Basic Graph Theory

Binary trees:

- a special case of graphs

- start with basic graph theory -
- a pictorial way to represent information.

References:

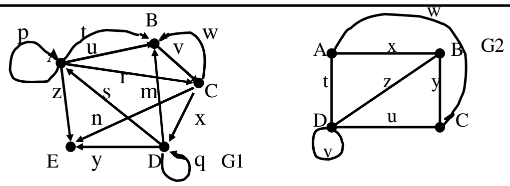
- Discrete Mathematics by Ross & Wright
- Discrete Mathematics by R. Johnsonbaugh
- Discrete Mathematics and Applications by S. Epp
- Schaum's Outline Series in Discrete Mathematics

Definition: Graph

A graph G=(V,E),an ordered pair, where V is a set of vertices (nodes), & E is a set of edges on the vertices.

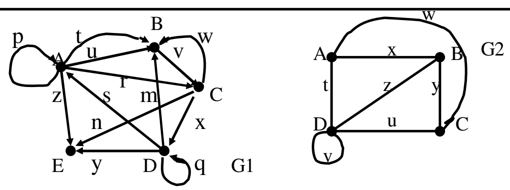
- V is not empty and has size n.
- E may be empty.

- Graphs are a modeling tool



- G1 has 5 vertices and 13 edges.
  V={A,B,C,D,E} & E = {m,n,p,q,r,s,t,u,v,w,x,y,z}
- G2 has 4 vertices and 7 edges.
  V={A,B,C,D} and E= {t,u,v,w,x,y,z}

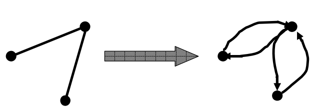
4



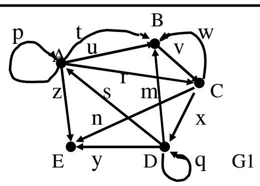
- Edge and vertex labels are arbitrary and optional.
- In G1 and G2, the vertices and edges are <u>each</u> labeled.

5

- G1 is a directed graph (arrowheads)
- G2 is an undirected graph (no arrows)
- All edges on a graph must be the same.
- To convert undirected graphs to directed graphs, replace each undirected edge with a pair of opposing edges.

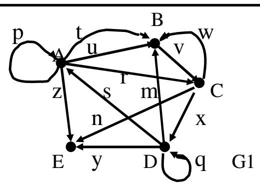


6



Edges can be named by the vertex pair they connect, e.g. in G1, AE, BC, DA are examples of edges. They are the same as edges **z**, **v**, and **s**, respectively.

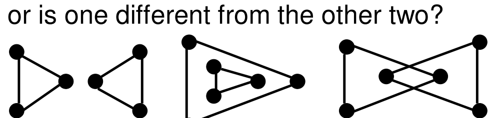
- Vertex B in G1 is said to be <u>adjacent</u> to vertex A. Vertex A is NOT adjacent to B
- B is also <u>adjacent</u> to vertices C and D.
- Vertex E is <u>adjacent</u> to vertices A,C and D.



<u>*Graphs* - Special edges</u>

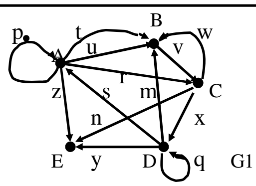
- Loops: on vertices A and D in G1, edge v in G2
- Parallel Edges: edges t and u in G1
- Note: edges v and w in G1 are not parallel

- Are graphs A, B and C
- all the same?
- all different?
- or is one different from the other two?



A B C

- This is an example of a <u>disconnected</u> graph with <u>connected</u> components.
- The way a graph is drawn is not important as long as the salient physical characteristics are preserved.



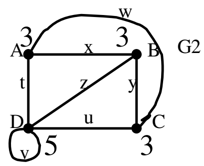
A **path** of length n is sequence of n edges from one vertex to another.

uvxy	length = 4
uvn	length = 3
z	length = 1
rxy	length = 3
ABCDE	length = 4

are examples of paths in graph G1.
The first and last path are the same.

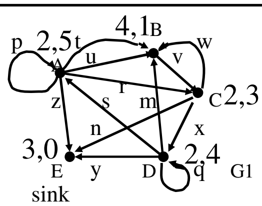
- As a rule interesting paths are **simple**, without repeated vertices or edges.
- A cycle or closed path begins and ends at the same vertex.
- uvxs is a cycle of length 4 in G1
- vxm is a cycle of length 3 in G1
- ACDA is a cycle of length 3 in G1
- xyut is a cycle of length 4 in G2
- An acyclic graph has no cycles.

- It is possible to attach additional information to an edge,
- e.g. time units required
  cost info in $
  capacity (e.g. gallons per minute).
- These values are called weights. All edges are weighted. If it is not relevant to the problem, then all the weights are 1 and are not shown.



- The **degree** of a vertex is the number of incident edges.

- In G2, vertices A, B, & C have degree 3.
- In G2, vertex D has degree 5.

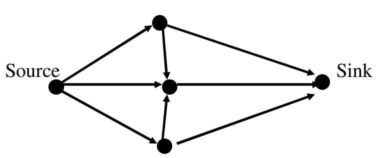


- In a directed graph, we distinguish between in-degree and out-degree.

- A has in-degree 2 & out-degree 5
- In G1, B has in-degree 4 & out-degree 1

- Vertex E is a sink. It has out-degree 0.

- A vertex with in-degree zero and positive out-degree is called a source.



- If the degree of each vertex is the same, the graph is <u>regular</u>.