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# A hierarchical compromise model for the joint optimization of recovery operations and distribution of emergency goods in Humanitarian Logistics

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ABSTRACT

The distribution of emergency goods to a population affected by a disaster is one of the most fundamental operations in Humanitarian Logistics. In the case of a particularly disruptive event, parts of the distribution infrastructure (e.g., bridges, roads) can be damaged. This damage would make it impossible and/or unsafe for the vehicles to reach all the centers of demand (e.g., towns and villages). In this paper, we propose and solve the problem of planning for recovery of damaged elements of the distribution network, so that the consequent distribution planning would benefit the most. We apply the model, called RecHADS, to a case study based on the 2010 Haiti earthquake. We also show empirically the importance of coordinating recovery and distribution operations optimization.


## 1. Introduction

The Emergency Events Database[1] (EM-DAT) contains essential core data on the occurrence and effects of mass disasters in the world from 1900 to the present. A wide array of different types of emergencies are recorded: from natural events like floods, pandemics and earthquakes to human activity based disasters like terrorism, train accidents, and nuclear power plant failures. According to the EM-DAT, from 2000 to 2010 exactly 8351 disasters occurred all over the world which, on average, amounts to more than two new disasters every day. The earthquakes in Haiti and Chile on January and February 2010, respectively, the Pakistan floods in July 2010, and the March 2011 Japan earthquake and tsunami are just some examples of the latest tragedies. When a disaster happens, it is easy to tell when a system response is inadequate. Luckily, contingency planning and disaster operations optimization can certainly help to reduce the likelihood of inadequate response to an emergency.

In the occurrence of particularly disruptive disasters, the distribution infrastructure can be seriously damaged, making it very difficult (if not impossible) to execute the response and recovery operations. That is the case of the Haiti 2010 earthquake where the airport, which already had low capacity, was destroyed, the port was badly damaged as the roads were in the country. Due to these bottlenecks, access of international aid to the victims was severely constrained [30]. In this paper, an infrastructure recovery model that tackles this kind of issues is proposed. The model concerns the optimization of recovery plans for damaged elements of the distribution network (e.g., bridges and roads) after the disaster, with the aim of supporting the distribution operations in the best possible way. In the literature, a number of models that coordinates different operations in the context of disaster management have been proposed. Brown and Vassiliou [6] illustrate a real-time decision support system for operational and tactical allocation of units to tasks – including repairing of damages – in disaster management. Fiedrich et al. [10] propose a model for the optimization response operations following an earthquake. The model optimizes the allocation of resources (e.g., cranes, trucks, dozers) to response tasks (e.g., search-and-rescue, stabilization, and rehabilitation of transportation lifelines) with the objective of minimizing the number of casualties. Yan and Shih [32] present a model that combines infrastructure recovery planning and aid distribution operations. This model minimizes the time necessary for both emergency repair and relief distribution and is solved heuristically. On the other hand, our model includes multiple optimization criteria that are generally involved in distribution operations in a Humanitarian Logistics context (i.e., time, cost, reliability, security, and demand satisfaction) and is solved to optimality. The importance of researching methodologies to incorporate maintenance of infrastructure to improve operations under disaster conditions has also been recognized by Altay and Green [2], who pointed out the lack of contributions in this research field.

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Efficient emergency management is certainly one of the primary issues in today's society, and developing tools for decision support is an extremely important and timely topic. In fact, when a disaster occurs, optimizing the use of scarce resources and efficiently managing the aids delivered by diverse relief organizations is of primary importance. Intelligent planning for distribution operations is certainly necessary as, as stated in the INSEAD report [30], "transportation is critical to deliver aid at the right time and to the right place." Despite that, not much research has been done along these lines: according to Altay and Green [2], up to 2005 humanitarian emergencies were generally not addressed in the OR/MS journals and more research needs to be done on disaster recovery. Recent events have brought this issue to the forefront of public concern in the last years, and the interest in these matters has been increasing. Van Wassenhove [29] pinpoints and compares characteristics of both disaster and private sector logistics. The main statement of the author is that collaboration between them would certainly be fruitful. In fact, Humanitarian Logistics performances would benefit from private logistics expertise on efficient managing. Similarly, the private sector could learn competencies belonging to humanitarian organizations, such as agility, adaptability, and preparedness.

Although research in Humanitarian Logistics is still relatively new, a certain number of models have been proposed in the last years. Haghani and Oh [12] present a large-scale multi-commodity multimodal network flow model with time windows in the context of disaster relief operations. Two different heuristics are proposed for the solution of the problem. Chang [7] proposes an accessibility based methodology for evaluating and enhancing the performance of urban transportation systems in the aftermath of a disaster. Barbarosoğlu and Arda [4] present a two-stage stochastic programming model for relief distribution that includes uncertain arc capacities and demands to represent the lack of knowledge following a disaster. Özdamar et al. [33] present a multi-period multi-commodity network flow model to address the problem of distributing emergency goods and resources. An exhaustive review of the literature on Humanitarian Logistics in disaster relief operations can be found in the paper by Kovács and Spens [14] that also provides an interesting comparison with business logistics. McCoy [21] surveys models of humanitarian response logistics, including distribution of relief supply decisions. In Humanitarian Logistics, traditional cost minimizing measures are not fundamental, as humanitarian operations have different objectives, such as speed and equity of the distribution, as well as safety and reliability of the distribution routes. Balcik et al. [3] optimize last mile distribution operations by considering both distribution costs and equity in the allocation of relief supplies from the local distribution centers to the destination points. A location-routing model is proposed by Ukkusuri and Yushimito [28]. The authors optimize the prepositioning of supplies while taking into account disruption in the transportation network given by the disaster, and the resulting routing scheme. Ortuño et al. [24] develop a decision support system for organizations in charge of the distribution of humanitarian aid, called HADS (Humanitarian Aid Distribution System). The main model of the decision support system presented by the authors consists of a multi-criteria distribution problem that designs distribution routes with respect to budget, aid supply, demand, and vehicles availability constraints. The best trade-off among the criteria is found by using a lexicographic goal programming approach. An extension to the model is proposed in a following article by Vitoriano et al. [31]. Building upon this preliminary research, our study takes a step forward in the development of distribution models in the context of Humanitarian Logistics by including an upper decision level dealing with the recovery of transportation infrastructure elements. The resulting problem is multi-criteria in nature and is formulated as a lexicographical model. More recently, Rath and Gutjahr [25] present the problem of designing a supply system with intermediate warehouse to distribute relief goods to people affected by a disaster. A three-objective problem considering medium-term economic, short-term economic, and humanitarian objective functions is proposed. The program is solved by means of a math-heuristic.

The rest of the paper is organized as follows. In the next section our recovery model is presented and a multi-criteria solution approach is proposed (Section 2). Section 3 is devoted to the computational experiments and to the analysis of the results. Next, we show empirically the importance of optimizing recovery and distribution operations in a coordinated fashion (Section 4). Finally, the paper is concluded with some insights and guidelines for future research (Section 5).

## 2. Recovery of damaged arcs in post-disaster operations

The Santa Cruz County Emergency Response and Recovery Plan [26] identifies within its Essential Support Function (ESF) the following activities:

- Emergency flood fighting operations.
- Emergency clearance of debris to allow for reconnaissance of the damaged areas and passage of emergency personnel and equipment for lifesaving.
- Temporary construction of emergency access routes which include damaged streets, roads, bridges, airfields and any other facilities necessary for passage of rescue personnel.

