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A multi-criteria optimization model for humanitarian aid distribution

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Abstract Natural disasters are phenomenons which strike countries all around the world. Sometimes, either by the intensity of the phenomenon or the vulnerability of the country, help is requested from the rest of the world and relief organizations respond by delivering basic aid to those in need. Humanitarian logistics is a critical factor in managing relief operations and, in general, there is a lack of attention on the development of mathematical models and solution algorithms for strategic and tactical decisions in this area. We acknowledge that in humanitarian logistics traditional cost minimizing measures are not central, and postulate that other performance measures such as time of response, equity of the distribution or reliability and security of the operation routes become more relevant. In this paper several criteria for an aid distribution problem are proposed and a multi-criteria optimization model dealing with all these aspects is developed. This model is the core of a decision support system under development to assist organizations in charge of the distribution of humanitarian aid. Once the proposed criteria and the model are described, an illustrative case study based on the 2010 Haiti catastrophic earthquake is presented, showing the usefulness of the proposal.

Keywords Humanitarian logistics · Distribution models · Goal programming

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1 Introduction

When a major disaster strikes a country, if local emergency services are overwhelmed by the consequences, help is requested from the rest of the world. Then, relief organizations respond by delivering basic aid (food, medical supplies, shelter) to those in need. Relief operations, mainly related to resources acquirement and delivery and warehousing of supplies to people in the affected area, are launched, and humanitarian logistics becomes a critical factor for success in preparing for and managing these operations.

Following [16], it can be acknowledged that humanitarian logistics, as well as business logistics, encompasses a range of activities including preparedness, planning, procurement, transport, warehousing, tracking and tracing and customs clearance. It can, therefore, be concluded that the basic principles of managing the flows of goods, information and finances present in the business field also remain valid for humanitarian logistics. The first and fundamental difference is in the motivation for improving the logistics process, i.e. in the case of humanitarian logistics it is required to go beyond profitability.

It is then worth mentioning that, despite the field of logistics and supply chain management in the business sector has attracted a lot of attention in the operational research community, not much has been done to transform these methodological advances into suitable tools to humanitarian aid organizations. A survey on the OR/MS literature to identify potential research directions in disaster operations can be found in [1]. For example, an important aspect of humanitarian logistics is to preserve connectivity in the involved networks (by identifying critical nodes/edges of the network). See the work described in [3]. Parallels of humanitarian logistics and business logistics have been stated in [6], where the specific characteristics of the humanitarian sector were highlighted. Some of these characteristics are:

- unpredictable demand in terms of timing, geographic location, type of commodity, quantity of commodity;
- short lead time and suddenness of demand for large amounts of a wide variety of products and services;
- lack of initial resources in terms of supply, human resource, technology, capacity and funding;
- deficient infrastructures and security concerns.

The INSEAD report [28] enumerates nine steps to consider in the humanitarian supply chain when responding to a major disaster. Step 5, in particular, refers to Transportation and Execution, being transportation critical to deliver aid at the right time and to the right place. The comparison between the commercial supply chain and the humanitarian relief chain addressed in [6] stresses that traditional performance measures focus on resources (maximizing profit or minimizing costs, for example), meanwhile the primary focus in humanitarian supply chains is on output performance measures, such as the time required to respond to a disaster, or the ability to meet the needs of the affected population.

In particular, when facing transportation and distribution problems in humanitarian logistics, criteria other than cost should guide the search for a solution. Because it is critical that the deliveries are fast, fair and safe, it is not clear that the classic cost-minimizing routing problems properly reflect the relevant priorities in disaster relief. Campbell et al. [7] take the first steps toward developing new methodologies for these problems, formulating two different objective functions for a TSP (Traveling Salesman Problem) and a VRP (Vehicle Routing Problem) in a disaster relief application and studying the corresponding models.

The importance of this problem has been made patent after the devastating earthquake in Haiti (January 2010), where one of the aftershocks facing the country has been the struggle to coordinate relief efforts in order to reach those in need most effectively. Transportation operations must take into account the actual road conditions, fuel availability, airports' and ports' capacity after the disaster, etc. and executing the response operation has been the most serious problem: the airport was destroyed and the port was badly damaged, as were the roads in the country. So, access of international aid to the victims was severely constrained in Haiti, causing an increase in tension, which in turn increased security problems, and rioting and violence became a real threat. Thus, although a natural disaster, security became an important bottleneck.

This paper pursues to compensate the general lack of attention on the development of mathematical models and solution algorithms for strategic and tactical decisions in humanitarian logistics. A multi-criteria optimization model for aid distribution problems is proposed, taking into account main criteria being involved in a disaster response operation (time, cost, reliability, security, equity, etc.). This model here presented is the core of a project that pursues a decision support system for the assistance of organizations in charge of the distribution of goods, providing help in the selection of vehicles and the design of routes to distribute the aid (see [21] for a preliminary version).

The rest of the paper is organized as follows. Section 2 presents the distribution problem addressed, together with a revision of previous work on related problems. An integer linear programming model based on flow models for load and vehicles, with operation conditions and without objective function, is presented in Sect. 3. New attributes and measures to compute them linearly are introduced in Sect. 4, and aggregated in a goal programming model in Sect. 5. Discussion on applying the model to the Haiti earthquake 2010 is shown in Sect. 6 and, finally, some conclusions are presented.

2 Problem description

The problem addressed in this paper focuses on the distribution of emergency aid in disaster relief operations. More specifically the problem consists of designing routes for vehicles among nodes that have an available quantity of goods (typically cities with airport, port, etc.) or have a demand of those goods, choosing the types of vehicles more adequate and determining the flow of the aid. In the design of such routes several criteria are considered, some of them closely related to the specific conditions of disaster-stricken zones in developing countries.

