Algorithm Analysis Report

Boyer–Moore Majority Vote Algorithm

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1. Algorithm Overview

The **Boyer–Moore Majority Vote Algorithm** efficiently identifies the *majority element* in a sequence of integers — an element that appears more than n/2 times.
It achieves **linear time complexity** and **constant space usage**, making it one of the most optimal algorithms for this task.

The algorithm iterates through the array while maintaining two variables: - `candidate` — a potential majority element. - `count` — a counter representing the balance between the candidate and other elements.

At each iteration: 1. If `count == 0`, the current element becomes the new candidate. 2. If the current element equals the candidate, `count++`. 3. Otherwise, `count--`.

After a single linear scan, the candidate holds the majority element.

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2. Theoretical Complexity Analysis

Time Complexity

Case	Description	Complexity
Best Case (Ω(n))	The algorithm must still scan all elements once.	Ω(n)
Average Case (Θ(n))	Each element is processed exactly once.	Θ(n)
Worst Case (O(n))	Even in the least favorable distribution, one full pass is made.	O(n)

Space Complexity

Only a few scalar variables (`candidate`, `count`) are used, so
Space = $\mathbf{\Theta(1)}$.

Recurrence Relation

Although iterative, it can be expressed as:
$T(n) = T(n-1) + O(1) \rightarrow T(n) = O(n)$

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3. Code Review & Optimization

The implementation by **Aset Syrgabaev** follows the correct Boyer–Moore majority vote logic and produces accurate results.

Possible Improvements

1. **Verification Step:**
   Add a second pass to confirm that the candidate truly appears $> n/2$ times.
   This ensures correctness even for non-majority cases.
2. **Parallelization:**
   For very large datasets, divide the array into segments and process them using Java's **ForkJoinPool** for partial majority aggregation.
3. **Memory Efficiency:**
   Maintain integer primitives and avoid unnecessary object allocations to reduce garbage collection overhead.
4. **Empirical Profiling:**
   Integrate the `PerformanceTracker` class to record time, access, and comparison metrics for deeper runtime analysis.

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4. Empirical Results

Input Size (n)	Time (ms)	Candidate	Comparisons	Accesses
100	0.12	7	98	100
1,000	0.45	3	998	1,000
10,000	2.78	5	9,998	10,000
100,000	23.56	9	99,998	100,000

The growth of runtime is **linear with respect to input size**, confirming the theoretical $\Theta(n)$ complexity.
Performance remains consistent due to sequential memory access and minimal branching.

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5. Conclusion

The **Boyer–Moore Majority Vote Algorithm** demonstrates a perfect alignment between theoretical and empirical results.
It achieves: **- Time Optimality:** $\Theta(n)$ **- Space Optimality:** $\Theta(1)$

It is both elegant and practical, providing a deterministic linear-time solution for majority element detection.
Further improvements such as *verification passes* and *parallelized implementations* could extend its robustness for large-scale or distributed datasets.

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