Generally, these operations involve public works and emergency engineering, which normally have a higher cost than the distribution operations and rely on different resources, such as clean-up teams and special machinery. Moreover, the agencies in charge of the recovery operations may not coincide with the ones in charge of the distribution. It is well-known that coordination and cooperation among agents is essential to a successful and efficient response, especially when facing large scale emergencies [5,15]. Therefore, it is important to develop tools that favor collaboration among agencies and allow for the coordinated optimization of different tasks. In this section we propose a model for the optimization of recovery and distribution operations in a coordinate fashion.

The model presented here is called RecHADS and concerns the optimization of recovery plans for damaged elements of the distribution network (e.g., bridges and roads) in an advanced response phase; i.e., when evacuation operations have been carried out, being the affected population located and settled, and distribution and recovery operations are running. We incorporated in the formulation a distribution model so that the recovery operations are designed to support aid distribution in the best possible way. Given the phase under consideration the recovery operations should benefit the distribution operations as a whole (or at least for an extended period of time), rather than taking into account the necessity of a single aid distribution mission.

A hierarchical model is proposed. The rationale behind RecHADS is that the highest priority should be assigned to helping the highest number of people in distress. In the model, this translates into maximizing the total reached demand, $Z_{DG}$. Once the maximum reachable demand is identified and fixed a model for medium-long term distribution planning is applied, including several performance attributes: cost, delivery time, security, and reliability. The model optimizes across all the criteria by minimizing the maximum distance of each attribute from its ideal value, $D_\infty$. Finally, the last level of optimization aims at minimizing the sum of the attribute distances, $D_1$, while constraining the total demand satisfied to be exactly $Z_{DG}$, and keeping the maximum attribute distance to its ideal value below $D_\infty$. The resulting model is lexicographic and the achievement function considered is the following:

$$\text{Lex min } a=[(-Z_{DG}),(D_{\infty}),(D_1)]. \quad (1)$$

In the rest of this section we present the elements constituting the model, explain how $Z_{DG}$, $D_{\infty}$, and $D_1$ are calculated, and show the algorithm used for the solution.

### 2.1. Load flow model

RecHADS involves long-term decisions concerning the very structure of the distribution infrastructure. Typically, the region considered is represented as a transportation network, where nodes correspond to cities and towns, and edges represent the roads and other transportation elements (e.g., bridges, tunnels). Recovering an edge (e.g., cleaning up a road from debris, or setting up a temporary bridge) generally necessitates a high expense of resources, specialized workers and machinery. Thus, the recovery plan should take into account a long distribution horizon, and not just the goals of a single mission. To this end, we propose a flow model where there is no restriction on the capacity of the supply centers.

The arrival and the size of supplies are often difficult to know with certainty and to forecast. Therefore, supply centers are characterized by a parameter that defines the size of a center relatively to the size of the whole set of supply centers. In other words, the parameter is a proxy for the importance of the center, and expresses the proportion of total flow that shall originate from it. Concerning the demand centers, the demand value does not represent the exact quantity of aids needed, but it characterizes the size of the center (e.g., population affected by the disaster). In conclusion, the distribution planning model proposed can be seen as an uncapacitated commodity flow model with proportionality constraints on the supplied flow.

*Parameters*

$N$ Set of nodes, indexed by $i$ and $i'$.
$N^t$ Set of transitional nodes, indexed by $i$ and $i'$.
$N^d$ Set of demand nodes, indexed by $i$ and $i'$.
$N^s$ Set of supply nodes, indexed by $i$ and $i'$.

Note that $N^s$, $N^d$, and $N^t$ define a partition of $N$.

$dem_i$ Demand at node $i \in N^d$. This parameter represents the size of the demand center.
$qav_i$ Size of supply node $i \in N^s$. This parameter is used to provide a term of comparison for the "importance" of the supply centers. The model assumes the vector to be normalized; i.e., $\sum_{i \in N^s} qav_i = 1$.

Although in the original network we consider that the nodes are connected by edges, for modeling purposes we replace each edge $\{i,i'\}$ with a pair of arcs $(i,i')$ and $(i',i)$.

$A$ Set of arcs, indexed by $(i,i')$ and $(i',i)$.
$\widehat{A}$ Subset of arcs, $\widehat{A}=\{(i,i') \in A \mid i<i'\}$.

*Variables*

$QTR_{ii'}$ Flow passing through arc $(i,i') \in \widehat{A}$ (when negative, it represents a reverse flow; i.e., a flow going from $i'$ to $i$).
$QF_i$ Flow staying at node $i \in N$ (when negative, it represents the quantity of flow supplied by the node).

*Constraints*

$$QF_i = \sum_{i' \in N \mid (i',i) \in \widehat{A}} QTR_{i'i} - \sum_{i' \in N \mid (i,i') \in \widehat{A}} QTR_{ii'} \quad \forall i \in N \quad (2)$$

$$QF_i = 0 \quad \forall i \in N^t \quad (3)$$

$$0 \le QF_i \le dem_i \quad \forall i \in N^d \quad (4)$$

$$-QF_i = qav_i \sum_{i' \in N^d} QF_{i'} \quad \forall i \in N^s \quad (5)$$

We formulate the flow model by considering a compact formulation that uses continuous and free arc variables that take a positive value to represent a flow on arc $(i,i') \in \widehat{A}$, while they take a negative value to represent a flow on arc $(i',i)$. Variables $QF_i$ are continuous. A positive value represents the flow delivered to the node, while a negative value represents the flow supplied by the node. This formulation seemed to be more efficient compared to other formulations that used two distinct variables, one for arc $(i,i')$ and one for arc $(i',i)$. In fact, the average improvement in the solution time was 7.97% in the instances considered in the computational tests (see Section 3). Constraints (2) compute for each node the quantity of flow delivered or supplied. Flow preservation is ensured by constraints (3). Constraints (4) enforce that the flow delivered at the demand nodes cannot exceed the demand. Finally, constraints (5) enforce that the ratio of total delivered flow originated from $i \in N^s$ is exactly $qav_i$.

### 2.2. Model attributes

Unlike distribution logistics, Humanitarian Logistics models must take into account different attributes other than time or cost. Following the philosophy behind HADS, we consider a multi-criteria distribution model that combines time, reliability, security, and demand satisfaction. This subsection is devoted to presenting the parameters, variables and constraints necessary for the calculation of the attributes values of a distribution scheme.

*Parameters*

$dist_{ii'}$ Length of arc $(i,i') \in A$ in terms of travel time.
$r_{ii'}$ Reliability of arc $(i,i') \in A$.
$p_{ii'}$ Ransack probability of arc $(i,i') \in A$.
$q_{ii'}$ Security of arc $(i,i') \in A$: $q_{ii'} = 1-p_{ii'}$.
$bd_1$ Constant value greater than or equal to the maximum possible flow per arc. A feasible value is: $bd_1 = \sum_{i \in N^d} dem_i$.
$bd_2$ Constant value greater than or equal to the maximum length of a path in the network. A feasible value is: $bd_2 = \sum_{(i,i') \in \widehat{A}} dist_{ii'}$.
$V$ Set of attributes, indexed by $v$ and $v'$: $V=\{TX,DG,PX,PG,RMN,RG\}$.

The meaning of each attribute is summarized in the following:

TX Maximum arrival time.
DG Total served demand.
PX Maximum ransack probability in the distribution plan.
PG Global security measure.
RMN Minimum used arc reliability in the distribution plan.
RG Global reliability measure.

*Variables*

$BLT_{ii'}$ Binary variable taking value 1 if arc $(i,i') \in A$ has non-null flow, 0 otherwise.
$TM_i$ Flow maximum arrival time at node $i \in N$.
$Z_v$ Current value of attribute $v \in V$.

