An Agent-Based Ramsey growth model with Brown and Green capital (ABRam-BG)

Aida Saraí Figueroa Alvarez *[1] and Sarah Wolf[1]

[1]Institute of Mathematics, Freie Universität Berlin

September 2024

The ODD protocol below includes changes in the model description across two ABRam (Agent-Based Ramsey growth) models. The original ABRam model (ABRam-T and black text throughout) was developed to investigate the nature of technical progress in a decentralized economy. The second model (ABRam-BG and blue text throughout) was developed to investigate transition dynamics from a "brown" to a "green" economy.

1 Overview

1.1 Purpose and Patterns

The purpose of the model **ABRam-T** is to analyze and test a decentralized economy composed of maximizing agents, with a particular focus on understanding the growth dynamics of the system. The model is built upon the well-known one-sector Ramsey growth model, with the introduction of endogenous technical progress through mechanisms of learning by doing and knowledge spillovers. In this setting, we aim to investigate farms that adopt different investment strategies based on different assumptions about the information available to them, in particular about the nature of technical progress.

The purpose of the **ABRam-BG** model is to study belief dynamics as a potential driver of green (growth) transitions and illustrate their dynamics in a closed, decentralized economy populated by utility maximizing agents with an environmental attitude. The model is built using the ABRam-T model and introduces two types of capital – green (low carbon intensity) and brown (high carbon intensity) – with their respective technological progress levels. ABRam-BG simulates a green transition as an emergent phenomenon resulting from well-known opinion dynamics [1] along the economic process.

The model is based on an analysis of three building blocks of Green Growth [6]; it is constructed directly from the first and second building block, which it extends from just two time steps, in order to explore long-term dynamics and growth of the economic system.

The model is abstract and does not aim to fit specific empirical data patterns for any given country or region. We evaluate ABRam-T and ABRam-BG based on its ability to reproduce two well-known theoretical results in economics:

1. **Decentralized Economy vs. Benevolent Planner:** This pattern illustrates the dilemma that a decentralized economy is not able to reproduce the optimal results of a benevolent planner in the Ramsey growth model.
2. **Reproduction of Kaldor Facts:** ABRam-T is capable of reproducing the Kaldor Facts, a set of stylized facts about economic growth.

Although with ABRam-BG we aim to study a scenario that has not occurred yet, we evaluate the model by its ability to reproduce a pattern described by [5]: in order for a green transition to successfully take place, it should be triggered in a timely manner.

*Corresponding author: aida.figueroa.alvarez@fu-berlin.de

[1]As is customary in the opinion and belief dynamics literature, we also refer to beliefs as opinions.

1.2 Entities, State Variables, and Scales

ABRam-T & ABRam-BG: The model's entities are:

1. **Agent Farm**: Agent that combines activities usually attributed to firms (like production) and households (like labour and consumption).

2. **Agent Statistician**: Agent that collects the system's information.

3. **Economy**: Entity that contains the agents and the simulation procedures.[2]

The models operate on a timeline of discrete time steps. As they are abstract, these steps do not correspond to specific real-world time periods.

1.2.1 Farms

Farms are agents that integrate the economic concepts of firms and households.

ABRam-T: They are characterized by their `capital` and `investment strategy`.

ABRam-BG: They are characterized by their `capitals` and `opinion` about the environment. These variables guide the farms in their `labour allocation`, `investment`, `production`, `consumption`, and their `utility` evaluation.

ABRam-T: We distinguish three types of farms depending on their investment strategy: *collaborative farms* (represented by value 0), *ignorant farms* (represented by value 1), and *witty farms*. Witty farms use expectations, they either follow an adaptive rule (represented by value 2), or a trend-follower rule (represented by value 3). When a witty farm follows an adaptive rule, the value of the adaptive constant can take two values. Similarly, the trend following rule can also take two values.

ABRam-BG: We distinguish two types of farms depending on their opinions: *brown farms* (represented by value 0) and *green farms* (represented by value 1).

The variables and parameters characterizing these agents are listed in the following tables.

Parameter	Description	ABRam-T	ABRam-BG
`alpha`	Output elasticity of the capital	0.33	0.33
`delta`	The capital stock decay rate	0.05	0.05
`rho`	The factor to discount future felicity	0.99	0.99
`adapt_cns`	Expectation weight factor for adaptive rule.	$\{0.65, 1.00\}$	
`trend_cns`	Extrapolation coefficient for the trend following rule.	$\{0.40, 1.30\}$	

Table 1: Farms' parameters.

