How the Laws of Logic Lie About Mathematical Objects

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> *One should avoid bewitchery by images of object-language domains.*
>
> —J. Azzouni, *Deflating Existential Consequence*.

A simple argument for realism about mathematical objects runs as follows:

(1) There are four prime numbers between 1 and 10.

(2) Therefore, there are prime numbers.

(C) Therefore, there are numbers.

Call a philosopher denying (C) a nominalist about numbers and one asserting (C) a realist about numbers.

Concerning this simple argument, the realist, it would seem, has the upper hand. After all, it is trivial to verify that there are indeed four prime numbers between 1 and 10, and so, (1) is true; (2) follows trivially from (1), and (C) follows trivially from (2).

Hence, (C) is true and there are numbers. The nominalist must therefore say, somewhat incoherently, that while it is true that there are prime numbers between 1 and 10, there are in fact no numbers. Meanwhile, the realist can take our discourse at face value and accept both the premises and the conclusion.[1]

In this paper, I will argue, contrary to what otherwise seems obvious, that it is open to the nominalist to claim that the simple argument is in fact invalid—and further, that this invalidity impacts other semantic arguments for realism about numbers, in particular arguments that claim that we should be ontologically committed to those objects our discourse quantifies over or to those objects that are referred to by singular terms in true statements. This in turn will make a novel nominalist position is available which I will refer to as 'deflationary nominalism'.[2] This view is composed of three interrelated claims, namely that

(i) Objects just are what singular terms refer to, and that reference is successful when those singular terms figure in true statements.

(ii) Reference to mathematical objects makes no demands on the world in Linnebo's sense, and is thus thin (Linnebo 2018).

(iii) Existence necessarily makes demands on the world, and hence that if *K*s exist, reference to *K*s is thick.

It follows from (i)–(iii) that (C) is false. From the perspective of the realist, this set of claims is of course inconsistent, since from (i) it follows, via simple arguments, that (C)

1. Or so it is claimed. See Clarke-Doane 2024 for an argument to the contrary.

2. I should stress, however, that my deflationary nominalism is not what Azzouni (2017) refers to by the same name. For Azzouni, what is deflated is *existential consquence* and hence that sentences like (C) are *true*, but do not entail that numbers exist. The view I'm arguing for is that *reference* to mathematical objects is deflated and that sentences like (C) are *false*.

is true. The goal of the paper is to show that (i)–(iii) is indeed a coherent view, by way of defending the claim that simple arguments are in fact invalid.

The structure of the paper is a follows: In §1, I will discuss some recent views in the philosophy of logic that play a role in my argument, namely what has been called *logical nihilism* and a related view, *logical particularism*.[3] I will also, following Gillian Russell's recent paper (2024), introduce the idea of idealisation in logic and argue that in virtue of certain idealisations, formalisation already builds into our models that reference requires existing objects. I conclude that if we reject what Thomas Hofweber has called "the ideal of deductive logic" (Hofweber 2007), the nominalist can coherently reject simple arguments as being invalid.

In §2, I will give an outline of deflationary nominalism and argue that by rejecting simple arguments and helping themselves to the resources of a family of views that I will refer to as 'easy ontology' (which includes the views of Amie Thomasson, as well as the neo-Fregeans and Øystein Linnebo, among others) the nominalist can provide a deflationary account of sentences like (1) that does not entail that ontological conclusions like (C) is true, provided that simple arguments are invalid. On this view, (1) genuinely refers to numbers, but does not entail ontological commitment to numbers, and further, given the assumptions derived from easy ontology, (C) is a *genuine* counterexample to simple arguments. The two strands of the argument therefore mutually support each other, the rejection of simple arguments and deflationary nominalism.

In §3, I will argue that the resulting view is indeed coherent, by comparing it with the situation in the philosophy of fiction. I will argue that the deflationary nominalism outlined in§2 is not only coherent, but also respects two intuitions that nominalists

3. See e.g. Cotnoir 2018, Russell 2018a and Russell 2018b for the former and Payette and Wyatt 2018 and Wyatt and Payette 2021 for the latter.

would like to have, both in the philosophy of fiction and in the philosophy of mathematics, namely that certain statements about numbers and fictional characters are true, but numbers and fictional characters do not exist—where both claims are read at face-value.

I will not be concerned with giving a fully sketched nominalist position, since at every point too much can be said. I will therefore only give a brief outline of what the deflationary nominalist position is, even if there are lots of objections to consider or different ways to go. The general claim of the paper is therefore more modest than otherwise might appear, namely that *if* one accepts the positions in the philosophy of logic outlined in §1, the deflationary nominalism presented in §§2–3 is a coherent option that should not be prematurely rejected.

1 Some preliminaries

What do I mean when I say that the simple argument given in the introduction is invalid? Merely what philosophers usually mean when they say that an argument is invalid, namely that the premises can be true and the conclusion false. This claim might seem preposterous, as it is trivial to show that the simple argument is valid, e.g. by translating the relevant sentences into the language of first-order logic and verifying that the conclusion follows from the premises by elementary logical laws. Alternatively, we might show that there is no model in which the premises are true and the conclusion false, or we could just intuitively *see* that accepting the premises and denying the conclusion commits us to a contradiction.

For the reasons just outlined, simple arguments seem like perfectly good inferences to make. But as Hofweber points out (2005a), it is open to the nominalist to reject them,

provided they can explain why we would be "fooled" into making such inferences and why they seem so compelling to us. As it so happens, recent (and some not so recent) debates in the philosophy of logic have opened up the possibility of doing just that.

In this section, I want out outline these positions, arguing that it is open to the nominalist to adopt a view in the philosophy of logic that makes their rejection of simple arguments coherent. This will allow the development of a deflationary nominalism that in turn can, given a few assumptions a nominalist is likely to accept, make it so that (C) should be considered a genuine counterexample to simple arguments. I will not, however, argue for the *truth* of such philosophical views about logic here, rather establishing a conditional claim, that if such views are coherent, then nominalist's rejection of simple arguments cannot be declared incoherent based on logic alone.

1.1 Logical nihilism and particularism

Since the publication of Beall and Restall's book on logical pluralism (Beall and Restall 2006, see also Beall and Restall 2000), there has been a lively debate in the philosophy of logic on the question of whether or not there is one true logic or many—and indeed what such a question even means. One strand of this debate has been framed around what has been called the "canonical application of logic" (Priest 2014) which is the idea that the job of logic to give an analysis of logical consequence in natural language.[4] Here is a much cited passage from Beall and Restall:

> Logic, in the core tradition, involves the study of formal languages, of course, but the primary aim is to consider such languages as interpreted: languages which may be used either directly to make assertions or denials,

4. See also Bueno and Colyvan 2004, Resnik 2004 and Cook 2010.

> or to analyse natural languages. Logic, whatever it is, must be a tool useful for the analysis of the inferential relationships between premises and conclusions expressed in arguments we actually employ. If a discipline does not manage this much, it cannot be logic in the traditional sense. (Beall and Restall 2006, p. 8)

Accordingly, logical monists say that there is one and only one formal system that correctly describes the consequence relation in natural language and pluralists say that many formal systems do.

In recent papers by a few authors, among them A. J. Cotnoir (2018) and Gillian Russell (2018a, 2018b), the possibility has been broached that there is a third option, *logical nihilism*—that *no* formal system does so.[5] Importantly, this should not be confused with the view that there are no valid arguments or the claim that there are no norms of reasoning that natural language arguments obey.[6] It is rather that no formal system can accurately capture natural language inference, and hence, even though Cotnoir and Russell do not emphasise this point too strongly, valid inferences are not valid in virtue of formal rules that a given inference is an instance of—otherwise, more than one formal system could capture such inferences and their view would be a kind of pluralism.

For that reason, logical nihilism is related to another view, logical particularism, articulated in a few recent papers by Nicole Wyatt and Gillman Payette.[7] According to logical particularism, there is no system of logical rules and principles that has a universal application and the validity of a valid argument is not to be explained in virtue of it

5. See also Franks 2015.

6. That view has, however, been defended. See e.g. Estrada-González 2021.

7. See Payette and Wyatt 2018, Wyatt and Payette 2021, as well as Wyatt and Payette 2018b and Wyatt and Payette 2018a.

exhibiting a certain form or being an instance of a valid rule. The relationship between formal logic and natural language arguments should rather be thought of, on Payette and Wyatt's view, in the same way as we think of the relationship between a mathematical model and some phenomenon to be explained: A meteorological simulation of hurricanes might, for example, explain why a given hurricane went the path that it did, but we would not say that it went the way that it did *in virtue* of any features of the simulation.