*Arcs utilization variables definition*

$$QTR_{ii'} \le bd_1 BLT_{ii'} \quad \forall (i,i') \in \widehat{A} \quad (6)$$

$$-QTR_{ii'} \le bd_1 BLT_{i'i} \quad \forall (i,i') \in \widehat{A} \tag{7}$$

$$BLT_{ii'} + BLT_{i'i} \le 1 \quad \forall (i,i') \in A \tag{8}$$

$$BLT_{ii'} \in \{0,1\} \quad \forall (i,i') \in A \tag{9}$$

Constraints (6) and (7) bind together the arc flow variables $QTR_{ii'}$ to the variables $BLT_{ii'}$, defined as binary (9). Constraints (8) are redundant in the Mixed Integer Program (MIP), but help to remove symmetric solutions and to tighten the linear relaxation of the model. Variables $BLT_{ii'}$ are used for the definition of some of the problem attributes.

*Demand satisfaction*: Variable $Z_{DG}$ represents the total demand served. It is evident that this measure is of primary importance, as the main objective of Humanitarian Logistics is to provide help and emergency goods to as many people as possible. Note that, given the structure of the model (i.e., no capacity at supply level), maximizing the demand satisfaction is equivalent to maximizing the number of people that can be reached from any supply center (i.e., by selecting those arcs that increase the connectivity among demand nodes and supply nodes as much as possible):

$$Z_{DG} = \sum_{i \in N^d} QF_i \tag{10}$$

*Time*: Time is usually a critical factor in first response operations, when delivering emergency supplies as quickly as possible is crucial. Afterward, it is generally sufficient to keep the distribution time below a target:

$$TM_i \ge TM_{i'} + dist_{i'i} - bd_2(1 - BLT_{i'i}) \quad \forall (i',i) \in A \tag{11}$$

$$Z_{TX} \ge TM_i \quad \forall i \in N \tag{12}$$

$$TM_i \ge 0 \quad \forall i \in N \tag{13}$$

Constraints (11) compute the visit time of each node $i \in N$. By assigning a univocal visit time these constraints eliminate flow sub-cycles [22]. Variable $Z_{TX}$ is defined as the maximum among the flow arrival times, $TM_i$ (12); i.e., the latest service time.

*Security*: Security is an important attribute in Humanitarian Logistics, as humanitarian organizations generally operate in an insecure environment. In fact, distribution operations are often run in war zones, where ransacks can happen. Recent events in New Orleans and Haiti showed that major disasters in under-developed areas may lead to anarchy [23,1]. It is of the utmost importance to care for the operators and identify itineraries that protect them from harm. The type of risk considered in RecHADS is ransacks of the vehicles carrying the aid. Assessing the risk is an important issue that should be considered during the decision making process [9] and the model is general enough to be applied to other kinds of dangers, as well.

Two different security measures are proposed. The first one, $Z_{PX}$, identifies the worst (maximum) arc ransack probability in the distribution scheme:

$$Z_{PX} \ge p_{ii'} BLT_{ii'} \quad \forall (i,i') \in A \tag{14}$$

The second one is a global measure of security that expresses the probability that the distribution will be completed without suffering any attack.

$$P(\text{complete distribution without attacks}) = \prod_{(i,i') \in A | BLT_{ii'} = 1} q_{ii'} \tag{15}$$

We obtain a linearization of this expression by applying the logarithm

$$Z_{PG} = \sum_{(i,i') \in A} \log q_{ii'} BLT_{ii'} \tag{16}$$

Note: $Z_{PG}$ is expressed as the logarithm of the global security measure.

*Reliability*: Disasters can damage elements of the distribution infrastructure. Moreover, smaller scale disasters can take place after a major one, such as aftershocks or landslides after an earthquake or a heavy precipitation, respectively. Therefore, distribution operations must take into account some degree of uncertainty regarding the state of the infrastructure. RecHADS considers $r_{ii'}$, $\forall (i,i') \in A$, probability that an arc can be crossed.

Two measures of reliability are considered; i.e., the worst (minimum) arc reliability and a global reliability:

$$Z_{RMN} \le r_{ii'} + 1 - BLT_{ii'} \quad \forall (i,i') \in A \tag{17}$$

The worst arc reliability $Z_{RMN}$ takes the value of the minimum among the reliabilities of the arcs used in the distribution plan ($BLT_{ii'} = 1$). To compute a global measure of reliability, we assume independence between the arcs and consider the probability of successfully crossing all the arcs used in the distribution plan:

$$Z_{RG} = \sum_{(i,i') \in A} \log r_{ii'} BLT_{ii'} \tag{18}$$

Note: $Z_{RG}$ is expressed as the logarithm of the global reliability.

### 2.3. Recovery model

The most innovative characteristic of RecHADS lies in the incorporation of recovery decisions to a multi-criteria distribution model. We now explain how this feature has been included in our model.

We define $\tilde{A} \subseteq A$ as the subset of damaged arcs that need to be recovered in order to be used and that require the investment of specific resources (i.e., teams, equipment) to be recovered.

Recovering an arc $(i,i') \in \tilde{A}$ has a cost, $rc_{ii'}$, and results in the complete restoration of the arc (i.e., the arc is fully reliable and $r_{ii'} = 1$) allowing for its use in the distribution operations. A budget $rb$, which cannot be exceeded, is available for recovery operations. This feature is enforced by the following constraint:

$$\sum_{(i,i') \in \tilde{A}} rc_{ii'} BLT_{ii'} \le rb \tag{19}$$

As a consequence, the reliability measures need to be slightly adapted.

$$Z_{RMN} \le r_{ii'} + 1 - BLT_{ii'} \quad \forall (i,i') \in A/\tilde{A} \tag{20}$$

$$Z_{RMN} \le 1 \tag{21}$$

$$Z_{RG} = \sum_{(i,i') \in A/\tilde{A}} \log r_{ii'} BLT_{ii'} \tag{22}$$

Constraints (20) and (22) exclude from the calculation of $Z_{RMN}$ and $Z_{RG}$, respectively, the unreliable arcs $(i,i') \in \tilde{A}$. In fact, by assumption, an unreliable arc $(i,i') \in \tilde{A}$ is either unusable, or recovered and completely reliable; i.e., either $BLT_{ii'} = 0$, or $r_{ii'} = 1$ and $\log r_{ii'} = 0$, respectively. Finally, constraint (21) has been included to preserve feasibility in the case that $\tilde{A} \equiv A$.

Note that, as a consequence of this new set of constraints, some of the demand nodes may not be reachable from any of the supply nodes. This reflects the fact that, after a major disaster, some urban areas may get isolated from the aid distribution centers. As a consequence, the demand satisfaction measure, $Z_{DG}$, acquires a new meaning as it represents the maximum demand that can be reached depending on the set of undamaged and recovered arcs.

### 2.4. Multi-criteria model

The different attributes presented in Section 2.2 represent different optimization criteria. The decision maker can use preference weights to devise distribution plans which are a trade-off among the criteria. Since RecHADS involves design decisions over the distribution infrastructure, the maximization of the demand satisfaction attribute is considered immeasurably preferred to the optimization of any other set of attributes. In fact, long-term decisions such as these should not only take into account the initial emergency necessities, but should aim at a rapid recovery of the system, which necessarily includes the possibility of providing assistance wherever is needed. This idea is supported by Article 159 of the UN Agenda for Development[2]: "Where emergency situations arise, rapid provision of humanitarian assistance by the international community remains, of course, imperative. However, this form of assistance must be planned with a view to an equally rapid transition to rehabilitation and reconstruction and be part of the continuum concept which aims at resuming development at the earliest opportunity."

To solve the multi-criteria model we propose a three-level lexicographic model. The steps of the solution methodology are illustrated in the following.