Regarding single-objective models, Knott [15] developed a linear programming model for the bulk food transportation problem and the efficient use of the truck fleet to minimize the transportation cost or to maximize the amount of food delivered. Haghani and Oh [12] introduced a multi-commodity, multi-modal network flow problem with time windows for disaster relief operations (they acknowledge the multi-objective nature of disaster relief operations, but choose to minimize costs, due to the fact that they are easier to quantify). Barbarosoglu and Arda [5] develop a scenario based, two-stage stochastic programming model for transportation planning in disaster response that extends the determinist model in [12]. Angelis et al. [2] consider a multi-depot, multi-vehicle routing and scheduling problem for air delivery of emergency supply deliveries for the World Food Programme in Angola (they develop and solve an integer linear programming model whose goal is to maximize the total demand satisfied). Balcik et al. [4] address the last mile distribution problem considering joint resource allocation and vehicle routing decisions (a mixed integer programming model that determines delivery schedules for vehicles and equitably allocates resources is proposed, based on supply, vehicle capacity, and delivery time restrictions, minimizing transportation costs and maximizing benefits to aid recipients). Ozdamar et al. [22] address an emergency logistics problem for distributing multiple commodities from a number of supply centers to distribution centers near the affected areas (they formulate a multi-period, multi-commodity network flow model to determine pick-up and delivery schedules for vehicles as well as the quantities of loads delivered on these routes, minimizing the amount of unsatisfied demand over time).

Each one of the above referenced models chooses an objective to optimize, the amount of satisfied or unsatisfied demand, overall cost, time of operation, etc. But in real life situations, usually several objectives are present, and should be considered jointly. Hence, Viswanath and Peeta [29] formulate a multi-commodity maximal covering network design problem for identifying critical routes for earthquake response with two objectives, minimize total travel time and maximize total population covered, in such a way that the problem is formulated as a two-objective integer programming model. Tzeng et al. [27] propose a fuzzy multiple objective model that concentrates on the effectiveness and fairness of the overall distribution system, to avoid the oversight of critical but difficult-to-reach areas in the real world. Finally, Nolz et al. [19] propose a multi-objective covering tour problem that aims at distributing emergency aid among the population, with respect to two main criteria, the first one related to distances between population and distribution points, and the second one related to cost of the chosen tour. In all these cases, main criteria are cost, time and sometimes fairness of the distribution system. But there are other elements of vital importance in the distribution of humanitarian aid which have not been taken into account, as the reliability of the routes or the security problems associated. There is a lack of approaches that take into account all these elements and allow to use some or all of them to guide the search for an efficient solution, and this is the objective of this paper.

The problem introduced in this work extends the bi-criteria approach applied to humanitarian aid distribution problems, by considering a number of different criteria, in terms of cost, time, reliability, security, and fairness that will be described in detail in Sect. 4.

More specifically, the problem here addressed is described through the following elements:

1. *Transport Network* Nodes representing the places of pick-up, delivery or connection, and main links characterized by distance and average velocity.
2. *Goods* They are considered as single commodities (they could be single commodities of packs of basic aid, for example). Information about quantity available at each node, and quantity required at each node (as a desired value to be achieved) will be provided.
3. *Vehicles* Several types are considered, characterized by capacity, average velocity, variable and fixed costs and availability in each node of the network.
4. *Operation elements* They include the global quantity to be distributed in the operation and the budget available.
5. *Other attributes* For each arc, reliability and ransack probability of the link. The last characteristic is defined for one vehicle, and will vary with the length of the convoy.

Some assumptions and simplifications are made in order to make the resulting model useful for the characteristics of the operation and users, that will be introduced in next sections, along with the main elements of the model. The problem consists of designing routes for vehicles among nodes and choosing the types of vehicles more adequate, in order to distribute a fixed quantity of humanitarian aid, allowing the use of different criteria, or any combination among them, to guide the search process.

3 Structural model: flow and operation constraints

In the core of the distribution model proposed in this work lay two Network Flow models, one for load and another one for vehicles, as well as the relations between them. Such a model has been chosen instead of a VRP (Vehicle Routing Problem) in order to obtain a solution in real life situations as fast as possible. Together with some constraints related to the resources assigned to perform the operation they constitute the basic set of restrictions of the model presented in Sect. 5.

3.1 Data model

Main data are related to the network, load, vehicles and operation characteristics.

A) Data related to the network infrastructure.

- (N, A) : transport network, where N is the set of nodes and A is the set of arcs
- i, i' : indices to refer nodes $i, i' \in N$
- $dist_{ii'}$: length of arc $(i, i') \in A$
- $velc_{ii'}$: maximum velocity through arc $(i, i') \in A$

B) Data related to load. Network nodes can be demand, supply or transfer nodes.

- dem_i : demand at node $i \in N$ in units of load
- qav_i : supply at node $i \in N$ in units of load
- Note that $dem_i = qav_i = 0$ for i : transfer node

C) Data related to vehicles. Several types of vehicles (from now on equivalent to lorries) are considered. Note that the cost of travelling through an arc is divided into two basic costs. Both depend on the distance travelled, but one of them is independent of the load, while the other depends on it. Note also that in real situations some types of vehicles can not use some links (big lorries across narrow tracks, for instance). An incompatibility 0–1 matrix will be used to reflect this fact.

- V : set of vehicle types, defined by their characteristics
- j, j' : indices to refer vehicle types, $j, j' \in V$
- cap_j : capacity of vehicle type $j \in V$, in units of load
- $velv_j$: normal velocity of vehicle type $j \in V$
- vav_{ji} : availability of vehicles type $j \in V$ at node $i \in N$
- $cf_{ii'j}$: cost of a vehicle type $j \in V$ through $(i, i') \in A$, per unit of length
- $cv_{ii'j}$: cost of a vehicle type j through $(i, i') \in A$, per unit of length and load
- $comp_{ii'j}$: 0 if arc $(i, i') \in A$ can not be used by vehicles type $j \in V$, 1 otherwise

D) Data related to the operation. These data include the budget and the global quantity to be distributed. Note that an operation usually has at a short term level the objective to distribute a specific quantity, even if demand and supply are greater, especially if they are operations repeated along time.

- $qglobal$: load to be distributed in the operation
- b : budget available to perform the operation

E) Additional data. Note that bounds on some functions will be needed when modelling several constraints. These bounds will be noted as bd throughout the paper.

Next, the two basic Flow Models will be presented. New variables will be introduced each time they are needed.

