Logic Synthesis & Verification, Fall 2023

National Taiwan University

Problem Set 1

Due on 2023/9/28 23:59 on NTU Cool.

1 [Characteristic Function]

(10%) Give a five-variable (including two current-state variables, one input variable, and two next-state variables) Boolean function that characterizes the state transition relation in the graph of Figure 1. Let states S_0, S_1, S_2, S_3 be encoded as 00, 01, 10, 11, respectively. Show the Boolean function with a truth table.

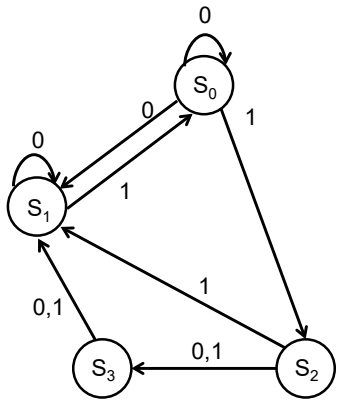

Fig. 1. A state transition graph.

2 [Boolean Algebra Definition]

(15%) Does the tuple $(D_n, \text{lcm}, \text{gcd}, 1, n)$ for $n = 90$ forms a Boolean algebra? Justify your answer by examining the conformance or violation of the axioms.

3 [Uniqueness of Complement]

(10%) Given any Boolean algebra $(\mathbb{B}, \cdot, +, \underline{0}, \underline{1})$, show that for any $a \in \mathbb{B}$, there is a unique $a' \in \mathbb{B}$ such that both equalities $a + a' = \underline{1}$ and $a \cdot a' = \underline{0}$ hold.

4 [Properties of Boolean Algebra]

(24%) Prove the following equalities using ONLY the five postulates of Boolean algebra (or other properties that you have proven using the postulates). Please specify clearly which postulate is applied in each step of your derivation.

(a) $(a \cdot b) + c = (a + c) \cdot (b + c)$
(b) $(a + b) \cdot a = a$
(c) $(a')' = a$

5 [Minterm Canonical Form]

(10%) Prove the theorem of minterm canonical form using Boole's expansion theorem.

6 [Number of Boolean Functions]

(a) (5%) How many functions (mappings) $f : \mathbb{B}^n \to \mathbb{B}$ are there for $|\mathbb{B}| = m$?
(b) (10%) How many Boolean functions of n variables are there under a Boolean algebra with $|\mathbb{B}| = m$? Reason your answer by the theorem of minterm canonical form.

7 [Boolean Functions]

(16%) Let $f(x, y)$ be a Boolean function for $\mathbb{B} = \{0, 1, a, a'\}$ with the following partial function table.

x	0	0	0	0	1	1	1	1	a	a	a	a	a'	a'	a'	a'
y	0	1	a	a'	0	1	a	a'	0	1	a	a'	0	1	a	a'
f	a	1			0						a'					

(a) How many Boolean functions are consistent with the above function table? Please explain.
(b) Please complete the above function table and list all possibilities if there is more than one.