#### 2.4.1. First lexicographic optimization level: maximization of the total served demand

As explained in Section 2.3, all the arcs $(i,i') \in \tilde{A}$ are classified as unreliable and cannot be used. In this first optimization level we compute the maximum demand that can be served with the actual set of unsafe arcs $\tilde{A}$ and the available recovery budget $rb$.

RecHADS-1:

$$
\begin{aligned}
Z^{\star}_{DG} = \max \quad & Z_{DG} \\
\text{s.t.} \quad & \text{Load flow model (2)–(5)} \\
& \text{Arcs utilization (6)–(9)} \\
& \text{Delivered flow attributes (10)} \\
& \text{Recovery model (19)}
\end{aligned}
$$

Objective function value $Z^{\star}_{DG}$ is the maximum demand that can be served with the current available resources. The proposed formulation for RecHADS-1 consists of $2|N|+3|A|+2$ constraints, $2|N|+6|A|+|A|/2$ continuous variables, and $|A|$ Boolean variables. The resulting model is a MIP problem that can be solved by using a generic MIP solver.

Normally, RecHADS-1 has several multiple optimal solutions; i.e., all the different paths that can be used to deliver a flow of size $Z^{\star}_{DG}$. Therefore, we fix the total delivered flow to $Z^{\star}_{DG}$, and we use the remaining lexicographic optimization levels to break the tie and identify the best combination of recovered arcs set and distribution plan, with respect to the preferences of the decision maker.

#### 2.4.2. Building the payoff matrix

Having computed the maximum served demand $Z^{\star}_{DG}$, we can use the following constraint to impose that the delivered flow will always be maximal, regardless of the criteria used in the second part of the optimization process:

$$Z^{\star}_{DG} = \sum_{i \in N^d} QF_i \tag{23}$$

We can now compute the payoff matrix which shows the attribute values of the distribution scheme obtained by optimizing individually for each of the criteria. The payoff matrix Z for RecHADS is pictured as

$$
Z = \begin{bmatrix}
Z^{TM}_{TM} & Z^{PX}_{TM} & Z^{PG}_{TM} & Z^{RMN}_{TM} & Z^{RG}_{TM} \\
Z^{TM}_{PX} & Z^{PX}_{PX} & Z^{PG}_{PX} & Z^{RMN}_{PX} & Z^{RG}_{PX} \\
Z^{TM}_{PG} & Z^{PX}_{PG} & Z^{PG}_{PG} & Z^{RMN}_{PG} & Z^{RG}_{PG} \\
Z^{TM}_{RMN} & Z^{PX}_{RMN} & Z^{PG}_{RMN} & Z^{RMN}_{RMN} & Z^{RG}_{RMN} \\
Z^{TM}_{RG} & Z^{PX}_{RG} & Z^{PG}_{RG} & Z^{RMN}_{RG} & Z^{RG}_{RG}
\end{bmatrix} \tag{24}
$$

Each element $Z^{\nu'}_{\nu}$ of Z shows the value of attribute $\nu' \in V/\{DG\}$ obtained when optimizing specifically for attribute $\nu \in V/\{DG\}$. Attribute DG is excluded from the payoff matrix as its value is already fixed. The payoff matrix is obtained by iteratively solving RecHADS-$\nu$ for each $\nu \in V/\{\text{DG}\}$, with variables $Z_{\nu'}$, $\forall \nu' \in V/\{DG\}$ renamed as $Z^{\nu'}_{\nu}$.

RecHADS-$\nu$:

$$
\begin{aligned}
\text{opt} \quad & Z^{\nu}_{\nu} \\
\text{s.t.} \quad & \text{Load flow model (2)–(5)} \\
& \text{Arcs utilization (6)–(9)} \\
& \text{Attributes (11)–(14), (16), (20)–(22)} \\
& \text{Maximum flow (23)} \\
& \text{Recovery model (19)}
\end{aligned}
$$

The proposed formulation for RecHADS-$\nu$ consists of $4|N|+6|A|-|\tilde{A}|+5$ constraints, $2|N|+6|A|+|A|/2$ continuous variables, and $|A|$ Boolean variables. As all the attributes constraints are linear, the model is still a MIP problem and, as for RecHADS-1, it can be solved by using a generic MIP solver. The direction of optimization depends on the attribute considered. We *maximize* the attributes belonging to set $V^{\max} = \{PG, RMN, RG\}$, and we *minimize* the attributes belonging to set $V^{\min} = \{TM, PX\}$.

#### 2.4.3. Ideal and anti-ideal points

Payoff matrices are useful tools that help the decision maker to identify dominated recovery plans and, more generally, to understand the relationships between the different attributes. The payoff matrix allows us to define the ideal point $Z^{+}$ and the anti-ideal point $Z^{-}$. These points represent the maximum and minimum obtainable values for the attributes in the payoff matrix, respectively.

It follows that the ideal point components are the payoff matrix diagonal elements:

$$Z^{+}_{\nu} = Z^{\nu}_{\nu} \quad \forall \nu \in V/\{DG\} \tag{25}$$

The anti-ideal point components are, for each attribute $\nu$, the worst value among the $Z^{\nu}_{\nu'}$:

$$Z^{-}_{\nu} = \min_{\nu' \in V/\{DG\}} Z^{\nu}_{\nu'} \quad \forall \nu \in V^{\max} \tag{26}$$

$$Z^{-}_{\nu} = \max_{\nu' \in V/\{DG\}} Z^{\nu}_{\nu'} \quad \forall \nu \in V^{\min} \tag{27}$$

As an example, consider that we solve one RecHADS-$\nu$ for each $\nu \in V/\{\text{DG}\}$, obtaining the following payoff matrix:

$$
Z = \begin{bmatrix}
52,000 & 0.3 & -2.33 & 0.75 & -2.05 \\
139,500 & 0.3 & -2.24 & 0.75 & -2.34 \\
139,500 & 0.3 & -1.79 & 0.75 & -2.05 \\
139,500 & 0.3 & -2.24 & 0.75 & -2.34 \\
139,500 & 0.3 & -2.11 & 0.75 & -1.9
\end{bmatrix}
$$

According to formulas (25)–(27) the ideal and anti-ideal points are

$$Z^{+}_{\nu} = (52,000; 0.3; -1.79; 0.75; -1.9),$$

$$Z^{-}_{\nu} = (139,500; ,0.3; -2.33; 0.75; -2.34).$$

[2] http://www.un.org/Docs/SG/human.htm, last accessed date April 11, 2012.

The points identified will be used in the next steps to define bounds for the values taken by the different attributes.

*2.4.4. Second lexicographic optimization level: minimization of the Chebyshev distance*

To obtain a solution to the multi-criteria problem we use Compromise Programming (CP) [34]. CP is a decision making technique that finds the best solution by minimizing a distance function, $d(Z,p,\alpha)$, defined through a norm of the weighted normalized distances to the ideal point:

$$d(Z,p,\alpha)=\left\|\frac{Z^{+}-Z}{Z^{+}-Z^{-}}\right\|_{p,\alpha}$$
$$=\left[\sum_{v\in V/\{DG\}|Z_v^+\neq Z_v^-}\left(\alpha_v\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-}\right)^p\right]^{1/p} \quad (28)$$

$Z$ is the current solution attribute vector, $p$ is the order of the norm, and $\alpha$ are the weights assigned by the decision maker to the attributes. Without loss of generality, we assume the $\alpha$ weights to be normalized; i.e., $\sum_{v\in V/\{DG\}}\alpha_v=1$. Function $d(Z,p,\alpha)$ expresses the total weighted distance between the ideal point and the current solution. This distance is normalized with respect to the distance between the ideal and the anti-ideal point to scale the attribute values between 0 and 1 so that they can be compared. The components of the weight vector $\alpha$ express the importance of each attribute when it comes to devise the recovery plan.