And so, for the logical particularist, it is possible to use formal systems to *explain* why certain arguments are valid—just as we can use a mathematical model to explain why hurricanes behave as they do, and that is compatible with the view that some arguments that the model is silent on are actually valid, or that the model deems certain arguments valid, even if they are not. For a logical particularist, validity is a property of a given argument, not of schemes or rules from which the arguments inherit their validity. Thus, there are valid arguments, and while formal systems can be useful in explaining why, validity and invalidity are largely independent of formalisation.

There isn't space here to canvass all of the arguments that have been brought forth in support of logical nihilism or particularism nor to evaluate their cogency. So, I am only presenting this as an antecedent in a conditional claim, namely, that if the nominalist accepts either logical nihilism or logical particularism, then it is not incoherent to deny the validity of simple arguments.[8]

The one style of argument that I am interested in here, however, concerns the idea of *counterexamples*. This is because, as Russell points out, logical laws are meant to be extremely general and so, the claim that there are logical laws is very strong, and thus the

8. Logical nihilists differ among themselves, as well as with logical particularist, but for our purposes here, they are close enough as to count as very similar views.

claims of the logical nihilist are correspondingly weak. We might, for example, think that McGee's (1985) counterexample to modus ponens is a genuine counterexample, giving us an argument where the premises are true and the conclusion false, while at the same time think that modus ponens is a perfectly good rule to use in most cases.[9] Similarly, most dialetheists think that dialetheia are rare and that disjunctive syllogism is a fine rule most of the time and most intuitionists think that the law of excluded middle is likewise perfectly acceptable in most cases. The nihilist can take the same attitude to *every* putative logical law.[10]

The logical nihilists and particularists thus reject the orthodox view that correct inference is a matter of following (or thinking in accordance with) logical laws that have universal application and no exceptions whatsoever—what Thomas Hofweber has called the *ideal of deductive logic* (Hofweber 2007). For the rest of the paper, I will argue that by rejecting this orthodoxy, it is open to the nominalist to resist many influential arguments for realism about mathematical objects. In particular, I will argue that formalising certain vernacular statements into a formal language necessarily entails a certain amount of idealisation and that such idealisation can have philosophical consequences—in our case making invalid arguments about mathematical objects seem valid.

The argument that I will outline is essentially twofold, (i) that by translating simple arguments into the language of first-order logic, the idealisation that is inherent in

9. For a counterexample against modus tollens, see Yalcin 2012.

10. There is not space to cover it here, but to demonstrate this point, Russell uses the device of self-reference to show how counterexamples could be generated against any putative logical law. For example, slightly adapting Russell's device, 'This sentence does not occur in the scope of a conjunction' is true, and hence, ' 'This sentence does not occur in the scope of a conjunction' and 2+2=4' ' is false. This is a counterexample to the otherwise solid rule of conjunction introduction since the latter does not follow from the former.

I agree with Russell in that trying to explain away such examples as spurious is *ad hoc*.

the very process of translation builds in the assumption that mathematical objects exist and (ii) that if we make certain plausible assumptions about reference to mathematical objects and existence, then the nominalist can view (C) as a *genuine* counterexample to simple arguments, one that should not be ruled out as *ad hoc*.

1.2 Idealisation and how the laws of logic lie

In a recent paper by Russell, "How the Laws of Logic Lie" (2024), which the title of this paper is an obvious nod to, she picks up on the idea that it is the job of logic (as a field of study) to analyse natural language consequence and that when, in the pursuit of this goal, logicians build their formal languages, they are doing something very similar to what other scientists are doing when they come up with models of empirical phenomena. But because natural languages are very complex, this process involves a certain amount of idealisation that is comparable to idealisation in physics. This has the result that many logical laws are strictly speaking false when applied to what they are models of, natural language consequence.

What does Russell mean when she says that the laws of logic are false? Consider her example from geometry: Geometry speaks of triangles and lines, but the triangles and lines that the theory speaks of are not the triangles and lines of this world; they are always at least a little bit crooked and a little bit skewed. It is therefore tempting to say that the Pythagorean Theorem is only true of the idealised triangles, and not of triangles in the real world. Similarly, the ideal gas law really only talks about ideal gases, and not real gases.

This does not, in itself, lead to any false statements. If the Pythagorean Theorem is true about idealised triangles and the ideal gas law is true about ideal gases, the universally quantified conditional resulting from quantifying over the idealised objects is

vacuously true about the real world, but not false: there are no idealised triangles nor ideal gases. This leads to a dilemma, however, which is that if we only quantify over idealised objects, the claim is vacuously true, but can hardly be said to be *about* the real counterparts of the idealised objects, but if we quantify over real triangles and real gases as well, the laws are strictly speaking false. The laws thus derived really only tell us something about the models, and not reality—strictly speaking.

In deriving these laws, we've made assumptions that simplify what we are reasoning about, without regard to whether they are true about the target phenomenon; we have *idealised*. This idealisation, however, despite resulting in laws that are strictly speaking false, can be very useful, since without it our models would be too complex and unwieldy to use. We can, for example, reason about a real triangle using the Pythagorean Theorem and get a good approximate result, even if the triangle isn't really a right triangle and the length of the hypotenuse isn't really the sum of the squares of the two other sides—close enough, though. The theorem is also extremely important in deriving other theorems, giving it a crucial theoretical role. And similarly for the ideal gas law.

Russell gives a number of examples of how idealisation in logic leads to laws of logic that lie in this sense; laws that are only true in the models or aren't really about what they purport to be about. There is not space here to go over all of them, but I will mention one. First-order models assume that every name in the language denotes one and only one object, and so, that there are no empty names. This is a useful idealisation, because in most cases, this is simply not relevant and saves us from deciding what to do with sentences that contain singular terms that fail do denote. Nevertheless, natural language is replete with names that we either know are empty or could very well be—"Vulcan" would be an example of the former and "Moses" could well be an example of

the latter.

Useful as this idealisation might be, however, it does lead to logical laws that are otherwise not obviously true. For example, it holds for all first-order models that $\forall x Fx \vDash Fa \vee \neg Fa$ and $\forall x Fx \vDash Fa$. But is it true that Vulcan either is a planet or it is not a planet? All planets revolve around a star, but should we therefore say that Vulcan revolves around a star? Another example, which raises similar questions, might be of the law $a = a \vDash \exists x(x = a)$. Since, presumably, every object is the same as itself, we might say that since Pegasus is Pegasus, it follows that Pegasus exists. But is the premise true? It certainly doesn't seem to be false! And if it is true, can't we therefore prove that everything we can coherently talk about exists, including Pegasus? But surely something has gone wrong with such reasoning? After all, Pegasus does not exist!

This is not to say that the study of such models has no interest. On Russell's view, any logic is just a model of the target phenomenon: the natural language entailment relation ("the distribution of truth over natural language sentences") and so, we should not necessarily abandon classical logic just because some of its laws are false, as it could still be the most appropriate model to use for many purposes, perhaps because it is simple, easy to work with and gives the right result most of the time.

1.3 Idealisation and the simple argument

Are there examples where such idealisation could lead our reasoning astray in actual cases? I believe so. Suppose for example that I specify a domain for my model which is in fact empty, but I do not realise (perhaps because the definition is too complex for me to grasp properly or because I believe that there are such things when there actually aren't). In such a case, the idealisation that the domain must not be empty might lead me to make invalid inferences. As a trivial example, suppose the domain is of all unicorns

and F denotes the property 'is a being with magical properties', I may conclude, since $\forall x Fx \vDash \exists x Fx$ is true for all models, that there are beings with magical properties, provided that the translation of $\exists x$ back into natural language as 'there are...' is correct.

However, the proposition that results from translating the conclusion back into natural language is false and the reason is simple to see: Our assumption that there are no empty domains means that $\forall x Fx \vDash \exists x Fx$ does not hold, and unless I already know that the domain is empty, I may reason wrongly. The assumption that the domain must not be empty might therefore be a good and useful idealisation (as Russell points out) but it can lead to falsehood when we translate arguments back into natural language. This particular example is, of course, such that it is unlikely that anyone would make this mistake, since the idealisation lies on the surface, as it were, but it does show that it is possible.

Now suppose we've formalised the simple argument into first-order logic and shown that it is valid. And since we have good reason to think that the premises are good translations of their natural language counterparts into the language of first-order logic and that the conclusion is likewise a good translation, it would be natural to suppose that we can therefore conclude that there are numbers. But how do we know that the conclusion is true of anything beyond the model?