[2]The model code uses the agent-based modelling framework agentPy [4], requiring the specification of a "model" entity. In other implementations, this might not be considered an entity in its own right.

Variable type	Variable	Domain	Description
State Variables	K	$\mathbb{R}^+$	(Total)Capital of the farm.
	InvStrg	$\{0,1,2,3\}$	Investment strategy of the farm.
	B	$\mathbb{R}^+$	Brown capital of the farm
	G	$\mathbb{R}^+$	Green capital of the farm
	O	$\{0,1\}$	Opinion of the farm about the environment
Supplementary Variables	P	$\mathbb{R}$	(Total) Production of the farm.
	I	$\mathbb{R}$	(Total) Investment of the farm.
	L	$\mathbb{R}^+/\{1\}$	(Total) Labour of the farm.
	C	$\mathbb{R}^+$	Consumption of the farm.
	u	$\mathbb{R}^+$	Instantaneous utility of the farm
	U	$\mathbb{R}^+$	Cumulative utility of the farm
	expected_I_others	$\mathbb{R}$	Farm's expectation on how much the other farms are going to invest.
	P_B	$\mathbb{R}$	Brown production of the farm.
	P_G	$\mathbb{R}$	Green production of the farm.
	I_B	$\mathbb{R}$	Brown investment of the farm.
	I_G	$\mathbb{R}$	Green investment of the farm.
	b	[0, 1]	Brown labour of the farm.
	g	[0, 1]	Green labour of the farm.
Information Variables	past_I	$\mathbb{R}$	Farm's investment from two time steps ago.
	past_total_I	$\mathbb{R}$	Statistician's variable Total_I from two time steps ago.
	eta	$\mathbb{R}$	Variable real_tech_prg from the Statistician
	beta	$\mathbb{R}$	Variable real_beta from the Statistician
	gamma	$\mathbb{R}$	Variable real_gamma from the Statistician
	tech_rat	$\mathbb{R}$	Ratio between the green and brown technical progress

Table 2: Farms' variables

1.2.2 Statistician

The **statistician** computes all the aggregate variables of the model and communicates to the farms those values that they need to know: aggregate capital, "Gross Domestic Product", and the technical progress of the economy at each time step. We introduce the statistician agent given the aggregate nature of the technological progress and the limited information the farms have about the economy. Table 3 lists the variables of the statistician.

1.2.3 Economy

The entity **Economy** represents the global environment. It contains the farms, the statistician, and the simulation procedures. It is in charge of initializing the agents, managing the model's schedule, and recording the variables associated with the agents. The attribute `parameters` contains the simulation parameters of the economy, i.e. the parameters that determine a certain simulation scenario. The parameters encompass the agents' parameters (Table 1) and the simulation's parameters (see Table 4)

The total number of farms in the simulation is given by the attribute `agents` and the attribute `steps` defines the simulation's length.

Variable	Domain	Description
Total_K	$\mathbb{R}$	Total capital of the system given by the sum of all farms' capital
GDP	$\mathbb{R}$	Gross domestic product of the system given by the sum of all farms' production.
Total_I	$\mathbb{R}$	Gross investment of the system given by the sum of all farms' investment.
Total_C	$\mathbb{R}^+$	Total consumption of the system given by the sum of all farms' consumption.
g_rate	$\mathbb{R}$	GDP growth rate
K_output_rate	$\mathbb{R}$	Capital to output ratio
real_tech_prg	$\mathbb{R}$	Technical progress of the economy.
Total_B	$\mathbb{R}$	Total brown capital of the system given by the sum of all farms' brown capital.
Total_G	$\mathbb{R}$	Total green capital of the system given by the sum of all farms' green capital.
Total_P_B	$\mathbb{R}$	Total brown production of the system given by the sum of all farms' brown production.
Total_P_G	$\mathbb{R}$	Total green production of the system given by the sum of all farms' green production.
Total_I_B	$\mathbb{R}$	Total brown investment of the system given by the sum of all farms' brown investment.
Total_I_G	$\mathbb{R}$	Total green investment of the system given by the sum of all farms' green investment.
real_beta	$\mathbb{R}$	Brown technical progress levels of the economy
real_gamma	$\mathbb{R}$	Green technical progress levels of the economy
C_CO2	$\mathbb{R}^+$	Carbon emissions produced by brown production in the system.
B_farms	$\mathbb{R}^+$	Number of farms that do not hold green capital.

