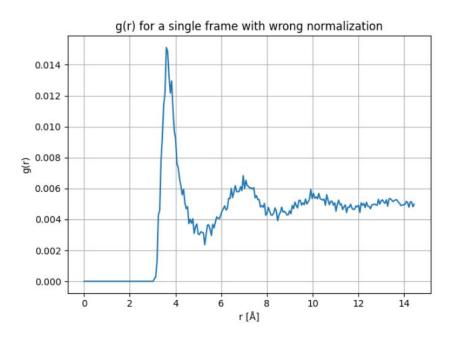
Hint: The volume of a sphere is

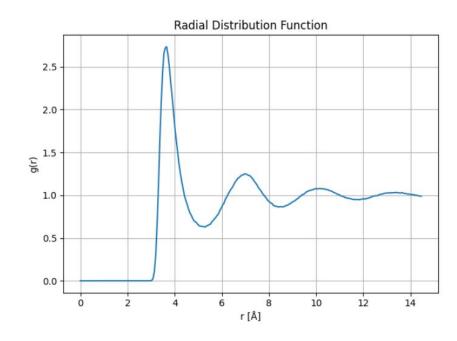


Hint: the radius can be written as L_bin * integer and contiguous radii are L_bin * integer & L_bin * (integer+1)

Normalization: Solution

```
g_normalization = np.zeros(N_bins)
for i in range(N_bins):
    g_normalization[i] = (4/3)*np.pi*((L_bin*(i+1))**3-(L_bin*i)**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_frames*(N_atoms/L_box**3)*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_atoms*N_ato
```





MD Visualization



- Property 1: MSD & Diffusion
- Property 2
- o Property 3

Produce MD data

MD Visualization

- Analyse MD data
 - o Property 1: MSD & Diffusion
 - Property 2: Radial distribution function
 - Property 3

Produce MD data

Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

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Average over time and over particles!

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Average over time and over particles!

For a given τ:

$$t=0 \\ v_{1}(\tau) \cdot v_{1}(0) \\ v_{2}(\tau) \cdot v_{2}(0) \\ v_{3}(\tau) \cdot v_{3}(0) \\ v_{4}(\tau) \cdot v_{4}(0)$$

Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

Average over time and over particles!

For a given τ:

t=0

$$v_1(\tau) \cdot v_1(0)$$

 $v_2(\tau) \cdot v_2(0)$ Particle
 $v_3(\tau) \cdot v_3(0)$ av. t=0
 $v_4(\tau) \cdot v_4(0)$

Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

Average over time and over particles!

For a given τ:

Definition

$$VACF(\tau) = \langle v(\tau) \cdot v(0) \rangle$$

Average over time and over particles!

Particle & time av.

```
def compute_vacf(vels, max_lag=N_frames):
    Compute the velocity autocorrelation function
    vacf = np.zeros(max_lag)
    for lag in range(max_lag):
        dot_sum = 0.0
        count = 0
        for t in range(N_frames - lag):
            v0 = vels[t]
            vlag = vels[t + lag]
            dot_sum +=  # Complete the function!
            count += N_atoms
            vacf[lag] = dot_sum / count
    return vacf
```

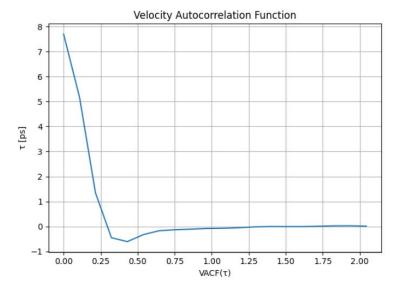
Particle & time av.

$$\begin{array}{c} t = 0 \\ v_1(\tau) \cdot v_1(0) \\ v_2(\tau) \cdot v_2(0) \\ v_3(\tau) \cdot v_3(0) \\ v_4(\tau) \cdot v_4(0) \end{array} \begin{array}{c} v_1(\tau+1) \cdot v_1(1) \\ \text{Particle} \\ v_2(\tau+1) \cdot v_2(1) \\ \text{av. } t = 0 \end{array} \begin{array}{c} v_1(\tau+2) \cdot v_1(2) \\ v_2(\tau+2) \cdot v_2(2) \\ \text{av. } t = 1 \end{array} \begin{array}{c} \text{Particle} \\ \text{av. } t = 1 \\ v_2(\tau+2) \cdot v_2(2) \\ \text{av. } t = 1 \\ v_3(\tau+2) \cdot v_3(2) \\ v_4(\tau+2) \cdot v_4(2) \end{array} \begin{array}{c} \text{Particle} \\ \text{av. } t = 2 \\ v_2(\tau+2) \cdot v_2(2) \\ v_3(\tau+2) \cdot v_3(2) \\ v_4(\tau+2) \cdot v_4(2) \end{array}$$

Hint: The missing line includes only one multiplication and two nested np.sum()'s

Particle & time av.