3.2 Load flow model

Variables :

$QTR_{ii'}$: load carried from i to i', $(i, i') \in A$
QF_i : load staying (stored or received) at node $i \in N$ at the end of the operation

Constraints

$$\sum_{i'/(i',i)\in A} QTR_{i'i} + qav_i = \sum_{i'/(i,i')\in A} QTR_{ii'} + QF_i \quad \forall i \in N \tag{1}$$

$$QF_i \le dem_i + qav_i \quad \forall i \in N \tag{2}$$

$$\sum_i QF_i = \sum_i qav_i \tag{3}$$

Conditions (1) are the load flow balance equations at the nodes. Conditions (2) ensure that load staying at a node do not exceed the original demand or supply (these constraints are upper bounds on the variables). Finally, constraint (3) ensures that total load staying at the nodes at the end of the operation is the available load.

3.3 Lorries flow model

Compared to the previous model, this one is more complex, as there is no demand of lorries. Then, it is not enough to assure the balance of flow, and sub-cycle elimination constraints must be added to avoid the displacement of load using nonexistent vehicles. As it is a flow model, it is assumed that vehicles travelling through an arc form a convoy and, then, the normal velocity of the convoy will be the minimum of the velocities of the types of lorries forming it.

Variables :

$NL_{ii'j}$: number of lorries type j travelling from i to i', $(i, i') \in A$
NLF_{ij}: number of lorries type j whose final destination is node $i \in N$

Constraints:

$$\sum_{i'/(i',i)\in A} NL_{i'ij} + vav_{ji} = \sum_{i'/(i,i')\in A} NL_{ii'j} + NLF_{i,j} \quad \forall j \in V, i \in N \tag{4}$$

$$\sum_i NLF_{i,j} = \sum_i vav_{ji} \quad \forall j \in V \tag{5}$$

$$NL_{ii'j} = 0 \quad \forall j \in V, (i, i') \in A \;/\; comp_{ii'j} = 0 \tag{6}$$

Conditions (4) are the flow balance equations for lorries. Conditions (5) ensure that total number of lorries staying at the nodes at the end of the operation equals the available lorries.

(6) forbids the use of some arcs to not compatible types of vehicles. Again, these conditions should be included as upper bounds on the variables with $comp_{ii'j} = 0$.

Sub-cycle elimination constraints: Following the modelling of [18], next constraints allow eliminating sub-cycles controlling the reaching times to nodes, which will be useful when dealing with criteria related to time.

The following variables will be added to the model:

TM_i : reaching time of node $i \in N$
$BL_{ii'j}$: binary variable taking value 1 if any lorry type j travels from i to i' (1 if and only if $NL_{ii'j} > 0 \quad \forall j \in V, (i, i') \in A$)
$BLT_{ii'}$: binary variable taking value 1 if any lorry travels from i to i' (1 if and only if $\sum_j BL_{ii'j} > 0 \quad \forall (i, i') \in A$)

And constraints:

$$TM_i \geq TM_{i'} + \frac{dist_{i'i}}{velv_j} - bd_1(1 - BL_{i'ij}) \quad \forall (i', i) \in A, j \in V \tag{7}$$

$$TM_i \geq TM_{i'} + \frac{dist_{i'i}}{velc_{i'i}} - bd_2(1 - BLT_{i'i}) \qquad \forall (i', i) \in A \tag{8}$$

$$NL_{ii'j} \leq bd_3 BL_{ii'j} \qquad \forall j \in V, (i, i') \in A \tag{9}$$

$$BL_{ii'j} \leq NL_{ii'j} \qquad \forall j \in V, (i, i') \in A \tag{10}$$

$$BL_{ii'j} \leq BLT_{ii'} \qquad \forall j \in V, (i, i') \in A \tag{11}$$

$$BLT_{ii'} \leq \sum_j BL_{ii'j} \qquad \forall (i, i') \in A \tag{12}$$

Conditions (7) and (8) are the subtour elimination constraints. They also reflect the fact that the velocity of a convoy through an arc is the minimum between the normal velocity of the convoy and the maximum velocity through the arc. Conditions (9)–(12) define the binary variables introduced. Note that for some criteria some of them can be redundant, but they are included in order to make the model as general as possible.

Rational use of lorries: To force a rational use of resources, next constraints ensure that a lorry will not travel if it is not going to be used. The basic idea is to allow the use of empty lorries through an arc only if they are used later in their route.

Let the binary variable

BLF_{ij} : binary variable taking value 1 if any lorry type j finishes at node i (1 if and only if $NLF_{ij} > 0 \; \forall j \in V, i \in N$)

and the constraints

$$\sum_{j'} cap_{j'} NL_{ii'j'} - cap_j \leq QTR_{ii'} + bd_4(1 - BLF_{i'j}) \quad \forall j \in V, (i, i') \in A \tag{13}$$

$$NLF_{ij} \leq bd_5 BLF_{ij} \quad \forall i \in N, j \in V \tag{14}$$

$$BLF_{ij} \leq NLF_{ij} \quad \forall i \in N, j \in V \tag{15}$$

Conditions (13) force lorries to carry load in the last connection of their routes. Conditions (14) and (15) bound the binary variable.

3.4 Relation load-lorries

To connect both submodels, it is necessary to show how the transported load is distributed among the different types of vehicles. Note that a routing model would assign load to specific vehicles. In our flow model only load assigned to types will be identified.

Variables:

$QLOR_{ii'j}$: load carried from i to i' using lorries type $j \in V, \forall (i, i') \in A$

Constraints:

$$QTR_{ii'} = \sum_j QLOR_{ii'j} \qquad \forall (i, i') \in A \tag{16}$$

$$QLOR_{ii'j} \le cap_j NL_{ii'j} \qquad \forall j \in V, (i, i') \in A \tag{17}$$

Conditions (16) assure that the transported load will be divided among the types of vehicles. Constraints (17) limit the load carried to the capacity of vehicles.

3.5 Operation characteristics

These characteristics refer to budget and global quantity to be distributed. The operation cost will be approximated assuming that the vehicles return to their original destination, empty and through the same route used to deliver the goods.

Variables:

$COST$: total operation cost

Constraints

$$COST = \sum_{j,(i,i') \in A} \left(2cf_{ii'j} dist_{ii'} NL_{ii'j} + cv_{ii'j} dist_{ii'} QLOR_{ii'j} \right) \tag{18}$$

$$COST \le b \tag{19}$$

$$\textstyle\sum_{i/dem_i > 0} QF_i = qglobal \tag{20}$$

Conditions (18) and (19) define and bound the operation cost, respectively. Condition (20) assures that the load to be distributed in the operation arrives to the demand nodes.