In this optimization level different norms could have been used, but we choose the infinity (or Chebyshev) norm, in order to minimize the maximum among the attributes distances from their ideal value:

$$d(Z,\infty,\alpha)=\max_{v\in V/\{DG\}}\left\{\alpha_v\left|\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-}\right|\right\} \quad (29)$$

Formula (29) can be expressed in terms of linear inequalities by introducing a variable $D_\infty$, whose value has to be minimized, and the following constraints:

$$D_\infty\geq\alpha_v\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-}\quad\forall v\in V/\{DG\}|Z_v^+\neq Z_v^- \quad (30)$$

$$Z_v=Z_v^+\quad\forall v\in V/\{DG\}|Z_v^+=Z_v^- \quad (31)$$

$$D_\infty\geq 0 \quad (32)$$

Constraint (31) has been introduced to preserve the feasibility of the model when, for a given attribute, the value of its corresponding components in the ideal and anti-ideal points are identical. In these cases, the attribute is forced to assume that specific value.

The complete formulation for the second optimization level of RecHADS is the following.

RecHADS-2:

$$\begin{aligned}
D_\infty^\star=\min\quad & D_\infty\\
\text{s.t.}\quad & \text{Load flow model } (2)-(5)\\
& \text{Arcs utilization } (6)-(9)\\
& \text{Attributes } (11)-(14),(16),(20)-(22)\\
& \text{Maximum flow } (23)\\
& \text{Recovery model } (19)\\
& \text{Chebyshev distance } (30)-(32)
\end{aligned}$$

The proposed formulation for RecHADS-2 consists of $4|N|+6|A|-|\tilde{A}|+11$ constraints, $2|N|+6|A|+|A|/2+1$ continuous variables, and $|A|$ Boolean variables. RecHADS-2 is a MIP problem and we solve it by using a generic MIP solver.

*2.4.5. Third lexicographic optimization level: minimization of the norm one distance*

Given its minimax structure, RecHADS-2 also has many multiple optimal solutions. For this reason we consider a further optimization level where we fix the Chebyshev distance to $D_\infty^\star$, while minimizing the norm one distance. The norm one distance is equal to the sum over the attributes of the weighted distances between the current attribute value and its ideal, scaled by the distance between ideal and anti-ideal values:

$$d(Z,1,\alpha)=\sum_{v\in V/\{DG\}|Z_v^+\neq Z_v^-}\alpha_v\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-} \quad (33)$$

Let us introduce the variable $D_1$ and the following constraints:

$$D_1=\sum_{v\in V/\{DG\}|Z_v^+\neq Z_v^-}\alpha_v\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-} \quad (34)$$

$$D_\infty^\star\geq\alpha_v\frac{Z_v^+-Z_v}{Z_v^+-Z_v^-}\quad\forall v\in V/\{DG\}|Z_v^+\neq Z_v^- \quad (35)$$

The model of the third hierarchical optimization level can be formulated as the following MIP problem.

RecHADS-3:

$$\begin{aligned}
\min\quad & D_1\\
\text{s.t.}\quad & \text{Load flow model } (2)-(5)\\
& \text{Arcs utilization } (6)-(9)\\
& \text{Attributes } (11)-(14),(16),(20)-(22)\\
& \text{Maximum flow } (23)\\
& \text{Recovery model } (19)\\
& \text{Maximum Chebyshev distance } (35),(31)\\
& \text{Norm one distance } (34)
\end{aligned}$$

The proposed formulation for RecHADS-3 consists of $4|N|+6|A|-|\tilde{A}|+11$ constraints, $2|N|+6|A|+|A|/2+2$ continuous variables, and $|A|$ Boolean variables.

*2.5. Solution algorithm*

Now that we have presented all the "ingredients" of our model, we can finally formalize the solution algorithm proposed.

**Algorithm REC.** Solution algorithm for the problem of coordinating arcs recovery and distribution operations.

//1 - Computing maximum demand that can be served
**solve** RecHADS-1 for optimal $Z_{DG}^\star$;
//2 - Computation of payoff matrix $Z$
**forall** $v\in V/\{\text{DG}\}$ **do**
  **solve** RecHADS-$v$ for optimal $Z_v^{v'}$, $\forall v'\in V/\{\text{DG}\}$;
//3 - Computation of ideal $Z^+$ and anti-ideal points $Z^-$
$Z_v^+=Z_v^v$, $\forall v\in V/\{DG\}$;
$Z_v^-=\min_{v'\in V|\{DG\}}Z_{v'}^v$, $\forall v\in V^{\max}$;
$Z_v^-=\max_{v'\in V/\{DG\}}Z_{v'}^v$, $\forall v\in V^{\min}$;
//4 - Solution of the multi-criteria model
**solve** RecHADS-2 for optimal $D_\infty^\star$;
**solve** RecHADS-3 for optimal $BLT_{ii'}$, $\forall(i,i')\in\tilde{A}$, and $Z_v$, $\forall v\in V/\{DG\}$;

The first step of Algorithm REC is the computation of the maximum demand that can be distributed with respect to the current network status and the recovery budget available. This is achieved by solving RecHADS-1. Next, we compute the payoff matrix $Z$ by solving five times RecHADS-$v$, optimizing with

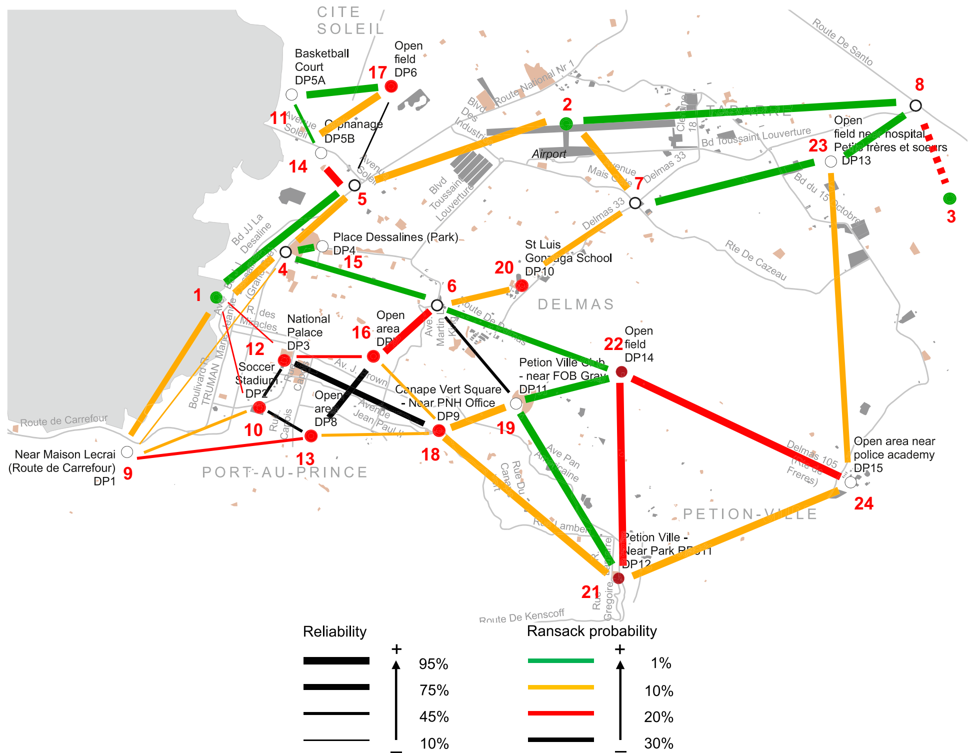


**Fig. 1.** Haiti dataset. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

respect to one of the attributes $v \in V/\{DG\}$ at a time, while imposing the delivery of the maximum quantity of demand. Once the payoff matrix has been filled, we proceed to computing the ideal and anti-ideal points, according to formulas (25)–(27). In the next step, we solve RecHADS-2 to find the solution that minimizes the maximum distance of the attributes from their best value. Finally, RecHADS-3 is solved to obtain the recovery plan (variables $BLT_{ii'}$, $\forall (i,i') \in \tilde{A}$) and the distribution scheme (variables $Z_v$, $\forall v \in V/\{DG\}$) that are the best compromise solution.