Table 3: State variables of the Statistician agent.

Name	Domain	Parameter description
agents	$\mathbb{N}$	Number of farms in the simulation
steps	$\mathbb{N}$	Length of the simulation
eta0	$\mathbb{R}^+$	Initial technical progress
InvStrg	$\mathbb{N}^{\texttt{agents}}$	List with each farm initial investment strategy
Tot_K0	$\mathbb{R}^+$	The system's initial capital
K0	$\mathbb{R}^{\texttt{agents}}$	List with each farm initial capital
beta	$\mathbb{R}^+$	Initial brown technical progress
gamma	$\mathbb{R}^+$	Initial green technical progress
Tot_B0	$\mathbb{R}^+$	The system's initial brown capital.
Tot_G0	$\mathbb{R}^+$	The system's initial green capital.
B0	$\mathbb{R}^{\texttt{agents}}$	List with each farm's initial brown capital.
G0	$\mathbb{R}^{\texttt{agents}}$	List with each farm's initial green capital.
interaction	{True, False}	To indicate whether the agents interact or not.
interaction_type	{'Voter', 'MajorityRule'}	Indicates type of interaction
interacting_farms	$[0, \texttt{agents}]$	Number of interacting farms under the Voter dynamics
network_topology	{'SW', 'FC'}	Network topology for the Voter dynamics. It can either be a Small World (SW) or a Fully Connected (FC) graph.
network_randomness	$[0, 1]$	The probability of rewiring each edge.
number_of_friends	$[0, \texttt{agents}]$	**Voter dynamics:** number of farms in the farm's neighbourhood. **Majority rule:** Number of the group size that meet randomly at each time step.
time_steps_in_1_year	$\{1, 4, 6\}$	How many simulation time steps are equivalent to one year for counting emissions.
seed	$\mathbb{R}$	Seed for the model's random number generator.

Table 4: Simulation parameters

1.3 Process overview and scheduling

1.3.1 Simulation setup

Upon initialization of the model, the *economy* is created. During this initialization the farms and the statistician are created, for more information see section 3.1.

1.3.2 Proceeding in time

In the model, time proceeds in discrete steps without there being an explicit real-world equivalent of a time step. In each time step, beginning with time $t = 1$ the **model's step function** is called. This function does the following.

1. **ABRam-T** and **-BG**: For each farm, the **farm's capital function** is called, which updates and computes the farm's capital $K_t^{(i)}$ for the current time step (for more information see Section 3.3.1).
2. **ABRam-T** and **-BG**: The statistician computes the technological progress `real_tech_prg` for the current time step using its function **technical_progress** (for more detail see Section 3.3.2).
3. **ABRam-T** and **-BG**: The statistician computes the system's total capital `Tot_K` by adding the farms' capital.

T.4 For each farm, the **farm's production function** is called (for more information see 3.3.5).

T.5 The farms' **investment** process starts.

- **T.**5.1. For each farm, the **farm's expectations formulation function** is called, which updates the farm's expectations about the other farms' investment (for more information see 3.3.6).
- **T.**5.2. For each farm the **farm's investment function** is called as described in section 3.3.7.

BG.4 For each farm, the **farm's read news function** is called (for more information see 3.3.3).

BG.5 For each farm, the **farm's labor function** is called as described in section 3.3.4.

BG.6 For each farm, the **farm's production function** is called as described in section 3.3.5.

BG.7 For each farm the **farm's investment function** is called as described in section 3.3.7.

8. **ABRam-T** and **-BG**: The statistician computes the gross investment of the system by adding the farms' investment.
9. **ABRam-T** and **-BG**: The statistician computes the GDP growth rate of the system (for more detail see Section 3.3.8).
10. **ABRam-T** and **-BG**: The statistician computes the GDP of the system by adding the farms' production.
11. **ABRam-T** and **-BG**: For each farm the **farm's consumption function** is called as described in Section 3.3.9.
12. **ABRam-T** and **-BG**: For each farm the **farm's utility function** is called (for more detail see 3.3.10).
13. **ABRam-T** and **-BG**: The statistician computes the system's gross consumption by adding the farms' consumption.
14. **ABRam-T** and **-BG**: The statistician computes the system's capital-to-output ratio by calculating the ratio of the total capital to the GDP.

15 **ABRam-BG:** The farms interact via the **opinion process** as described in section 3.3.11, and update their opinion using the **farm's update opinion function**.