Here, the nominalist can point out that it is already a part of the idealisation of first-order logic that we are quantifying over objects since we *stipulate* when we are doing the translation that the truth-value of the formalised proposition depends on what objects there are in a domain we specify—a domain of *objects*. It is therefore open to the nominalist to say that while our translation of (1) is a perfectly good formalisation for a lot of purposes, it cannot tell us anything about ontology because the very idealisation we have chosen when formalising the argument already builds in the conclusion that there

are numbers; the formalised counterpart of the conclusion is therefore stipulated to be true if and only if there are such objects as numbers. It follows that the formalised version of the argument can only have ontological import if we build into our translation of (1) that numbers exist.

So, if even we assume, as I will do, that there is no better translation of (C) into the idiom of first-order logic than $\exists x Fx$, it is still open to the nominalist to deny that the conclusion is true while accepting that the formal argument is valid—i.e. admitting that there is no model where the premises are true and the conclusion false. From the nominalist perspective, then, the translation into a formal language doesn't have (and should not have) much force in ontological debates. After all, if one wants to say that the argument is valid based on a translation into the language of first-order logic, the conclusion is already built-in, given how the logical apparatus works.

Or, to put it in Russell's terms, either (C) is just meant to be true in the model and we have no reason to think that it tells us anything about the world beyond the model, or it is supposed to tell us something about something more than the model, in which case we have no particular reason to think that (C) is true in the first place—since, as the foregoing shows, what is at stake is built into the very way the argument is formalised. It is thus no great surprise that that a sentence in the language of first-order logic that says that numbers exist comes out true in that model, and since any such model conceives of domains as consisting of objects, the realist position naturally, but misleadingly, suggests itself.[11]

11. One might think that the kind of ontological commitment a theory has depends on what kind of quantification one has in mind. For example, my argument against the formalised version of simple argument took quantification to be objectual, i.e. it assumed that the correct semantic clause for the existential quantifier stipulates that sentences of the form $\exists x \varphi(x)$ are true iff there is an object in the domain that satisfies φ. A nominalist might want to adopt a substitutional reading according to which such a sentence is true iff there is some term t such that $\varphi(t)$ is true.

Here, I'm not going to be concerned with such issues and just take quantification to be objectual. This

The further step of claiming that therefore nominalism is false would, however, simply beg the question against the nominalist. The nominalist can accept that the argument is valid in the language of first-order logic in the sense that there is no first-order model where the premises are true and the conclusion is false or that there is a proof in the language of first-order logic, while at the same time denying that the natural language argument is valid—i.e. that the premises, as expressed in natural language, are all true while the conclusion is false. The nominalist would not, as it were, fail a logic class, but rather doubts that the formalisation of the simple argument establishes its conclusion beyond its models. The idealisation inherent in formalisation illegitimately builds in the ontological conclusion.[12]

This does not, as I hope to show later, mean that the nominalist must therefore deny that singular terms that seem to refer to mathematical objects genuinely do so refer. The claim is rather that the temptation to think that such reference establishes an ontological conclusion is induced by philosophers constantly having the example of formal models before their eyes. We can, the nominalist accepts, translate the simple argument into the language of first-order logic and show that the conclusion therefore holds in all first-order models (or alternatively derive the conclusion via accepted formal rules), but that this is so, it is still open to the nominalist to claim, is because of the inevitable idealisation that such formalisation requires.

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should not create problems for the nominalist position, since the objectual interpretation is one where the realist is on stronger ground anyway.

12. Philosophers are thus guilty of a philosophical analogue of the Ricardian vice, where reliance on overly simplified formal models leads economists to flawed predictions and conclusions—by ignoring the theoretical assumptions of the model itself when applying it to reality. The term is due to Schumpeter 1954/2006, p. 448.

1.4 The Fregean argument

The foregoing argument only concerns a formal version of the simple argument and has thus only shown that formalising the argument into the language of first-order logic to assess its validity is not convincing to the nominalist. At this point, could it not be objected that the lens through which I've been discussing simple arguments is too Quinean; that the real issue is not what follows from a translation of our discourse into a regimented language and back, but the more basic fact that (1) would only be true if there are numbers, even on a more intuitive construal of what validity means (i.e. not a formal one) and hence there is no way for the premises true and the conclusion false?

That is to say, it could be maintained that the proposition that there are four prime numbers between 1 and 10 could only possibly be true if there are numbers, and hence that the conclusion should be true after all. On this view, the correct *translation* of (1) requires objects in the domain, and hence, it is not an artefact of the translation that the conclusion comes out true, but just a logical consequence of the premises, a fact which the translation merely demonstrates. A proponent of this view would say that it already follows from the truth of (1)—without any formalisation—that there are numbers and the translation into first-order logic merely registers that fact. The requirement that a domain must contain objects is therefore not misrepresenting the argument, but merely reflecting what is already there.

This objection requires that for (1) to be true, quantification (understood non-formally) over a domain of discourse ontologically commits us to what is quantified over. For that reason, I believe, the objection is equivalent to the most influential semantic argument for realism about mathematical objects, the so-called Fregean argument. It has been endorsed by a who's who of philosophers of mathematics and a modern

version of it (not Frege's!) runs as follows:

(1′) The singular terms of mathematical statements refer to mathematical objects, the predicates to properties of such objects and the first-order quantifier range over the objects.

(2′) Mathematical statements are capable of being true or false, and those accepted as theorems are true.

(3′) If a statement is true and quantifies over a range of objects, it ontologically commits us to the kind of objects its quantifiers range over.

(4′) There are mathematical objects.

The idea would then be that since premise (1) of the simple argument already quantifies over natural numbers, we do not need to formalise the simple argument to get to the conclusion that there are numbers, or indeed rely on the simple argument at all. It is enough that we quantify over mathematical objects in true statements.

So far, I've argued that one reason to think that there are numbers is that when we translate simple arguments into a formal language, we can derive existentially quantified statements that assert the existence of numbers when translated back into natural language. That would be one reason to accept premise (3′)—the premise from which the rest of the argument derives its ontological force. In response, I've argued that idealisation in the translation process builds in the conclusion that there are numbers, and hence that the nominalist should not be much moved by such reasoning.

The question that will concern me for the rest of the paper will be if the realist response that such a detour into a formal language is not necessary and whether the claim that the ontological import of sentences like (1) is more direct can be resisted. What I

will try to show is that without simple arguments, the realist has a hard time defending this position and that there is little reason to believe in simple arguments if it were not for formalisation. In the course of my argument, I will outline a position I will refer to as 'deflationary nominalism' and argue that the deflationary nominalist can coherently accept that (1) is true and genuinely refers to numbers while denying that we need to be ontologically committed to numbers, and further, that given the nominalist's commitment, (C) can be seen as a genuine counterexample to simple arguments.

2 Deflationary nominalism and the invalidity of simple arguments

The Fregean argument, as I've presented it above, does not rely on simple arguments to establish its conclusion, at least not directly. The idea is rather, that if we take our discourse at face value, then it seems that the very fact that (1) is true means that the singular terms that figure in (1) and purport to refer, do refer. And if they refer, they refer to objects. And what are objects if not something that exists?

Very broadly speaking, there have been two options available to the nominalist to resist this conclusion, which are to deny that singular terms in true statements genuinely refer or to deny that the relevant statements are true. Both options have been extensively explored in the literature, but are are uncomfortable for the nominalist for various reasons. In this section, I want to outline a third option which I will refer to as 'deflationary nominalism'. The core of this position is that reference *just is* for a single term to figure in a true statement and that in certain contexts, being an object is nothing above that referential role. The idea is thus not that the truth of certain statements can be reduced to the truth of others, showing that certain singular terms, despite appear-

ances, do not refer; no, it is rather that reference *just is* when singular terms figure in true statements. The notion of reference is thus deflated and the move from reference to existence is blocked via the rejection of simple arguments.

At the same time, however, the deflationary nominalist does not deny, contra e.g. Azzouni, that "There are Fs" or "Fs exist" incur ontological commitment to *F*s and thus, the deflationary nominalist takes our discourse at face value both in the sense that singular terms in true statements really refer (by deflating the notions of 'reference' and 'object') and in the sense that statements of the form "There are Fs" in natural language carry ontological commitment. It's just that for the deflationary nominalist, statements asserting the existence of numbers are false.

2.1 Easy ontology and reference

There are a few different approaches to ontology that we might characterise as "easy" (and for my purposes here, that is any view where reference to objects is secured by stipulation, broadly construed, and where the existence of the objects so referred to is implied by the truth of the statements in which the terms referring to them figure; in a sense, a view where reference to certain objects just cannot fail). For easier exposition, it is useful to pick one as a representative example, and for our purposes neo-Fregeanism is the most convenient, both because of the close connection with the Fregean argument and because the deflationary nominalist position is in many ways inspired by it.

