Logic Synthesis & Verification, Fall 2023

National Taiwan University

Problem Set 3

Due on 2023/10/28 (Saturday) 23:59.

1 [Cofactor and Generalized Cofactor]

(18%) Let f and g be completely specified functions. Prove the following equalities:

(a) (3%) $f = xf_x \oplus (\neg x)f_{\neg x}$
(b) (3%) $f = f_{\neg x} \oplus x(f_{\neg x} \oplus f_x)$
(c) (3%) $f = f_x \oplus (\neg x)(f_{\neg x} \oplus f_x)$
(d) (3%) $f = g \wedge co(f, g) \vee \neg g \wedge co(f, \neg g)$
(e) (3%) $co(co(f, g), h) = co(f, gh)$
(f) (3%) $co(f + g, h) = co(f, h) + co(g, h)$

2 [Operation on Cube Lists]

(6%) Consider the following orthogonal cube list.

$$\begin{pmatrix} 0 & - & 0 & - & 1 & 1 & 0 \\ - & 0 & - & 1 & 0 & - & 0 \\ - & 1 & 1 & 0 & - & - & - \end{pmatrix}$$

Add the cube $(1{-}0{-}{-}{-}0)$ to the above list with orthogonality being maintained.

3 [Symmetric Functions]

(20%) Given a Boolean function $f(x_1, \ldots, x_n)$, consider the following symmetry definitions.

S_1: $f(x_1, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, x_j, x_{i+1}, \ldots, x_{j-1}, x_i, x_{j+1}, \ldots, x_n)$
S_2: $f(x_1, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, \neg x_j, x_{i+1}, \ldots, x_{j-1}, \neg x_i, x_{j+1}, \ldots, x_n)$
S_3: $f(x_1, \ldots, x_n) = \neg f(x_1, \ldots, x_{i-1}, x_j, x_{i+1}, \ldots, x_{j-1}, x_i, x_{j+1}, \ldots, x_n)$
S_4: $f(x_1, \ldots, x_n) = \neg f(x_1, \ldots, x_{i-1}, \neg x_j, x_{i+1}, \ldots, x_{j-1}, \neg x_i, x_{j+1}, \ldots, x_n)$
S_5: $f(x_1, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, \neg x_j, x_{i+1}, \ldots, x_{j-1}, x_i, x_{j+1}, \ldots, x_n)$
S_6: $f(x_1, \ldots, x_n) = f(x_1, \ldots, x_{i-1}, x_j, x_{i+1}, \ldots, x_{j-1}, \neg x_i, x_{j+1}, \ldots, x_n)$
S_7: $f(x_1, \ldots, x_n) = \neg f(x_1, \ldots, x_{i-1}, \neg x_j, x_{i+1}, \ldots, x_{j-1}, x_i, x_{j+1}, \ldots, x_n)$
S_8: $f(x_1, \ldots, x_n) = \neg f(x_1 \ldots, x_{i-1}, x_j, x_{i+1} \ldots, x_{j-1}, \neg x_i, x_{j+1}, \ldots, x_n)$

(a) (16%) For each S_i, for $i = 1, \ldots, 8$, find the necessary and sufficient condition of f to be S_i-symmetric on variables x_1 and x_2.
(b) (4%) Which of the above definitions $S_1, \ldots, S_8$ satisfy transitivity, that is, if f is S_i-symmetric on (x_1, x_2) and (x_2, x_3), then f is S_i-symmetric on (x_1, x_3)?

4 [Unate Functions]

(16%) Prove or disprove the following statements.

(a) (8%) A unate cover without having any single-cube containment is a prime cover. (As an example of single-cube containment, the cube abc is contained by the single cube ab.)
(b) (8%) Show that a prime cover of a unate function must be a unate cover.

5 [Threshold and Unate Functions]

(12%)

Definition 1. *A* threshold function f *over Boolean variables* $x_1, x_2, \ldots, x_n$ *is defined by*

$$f = \begin{cases} 1, & \text{if } \sum_{i=1}^{n} w_i x_i \geq T, \\ 0, & \text{otherwise,} \end{cases}$$

for some constants w_i*'s and* T *in* $\mathbb{R}$.

(a) (4%) Express a three-input AND gate as a threshold function.
(b) (8%) Show that a threshold function must be a unate function.

6 [Unate Recursive Paradigm: Complementation]

(8%) Complement the function

$$f = a'b'ce + a'cd + ab'd' + bce' + bc'd + b'd',$$

in the sum-of-products (SOP) form by the unate recursive paradigm. Apply the *binate select heuristic* for branching and show your detailed derivation.

7 [Quine-McCluskey]

(20%) Given two incompletely specified functions f and g over variables a, b, c, d, let f be of onset minterms

$$\{0100, 0101, 1010, 1110\}$$

and don't care set minterms

$$\{0001, 0010, 1101\},$$

and g have onset minterms

$$\{0100, 0101, 0111, 1000, 1010, 1100, 1111\}$$

and don't care set minterms

$$\{0110, 1110\}.$$

Apply the Quine-McCluskey procedure to minimize the multi-output cover with the following steps.

(a) (5%) Derive all prime implicants for the multi-output cover by pairwise minterm merging.
(b) (5%) Build the Boolean matrix for column covering.
(c) (5%) Simplify the Boolean matrix to its cyclic core.
(d) (5%) Compute the minimum column covering and obtain the minimum multi-output cover.