16 **ABRam-BG:** The statistician computes the emissions of the brown production (for more details see section 3.3.12).

After each time step the **model's update function** (including at $t = 0$) is called. This function records the dynamic variables of the system.

2 Design Concepts

2.1 Basic Principles

The models build on the well known **one sector Ramsey growth model** with endogenous and directed technical progress.
ABRam-T: Endogenous technical progress is introduced via learning by doing and knowledge spillovers. Each agent **maximizes its intertemporal utility** function for its present and next time step, to decide how much to invest in the current time step.
ABRam-BG: Directed technical progress is introduced by incorporating two types of capital: brown (high carbon intensity) and green (low carbon intensity) with their respective technological progress levels. Each agent invests in either green or brown capital depending on their beliefs.

2.2 Emergence

In an economy of n farms, where $n \in \mathbb{N}$, the key outcome is its growth pattern, in terms of Gross Domestic Product (GDP) and its growth rate, and a transition pattern to a lower carbon intensity green economy. The economy can grow, stagnate, or contract, depending on the initial capital and parameters of the farms. The transition pattern is given by the green output share in the economy, in particular, we consider a "green transition" the case that this share is larger than 85% [5].

2.3 Adaptation

ABRam-T: The model includes no adaptation.
ABRam-BG: The model's farms have one adaptive behavior: deciding whether to invest in green or in brown capital. This decision is modeled as indirect objective seeking: a farm changes its opinion if she interacts with farms with opinions different to hers. This behavior is modeled using opinion dynamics, two simple dynamics are used: the Voter model and Majority rule dynamics, for more information see section 3.3.11.

2.4 Objectives

Each farm aims to maximize her intertemporal utility over two time steps. The utility of a farm depends on her current and future consumption. By solving this maximization problem, farms can determine the optimal investment for the present time period that will maximize their utility. How the farms solve their maximization problem depends on various assumptions about the information available to them. These assumptions form the basis of the farms' investment strategy (for more information see 3.3.7).

2.5 Learning

The model includes no learning.

2.6 Prediction

ABRam-T: To formulate the **witty farm's** investment, the farm needs to predict how much the other farms are going to invest. This is done by formulating expectations using one of two rules: an **adaptive rule**, or a **trend following rule**, for more detail see Section 3.3.6.
ABRam-BG: The adaptive behaviour of the farms is based on the implicit prediction that eventually a green transition will be necessary.

2.7 Sensing

The **farms** are aware of the system's technological progress level, or levels, to produce. This information is provided to them by the statistician.

ABRam-T: In addition, the **witty farms** need the system's gross investment and the total capital to formulate their expectations. Hence, they sense the system's total investment from the statistician.

ABRam-BG: In addition, the **farms** are able to sense the others farm's opinion to modify their investment behaviour. The sensing mechanisms depend on the selected belief dynamics for the simulation, for more information see section 3.3.11.

2.8 Interaction

There are two types of interactions in the model: the interaction between the farms and the statistician, and the interaction among the farms themselves. The farms directly interact with the statistician when the statistician gathers their data and shares information about the current technological progress (and total investment).

The farms interact with one another through technological progress, resulting from all farms' investment. This can be seen as a mediated interaction.

ABRam-T: The farms do not interact with one another directly.

ABRam-BG: The farms interact with one another directly to communicate their opinion. The interaction mechanism depends on the selected belief dynamics (section 3.3.11).

2.9 Stochasticity

In the model, stochasticity can be used to initialize the system by providing different capitals to the farms.

ABRam-T: The total initial capital of the system is distributed among the farms using a Pareto distribution. By providing different capitals to the farms, we create a heterogeneous economy to study effects of technological progress in the system. Other than at initialization, the model is deterministic.

ABRam-BG: Stochasticity is used during the simulation in the interaction process among farms, accordingly to the selected the belief dynamics, where the farms that interact at each time step are randomly selected (for more information see section 3.3.11).

2.10 Collectives

The model does not include collectives.

2.11 Observation

In the model, we collect all the agents' variables at each time step. The statistics of the system are computed by the statistician.

3 Details

3.1 Initialization

The initialization of the model is done when the model is created (see Section 1.3.1 for scheduling). A `parameters` dictionary is set up to run scenarios with differing initial conditions. The simulation `parameters` that can be changed upon initialization are listed in Table 4. We describe the setup of the agents in the following.

3.1.1 Initialization of the farms

The simulation starts with the initialization of the network, the farms and their state variables.