4 Attributes

Zeleny [31] defines attributes as descriptors of an objective reality which represent values of the Decision Makers. These values are measurable, independently of the decision maker preferences, and can usually be expressed as a mathematical function of the decisional variables. But as it has been previously stated, one characteristic of humanitarian logistics is that usual cost or time functions are not the most relevant issue, at least by themselves. Other attributes as reliability, security or equity must be considered. Hence, this section is devoted to the proposal of some new attributes to be considered in these operations, as well as the way to measure them through the elements of the model introduced in the previous section.

4.1 Cost

Cost is the classical criterium in distribution systems, and it cannot be forgotten. Although a condition regarding budget has already been included as a hard constraint, the *cost* can be included also as a attribute. The function to measure this attribute has already been defined in constraint (18):

$$COST = \sum_{j,(i,i')\in A} \left(2cf_{ii'j}dist_{ii'}NL_{ii'j} + cv_{ii'j}dist_{ii'}QLOR_{ii'j}\right)$$

4.2 Time

Time of operation is usually a critical issue in humanitarian logistics. Nevertheless, for operations repeated along time, keeping the time under a target is the most common criterium. In any case, *time of operation* should be included as an attribute.

Introducing the variable TX, maximum time to reach a node (which is the time of operation), it can be linearly computed as follows.

$$TX \geq TM_i \quad \forall i \in N \tag{21}$$

4.3 Equity

Taking into account the cost or time attributes, solutions delivering aid to the more accessible nodes and oversighting difficult-to-reach areas can be obtained. Equity in distribution, as a "fairness" criterium, becomes in this way a central issue.

The attribute in this case is the maximum deviation proportional to the demands. Naming DX to proposed variable, it can be linearly computed as follows.

$$DX \geq 1 - \frac{QF_i}{dem_i} \quad \forall i \in N/dem_i > 0 \tag{22}$$

4.4 Priority

Sometimes our aim regarding demand is to attend primordially the special requirements of a specific place. So, together with other criteria, the attribute *proportion of satisfied demand in a specific node* can be considered. Let L be the prioritized node. The value to be maximized or bounded is

$$PRI_L = \frac{QF_L}{dem_L} \tag{23}$$

4.5 Reliability

Disasters usually cause hard damages on the infrastructure. Moreover, some natural disasters continue along the time of operation, or cause new disasters, forcing the operation to be developed under adverse natural conditions. For instance, a strong earthquake strikes in an instant but aftershocks can produce new damages. Or after-disaster weather conditions, as rains, can produce landslides after the earthquake.

In such conditions the operation has to be developed with uncertainty about the damage in the infrastructure. A way to model this uncertainty is through reliability analysis. Reliability can be defined as the probability to perform successfully some activity. In our model, we have considered as basic data the reliability or probability of crossing completely an arc of the network:

$r_{ii'}$: probability of crossing the arc (i, i')

These data may be difficult to estimate, and quite often their values are based on subjective perceptions rather than on observed data. In any case, more significant than the true value is the scale.

Two measures are introduced for the reliability of an operation, one considering the worst arc used in the operation in terms of reliability, and another one considering the whole set of arcs used in the operation.

For the worst case, let *RMN* be the minimum reliability of arcs travelled. The function proposed is

$$RMN \leq r_{ii'} + 1 - BLT_{ii'} \quad \forall (i, i') \in A \tag{24}$$

To compute a linear global measure for reliability we need nevertheless a more extended study. Since reliability of a route is the probability of completing each arc of the route, assuming independence between arcs, for a route R we have

$$P(\text{complete a route } R) = \prod_{(i,i')\in R} P(\text{complete the arc } (i, i')) = \prod_{(i,i')\in R} r_{ii'}$$

Moreover, we also need to compute the reliability of each route defined by a solution, taking into account that they are not independent in case they share some arc. Therefore, an approximated measure is proposed considering the reliability of each arc used in the solution. This measure represents the probability of crossing all the arcs included in the solution;

$$\text{Reliability measure } = \prod_{(i,i')/BLT_{ii'}>0} r_{ii'}$$

And applying logarithms to linearize this expression we obtain the following function, being RG the attribute variable:

$$RG = \sum_{(i,i')\in A} \log r_{ii'} BLT_{ii'} \tag{25}$$

4.6 Security

Last, but not least, an important criterium to consider is the security. As it has been shown recently in the Haiti earthquake, when there is a lack of resources, sometimes the operation has to be developed in an unsecured atmosphere. Other times the operation has to be developed in war zones, etc. Then, it is relevant to have a model that can choose itineraries as safe as possible.

The basic data for this attribute is the probability of a vehicle to be ransacked when travelling through an arc, $p_{ii'}$. An assumption we are making is that a convoy is less probable to be assaulted than a vehicle alone, and that the probability decreases with the number of vehicles travelling as convoy. Then, the following function is proposed to obtain the probability of being ransacked when travelling through an arc. Note that, in order to be consistent, the function is only defined for arcs used (if certain arc is not used, the probability is 0, but the result of the function would be 1).

$$P(\text{being ransacked in arc } (i, i')) = p_{ii'}^{\sum_j NL_{ii'j}} \quad \forall (i, i') \in A/BLT_{ii'} = 1$$

As when dealing with the reliability, two measures can be developed: the worst case situation and a global measure. Considering the worst case, the model proposed after applying logarithms is shown below, where PX is the logarithm of the maximum probability of suffering an assault across an arc:

$$PX \geq \log p_{ii'} \sum_j NL_{ii'j} - bd_6(1 - BLT_{ii'}) \quad \forall (i, i') \in A \tag{26}$$

To develop a global measure is difficult in this case, because we need to consider the probability of not being ransacked in any arc, $1 - p_{ii'}^{\sum_j NL_{ii'j}}$. After applying logarithms the resulting function is non linear. To overcome this difficulty, lets define a new parameter as the probability that a convoy of maximum size with T vehicles will not be ransacked, $q_{ii'} = 1 - p_{ii'}^T$, varying with the number of lorries as follows:

$$P(\text{not being ransacked in arc}(i, i') \text{ travelling } \sum_j NL_{ii'j} \text{ lorries }) = q_{ii'}^{2 - \frac{\sum_j NL_{ii'j}}{T}}$$

