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Bridging the Gap

Integrating Linear Algebra in the Development and Understanding of Large Language Models for Software Engineering Applications.

by

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Preface

This bachelor's thesis was written for the Department of Electrical and Computer Engineering, Aarhus University. It is part of the Computer Engineering study program, and was written in the spring semester of 2024.
All source files associated with this thesis are found at: `https://github.com/asgersong/BSc`

Asger Poulsen, June 2, 2024

Abstract

Acknowledgements

Contents

Chapter 1

Introduction

Large Language Models (LLMs) have become a cornerstone in the field of natural language processing (NLP) and artificial intelligence (AI), driving significant advancements and innovations. These models are designed to understand, generate, and interpret human language at a level that is increasingly indistinguishable from that of a human being. The development and evolution of LLMs mark a pivotal shift in how machines can learn from and interact with textual data, enabling a plethora of applications ranging from automated text generation to sophisticated conversational agents.

Chapter 2

Theoretical Foundations

2.1 Linear Algebra in Deep Learning

Linear algebra forms the cornerstone of deep learning, providing the necessary mathematical framework to model and understand complex relationships within data. It is instrumental in defining the operations and transformations that occur within deep neural networks, including those underlying LLMs.

2.1.1 Vectors, Matrices, and Tensors

Vectors and matrices are fundamental to representing data and parameters in neural networks. A vector $\mathbf{v} \in \mathbb{R}^n$ can represent a point in n-dimensional space or a single data instance with n features. Matrices $A \in \mathbb{R}^{m \times n}$ facilitate linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$, and tensors generalize these concepts to higher dimensions, accommodating the multi-dimensional data structures processed by neural networks.

A typical case, representing a basic neural network operation, can be expressed as:

$$\mathbf{y} = A\mathbf{x} + \mathbf{b} \tag{2.1}$$

where A is the weight matrix, $\mathbf{x}$ is the input vector, $\mathbf{b}$ is the bias vector, and $\mathbf{y}$ is the output vector of the transformation.

2.1.2 Matrix Operations

Matrix operations such as addition, multiplication, and transposition are essential in neural network computations. Matrix multiplication, in particular, plays a crucial role in transforming data between layers, capturing the relationships between input and output features. The dot product of two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$ is defined as:

$$C = AB = \sum_{i=1}^{n} A_{ij} B_{jk} \tag{2.2}$$

where $C \in \mathbb{R}^{m \times p}$ is the resulting matrix. Matrix multiplication is a key operation in neural networks, enabling the transformation of input data through multiple layers of weights and biases.

Flops

Floating-point operations (FLOPs) are a measure of the computational complexity of matrix operations. The number of FLOPs required for matrix multiplication is proportional to the product of the dimensions of the matrices involved. For example, the number of FLOPs for multiplying two matrices $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times p}$ is $2mnp$.

2.1.3 Singular Value Decomposition

Singular Value Decomposition (SVD) is a powerful technique for decomposing a matrix into singular vectors and singular values, providing insight into the structure of the data. For any matrix $A \in \mathbb{R}^{m \times n}$, SVD is given by:

$$A = U \Sigma V^T \tag{2.3}$$

where U and V are orthogonal matrices containing the left and right singular vectors, respectively, and Σ is a diagonal matrix with singular values. SVD is essential in many machine learning tasks, including noise reduction, data compression, and the analysis of neural network layers.

2.1.4 Low-Rank Approximation

Low-rank approximation is a powerful technique in linear algebra that provides significant benefits in terms of computational efficiency and storage requirements. This method is particularly useful in the context of deep learning, where large matrices are frequently encountered, and computational resources are often a limiting factor.

Concept of Low-Rank Approximation

Given a matrix A of dimensions $m \times n$, low-rank approximation aims to approximate A by another matrix A_k of lower rank k, where k is much smaller than both m and n. This approximation leverages the Singular Value Decomposition (SVD) of A:

$$A \approx A_k = U_k \Sigma_k V_k^T$$

Here:

- U_k is an $m \times k$ matrix containing the top k left singular vectors.
- Σ_k is a $k \times k$ diagonal matrix containing the top k singular values.[1]
- V_k^T is a $k \times n$ matrix containing the top k right singular vectors.

This decomposition allows A_k to capture the most significant components of A, effectively reducing its rank while preserving its essential characteristics.

[1] While it is possible to represent Σ_k as a k-element vector, choosing a $k \times k$ matrix allows for more flexibility in mathematical operations and maintains consistency in the representation of the decomposition, which can simplify implementation and theoretical analysis.

Reduction in Storage

The primary benefit of low-rank approximation is the significant reduction in storage requirements. Instead of storing the original matrix A with $m \times n$ elements, the low-rank approximation stores the three smaller matrices U_k, Σ_k, and V_k^T:

- U_k with $m \times k$ elements.
- Σ_k with $k \times k$ elements.
- V_k^T with $k \times n$ elements.