For the **ABRam-T model** the attribute `InvStrg` of the `parameters` dictionary determines how many farms of each type are created. The `InvStrg` is a list that contains the investment strategies of the farms. In simulations that include witty farms, both the expectation weight factor for the adaptive rule and the extrapolation coefficient for the trend follower rule are restricted to a single value.

For the **ABRam-BG model** two capital variables are initialized: $B_0^{(i)}$ and $G_0^{(i)}$. The farms initially can only hold one type of capital. The initial capital variable is either the same constant $K_0^{(i)} = B_0^{(i)} + G_0^{(i)}$ for the farms or is set randomly using a Pareto distribution. The farms' initial capital(s) is/are saved to the list(s) `k0`/`G0` and `B0`. After the capital initialization, the state variables for labour (`b` and `g`) are computed according to the farms' capital holding. The state variable for production (`P`/`P_B` and `P_G`) is initialized by computing the farms' output using the farm's initial capital and the production function (for details see Section 3.3.5). For the farm's state variable for investment (`I`/`I_G` and `I_B`) we use as well the initial capital and the farm's investment function given by their investment strategy (for details see Section 3.3.7). All the farms' expectations on how much the others are going to invest are set to zero, i.e. `expected_I_others = 0`, and the same holds for the variables that indicate past investment and gross past investment (`past_I` and `past_gross_I`) since we assume no prior history or information before time zero. The farms' initial consumption (`c`) is obtained by the farm's consumption function (for more detail see 3.3.9) and the farms' utilities are computed using the utility function (for details see Section 3.3.10).

3.1.2 Initialization of the statistician agent.

Then one agent statistician is created for the simulation. Since the statistician agent computes aggregate variables, it can access the values of all farms. The statistician computes `Total_B`, `Total_G` and `Total_K` which is the sum of the agent's respective capitals. Technical progress is given by the parameter `eta0`/`beta` and `gamma`. The state variables `Total_P_G`, `Total_P_G` , `GDP`, `Total_I_B`, `Total_I_B`, `Total_I` and `Total_C` (see Table 3) are initialized by summing the corresponding farm's variables. We initialize the growth rate variable `g_rate` by setting it equal to zero (since we assume no prior history or information before time zero). The variable for the capital-to-output ratio is initialized by the ratio between the total capital and the GDP. Finally the variables `C_CO2` and `B_farms` are initialized.

3.1.3 Finalize Initialisation

In the last part of the initialization, the **model's update function** is called, this function records the variables of interest of the system.

3.2 Input data

The model does not use input data to represent time-varying processes.

3.3 Submodels

In this section we describe the submodels, these are methods either used by the **farms** or by the **statistician**.

3.3.1 Capital (Farm)

The farm's capital at the current time step $K_t^{(i)}$ is computed using the following recursive function:

$$K_t^{(i)} = (1 - \delta)K_{t-1}^{(i)} + I_{t-1}, \tag{1}$$

where $K_{t-1}^{(i)}$ is the capital from the previous time step, δ is the depreciation rate (see Table 1), and $I_{t-1}^{(i)}$ is the investment from the previous time step. For ABRam-BG farms, initially, the capital is either the farm's brown ($B_{t-1}^{(i)}$) or green ($G_{t-1}^{(i)}$) capital. Once a brown farm switches to green investment, the capital equation holds for its green capital stock, while the brown capital stock evolves via depreciation only, i.e.,

$$B_t^{(i)} = (1 - \delta)B_{t-1}^{(i)}.$$

3.3.2 Technical Progress (Statistician)

Technical progress η grows proportionally to aggregate capital. We assume learning-by-doing which goes hand in hand with capital accumulation. The state variable `real_tech_prg` is calculated as a recursive function. It is the product of the previous technical progress η_{t-1} and the ratio of current total capital to previous total capital:

$$\eta_t = \frac{K_t}{K_{t-1}} \eta_{t-1}. \tag{2}$$

For the state variables `beta` and `gamma` equation (2) is also used but either for the system's brown (B_t) or green (G_t) capital.

3.3.3 Read news (Farm)

While in the ABRam-T version, the information from the statistician is passed to the farms implicitly, in ABRam-BG, the farms collect the current levels of the green and brown technical progress from the statistician via this explicit function, and compute the ratio between them.