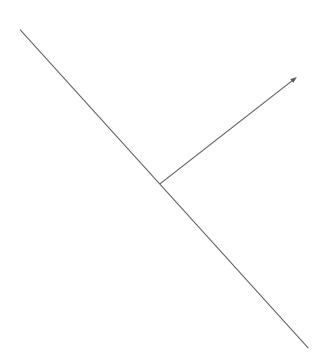
Velocity autocorrelation function: Solution

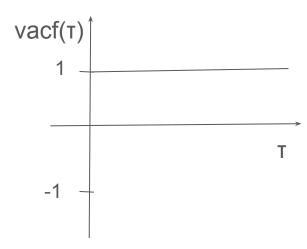


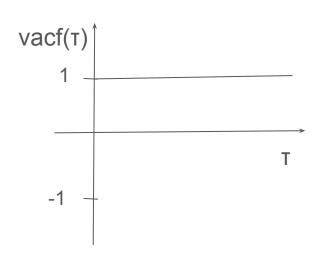
When the velocity autocorrelation function is negative?

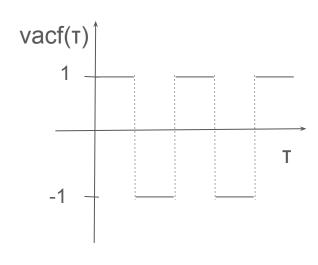


When the velocity autocorrelation function is negative?









Area below VAFC

Estimate the integral of the VACF

np.trapezoid(vacf, time)/3

Area below VAFC

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This is an estimation of a value that you obtained before, which one?

MD Visualization

- Analyse MD data
 - o Property 1: MSD & Diffusion
 - Property 2: Radial distribution function
 - o Property 3

Produce MD data

MD Visualization

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Results comparison

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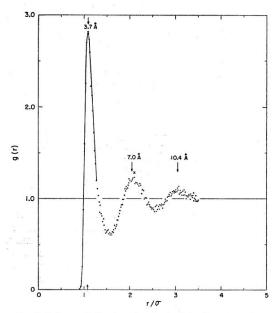


Fig. 2. Pair-correlation function obtained in this calculation at 94.4°K and $1.374~\text{gcm}^{-3}$. The Fourier transform of this function has peaks at $\kappa\sigma=6.8,\,12.5,\,18.5,\,24.8$.

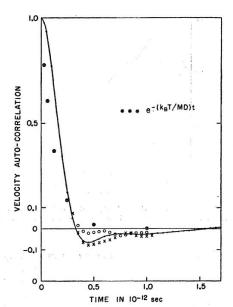


Fig. 4. The velocity autocorrelation function. The Langevintype exponential function is also shown. The continuous curve, the circles, and the crosses correspond to the curves shown in Fig. 3.

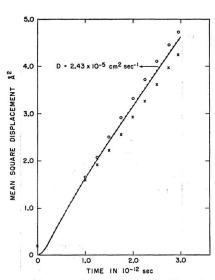


Fig. 3. Mean-square displacement of particles. The continuous curve is the mean of a set of 64 curves; the two members of the set which have maximum departures from the mean are shown as circles and as croses. The asymptotic form of the continuous curve is 6Dt+C, with D as shown on the figure and C=0.2 Å².

Results comparison

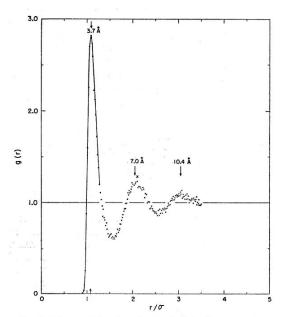


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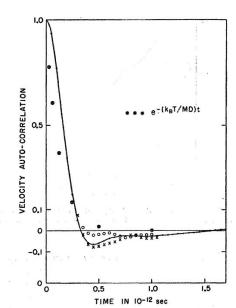


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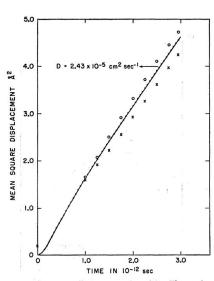


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Question: The temperature of our system is higher or lower than the system in the paper? How do you know it?

About the paper



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