This is no place to give a full treatment of neo-Fregeanism, but I nevertheless want to highlight a few things. The neo-Fregean view in the philosophy of arithmetic is composed of two main theses. The first (i) is the realist claim that arithmetical truths are about independently existing objects, the numbers, and the second (ii) is the logicist view that such truths are analytic, by which they mean that they can be known only by

the application of logical laws and definitions, and so are knowable *a priori*. Accordingly, numbers are *logical objects*—that is, objects that are knowable by nothing more than logic plus definitions (and presumably, only by such means).

Those of a nominalist bent might be inclined to dismiss talk of logical objects as a mere *façon de parler* or a term of art—claiming that such objects do not really exist, but merely reflect our way of speaking. This is not the neo-Fregean view, as these objects are taken to be "furnishings of the world every bit as objective as mountains, rivers and trees" (Wright 1983, p. 13). For the neo-Fregean, objects are what singular terms refer to and such reference is successful when those singular terms figure in true statements (see e.g. Wright 1990, p. 74). It would thus not be explanatory to say that singular terms in true statements are those expressions which refer to objects, even though that is true, but rather that objects are what singular terms in true statements refer to. Therefore, the notion of 'singular term' is a syntactic one, and if a term functions syntactically as a singular term, it must also function semantically as a singular term, i.e. refer to an object (see e.g. Wright 1983, p. 13).

How is such reference secured? On the neo-Fregean view, this is done via abstraction principles that are taken to be a kind of implicit definition of the relevant concept, e.g. 'number', and hence are analytic truths (see e.g. Wright and Hale 2001, p. 4 or Wright and Hale 2009, p. 179). From this meagre starting point it can be shown, even if we will not get further into that here, that the axioms of full second-order Peano arithmetic can be derived (see e.g. Parsons 1964 and Boolos 1990) and so, the neo-Fregean can explain how we can attain knowledge of the numbers in a way that avoids epistemological problems that have been associated with realism about numbers, at least on the surface, since our knowledge of numbers is *a priori*.

The important point for our purposes is, however, that by introducing such an

abstraction principle we can give meaning to a whole bunch of novel sentences that contain reference to numbers whose truth is guaranteed by the abstraction principle, one side of which is itself innocent of reference to numbers, at least on the surface. And since reference to objects in true sentences requires the existence of those objects, the truth of the new sentences we've obtained via our abstraction principle is enough to show that numbers exist.

The worry that the world might fail to live up to what the new sentence requires is then thought to be incoherent, requiring that we could have "some sort of independent, language-unblinkered inspection of the contents of the world, of which the outcome might be to reveal that there was indeed nothing there capable of serving as the referents of what Frege takes to be numerical singular terms" (Wright 1983, p. 14). The idea is that when

> ...certain appropriate sentences containing them are, *by ordinary criteria*, true, then it follows that those terms do genuinely refer. And, being singular terms, their reference will be to objects.

Accordingly,

> There is to be no further, intelligible question whether such terms really have a reference, whether there really are such objects.[13]

On this view, there is, as McBride puts it, no "ontological gap" to be bridged between what is truly said and what the world is like (MacBride 2003).

There is room to argue over what exactly the criteria for successful reference should be (and hence objecthood). For Thomasson, for example, who holds a different easy

13. I must register one caveat here: The wording of the passage indicates that Wright puts some distance between himself and what the neo-Fregean requires, although it is hard to say.

view of ontology, reference to *K*s is secured when the application (and co-application) conditions associated with the term *K* are fulfilled, and for Linnebo, "being a possible referent of a singular term suffices for objecthood" (Linnebo 2018, p. 25) and that "singular reference can be explained in terms of the concept of a criterion of identity" (p. 22). The important point is that, for the easy ontologist, these ordinary criteria that explain the truth of statements in which singular terms that refer to abstract objects figure can be such that it is a matter of stipulation, definition or analytic entailment from nothing or very little. We don't need, to put the point bluntly, to have any acquaintance with mathematical objects in order to be able to refer to them or have criteria of identity in talking about them. And so, reference is in this sense *easy*—and existence correspondingly so.

2.2 The 'magic objection' and thin objects

But how could it possibly be that by simple definitions like abstraction principles, we can derive the existence of numbers? After all, the criteria by which we judge that a statement of mathematics is true (or even made true) all seem internal to mathematics without seeming to point outside themselves—how, for example, do we determine that a number is prime? According to the neo-Fregean, such truths follow from nothing more than definitions and logic, going from truths that do not refer to numbers, and seemingly have nothing to do with numbers, to truths about numbers that themselves require that numbers exist. But aren't objects essentially something in the world? And if they are not, do they then exist?

How can, as Matti Eklund has asked, so much be had for so little (Eklund 2006)? Eklund goes on to compare the philosophical move that has been made with pulling a rabbit out of a hat, as does Yablo, adding that our feeling of "hocus-pocus" regarding

such arguments is "really very strong and has got to be respected" (Yablo 2000). Likewise, Thomas Hofweber asks, not about neo-Fregeanism in particular, but rather other easy approaches: "How could it be that the substantial ontological questions have an immediate trivial answer?" (Hofweber 2005b).

One popular strategy among easy ontologists for responding to the magic objection is to say that existence can, after all, just be easily had; existence just is the sort of thing that we can discover by analytic or logical means. After all, what it 'takes' for one side of an abstraction principle to be true is exactly what it takes for the other side to be true, and so, 'there is no gap for metaphysics to plug' (Hale and Wright 2009, p. 193).

One proponent of this view is Øystein Linnebo. For Linnebo, such existence simply just doesn't amount to very much. He defines an object to be 'thin' if its existence does not make any demands on the world—where the claim that the existence of *F*s makes no demands on the world simply means that there is some true statement φ that suffices for the existence of *F*s that itself does not require ontological commitment to *F*s (Linnebo 2018, p. 42).[14]

Even if, however, Linnebo thinks that not much is required for thin objects to exist, he nevertheless thinks that they do exist. It's just that their thinness is meant to take the sting out of this claim. Our stipulation, does not, on Linnebo's view *create* thin objects, and it is worth it to see why. First of all, as we've seen, Linnebo thinks, like other easy ontologists, that if we refer to objects in true statements, then these objects exist. But, if the true statements that refer to such objects are true necessarily, like mathematical statements plausibly are, then they are true in every possible world, and hence also in worlds where there are no human beings and no language—and no such stipulation to

14. It should be noted that Wright credits Dummett with this idea of 'thin reference' (see Wright 1992, p. 179 and Dummett 1973, p. 505.).

these objects made. And so, for that reason, Linnebo thinks that mathematical objects are indeed independent of human practices (Linnebo 2018, p. 189–190).

Likewise, Amie Thomasson thinks that if the question of how the "world can live up to" our stipulation is misguided—the simple arguments are inferring what really exists, prior to any stipulation, and not calling those objects into existence. We can introduce new terms by introducing new concepts, but what was introduced was just the concept, not the entities themselves. On this view, if the application conditions for the term are fulfilled, then reference to the given entity is guaranteed and hence we can infer, via simple arguments, that the objects so easily referred to, do exist (Thomasson 2015, p. 217).

2.3 'Existence' as a thick concept

These responses do not, in my view, really touch the intuition behind the magic objection. It's simply this: Existence seems to make demands on the world, in Linnebo's sense of the phrase; to say that something exists or does not exist seems to entail that the world would somehow be different if it did or did not exist, that to be real is to be part of the world. This intuition seems to be as much based on semantic considerations as the claim that reference is easy: it feels as if that just is what it is for something to exist—not in the sense that it is a definition, with necessary and sufficient conditions, but rather a truism about what it is to exist, the *meaning* of the term.

Hence, one feels that it comes close to a contradiction in terms to say that an object exists without making any demands on the world—almost as if the easy ontologist is telling us that certain kinds of objects exist, but then immediately taking it back by explaining that the kind of existence they have in mind is such that it just cannot fail to obtain. Thin existence, the proponent of the magic objection wants to insist, just isn't

existence. This feeling is captured quite well, I believe, in Chalmer's insistence that "no existence claim is trivially true" and that while it may be analytic that if a given object exists, then there is a further object (e.g. if there are particles arranged in a heap, then there is a heap), the claim that

> there is an object with those properties is never trivial or analytic, and is never trivially or analytically entailed by a sentence that does not make a corresponding existence claim. (Chalmers 2009, p. 79)

Chalmers goes on to argue that we can make a distinction between *ordinary* existence claims and *ontological* existence claims. This distinction, which is quite intuitive and inspired by Carnap's distinction between internal and external questions (Carnap 1950), is simply that we assert ordinary existence claims by ordinary criteria internal to a given subject matter, uninformed of ontology, and assert ontological existence claims when we are doing ontology. The distinction is, quite broadly, that the typical nominalist would assent to the claim that there are infinitely many prime numbers when doing mathematics, with questions of ontology far from their mind, but might give a negative answer to the question when doing philosophy because if there were prime numbers, then they would be abstract objects, and since there are no abstract objects, there are not infinitely many prime numbers.