This function has the following properties:

- for the convoy of maximum size T, the probability is $q_{ii'}$, as it was defined
- it is increasing with the number of vehicles
- the minimum value is $q_{ii'}^2$

A global measure to approximate the value can be defined now as the probability of not being ransacked in any used arc:

$$\prod_{(i,i') \in A / BLT_{ii'} = 1} q_{ii'}^{2 - \frac{\sum_j NL_{ii'j}}{T}}$$

and applying logarithms to linearize the function, and introducing the variable PG, we obtain

$$PG = \sum_{(i,i') \in A} \log q_{ii'} \left(2 - \frac{\sum_j NL_{ii'j}}{T} - 2(1 - BLT_{ii'}) \right) \tag{27}$$

5 Criteria aggregation: a goal programming model

Several attributes have been defined in Sect. 4, and the objective has to be able to deal with all (or a subset) of them together.

Multi-criteria optimization is a very successful research area which includes different approaches for dealing with several criteria. A recent survey can be found in [9], see also Ehrgott and Gandibleux [10] for a good review of main approaches, [30] for a recent application. See also [23], and the handbook [32]. In addition, some recent related articles can be found in [11].

Among all the methods and techniques developed in this context, Goal Programming (see, e.g., Charnes and Cooper [8] and Romero [26]) has been chosen because of its flexibility, capacity to deal with many criteria (until 8 attributes have been introduced in our work) and the success of its applications to real life problems. The main reason of the GP success is in our opinion the underlying Simonian "satisficing" philosophy (see, e.g., [13,17]). The overall proposal is the simultaneous satisfaction of several goals relevant to the decision maker, more than the proximity to the optimal of the objectives, an approach that fully fits our humanitarian framework (see also [14] for a good survey about goal programming).

5.1 Goal constraints

A goal constraint is defined on an attribute and a target for the attribute. Also, since it is not possible to ensure the achievement of the goal, deviational variables must be included, which will be penalized in the objective function.

Crucial data for this model are targets and penalties. Targets are defined by the decision maker from previous information and requirements, from the information obtained in the pay-off matrix, or both.

For instance, for operation time and cost, a target usually can be proposed from the operations requirements. Equity and priority targets usually will be proposed taking into account both operation conditions and the information obtained in the pay-off matrix. As for reliability and security targets, due to the subjective nature of the basic data, usually will relay on the information given by the ideal of the pay-off matrix.

We will refer below to targets of each attribute as t_v, where v is the corresponding variable which gathers the value of the attribute. So, the following targets must be defined, depending on the attributes considered: t_{COST}, t_{TX}, t_{DX}, t_{PRI_L}, t_{RMN}, t_{RG}, t_{PX} y t_{PG}.

Then, deviational variables must be defined, which will be denoted by DV_v, in the same sense as the targets, and they will be positive variables. Thus, the following goal constraints are included in the model:

$$COST - DV_{COST} \leq t_{COST} \tag{28}$$

$$TX - DV_{TX} \leq t_{TX} \tag{29}$$

$$DX - DV_{DX} \leq t_{DX} \tag{30}$$

$$PRI_L + DV_{PRI_L} \geq t_{PRI_L} \tag{31}$$

$$RMN + DV_{RMN} \geq t_{RMN} \tag{32}$$

$$RG + DV_{RG} \geq t_{RG} \tag{33}$$

$$PX - DV_{PX} \leq t_{PX} \tag{34}$$

$$PG + DV_{PG} \geq t_{PG} \tag{35}$$

Finally, the objective function consists of the deviational variables of the attributes considered penalized in order to make them as small as possible. A first term to include in these penalties is the target of the attribute considered (if not zero) dividing the variable, in order to normalize units and work with percentage deviations. The second term that penalties must include is a weight of importance of any attribute showing the preferences of the decision maker, called α_v. There are different methods in the literature to obtain the preferences, most of them valid for this problem.

Then our objective function to be minimized (assuming $t_v \neq 0$) is:

$$\sum_v \frac{\alpha_v}{t_v} DV_v$$

5.2 Goal programming model

The proposed goal programming model to decide the distribution of humanitarian aid is the following:

$$\min \frac{\alpha_v}{t_v} DV_v$$

subject to the constraints

Load Flow model	(1)–(3)
Lorries Flow model	(4)–(15)
Relation Load Lorries	(16), (17)
Operation Characteristics	(18)–(20)
Attribute Measures	(21)–(27)
Goal Constraints	(28)–(35)

and the character of variables

$$QTR_{ii'}, QF_i, TM_i, QLOR_{ii'j}, COST, TX, DX, PRI_L, RMN, DV_v \geq 0$$
$$NL_{ii'j}, NLF_{ij} \geq 0 \text{ and integer}$$
$$BL_{ii'j}, BLT_{ii'}, BLF_{ij} \in \{0, 1\}$$

Note that it results a linear mixed integer programming model, which can be solved with standard commercial MP optimizers.

6 Case study

The proposed humanitarian aid distribution model has been applied to the Haiti catastrophic earthquake, January 12th, 2010. Its epicentre was 25 km west of Port-au-Prince, and it had a devastating magnitude of 7.0 Richter scale, with at least 52 aftershocks measuring 4.5 or greater. More than 270,000 people died, thousands were injured and homeless people sum up to more than one million. Besides, many important infrastructures, as roads and bridges, needed for transportation, were damaged. This case study presents the application of the proposed approach to the planning of an operation of humanitarian aid distribution in Port-au-Prince, Haiti's capital, and its surroundings.

The model proposed in Sect. 5 has been implemented in GAMS using CPLEX as optimizer. Data of basic model have been mostly estimated from the information contained at the web sites [20] and [24].

Transport network consists of 24 nodes and 42 available links between locations. Nodes are classified in 9 settlements demanding a total of 250 tons of load, 3 depots (the port with 60 tons of available aid, the airport with 80 tons, and Jimaní, border city of Dominican Republic, with 140 tons) and 12 intermediate nodes.