As explained at the beginning of this section, our model can be used by a decision maker to identify the arcs that should be recovered to improve the quality of the distribution the most in an advanced response phase. Given the emergency phase considered, the distribution model does not generate a strictly defined distribution plan (i.e., aid scheduling and routing), but it identifies the arcs that should be used during the distribution operations, according to the decision maker's preferences on the attributes.

## 3. Case study: infrastructure recovery and distribution planning in Haiti

RecHADS has been applied to the Haiti case study presented in [24].

The dataset, shown in Fig. 1, represents the transportation network in the area around Port-au-Prince after the January 12, 2010, earthquake. Specifically, the network has been built based on the information provided by OCHA[3] and RED HUM[4] available on January 31, 2010. The network consists of 42 arcs and 24 nodes. In the picture, the green nodes are the supply nodes (i.e., 1 is the port, 2 the airport, and 3 represents aids coming from the Dominican Republic, the country neighboring Haiti), while the red nodes are the demands. As indicated in the legend, different thicknesses and colors are assigned to the arcs, depending on their reliability and assigned ransack probability, respectively.

RecHADS has been modeled using the mathematical programming language GAMS 22.5 [11] and solved with CPLEX 10 [13]. The tests have been run on a computer equipped with a Genuine Intel(R) CPU T2500 at 2.00 GHz and 1 GB of RAM.

In our tests we used the reliability $r_{ii'}$ of an arc $(i,i') \in A$ as a proxy for the condition of highways, roads, bridges, and other components of the transportation network. We included in the set of damaged arcs all those with a reliability less than or equal to 45%:

$$\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \tag{36}$$

Set $\tilde{A}$ includes 12 arcs; i.e., $|\tilde{A}| = 12$. Fig. 2 shows the network without the damaged arcs. It can be easily seen that node 10 is now isolated from the rest of the network. We assumed that all the unreliable arcs have the same recovery costs $rc_{ii'}$; i.e., $rc_{ii'} = 1$. This may represent the case where the roads are covered by debris and the decision maker has exactly $rb$ cleanup teams to dispose of them.

Table 1 shows the scale of the models solved in terms of number of constraints, continuous variables, and Boolean variables.

We now provide insights on how the increase in recovery budget influences the value of the distribution plan attributes. This is an important analysis since it allows the decision maker to have a better understanding of the trade-off between distribution plan quality and recovery cost. Thanks to this tool, the decision maker can objectively measure the improvements and the worsening in the distribution plan quality given by increases or reductions to the investment in the recovery operations. For this analysis we considered all the values of recovery budget between 0 and $|\tilde{A}|$; i.e., $rb = 0, \ldots, |\tilde{A}|$. In the instances considered, the solution of Algorithm

[3] United Nations Office for the Coordination of Humanitarian Affairs. http://ochaonline.un.org/, last accessed date April 11, 2012.

[4] Red de Información Humanitaria para América Latina y el Caribe. http://www.redhum.org/, last accessed date April 11, 2012.

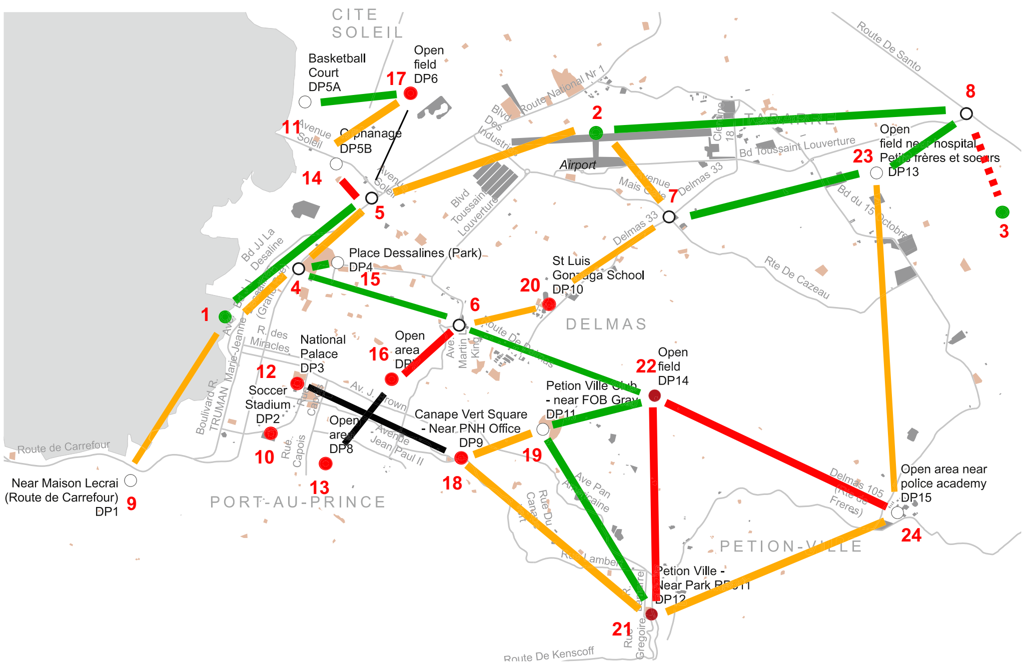


**Fig. 2.** Haiti dataset without arcs $(i,i') \in \tilde{A}$.

**Table 1**
Models scale in terms of number of constraints, continuous variables, and Boolean variables.

| Model | # Constraints | # Continuous Variables | # Boolean variables |
|---|---|---|---|
| RecHADS-1 | 176 | 321 | 42 |
| RecHADS-$v$ | 341 | 321 | 42 |
| RecHADS-2 | 347 | 322 | 42 |
| RecHADS-3 | 347 | 323 | 42 |

REC took 69 s on average and 190 s at most.[5] The solution times are modest so time is not an issue.

### 3.1. Total served demand

When no arc can be recovered ($rb=0$), the maximum demand that can be served is 220. As soon as one arc is recovered ($rb=1$), the satisfied demand increases to 250, which is the total demand in the system. This is due to the topology of the network and the reliability threshold $\tilde{r}$ chosen. In fact, as shown in Fig. 2, when the arcs in $\tilde{A}$ are forbidden node 10 is isolated. The demand associated to node 10 is exactly 30. As soon as at least one arc is recoverable, the program opens a path to node 10 that becomes reachable. As a consequence, now all the demand can be met.

### 3.2. Ideal and anti-ideal points

Fig. 3 illustrates the trend of the ideal and anti-ideal values colored in dark and light gray, respectively, and shows their values in the tables below the graphs. In each graph, the $x$-axis reports the available recovery budget, $rb$. One graph for each of the attributes $TX$, $PX$, $e^{PG}$, and $e^{RG}$ is presented.[6] Attribute $RMN$ is not reported as both the ideal and anti-ideal values are always equal to 75% regardless of the recovery budget available. From the observation of the graphs we can infer some interesting considerations. First of all, it is not necessary to recover all the unreliable arcs to achieve the best attribute values. Specifically, the minimum worst arc ransack probability $PX$ is reached when $rb=3$, the minimum total service time $TX$ can be obtained with just four recovered arcs, the global reliability probability $e^{RG}$, and the global security probability $e^{PG}$ reach the best values when five arcs can be recovered. Second, while $TX$, $PX$, and $e^{RG}$ ideal trends are monotonic, the $e^{PG}$ trend gets worse (decreases) when $rb=1$. This is due to the increase in the total demand served, $DG$, from 220 to 250, which forces the distribution plan to use at least one more arc (e.g., one of the arcs that connects node 10) and reorganize the flows to satisfy the flow proportionality constraints (5).