The difficulty, as Chalmers points out, is that both sorts of questions need to be subject to some kind of assertability criteria or norm of correctness, and those criteria seem hard to separate from the truth of the relevant statements. It might be more plausible that the ordinary statements might have a different norm of correctness than the ontological ones, but nevertheless, it seems that satisfying the ordinary criteria just is what it means for the relevant statement to be true—and if so, separating truth from

correctness means that we cannot take our discourse at face value. The ontological sense of 'exists' would then seem to be secondary to the first (see Thomasson 2015, for this point) and that seems hard to swallow.[15]

Nevertheless, the feeling that Chalmers is expressing, however, seems just as hard to dislodge and is the reason, I believe, why the literature is replete with attempts to spell out what kind of property 'existence' really is (even if I agree with easy ontologists that such attempts largely fail). Notice, however, that in propounding his dictum, Chalmers gives no arguments; he simply asserts that no existence claim follows analytically from nothing. If pressed by an easy ontologist, I don't even believe that he *could* give any arguments and that the best thing he could say would be that that's just part of what it means to exist, namely that we can't discover from our armchairs what the world is really like. But isn't that just intuitively right?

The difficulty here is that this seems to entail that there is a difference in kind between the existence of thick objects, whose reference is not stipulated, and the existence of thin objects, whose reference is stipulated. There are many reasons for be sceptical of such a distinction, and there is not space to go over them here.[16] However, while the nominalist might be satisfied by such explanations, that thin reference doesn't really entail *existence* in the way that they find problematic, most philosophers would conclude that this is a kind of existence, after all, even if we would explain that reference to the objects said to exist in this way is in a certain sense stipulated.

A good example is Stephen Schiffer, another easy ontologist. For Schiffer, if we can

15. Another philosopher who tries to resist the ontological conclusions of easy ontologists by making a similar distinction is Thomas Hofweber (see Hofweber 2005b). For Hofweber, the stipulated statements and the derived ones do indeed have the same truth-conditions, and hence simple arguments are valid, but the single terms in the derived statements don't really refer, and hence the quantified statement in (C) doesn't really entail an ontological conclusion.

16. See for example Inwagen 1998 and Inwagen 2009 for arguments to that effect.

infer the existence of objects from statements that do not contain reference to those objects, via what he calls "something-from-nothing transformations", we should conclude that they exist. Such objects he calls "pleonastic entities" and they include fictional entities, who are

> mere shadows of the pretending use of their names; they come softly into existence, without disturbing the pre-existing causal order in any way. (Schaffer)

For Schaffer, this is a form of realism, because pleonastic entities really exist, even if they come into existence "softly", but for the nominalist, it just sounds like a roundabout way of saying that pleonastic entities do not exist. From the nominalist perspective, the whole thing seems a muddle—but how to clear it up?

Because of such considerations, I will follow Chalmers in insisting that no existence statement follows trivially from nothing and insist on existence making demands on the world. I will take it as axiomatic for the rest of the paper that there is only one meaning of the existential quantifier and that this meaning is "thick" in Linnebo's sense—that is to say, that existence makes demands on the world. I take this to be what it in fact *means* to say that something exists and that it is this semantic function of existential quantifiers that lies at the root of the feeling that easy ontologists are somehow pulling off a trick.

I am therefore not trying to explicate the notion of existence or offering a substantial definition of what it is to exist, but rather merely trying to respect what I diagnose as a semantic intuition about claims such as 'Fs exist'. I do not pretend to think that everyone will agree with this feeling, but since my goal is to try to spell out a nominalist position that can resist easy arguments, it is hardly a drawback that it is made to respect

the feelings of those with nominalist inclinations and not these who lack them.

2.4 An outline of deflationary nominalism

With all this in place, we can finally outline the deflationary nominalist position, which consists of three related claims: The first is (i) the neo-Fregean claim that objects are what singular terms refer to and that reference is successful when those singular terms figure in true statements; (ii) the claim that reference to mathematical objects makes no demands on the world and is thus thin; (iii) and the idea that existence necessarily makes demands on the world, and hence that if *K*s exist, reference to *K*s is thick.

An easy ontologist will of course argue that the conjunction of (i)–(iii) is inconsistent: After all, a true sentence that refers to thin objects will entail (via simple arguments) that there are thin objects, and thus existence cannot be thick. From the deflationary nominalist perspective, however, the two assumptions, that (ii) reference to mathematical objects is thin and (iii) existence is thick show why such arguments are invalid; after all, an argument is invalid if it is possible for the premises to be true and the conclusion false. From the deflationary nominalist perspective, that is the situation here, and hence, from (ii) and (iii) it follows that simple arguments are invalid. And if that's the case, then the deflationary nominalist has no particular reason to accept the charge that the conjunction of (i)–(iii) is inconsistent.

The deflationary nominalist can, however, get an even stronger result. If reference to mathematical objects is thin, then it is necessarily thin. This is because if reference to a purported object *F* was thick, then *F* is *eo ipso* not a mathematical object. Take (1) as an example: (1) is a true statement where the reference to numbers is a canonical example of thin reference; no matter what your favourite account of mathematical truth is, the reference occurring in (1) is thin. But if that is so, take any possible thick object

and check if that object is the real referent of any of the singular terms in (1) and you will find that this object is not the referent—after all, what possible thick object can be the referent of say "four"? It seems that any purported thick object, say, Julius Caesar, would be ruled out because that object was not a number. It then follows from the conjunction of (i)–(iii) that not only do mathematical objects not exist, they do not exist *necessarily*.[17]

The easy ontologist can of course always claim that this purported counterexample to simple arguments is not genuine and perhaps *ad hoc*. It is unclear, however, why the dialectical situation should be construed in that way. It is true that the conjunction of (i)–(iii) is incompatible with the validity of simple arguments, and that if simple arguments are valid, then the deflationary nominalist's position is incoherent. But it goes the other way, too.

The deflationary nominalist thus offers an alternative picture, and whether or not one will accept it, depends on which one thinks is more plausible: The idea that we can easily reach ontological conclusions or that some arguments that seem valid are actually invalid. The plausibility of the second disjunct depends on the metalogical positions that the deflationary nominalist can adopt, and so, if one rejects the ideal of deductive logic, that does not seem that hard to swallow. But as matters stand, the two assumptions that lead the deflationary nominalist to the conclusion that simple arguments are invalid—namely that reference to mathematical objects is thin and that existence is thick—seem well-motivated.

Notice, however, that if (i)–(iii) describe a coherent view, then the deflationary nominalist is able to take our discourse literally and at face value—(1) is literally true

17. Crispin Wright and Bob Hale have made a more sophisticated version of this argument against Field (see Wright 1992). In their hands, it is an argument for the existence of numbers, rather than against.

and its singular terms refer to numbers—while denying that numbers exist. They key element of this view is the rejection of simple arguments, since it is first and foremost via such arguments that we can infer the existence of thin objects. Thus, the purported counterexample is not a genuine one unless deflationary nominalism is true and deflationary nominalism is not true unless the counterexample is genuine; the two are inextricably linked.

The deflationary nominalist position is thus that one cannot infer from a thin context to a thick one; that there is, to borrow a phrase from Russell's recent book, *a barrier to entailment* from thin contexts to thick.[18] So, just as philosophers have thought that one cannot derive an ought from an is, a general statement from only particular ones or indeed a statement about the past from a statement about the future, the deflationary nominalist thinks that one cannot derive a conclusion that makes demands on the world only from premises that do not.

3 Is deflationary nominalism coherent?

The deflationary nominalist and the easy ontologist agree that reference is easy, that singular terms occurring in true statements is enough for genuine reference. The difference is that for the deflationary nominalist, such reference does not imply existence. The easy ontologist will of course think that such a view is inherently incoherent, since on their view, reference in true statements implies existence. The deflationary nominalist response is that the inference that leads to that conclusion is invalid.