In Fig. 1 a map of the region is presented, showing the network nodes (depots labeled 1-3, demanding nodes labeled by 10, 12, 13, 16, 17, 18, 20, 21, 22, and the rest for intermediate nodes) and all available links. Links are shown in different color depending on their reliability and different thickness depending on their quality (determining the maximum speed of the lorries travelling through them). Note that in the figure thicker means faster.

The lorries used for transportation are of 3 types (135 big lorries, 95 medium size and 70 small size), and they are sited at the depots and at a few intermediate nodes, and when they go through the same link they always travel together forming convoys.

The planning operation consists of delivering 150 tons of humanitarian aid with a budget of 80,000 dollars.

The *pay-off matrix*, shown in Table 1, is obtained considering independently each of the attributes introduced in Sect. 4. Each row shows the value of the attributes obtained when optimizing individually each attribute (fist column). The ideal point containing the optimal values of each attribute, highlighted in bold, is shown in the diagonal of the pay-off matrix.

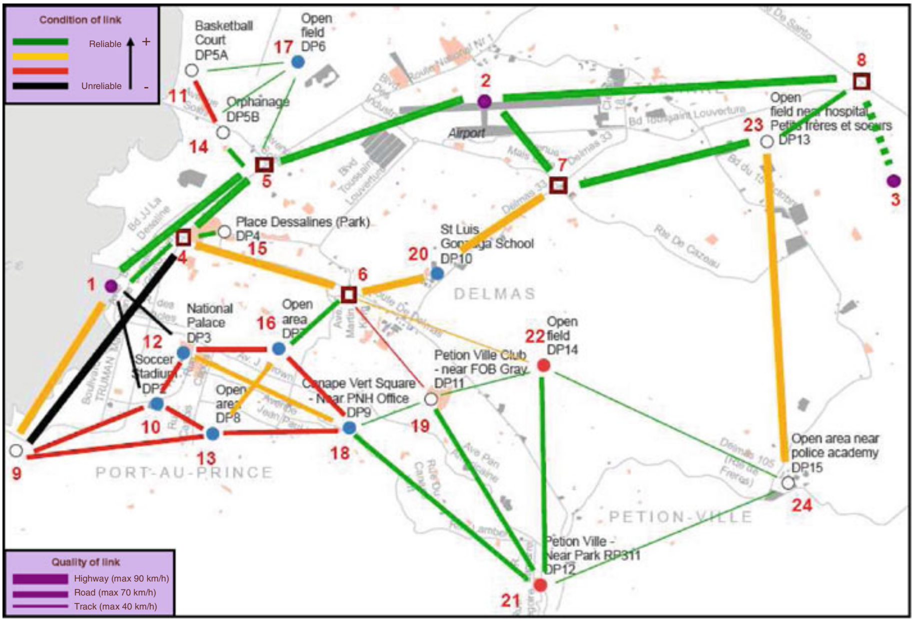

Fig. 1 Transport network at Port-au-Prince

Table 1 Pay-off matrix

Attribute	COST($)	TX(min.)	DX	PRI_{13}	RMN	RG(- ln)	PX	PG(- ln)
COST	**35,835.0**	141.75	1.0	0.00	0.1	5.83	0.85	11.76
TX	79,204.5	**83.85**	1.0	0.13	0.1	7.28	0.92	19.32
DX	42,453.0	146.25	**0.4**	0.60	0.1	7.76	0.92	15.37
PRI_{13}	39,498.0	141.75	1.0	**1.00**	0.1	4.32	0.88	12.32
RMN	79,931.5	159.75	1.0	0.90	**0.75**	2.07	0.92	19.60
RG	69,692.0	124.50	1.0	0.00	0.75	**1.43**	0.85	11.87
PX	80,000.0	117.00	1.0	0.00	0.1	4.73	**0.19**	9.08
PG	79,966.0	156.00	1.0	0.00	0.1	4.22	0.66	**7.80**

The meaning and units of the attributes are: the cost (in dollars), the time of response (in minutes, not including loading and unloading operations), the equity of the solution (maximum of deviations to the demands), the aid delivery to the priority node $L = 13$, the worst case and the global measure for reliability (measured as a probability and as the logarithm of a probability, respectively), and the worst and the global value of security (similarly to reliability).

The pay-off matrix given in Table 1 shows the hard conflict degree between attributes. For instance, solutions obtained for attributes different to equity only serve the nodes properly located for the considered attribute, ostracizing some demanding nodes that receive no aid at all (DX value equals 1). Nevertheless, it is possible to serve for all nodes at least 60% of their demand (ideal value of DX is 0.4, meaning that all nodes receive at least 60% of their demand).

Table 2 Aggregating goals

Attributes	COST($)	TX(min.)	DX	RMN	RG(- ln)	PX	PG(- ln)
C1: Reliability	69,692	124.50	1.0	**0.75**	**1.43**	0.85	11.87
C2: Security	79,991	123.00	1.0	0.1	4.73	**0.21**	**8.11**
C3: Equal weights	60,012	83.94	0.42	0.45	3.41	0.54	16.57
C4: Double reliab	56,115	84.44	1.0	**0.75**	**1.66**	0.54	11.17
C5: Double equity	59,558	83.94	**0.4**	0.45	3.41	0.66	16.64

Table 3 Distribution of humanitarian aid

Nodes	n10	n12	n13	n16	n17	n18	n20	n21	n22
Demands	30	40	30	30	10	30	40	20	20
COST	100%	75%	0%	100%	100%	0%	100%	0%	50%
TX	100%	65%	13%	100%	0%	0%	100%	0%	100%
DX	60%	60%	60%	60%	60%	60%	60%	60%	60%
PRI_{13}	97%	0%	100%	100%	90%	0%	100%	0%	60%
RMN	0%	0%	90%	50%	100%	100%	70%	100%	100%
RG	0%	0%	0%	100%	100%	100%	100%	100%	100%
PX	93%	80%	0%	100%	0%	67%	100%	0%	0%
PG	0%	100%	0%	100%	0%	97%	100%	0%	55%
Reliability	0%	0%	0%	100%	100%	100%	100%	100%	100%
Security	0%	100%	60%	100%	0%	73%	100%	0%	0%
Equal weights	58.5%	58.5%	58.5%	58.5%	90%	58.5%	60%	58.5%	58.5%
Double reliab	0%	0%	100%	100%	0%	100%	100%	0%	100%
Double equity	60%	60%	60%	60%	60%	60%	60%	60%	60%

From the information collected at the pay-off matrix and the operation requirements, the decision maker could set targets for the attributes.