Another important information provided by the charts in Fig. 3 is the graphical representation of the distance between the ideal and the anti-ideal values for each attribute. This distance represents the range of possible solutions for each criterion in the Pareto frontier. In our study, this information describes how increasing the budget affects the range of possible solutions of each attribute when varying the criteria weights.

### 3.3. Multi-criteria solutions

We now study the effect of increasing the recovery budget in the solutions of the compromise programming model RecHADS-3. In these tests, because of the lack of an expert decision maker, we opted for assigning the same importance weight to the attributes; i.e., $\alpha_v = 0.2$, $\forall v \in V$.

Table 2 shows the results of the analysis. The first column reports the number of arcs that can be recovered. The next columns illustrate the values of the attributes in the solution obtained. The decision maker can take advantage of the information provided by Table 2 to decide how much to invest in the recovery operations. For instance, an interesting insight is that while the best ideal values were obtained for $rb=5$, the best compromise solution is reached when six arcs can be recovered. Another observation is that, since the program optimizes for the best compromise, the attribute value trends may not be monotonic (as for TX and $e^{PG}$ in Table 2). This is

[5] Note that the solution times include data loading, as well as creation and population of the model matrices.

[6] Note that, as shown by (16) and (22), $PG$ and $RG$ are expressed logarithmically. Therefore, the exponential function is used to show the results as probabilities.

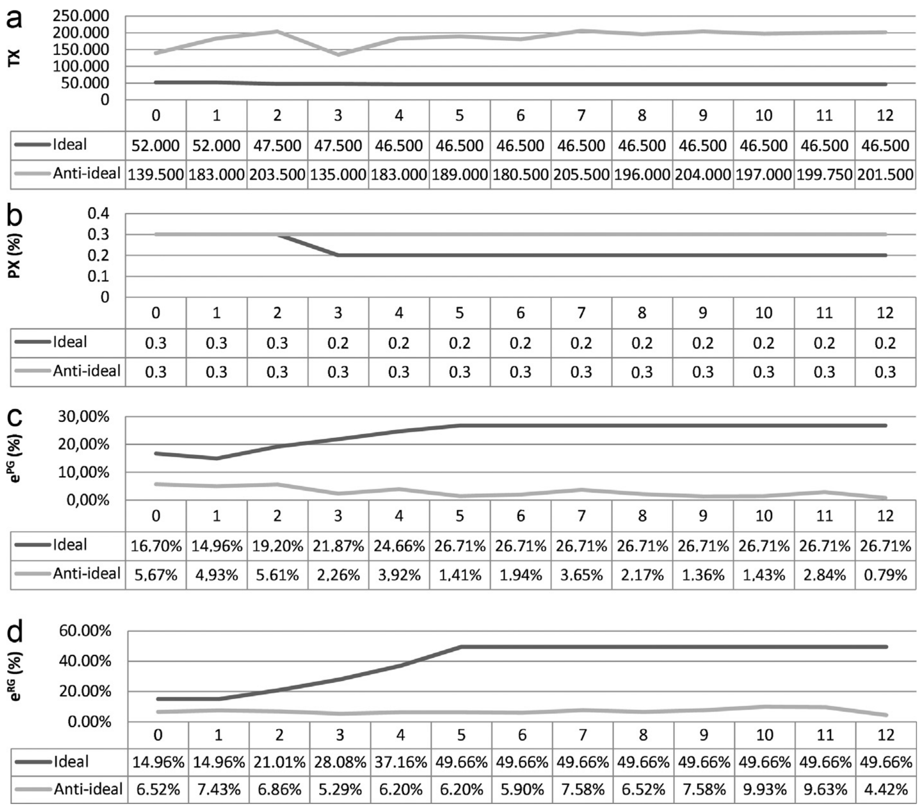


**Fig. 3.** Attributes ideal and anti-ideal values for $rb = 1, \ldots, |\tilde{A}|$. (a) Maximum service time, *TX*. (b) Worst (maximum) arc ransack probability, *PX*. (c) Probability that the distribution will be completed without suffering any attack, $e^{PG}$. (d) Probability of successfully crossing all the arcs used in the distribution plan, $e^{RG}$.

**Table 2**
Optimal attributes values of the multi-criteria solutions for $rb = 1, \ldots, \tilde{A}$.

| *rb* | *TX* | *PX* (%) | $e^{PG}$ (%) | *RMN* (%) | $e^{RG}$ (%) |
|---|---|---|---|---|---|
| 0 | 57,000 | 30 | 14.96 | 75 | 14.23 |
| 1 | 55,000 | 30 | 13.00 | 75 | 14.23 |
| 2 | 55,000 | 30 | 16.70 | 75 | 18.09 |
| 3 | 56,500 | 20 | 18.09 | 75 | 23.93 |
| 4 | 52,500 | 20 | 19.99 | 75 | 30.42 |
| 5 | 58,500 | 20 | 19.79 | 75 | 42.74 |
| 6 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 7 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 8 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 9 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 10 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 11 | 58,500 | 20 | 21.65 | 75 | 42.74 |
| 12 | 58,500 | 20 | 21.65 | 75 | 42.74 |

due to the Compromise Programming structure of the model, which looks for the best compromise solution among all the criteria. Therefore, a small worsening to a subset of the attributes can still lead to a better solution if the overall solution value improves.

## 4. Sequential optimization VS. coordinated optimization

As explained in Section 2, recovery and distribution operations may be carried out by different agencies with little (or no) coordination. One of the purposes of this work is to show the importance of cooperation among agents. To accomplish our goal, we use the case study to illustrate empirically the loss in solution quality when the recovery and the distribution operations are executed sequentially; i.e., the decision regarding which arcs to recover aims exclusively at improving the connectivity of the network.

The solutions for the sequential model (from now on, SM) are obtained by executing the following procedure:

1. Solve RecHADS-1 and fix the value of the variables $BLT_{ii'}$, $\forall (i,i') \in \tilde{A}$, to that of the optimal solution. In this first step, the recovery operations are executed by taking into account only the necessity of connecting all the demand nodes to the supply nodes. No further improvement to the network can be accomplished in the next steps.
2. By using the transportation network resulting from step 1, solve RecHADS-$\nu$ without constraint (19) to compute the payoff matrix and the ideal and anti-ideal points.
3. Once obtained the ideal and anti-ideal points, optimize RecHADS-2 and RecHADS-3 without constraint (19).

The solution procedure for the SM reduces the size of RecHADS-$\nu$, RecHADS-2, and RecHADS-3 by one constraint (19) and $|\tilde{A}|$ Boolean variables, compared to that of the coordinated model. Thus, as the set of damaged arcs $\tilde{A}$ increases the dimensionality of the SM decreases.

The procedure has been executed for the instances obtained from the following sets of damaged arcs:

(a) $\tilde{A} = \{(i,i') \in A | r_{ii'} \le 0.45\}$, node 10 disconnected.
(b) $\tilde{A} = \{(i,i') \in A | r_{ii'} \le 0.45\} \cup \{(12,18)\}$, nodes 10 and 12 disconnected.
(c) $\tilde{A} = \{(i,i') \in A | r_{ii'} \le 0.45\} \cup \{(12,18),(13,16)\}$, nodes 10, 12, and 13 disconnected.
(d) $\tilde{A} = \{(i,i') \in A | r_{ii'} \le 0.45\} \cup \{(12,18),(13,16),(6,16)\}$, nodes 10, 12, 13, and 16 disconnected.