There are, however, potentially other ways in which the deflationary nominalist position could fall into incoherence. For example, an important part of the deflationary

18. See Russell 2023.

nominalist position is that numbers are thin objects. But if numbers are thin objects and thin objects are referred to in true statements, surely there are thin objects? And how can something be an object and not exist, thin or not? Don't we likewise say of numbers, in true statements, that they are odd or even, prime or not prime, thus assigning properties to things that the deflationary nominalist thinks do not exist? How can something that does not exist have properties?

I believe, however, that by being thoroughly deflationary the view escapes incoherence. This may perhaps best be seen by looking at how these issues crop up in the philosophy of fiction. Consider for example the following statement:

(M) Mickey Mouse is a fictional mouse.

This statement, which is about a fictional character, Mickey Mouse, seems true. Furthermore, that it is true seems to depend on stipulation—loosely understood. After all, (M) is true because there are cartoons and animated films depicting a mouse called Mickey, etc. Those are the neo-Fregean's 'ordinary criteria'.

One strategy that the nominalist might consider is that of a paraphrase: If (M) is true in virtue of there being certain cartoons and animated films, then we might think that we could find a paraphrase of (M) that is reducible to nominalistically acceptable facts about these cartoons and films. However, it would be difficult to find a paraphrase that (a) does not mention Mickey Mouse at all, and (b) still preserves the meaning of (M). In other words, a paraphrase that refers only to the nominalistically acceptable facts in virtue of which (M) is true would not be a genuine replacement—it would simply be a different statement with the same truth-conditions.[19]

19. And this strategy could turn out to be even harder in the case of mathematics. Famously, Goodman and Quine's project of providing an austere nominalist paraphrase of mathematics ended in failure. See e.g. Goodman and Quine 1947.

For that reason, the deflationary nominalist prefers to take (M) at face value—insisting that it is true in virtue of the existence of certain cartoons and films, while also maintaining that it is genuinely about Mickey Mouse and that the term 'Mickey Mouse' genuinely refers to Mickey Mouse. It is important to note, however, that even if (M) is true in virtue of certain made-up stories in cartoon form about a mouse named Mickey, the truth of (M) is itself not stipulated and cannot be explained by relativising it to these stories ("Mickey Mouse has a dog named Pluto" is true *in* the stories, that Mickey is fictional is true *because* there are such stories).

Accordingly, reference to Mickey is thin (as it makes no demands on the world and reference to Mickey cannot fail, given that there are stories about him). For the deflationary nominalist, a fictional analogue of simple arguments is thus blocked, even if 'Mickey Mouse' refers to Mickey Mouse and the sentence says of Mickey that he is a fictional mouse, because arguments with premises that only contain thin reference and a thick conclusion are invalid. It just doesn't follow on the deflationary nominalist view that Mickey Mouse exists.[20]

Nevertheless, (M) refers to Mickey Mouse, because for the deflationary nominalist, that's just what reference *is* and for genuine reference to occur, it is enough that a singular term occurs in a true statement—with perhaps other syntactic requirements being present, for example, that there is a criterion of identity for the relevant objects.[21]

20. There is not space here to develop any substantial philosophy of fiction. There are however, important analogies between the case of fiction and the case of mathematics, namely that we seem to be forced into realism about fictional characters by the very fact that statements about fictional characters in true statements like (M) seem to contain reference to those same characters. Friend 2007 is an helpful overview of the issues.

Among those who endorse the realist position are e.g. Abell 2020, Kripke 2013,Salmon 1998, Thomasson 1998 and Inwagen 1977.

21. There is not space here to give an account of what our criteria of identity for fictional objects are, but I do take it for granted that we do have them. For example, "The main character in *A Study in Scarlet* is the same as the main character in *The Sign of the Four*" is seems true, even if Sherlock Holmes doesn't actually exist.

However, if the reference is thin, then the objects are correspondingly thin and because existence is thick, thin objects do not exist. Thus, the deflationary nominalist denies the last step, going from objecthood to existence. The view thus begins with a deflationary view of truth that is carried all the way through to a deflationary view of objects, where "object" is not conceived of as an ontological category. The deflationary nominalist therefore rejects what Eklund has called a 'correspondance theory of reference' and adopts a deflationary one.[22]

Thus, on the deflationary nominalist view, the truth of (M) does not require that there is an entity with certain properties that explain the truth of the statement in which it figures—that (M) is true because there is a mouse with the property of being fictional. To think that, I believe, is to conceive of the truth of a natural language statement on the model of a statement in a formal language, where truth is defined in such a way that it does require objects in a pre-specified domain to be present. After all, the statement (M) would best be analysed to be of the form *Fa* where *a* refers to Mickey Mouse and *F* is the predicate "...is a fictional mouse". This statement is then said to be true iff there is an object in the domain that *a* picks out and that object is in the subset of the domain that has fictional mice in its extension. This model would make the statement "Mickey Mouse exists" true, if we take that to be a good translation of $\exists x(x = a)$.

22. See e.g. Field 1994, Horwich 1998 and Båve 2009 for three different flavours of a deflationary theory of reference.

Field endorses a schema of the form

(R) If *b* exists then "b" refers to *b* and nothing else; if *b* doesn't exist then "b" doesn't refer to anything.

The deflationary nominalist cannot accept this, since existence is taken as a prerequisite for successful reference and their theory requires that "Mickey Mouse" refers to Mickey Mouse *and* that Mickey Mouse does not exist.

Båve's theory is more promising for the deflationary nominalist, according to which a speaker *a* refers to *b* iff *a* says something (which is) *about b*. Here, saying that a speaker refers is taken to be more basic than saying that an expression refers, which is taken to be derived. This, as Båve points out, has its roots in Strawson 1950, Austin 1962 and, in particular, Searle 1969.

In contrast, the deflationary nominalist can accept that (M) is true because there are stories about Mickey Mouse, depictions of him, etc. without thereby accepting that it follows that Mickey Mouse exists (in fact, it means that he does *not* exist)—exactly because they reject simple arguments. The formal model thus gets at least one inference wrong, and from the deflationary nominalist point of view, it is easy to see why: the idealisation of the model collapses the distinction between thin objects and thick objects, taking the presence of both in the domain to entail the existence of such objects—in the sense that a statement that says that these objects exist are made true by the model. The deflationary nominalist thus rejects formal modelling as a good guide to ontology and thinks that if the truth of (M) can be explained without Mickey Mouse existing, then reference to Mickey can too.

If, however, one has the mental model of reference that formal modelling suggests, it might seem that reference without an entity is impossible, because in formal models, the object must be in the domain to figure in true statements, and what is it to exist if not be in the domain? But that's just a part of the idealisation of first-order logic, and the deflationary nominalist does not accept that the translation of the corresponding existence claim is true when translated back into the vernacular. We could, if we wanted, model our discourse about fictional objects and get some results, but that's all that it is, a model (after all, we can stipulate what we want in a model and make whatever deductions we want).[23]

23. Consider for example van Inwagen's definition of object: x is an object $=_{\text{df}} \exists y(y = x)$. For the deflationary nominalist, this definition is a prime example of how formal idioms mislead philosophers—after all, how can van Inwagen give his formal definition of object without taking the notion of object, as it is defined for first-order models, for granted? After all, $\exists y(y = x)$ is true iff there is an object y in the domain such that $x = y$!

For van Ingwagen, this definition works because he specifies the domain as everything there is, and so, this sentence just registers that x is in the domain, i.e. exists. For the deflationary nominalist, however, the domain van Ingwagen has specified is not that of everything there is, but the set of everything that is referred to by singular terms in true statements—i.e. the set of all thin and thick objects.

But how can something be an object and not exist? Isn't the deflationary nominalist committed to the view that there are objects that do not exist? Isn't that incoherent and thus deflationary nominalism false? It is at this step that the deflationary nominalist rejection of simple arguments is crucial. Consider (M) again. For the deflationary nominalist, an object just is whatever is referred to in a true statement by singular terms, and thus, (M) refers to Mickey and when we utter (M) we are in fact talking about Mickey. But since reference to Mickey is stipulated in the sense that (M) is true in virtue of stories being told about Mickey and so on, not the fact that there is an entity with certain properties, Mickey is a thin object. And thin objects do not exist—but that doesn't mean that we cannot talk about them.

The claim is thus not that there objects that do not exist, but rather that there are true sentences which refer to objects that do not exist—the further inference that these objects therefore exist is blocked via the rejection of simple arguments. And isn't that just intuitively right? That (M) is about Mickey Mouse and that Mickey Mouse does not exist? The further step of saying that "Mickey Mouse" therefore refers to Mickey is thus deflated and taken to be nothing above and beyond the fact that 'Mickey Mouse' appears as a single term in a true statement about Mickey Mouse—which in turn is true by our 'ordinary criteria', namely because there are such stories.