3.3.4 Labour allocation (Farm)

In ABRam-BG, farms that hold two capital stocks need to optimally allocate their labour (we assume a fixed amount of $L_t^{(i)} = 1$ for all t and all farms i) to the two production processes. This is done by maximizing total production:

$$\max_{b_t^{(i)}, g_t^{(i)}} P_t^{(i)} = \left(P_t^{G^{(i)}} + P_t^{B^{(i)}} \right) \text{ such that } g_t^{(i)} + b_t^{(i)} = L_t^{(i)}$$

which means that for all t,

$$g_t^{(i)} = \frac{G_t^{(i)}}{B_t^{(i)}} \left(\frac{\gamma_t}{\beta_t} \right)^{\frac{1-\alpha}{\alpha}} b_t^{(i)} \tag{3}$$

$$b_t^{(i)} = \frac{L_t^{(i)}}{\frac{G_t^{(i)}}{B_t^{(i)}} \left(\frac{\gamma_t}{\beta_t} \right)^{\frac{1-\alpha}{\alpha}} + 1} \tag{4}$$

3.3.5 Production (Farm)

The farms in ABRam-T produce an output using the Cobb-Douglas production function:

$$P\left(K_t^{(i)}, \eta_t\right) = \left(K_t^{(i)}\right)^{\alpha} (\eta_t)^{1-\alpha}, \tag{5}$$

where $K_t^{(i)}$ is their current capital given by `K`, η_t is the system's current technical progress given by the statistician's variable `real_tech_prg` , and α is the output elasticity of the capital (see Table 1). Note that in this model we consider that farms always employ their constant labor amount of $L = 1$.

For ABRam-BG, farms with a single capital stock use the corresponding formula based on brown ($B_t^{(i)}$) or green ($G_t^{(i)}$) capital given by `B` and `G` correspondingly and the statistician's variables `beta` (brown technical progress) or `gamma` (green technical progress). If a farm holds both types of capital, the optimal allocation of labour is used and production is the sum of output from both production processes, i.e.,

$$P_t^{(i)} = (G_t^{(i)})^{\alpha} (\gamma_t g_t^{(i)})^{1-\alpha} + (B_t^{(i)})^{\alpha} (\beta_t b_t)^{1-\alpha}$$

3.3.6 Formulation of expectations (Farm)

The farms state variable `expected_I_others` is updated depending on the farms' investment strategy. Farms that are *collaborative* or *ignorant* do not need expectations, hence the value of their is zero and needs no updating. When the farms are *witty*, they generate their expectations about how much the other farms invest following either an **adaptive rule** or a **trend follower rule**.

If the witty farm follows an **adaptive rule**, to compute her expectations on how much the other farms invest $I_t^{(\sim i),e(i)}$, she uses:

$$I_t^{(\sim i),e(i)} = (1-\lambda_A) I_{t-1}^{(\sim i),e(i)} + \lambda_A (I_{t-1} - I_{t-1}^{(i)}), \tag{6}$$

where $I_{t-1}^{(\sim i),e(i)}$ is the agent's last forecast, $I_{t-1}^{(i)}$ is the agent's last time investment, I_{t-1} is the system's past gross investment, and λ_A is the expectations weight factor, given by the parameter `adapt_cns` (Table 1).

If the witty farm uses a **trend following rule**, to compute her expectations on how much the other farms invest, $I_t^{(\sim i),e(i)}$, the farm uses:

$$I_t^{(\sim i),e(i)} = I_{t-1} - I_{t-1}^{(i)} + \lambda_T (I_{t-1} - I_{t-2} - (I_{t-1}^{(i)} - I_{t-2}^{(i)})). \tag{7}$$

where $I_{t-1}^{(i)}, I_{t-2}^{(i)}$ correspond to the farm's previous investment and the farm's investment from two-time steps ago ($I_{t-2}^{(i)}$ is given by `past_I`, see Table 2), I_{t-1}, I_{t-2} correspond to the system's previous gross investment and the gross investment from two-time steps ago given (I_{t-2} is given by the variable `past_total_I` see Table 2) , and λ_T is the extrapolation coefficient for the trend following rule, given by `trend_cns` (Table 1).

3.3.7 Investment (Farm)

The investment functions are the farm's solution to the maximization problem of her intertemporal utility for two times steps, i.e.,

$$\max U = \max(\ln(C_t^{(i)}) + \rho \ln(C_{t+1}^{(i)}) \tag{8}$$

given:

$$C_t^{(i)} = P_t - I_t \tag{9}$$

$$K_{t+1}^{(i)} = (1-\delta) K_t + I_t \tag{10}$$

$$C_{t+1}^{(i)} = P_{t+1}, \tag{11}$$

where ρ is the factor to discount future felicity, δ is the rate of capital depreciation, and the farm's production P_t is given by the Cobb-Douglas production function from equation (5).