Instances (b)–(d) have been generated by incrementally adding one new arc to the set of damaged arcs $\tilde{A}$ in instance (a). In each instance, we considered $rb = 1, \ldots, d$, where $d$ is the number of disconnected nodes. In fact, in this dataset the recovery of $d$ arcs is sufficient for connecting all the demands to the supply nodes. Allowing for the recovery of more arcs would not be significant, as the choice of which arcs to restore would be completely random. The results for the SM are reported in Table 3.

We compare the quality of the solutions found by the SM and RecHADS by considering the gap between the solutions attributes. The gaps are calculated as follows:

$$G_v = \frac{Z_v - Z_v^{seq}}{|Z_v|} \quad \forall v \in V^{\max}$$

$$G_v = \frac{Z_v^{seq} - Z_v}{|Z_v|} \quad \forall v \in V^{\min}$$

where $v$ is the attribute considered, $Z_v$ is the RecHADS attribute value, $Z_v^{seq}$ is the attribute value of the SM, and $G_v$ is the gap. Note that the gap is positive when RecHADS performs better than the SM, while it is negative when the solution of the SM is better than that of RecHADS. All the gaps relative to the instances considered are shown in Table 4.

The first column shows the recovery budget $rb$ available and the remaining columns display the gaps $G_v$. We show in the last column the total gaps between the solutions found by the two programs to provide a general idea of which is a better compromise. It can be easily seen that in instance (a) (Table 4(a)), RecHADS finds a solution that is as good as that of the SM in terms of reliability and maximum security, but is better in terms of time and global security. The overall compromise improvement is 1.61%. When two nodes are disconnected (Table 4(b)) the solutions found by RecHADS dominate those of the SM. This is not the case in the instance with three isolated nodes (Table 4(c)). In fact, when $rb = 1$ the SM solution has a better global reliability, and when $rb = 2$ the distribution time is slightly better. Despite that, the solutions found by RecHADS are better in terms of compromise among all the attributes, as pointed out by the solution gap. Finally, RecHADS dominates completely the SM in the instances with four nodes isolated (Table 4(d)), and scores an impressive 20.77% solution gap when $rb = 4$. Another interesting observation is that, in the instances considered, the improvement in the quality of the solutions of RecHADS seems to increase with respect to the number of disconnected demand nodes in the network.

We acknowledge that these results do not state an undisputed superiority of our coordinated approach against the sequential approach. In fact, not all the solutions of the coordinated model dominate the corresponding solutions to the sequential one. Nevertheless, we believe that the overall improvement (expressed by the solution gap) can justify the solution of a more complex and sophisticated model considering that recovery problems are strategic in nature. Another important consideration is that the

**Table 3**
Sequential model solutions.

| $rb$ | $TX^{seq}$ | $PX^{seq}$ (%) | $e^{PG^{seq}}$ (%) | $RMN^{seq}$ (%) | $e^{RG^{seq}}$ (%) |
|---|---|---|---|---|---|
| (a) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\}$, *node 10 disconnected* | | | | | |
| 1 | 57,000 | 30 | 11.88 | 75 | 14.23 |
| (b) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18)\}$, *nodes 10 and 12 disconnected* | | | | | |
| 1 | 55,000 | 30 | 16.20 | 75 | 18.09 |
| 2 | 54,500 | 30 | 13.00 | 75 | 18.09 |
| (c) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18),(13,16)\}$, *nodes 10, 12, and 13 disconnected* | | | | | |
| 1 | 56,500 | 20 | 25.16 | 75 | 23.93 |
| 2 | 57,000 | 30 | 17.03 | 75 | 25.16 |
| 3 | 55,000 | 30 | 12.87 | 75 | 18.09 |
| (d) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18),(13,16),(6,16)\}$, *nodes 10, 12, 13, and 16 disconnected* | | | | | |
| 1 | 52,500 | 20 | 31.04 | 75 | 30.42 |
| 2 | 56,500 | 20 | 25.16 | 75 | 25.16 |
| 3 | 56,500 | 20 | 22.54 | 75 | 25.16 |
| 4 | 56,500 | 30 | 13.13 | 75 | 23.93 |

**Table 4**
Gaps (%) between sequential problem and coordinated problem solutions attributes.

| $rb$ | $G_{TX}$ | $G_{PX}$ | $G_{PG}$ | $G_{RMN}$ | $G_{RG}$ | Solution gap |
|---|---|---|---|---|---|---|
| (a) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\}$, *node 10 disconnected* | | | | | | |
| 1 | 3.64 | 0.00 | 4.41 | 0.00 | 0.00 | 1.61 |
| (b) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18)\}$, *nodes 10 and 12 disconnected* | | | | | | |
| 1 | 0.92 | 0.00 | 8.33 | 0.00 | 0.00 | 1.85 |
| 2 | 0.00 | 0.00 | 6.81 | 0.00 | 0.00 | 1.36 |
| (c) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18),(13,16)\}$, *nodes 10, 12, and 13 disconnected* | | | | | | |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.88 | 50.00 | 18.79 | 0.00 | −3.50 | 13.24 |
| 3 | −2.65 | 50.00 | 19.88 | 0.00 | 19.58 | 17.36 |
| (d) $\tilde{A} = \{(i,i') \in A \mid r_{ii'} \le 0.45\} \cup \{(12,18),(13,16),(6,16)\}$, *nodes 10, 12, 13, and 16 disconnected* | | | | | | |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 7.62 | 0.00 | 7.81 | 0.00 | 15.97 | 6.28 |
| 3 | 7.62 | 0.00 | 7.97 | 0.00 | 15.97 | 6.31 |
| 4 | 7.62 | 50.00 | 26.09 | 0.00 | 20.17 | 20.77 |

results in Table 4 show that the usefulness of coordinating recovery and distribution operations increases with the number of disconnected nodes, meaning that the effectiveness of the coordinated model increases as the problem at hand becomes more complicated. This behavior provides a strong argument that supports the coordinated optimization of recovery and distribution operations as, in the context of disaster management, decision makers will have to face situations that are, indeed, disastrous and, therefore, extremely complex.

## 5. Conclusions

The purpose of this work is to introduce a model that combines recovery operations of transportation infrastructure elements with aid distribution planning in Humanitarian Logistics. Another important feature of this model, called RecHADS, is that the distribution model considered is multi-criteria and includes useful criteria such as reliability and security. These are fundamental to devise distribution operations in developing countries affected by disasters. The problem has been modeled as a three level lexicographic problem, where the first level maximizes the total demand served, while the second and the third levels optimize the underlying multi-criteria model by using a compromise programming approach.

The model has been tested in a realistic case study, firstly introduced in [24], where RecHADS has been used to plan for recovery of roads in Haiti, with the aim of enhancing aid distribution. The case study shows how RecHADS can be used by agencies to obtain relevant information concerning the investment in recovery operations such as insights on the trade-off between recovery budget and quality of the resulting distribution plan. Finally, we illustrated empirically how useful it is to optimize recovery and distribution in a coordinate fashion.

Our future research plans include the use of RecHADS to devise recovery operations in Jipijapa, a canton of Ecuador frequently affected by heavy rains and landslides, in collaboration with an international NGO. This may require extending RecHADS to capture other complex features arising from the necessities of the case considered, such as identifying the best locations for distribution centers [8].

Also, we are planning to take a step forward and develop a stochastic model based on RecHADS for the protection of the transportation infrastructure prior to disasters [17,16,18–20,27].

## Acknowledgments

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