At this point, it is very useful to look at Jody Azzouni's criticism of a view that is committed there being non-existent objects, Routley's noneism (see Routley 1980). I believe that with this contrast in mind, it can be shown that deflationary nominalism has the resources to avoid what would amount to an unpalatable Meinongianism (in the sense that a non-existent object nevertheless exists in some sense, rather than the idea that we can refer to objects that do not exist, a notion the deflationary nominalist accepts).

What Azzouni finds problematic with Routley's account is that it is explicitly committed to the view that there are objects that do not exist and that such objects have properties. Azzouni writes:

> ...Routley (and Landesman) clearly claim that such objects ("items") *have properties*, and I don't. And by "properties" they don't mean the mere truthful concatenation of predicates with singular terms that pick out *nothing*. When *I* describe how talk involving the nonexistent comes up, I speak of *stipulation* and *making it up*. There's nothing to *discover* about the properties of the nonexistent—there are no general (metaphysical) claims that can be made about the nonexistent except that they don't have any properties, and don't exist (Azzouni 2004, p. 56)[24]

Nevertheless, Azzouni is firm that sentences like (M) are true, even if he himself thinks that Mickey Mouse does not exist in any sense at all. Azzouni wants to explain this by the fact that the truth-values of such statements can be determined without there being any objects to which the single term 'Mickey Mouse' refers to. Instead of rejecting simple arguments, Azzouni just wants to say that the corresponding statement asserting the existence of Mickey Mouse does not carry ontological commitment, despite being true. Thus, Mickey Mouse does not, on Azzouni's view, have the property of being a fictional mouse, even if the corresponding statement that Mickey Mouse is a fictional mouse is true. However, this leads to what Azzouni calls a 'terminological loose end'.

After all, what we are doing in statements like these, is something *like* referring to objects, but because 'referring' is often used in an ontologically committing way, he

24. Charles Landesman defends Meinong's idea of "Aussersein", which according to him, is "just a technical way of asserting that [an] object is something that can be thought about or referred to" (Landesman 1975). This certainly has affinities with the deflationary nominalist position.

introduces a new term, 'referring*' which is intended to be a more general notion than referring, but includes it. So, when we say that "Mickey Mouse is a fictional mouse", Azzouni prefers to say that we are referring* to Mickey, but not referring to him.

From the deflationary nominalist perspective, however, this amounts to a mere terminological difference. Like Azzouni, the deflationary nominalist speaks of 'stipulation' and 'making it up'; but what Azzouni calls 'reference*', the deflationary nominalist just calls 'reference', and what Azzouni calls 'reference', the deflationary nominalist calls 'thick reference'. And to say that a thin object like Mickey (who does not exist and is made up) has the property of being fictional is not to say something extraordinary or incoherent about Mickey, because for the deflationary nominalist to say that Mickey Mouse has that property is for them not to say anything more than that the statement "Mickey Mouse is a fictional mouse" is true—that *just is what it is* for Mickey to have that property. Azzouni's use of locutions of the form '*x* has property *y*' is thus substantial and applies only to thick objects, while the nominalist's use is deflationary all the way through.

Thus for Azzouni, saying that Mickey has properties would be to say that there is an object in the world which has properties that determine the truth-value of statements where 'Mickey' appears as a singular term. And since the truth-value of such statements depends on other factors, including stipulation and make-belief, this is just false.[25] It is understandable that Azzouni finds that kind of talk incoherent when applied to objects that are said, in the very same breath, to not exist. That model, the deflationary nominalist thinks, might be appropriate for thick objects, reference to which is not just stipulated, and so the claim is *not* that there exist an entity, Mickey Mouse, that deter-

25. We should, however, probably say that (M) is stipulated *or* analytic, since it is not stipulated that Mickey is fictional. All kinds of things about Mickey were stipulated by Disney, but that just *means* that Mickey is fictional. That part is not stipulated.

mines which sentences in which 'Mickey Mouse' figures as a singular term are true; it is the opposite: the true sentences in which 'Mickey Mouse' figures as a singular term are what determines what properties Mickey has, where "having a property" is read in a deflationary manner.

If read in that deflationary way, the thought that an object that does not exist has properties is not as strange as it might seem. Suppose for example that sometime after Tarski's death, people had come to believe that he, not Gödel, had proved the incompleteness theorems. The proposition

(T) Tarski is famous for having proven the incompleteness theorems

would now be true. By stipulation, however, there would not be a Tarski to have the property of being famous for having proved the incompleteness theorems and there would not even be a causal chain linking Tarski to the proving and publication of the theorems—after all, Tarski had already died and Gödel did those things. Further, (T) would be true of Tarski *now*, not in the past when he was alive, because before his death, people truly believed that Gödel had proved the incompleteness theorems.

How to respond to this case? Should way say that (T) is false, on the grounds that there is no Tarski to be referred to in (T)? Or is it better to say that (T) is true, but Tarski does not have the corresponding property of being famous for having proven the incompleteness theorems, on the grounds that he no longer exists? The first option seems just unreasonable—after all, (T) is *about* Tarski in the present and says of *him* that he is famous for having proven the theorems in the past. Perhaps a clever paraphrase could be found so that (T) doesn't really talk refer to Tarski in the present, while having the same meaning, but should we not just take (T) at face value?

If we take the second option, the deflationary nominalist can just stipulate that

for Tarski to have that property is nothing above and beyond (T) being true. Here, it would be possible to make a distinction between real properties that require some kind of change in an object to be acquired and "Cambridge properties" that do not. On this view, "...is red" would be a real property but "...is famous" a Cambridge one.[26] The discomfort with saying that a non-existent object can have properties might thus be lessened by stipulating that non-existent objects can only have Cambridge properties, while existent objects can have real properties.[27]

The point is that, however, that no matter how we spell the view out, it is open to the deflationary nominalist to say that Tarski could have the property of being famous for having proven incompleteness theorems by adopting a deflationary view of properties. The alternative would be to admit that (T) could be true without Tarski having that property and without referring to Tarski—and is it really stranger that we can refer to things that do not exist than to suppose that we can refer to things that *used* to exist? (And here it is important to remember that the case is such that (T) became true after Tarski had died, that there is no causal chain from Tarski himself to the truth of (T).)[28]

The substance of the deflationary nominalist view of properties and Azzouni's is thus largely identical. For Azzouni, we can successfully refer* to Mickey because there are true statements about Mickey where Mickey appears as a singular term. In such a case, the deflationary nominalist, conceiving of reference primarily as a syntactic phenomenon, prefers to simply say that we refer to Mickey in such statements; after all, for

26. See Miller 1982 for this distinction and a similar point. The original distinction between a real change and a Cambridge change is due to Geach.

27. We thus also have a neat solution to the problem of negative existentials: "Mickey Mouse does not exist" is true because there is no such thick object, but by rejecting simple arguments, we need not ascribe any real property to an existing object to make that claim.

28. Or perhaps the easy ontologist thinks that via easy arguments we have a straight-forward argument for the immortality of the soul? After all, if (T) is true, then 'Tarski' refers to Tarski, and hence Tarski must exist—not merely *existed*.

the deflationary nominalist, reference just is when singular terms figure in true statements. The deflationary view of properties, however, is just that for *a* to have a property *F* is for there to be a true statement saying that *a* is *F*. Some properties, we could nevertheless still grant, are real and others are merely Cambridge. The difference would merely be found in the truth-conditions of the relevant statements.

Another possible reason, however, that Azzouni might find Routley's claim that some objects do not exist to be incoherent is that it seems to be tantamount to saying that there are objects that do not exist—a clear contradiction, if "there are..." is read in an ontologically committing manner. The deflationary nominalist does not need to be committed to this, however. For the deflationary nominalist, the statement

> There are objects that do not exist

is either false and only follows from the deflationary position via a simple argument, the validity of which the deflationary nominalist rejects or is just a paraphrase of the claim that there are true statements in which singular terms figure that refer to objects that do not exist.

The realist might nevertheless reason as follows: There are true statements in which a singular term refers to Mickey Mouse, and hence, even by the deflationary nominalist standards, Mickey is an object. It follows, since Mickey is a thin object, that there are thin objects. The deflationary nominalist response is this: As the argument itself assumes, Mickey Mouse is an object just means that a singular term referring to Mickey occurs in a true statement—and nothing more. We can thus either explain the conclusion as one that follows from a simple argument or just a paraphrase of the deflationary nominalist position; there are no thin objects, unless one means by that that there are true statements in which singular terms with thin reference figure.