In order to illustrate the sensitivity of the solutions to different attribute weights, in Table 2 the attributes of some solutions obtained with different sets of preferences are shown. Targets have been fixed to the ideal point provided by the pay-off matrix. As the attributes *equity* and *priority* refer to different ways to satisfy the demand, one of them should be chosen, being in this case *equity*.

Solution in the first row (*C1*) is the one obtained aggregating the two proposed reliability measures, obtaining the same solution when minimizing only the global reliability measure. However, if the two proposed security measures are aggregated (second row, *C2*) a new solution is obtained with both security measures very close to the optimal values. From now on, the relative importance of reliability (security) will be divided between both reliability (security) criteria, assigning half the weight to each attribute.

In the last three rows solutions obtained including all the attributes with different sets of preferences are presented. In the first combination, *C3: Equal Weights*, the same relative importance is given to each criterium; in the second combination, *C4: Double reliab*, the reliability weight is doubled, obtaining a solution whose reliability is very close to its optimal and the rest of attributes are improved with respect to the solution of first row; finally

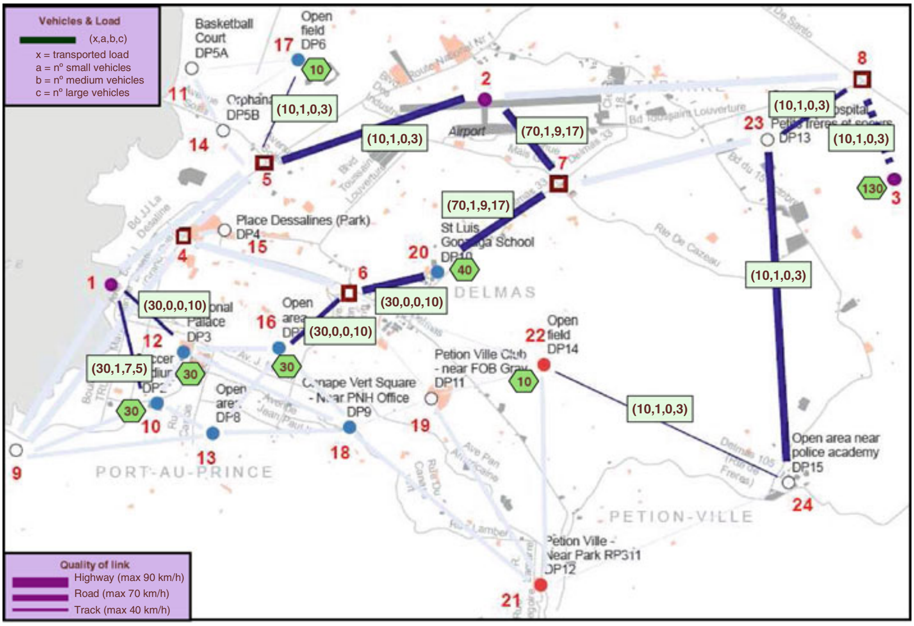

Fig. 2 Solution minimizing total cost

in the last row (*C5*) the equity weight is doubled and as a result a solution with an optimal value for equity is obtained, showing a reasonable performance in the rest of the attributes.

Table 3 details the amount of humanitarian aid delivered to each demanding node in all the solutions presented in Tables 1 and 2. In the first row the demanding nodes are listed and in the second row their demands are given. The rest of the rows presents the percentage of the demand satisfied on each node in each proposed solution.

It can be observed that there are some nodes, as for example 16 and 20, that receive at least 50% of their demand in all proposed solutions, because they are centrally located and thus they are always served no matter what criterium is chosen. On the other hand there are some nodes, as for example 21, that receive nothing in more than half of the proposed solutions, highlighting the importance of considering alternative attributes regarding the fairness of the distribution of the humanitarian aid.

Figures 2, 3, 4 and 5 illustrate the distribution of humanitarian aid corresponding to some of the previously proposed solutions. The links that are actually used in each case are enhanced with a darker color and have an associated label (x, a, b, c), where x is the amount of humanitarian aid being transported by the link and a, b and c represent the number of small, medium and large lorries, respectively, used to transport the corresponding load.

The depots with unused aid remaining and the demanding nodes that actually receive humanitarian aid have an hexagon-shaped box with these values.

In Fig. 2 it can be observed that minimizing total cost only the nodes closest to the depots are visited. Moreover, in the solution given by Fig. 3 where only equity is taken into account, all demanding nodes are visited and at least 60% of their demand is satisfied. However, in both solutions some potentially unreliable links such as (1–10) and (1–12) are used. The solution illustrated in Fig. 4, maximizing reliability, does not include these links, using only the most reliable available ones, but again some nodes are unattended.