To solve this maximization problem, we consider different assumptions about the information available to the farms. This leads to different solutions and, consequently, different investment strategies. (for more detail see [3]).

We say a farm is **collaborative** when the farm acts like a benevolent planner in a closed economy. Then, the investment of farm i in a decentralized economy, $I_t^{\text{Co(i)}}$, at time t is

$$I_t^{\text{Co(i)}} = \frac{1}{1+\rho} \left(\rho \left(K_t^{(i)} \right)^{\alpha} \eta_t^{1-\alpha} - (1-\delta) K_t^{(i)} \right). \tag{12}$$

When the farm is **ignorant**, she neglects her contribution to the aggregate capital stock, therefore she considers the technical progress of the future, η_{t+1}, as a given constant. Farm i's investment, $I_t^{\text{Ig}(i)}$, is given by

$$I_t^{\text{Ig}(i)} = \frac{1}{1+\alpha\rho} \left(\rho\alpha \left(K_t^{(i)} \right)^{\alpha} \eta_t^{1-\alpha} - (1-\delta) K_t^{(i)} \right). \tag{13}$$

The **witty** farm explicitly considers that the future technical progress η_{t+1} contains her investment. The solution to her maximization problem leads to a quadratic equation:

$$\begin{aligned} &- (1+\rho) \left(I_t^{(i)} \right)^2 + \left[\rho P_t^{(i)} - (2+\rho) \hat{K}_t^{(i)} - (1+\rho\alpha) \left(K_{t+1}^{(\sim i),e} \right) \right] I_t^{(i)} \\ &+ \rho P_t^{(i)} \hat{K}_t^{(i)} + \rho\alpha P_t^{(i)} K_{t+1}^{(\sim i),e} - \left(\hat{K}_t^{(i)} \right)^2 - \hat{K}_t^{(i)} K_{t+1}^{(\sim i),e} = 0 \end{aligned} \tag{14}$$

where $K_{t+1}^{(\sim i),e} = (1-\delta)\sum_{j\neq i} K_t^{(j)} + I_t^{(\sim i),e(i)}$ is the expected future capital of the others, $\hat{K}_t^{(i)}$ is the farm's depreciated capital. The solution of this quadratic equation provides two critical points. The farm chooses the solution that belongs to the feasible domain, i.e. $-(1-\delta)K_t^{(i)} < I_t^{\mathrm{Wi}(i)} < P_t^{(i)}$.

In the **ABRam-BG** model, the farms' investment strategy is generally **ignorant**, i.e. they invest according to equation (13). Whether the investment augments brown or green capital depends on the farm's environmental belief; farms with opinion 1 invest into green capital, farms with opinion 0 into brown capital. The existing capital $K_t^{(i)}$ in equation (13) is initially either the agent's brown ($B_t^{(i)}$) or green ($G_t^{(i)}$) capital, while for farms that have switched from 0 to 1 in the opinion dynamics process – i.e., farms that initially held brown capital but changed their opinion and hence begin to invest in green capital – the sum of both capital stocks is used, such that

$$I_G^{(i)} = \frac{1}{1+\alpha\rho}\left(\rho\alpha P_t^{(i)} - (1-\delta)(B_t^{(i)} + G_t^{(i)})\right), \tag{15}$$

$$I_B^{(i)} = 0. \tag{16}$$

3.3.8 GDP growth rate (Statistician)

The gross domestic product growth rate g_t is given by

$$g_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}. \tag{17}$$

Where GDP_t is the sum of the farms' production and GDP_{t-1} is the system's previous time step gross domestic product.