There is no incoherence here: The deflationary nominalist maintains that if a given statement in which a a singular term occurs is true, then that is what is what it is for that singular term to refer, and so, in the case of mathematical or fictional objects, it does not matter if reference to the objects is established via stipulation, abstraction principles or constitutive rules determining application conditions—as long as the truth of the statement makes no demands on the world, this reference is thin, and thus does not incur ontological commitment, as the corresponding existence claim is just false.

It should be noted, however, that there might be cases in everyday speech where certain statements are made that can hardly be taken at face-value, for example some statements about fictional characters. One might, for example, be asked "What fictional characters are there?" and give an answer like "There is Mickey Mouse, Sherlock Holmes, Captain Ahab, ...". In such cases, I do not believe that it would be inappropriate to reply to the incredulous metaphysician asking if one really thinks that Mickey Mouse exists that that is quite obviously not what was meant—and to subsequently give a paraphrase that makes the same statement without ontological commitment. It is therefore not a part of the deflationary nominalist view that paraphrase is *never* appropriate to get rid of what seem to be problematic ontological committments—it is just not successful in some important cases, including mathematical language and some statements about fictional characters, such as "Mickey Mouse is a fictional character"—which is not a part of the fiction in which Mickey appears (*in* Mickey Mouse comics, Mickey is not a fictional mouse).

Nor should the deflationary nominalist reject Azzouni's claim that there are cases where phrases like 'There are *F*s' is not ontologically committing, even if a paraphrase is not readily available. It is rather that some important cases do not seem to be like that, such as (C) or (M). In fact, if we accept the deflationary nominalist account of reference,

the statement 'there are thin objects' simply means that there are true statements in which objects figure that make no demands on the world—and that way of putting it is something that the easy ontologist should accept. There is thus no particular reason to think that the claim that there are thin objects is ontologically committing, given that this just means that statements like (M) are true.

Referring to statements There's another possible incoherence lurking, however. Haven't I said again and again that an object is just whatever is referred to by a singular term in a true statement, and thus talked about statements? And doesn't it follow from what I've said that there are statements? And doesn't "statement" here mean 'statement-type' (where the statement: "There are statements" is just a token of a statement of that type)? And if there are statement-types, surely they must be abstract objects, and nominalism entails that there are no abstract objects. The deflationary nominalist position can thus not be coherently maintained, since it denies the existence of abstract objects while relying on their existence. As is well known, arguments against nominalism of roughly this shape are common.

Putting the fact that nowhere have I claimed that the deflationary nominalist must be a nominalist about all abstract objects aside, the most obvious answer to this objection would be to simply point out that by denying the validity of simple arguments and denying the model of reference suggested by formalisation, the deflationary nominalist thinks it is possible to talk about things that do not exist, and hence there is no reason to accept that the statement "there are statements" is true, while continuing to talk about them; just as the deflationary nominalist can say that Mickey Mouse is a fictional mouse, without being ontologically committed to Mickey, they can say that (M) is a statement without being ontologically committed to statements.

However, I do not think that the deflationary nominalist must say this (or perhaps rather, that they must not say *only* this). The deflationary nominalist position is based on the distinction between thick objects and thin objects, and that distinction is between reference which makes demands on the world and that which does not. But as Linnebo notes, this is not an absolute notion, as it is possible to speak of *relatively* thin objects, which are considered thin relative to some other objects, which in turn do make some demands on the world (Linnebo 2018, p. 4). An object which is thin relative to some other objects thus exists insofar as the latter exist, but exists nonetheless.

The thought would thus be that statement-types exist insofar as their tokens exist, not in the sense that the type is reducible to their tokens, but in the sense that just as easy ontologists can give meaning to statements where reference to numbers occurs by using abstraction principles or application conditions, we could e.g. give an account of statement-types (or propositions) that was based on a similar abstraction—the difference would just be that on one side of that abstraction principle there would be thick objects, the tokens, instead of no objects—and *mutatis mutandis* for other ways of doing easy ontology.[29] They claim would thus be, even if there is not space to spell this suggestion out further here, that statements or propositions are only relatively thin and thus do exist, also by the deflationary nominalist's lights—even if it is just insofar as they are abstractions of thick objects, and hence that there is no incoherence for the deflationary nominalist to talk about them in their theory.

This, it should be added, goes as well for other types of abstract objects that realists have complained that nominalists want to outlaw from our language, ranging from

29. For an example of an account that tries to draw a distinction between abstract objects that can be located, like impure sets and 'repeatable artworks' (i.e. *Anna Karenina* itself, not its copies) and those that can not, e.g. mathematical objects, see Davies 2019. Davies attempts to do this via the method I've described, abstraction principles that work on tokens.

"numbers to functions to sets, from species to genera to phyla, from shapes to books to languages, from games to corporations to universities".[30] On the contrary, the deflationary nominalist need not have any qualms about talking about any of these things—without ontological commitment to them as abstract objects that goes any further than believing that their existence is fully explained by the tokens. There is thus no revision of our language required on the deflationary nominalist view.

The preceding discussion shows, I believe, that not only is it not incoherent to refer to objects that do not exist in true statements, provided that simple arguments are actually invalid. I've focused here on examples concerning fictional objects, but the kinds of arguments realists have brought to bear in the case of mathematics can be mirrored in the case of fiction. And while it is perhaps reasonable to say about sentences like (1) that the fact that it is true and refers to numbers means that numbers exist, the same result is extremely counterintuitive in the case of sentences like (M) "Mickey Mouse is a fictional mouse". After all, Mickey Mouse does not exist, and so, that it his existence follows logically from (M) should be seen as something of an embarrassment—after all, "*x* is fictional" analytically entails "*x* does not exist". The deflationary nominalist can easily accomodate both intuitions, that the sentence is true and about Mickey Mouse and that Mickey Mouse does not exist, via their rejection of simple arguments.

4 Concluding remarks

In metaphysical debates on the existence of mathematical objects it has long been taken for granted that reference to objects in true statements requires the existence of those same objects. In this paper, I've argued that it possible for nominalists to challenge

30. This list of nominalistically suspect entities is borrowed from Burgess and Rosen 1997, p. 34.

this assumption by rejecting the idea that deductive inference is a matter of following inference rules that correctly capture or describe the laws of logic. The resulting view is one where terms that purport to refer to objects in true statements really do refer, but at the same time, it does not follow that the objects referred to exist—the relevant arguments are, on the nominalist view, simply invalid.

Importantly, by adopting this view, the nominalist is able to take our discourse at face value and accept that (1) there really are four prime numbers between 1 and 10 or that Mickey Mouse is a fictional mouse, without trying to argue that the semantics of such statements is different from the semantics of other areas of our discourse: '1' genuinely refers to the number one and "Mickey Mouse" genuinely refers to Mickey Mouse, just as "Angela Merkel" refers to Angela Merkel in the sentence "Angela Merkel was the eighth chancellor of Germany", etc. The account thus implicitly offers a solution to Benaceraff's dilemma, of explaining how the we can have a uniform semantics over our language while at the same time offer a reasonable epistemology of mathematics (see Benacerraf 1973).

In this respect, deflationary nominalism can inherit the epistemological benefits of easy ontology without demanding a change in the semantics of our mathematical discourse, since singular terms that occur in thin contexts genuinely do refer, just as much as singular terms in thick contexts. It's just that reference is understood as nothing more than playing a syntactic role. There is thus nothing preventing the deflationary nominalist from offering an account of the semantics of mathematical statements where single terms refer, objects can be said to have properties and stand in relations, and so on. It is just that their understanding of what it is to refer, have properties and stand in relations is deflated.[31] There is likewise no epistemological problem because sentences

31. This of course depends on what we mean by 'uniform semantics'. If we mean that the nihilist can

like (1) are true according to our ordinary criteria and reference to mathematical objects is correspondingly thin and cannot fail.

What is different, however, is the metasemantics—what makes the different statements true, the neo-Fregean's 'ordinary criteria'. These criteria vary from one area of our discourse to another, without requiring a corresponding change in the first-order semantics. Statements about mathematical objects thus refer to objects, just as statements about chancellors. The further ontological step of requiring the existence of the mathematical objects is resisted, however, by rejecting inferences that go from thin contexts to thick.

This move is possible without contradiction or incoherence by adopting a view in the philosophy of logic whereby formal logic is just a model of our discourse, and hence not the reliable guide to ontology it has been taken to be.

Declarations

Conflict of interest

The author declares no conflict of interest.

now provide an accurate formal model for both kinds of objects, thick and thin, the claim would be false. But demanding such a model would beg the question against the nihilist who could just simply deny that the semantic horn of Benaceraff's dilemma has any force.

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