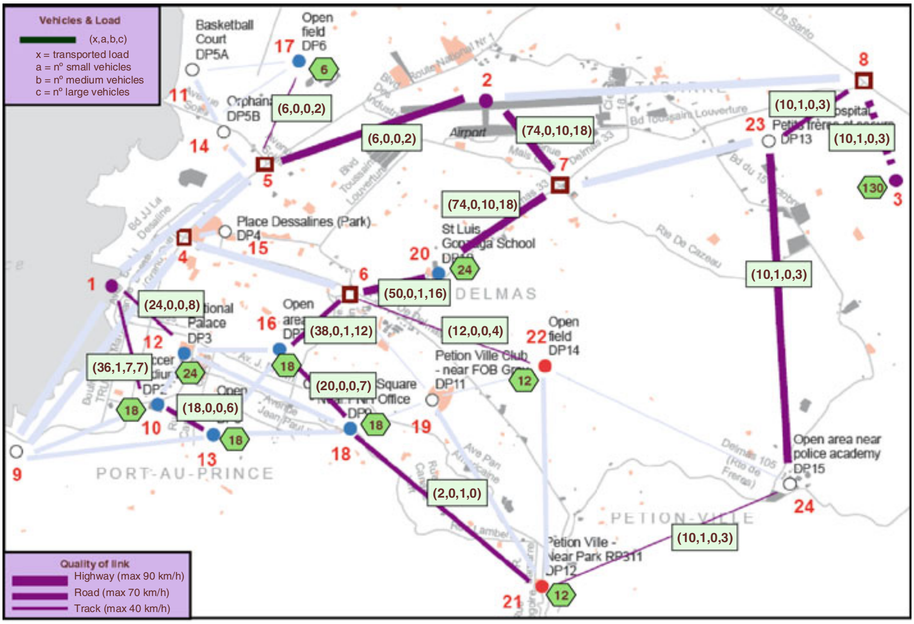

Fig. 3 Solution maximizing equity

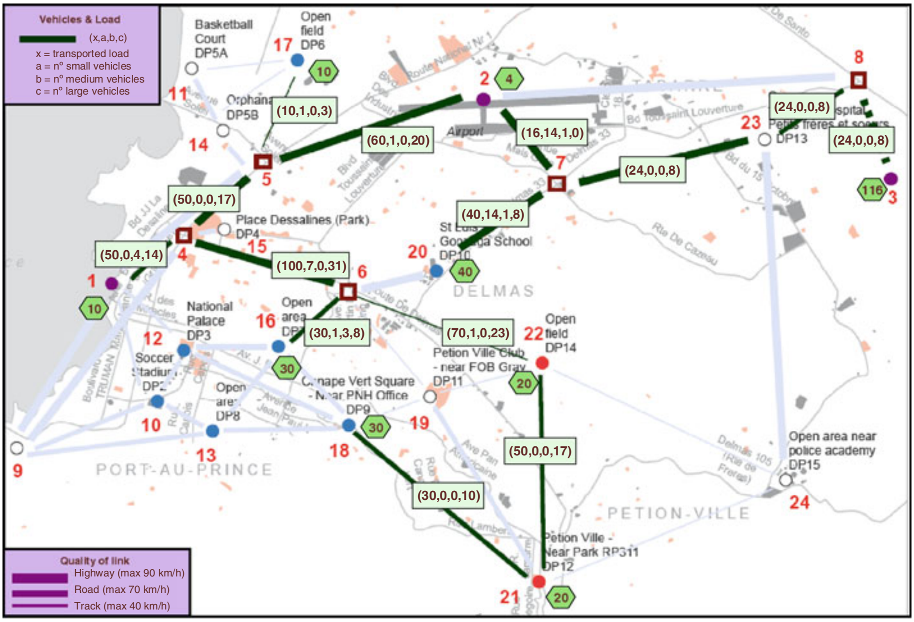

Fig. 4 Solution maximizing reliability

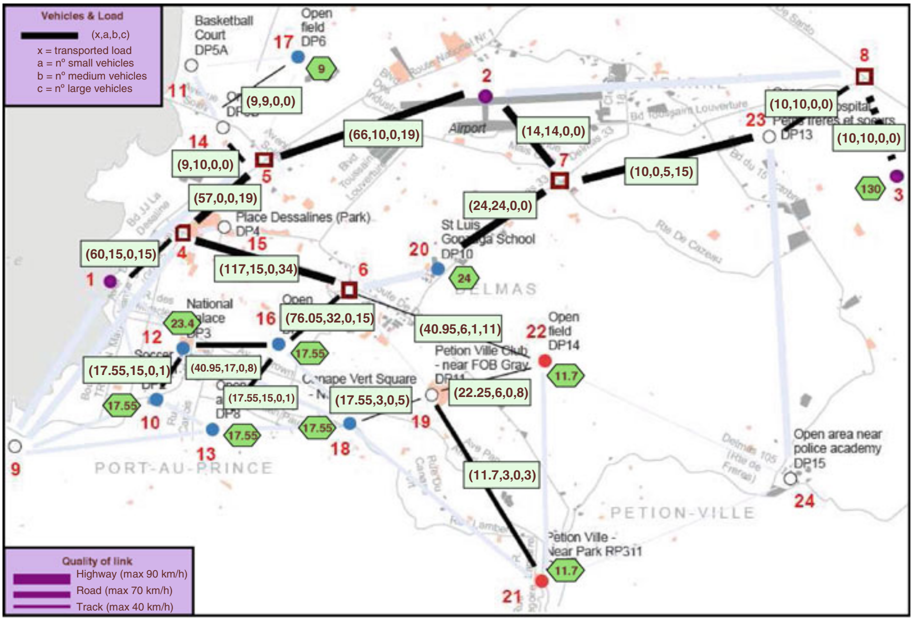

Fig. 5 Solution aggregating all attributes with equal weights

Finally, the solution illustrated in Fig. 5 takes into account all considered attributes with equal weights, (Combination 3 of Table 2), visiting all demanding nodes, not using the most unreliable links and yielding reasonable values for the rest of attributes. Note that only the fastest lorries are used through the longest arc (linking nodes 3 and 8), yielding an operation time that is very close to the optimal. In fact, this is the case in all solutions with a nonzero time weight (see Combinations 3, 4 and 5 of Table 2), while the solutions shown in Figs. 2, 3 and 4 with zero time weigh give larger times, due mainly to the fact of slower lorries being used along the arc (3–8), concecting the frontier with the city.

7 Conclusions

Humanitarian logistics is an emergent area demanding tools to support decisions made under adverse conditions. Although some studies provide insight into various disaster recovery efforts, the design of models for distribution of emergency aid considering several objectives has not been addressed in literature until recently.

These works devoted to quantitative models for humanitarian aid distribution stress that developing models to compute the most difficult and relevant attributes dealing with several criteria at the same time is a hard problem that deserves attention.

This paper presents an original model for humanitarian aid distribution, proposing a new approach to the problem based upon cost, time, equity, priority, reliability and security. Measures to compute these attributes through a linear programming model have been introduced. These measures are obtained from a structural model based on a flow model for load, another one for vehicles, and some operation requirements as budget and quantity to be distributed.

Even though the model is specifically designed for humanitarian logistics, it could be extended to business logistics under adverse conditions.

A goal programming model able to deal with the criteria all together has been presented. It results a mixed integer linear programming model, whose solution can be reached quickly.

The model has been applied to a case study based on the Haiti earthquake 2010 to illustrate its behavior with promising results.

Future research is related to the development of a decision support system based on this core paper and other related issues within a wider project (see, e.g., [25]), to be offered to humanitarian organizations in a free access web site.

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