3.3.9 Consumption (Farm)

Consumption $C_t^{(i)}$ of each farm is calculated as the difference between the farm's production and investment values

$$C_t^{(i)} = f(K_t^{(i)}, \eta_t L) - I_t^{(i)}.$$

3.3.10 Utility (Farm)

The farm computes her instantaneous utility $u(C_t^{(i)})$, given by

$$u(C_t^{(i)}) = \ln(C_t^{(i)}). \tag{18}$$

The farm's lifetime (or cumulative) utility $U(C_0^{(i)}, \ldots, C_t^{(i)})$ is given by adding the farm's instantaneous utility to the previous value,

$$U(C_0^{(i)}, \ldots, C_t^{(i)}) = \sum_{T=0}^{T=t-1} U(C_T^{(i)}) + u(C_t^{(i)}). \tag{19}$$

3.3.11 Opinion process

To update the farms' beliefs we consider two types of belief dynamics: the Voter model and the Majority rule. Each farm is endowed with a binary belief/opinion $\mathcal{O}_t^{(i)} \in \{0,1\}$, i.e. at time t, the farm either cares ($\mathcal{O}_t^{(i)} = 1$) or does not care ($\mathcal{O}_t^{(i)} = 0$) about the environment, where the latter is the default. The farm's belief can change by interaction with a neighboring agent who has a different belief, however, we consider agents that do care stubborn, that is, they will not revert to $\mathcal{O}_t^{(i)} = 0$. When a farm's belief changes, she starts to care for the environment. Therefore her investment behaviour changes, i.e., she is going to start investing in green capital.

The voter model (Farm): At each time step t, a random agent i is drawn and randomly chooses an agent j from its neighborhood n_i, so for the farm i we have that her belief is update as follows:

$$\mathcal{O}_{t+1}^{(i)} = \begin{cases} \mathcal{O}_t^{(j)} \text{ if } \mathcal{O}_t^{(j)} = 1 \text{ where } j \in n_i \\ \mathcal{O}_t^{(i)} \text{ otherwise} \end{cases} \tag{20}$$

The majority rule (Economy): At each time step t, when a group of r random agents meet and discuss the issue, the individuals update their beliefs to match the beliefs of the majority [2]. For our set-up, green agents are stubborn, therefore if the majority's belief is $\mathcal{O}_t = 0$, all the discussion group's farms keep their previous belief. For a group $G_r = \{1, ...r\}$, of r random agents, we define the group's average belief at time t,

$$\hat{\mathcal{O}}_t = \frac{1}{r} \sum_{j \in G_r} \mathcal{O}_t^{(j)}.$$

So at each time step t, for every farm in the discussion group, the beliefs are updated as follows:

$$\mathcal{O}_{t+1}^{(k)} = \begin{cases} 1 & \text{when } \hat{\mathcal{O}}_t > 1/2 \\ \mathcal{O}_t^{(k)} & \text{when } \hat{\mathcal{O}}_t \leq 1/2 \end{cases} \tag{21}$$

3.3.12 Count emissions (Statistician)

To account for emissions produced by the brown production process, we use the equation presented in [1], without taking into account land-use, non-abatable emissions, or emissions produced by green capital.

$$\text{Em } CO_2 = \sigma(t) P_{B,t}, \tag{22}$$

where $P_{B,t}$ is the economy's total brown production at time step t, and $\sigma(t)$ the carbon intensity. This carbon intensity evolves according to

$$\sigma(t+1) = \sigma(t)(1 + [\sigma(t_0)(1 - C_{yrate})]^{1/t}). \tag{23}$$

Here $C_{yrate} = -1.5\%$ is the yearly rate of carbon intensity [1], and t indicates how many time steps of a simulation are interpreted as being equivalent to a year. It is given by the parameter `time_steps_in_1_year` (see Table 3).

Acknowledgements

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – The Berlin Mathematics Research Center MATH+ (EXC-2046/1, project ID: 390685689).

References

[1] Lint Barrage and William D Nordhaus. Policies, projections, and the social cost of carbon: Results from the dice-2023 model. Working Paper 31112, National Bureau of Economic Research, April 2023.

[2] Claudio Castellano, Santo Fortunato, and Vittorio Loreto. Statistical physics of social dynamics. *Rev. Mod. Phys.*, 81:591–646, May 2009.

[3] Aida Sarai Figueroa Alvarez and Sarah Wolf. Exploring heterogeneity and expectations in technical progress – an exercise in agentization. February 2024.

[4] Joël Foramitti. Agentpy: A package for agent-based modeling in python. *Journal of Open Source Software*, 6(62):3065, 2021.

[5] F. Lamperti, G. Dosi, M. Napoletano, A. Roventini, and A. Sapio. Climate change and green transitions in an agent-based integrated assessment model. *Technological Forecasting and Social Change*, 153:119806, April 2020.

[6] Gesine Steudle Sarah Wolf Jahel Mielke and Carlo Jaeger. Green growth mechanics: The building blocks. Climate Global Forum, January 2018.