Thus, making the total number of elements required to store these matrices:

$$m \times k + k \times k + k \times n = k(m + k + n)$$

For k much smaller than m and n, this results in a substantial reduction in storage. For example, consider a matrix A of size 1000×1000. If we approximate A with rank $k = 50$ we have:

- Original Matrix: $1000 \times 1000 = 1000000$ elements.
- Low-rank approximation: $1000 \times 50 + 50 \times 50 + 50 \times 1000 = 102500$ elements.

which is a substantial reduction in storage requirements.
But if we choose $k = 500$, the low-rank approximation would require

$$1000 \times 500 + 500 \times 500 + 500 \times 1000 = 1250000 \text{ elements},$$

which is more than the original matrix A. Thus, the choice of k is crucial in achieving storage reduction.
More specifically, we want

$$\begin{gathered} k(m + k + n) = k^2 + k(m + n) < mn \\ \Updownarrow \\ k^2 + k(m + n) - mn < 0 \\ \Updownarrow \\ k < \frac{\sqrt{4mn + (m + n)^2} - m - n}{2} \end{gathered}$$

This inequality provides a guideline for selecting an appropriate rank k to achieve storage reduction.

Reduction in Computation for Matrix-Vector Multiplications

Low-rank approximation can also reduce the computational complexity of various matrix operations. In the case of a matrix-vector multiplication we have:

- **Original Matrix**: Multiplying A (size $m \times n$) with a vector requires $O(mn)$ operations.

- **Low-Rank Approximation**: Multiplying $A_k = U_k \Sigma_k V_k^T$ with a vector involves three steps:
  - V_k^T with the vector: $O(kn)$ operations.
  - Σ_k multiplication: $O(k^2)$ operations.
  - U_k multiplication: $O(mk)$ operations.
- **Total**: $O(kn) + O(k^2) + O(mk) = O(k(k + m + n))$ operations.

Therefore, the appropriate choice of rank k to achieve computational efficiency for matrix-vector multiplications is also

$$k < \frac{\sqrt{4mn + (m+n)^2} - m - n}{2}$$

Reduction in Computation for Matrix-Matrix Multiplications

In the case of a matrix-matrix multiplication we have:

- **Original Matrix**: Multiplying A (size $m \times n$) with another matrix of size $n \times p$ requires $O(mnp)$ operations.
- **Low-Rank Approximation**: Multiplying $A_k = U_k \Sigma_k V_k^T$ with a matrix of size $n \times p$ involves:
  - V_k^T with the matrix: $O(knp)$ operations.
  - Σ_k multiplication: $O(k^2 p)$ operations.
  - U_k multiplication: $O(mkp)$ operations.
- **Total**: $O(knp) + O(k^2 p) + O(mkp) = O(k(k + m + n)p)$ operations.

Therefore, the appropriate choice of rank k to achieve computational efficiency for matrix-matrix multiplications is also

$$k < \frac{\sqrt{4mn + (m+n)^2} - m - n}{2}$$

By choosing a sufficiently small k, these operations can be performed much more efficiently compared to using the full-rank matrix A.
Thus, making low-rank approximation via SVD a powerful technique that offers significant benefits in terms of storage reduction and computational efficiency. By focusing on the most important components of a matrix, low-rank approximation makes it feasible to handle large datasets and complex computations more effectively, which is crucial in the field of deep learning and beyond.

2.1.5 Neural Networks in Deep Learning

A Neural network consist of interconnected nodes or "neurons" arranged in layers, with each layer designed to perform specific transformations on its inputs to capture and transmit increasingly abstract features to subsequent layers.

Architecture of Neural Networks

The architecture of a neural network is defined by layers, each comprising a set of neurons connected by weights. These weights are adjusted during the training process to minimize the difference between the actual output of the network and the desired output. A typical feedforward neural network can be mathematically represented as:

$$\mathbf{h}^{(l)} = f(W^{(l)}\mathbf{h}^{(l-1)} + \mathbf{b}^{(l)}) \tag{2.4}$$

where $W^{(l)}$ and $\mathbf{b}^{(l)}$ are the weight matrix and bias vector for the l-th layer, $\mathbf{h}^{(l-1)}$ is the output from the previous layer, and f is a non-linear activation function such as ReLU or sigmoid.

Learning Process

The learning process in neural networks involves adjusting weights and biases to reduce a loss function, commonly through backpropagation. This method efficiently computes gradients using calculus' chain rule:

$$W^{(l)} \leftarrow W^{(l)} - \eta \frac{\partial \mathcal{L}}{\partial W^{(l)}} \tag{2.5}$$

where η is the learning rate and $\mathcal{L}$ is the loss function.

Optimization and Regularization

Optimization algorithms like Stochastic Gradient Descent (SGD), Adam, and RMSprop are crucial for weight updates. Regularization techniques such as dropout and weight decay help prevent overfitting, ensuring the network generalizes well to new data.

2.2 Understanding LLMs

Large Language Models (LLMs) such as GPT (Generative Pre-trained Transformer) and BERT (Bidirectional Encoder Representations from Transformers) have revolutionized the field of natural language processing (NLP) by leveraging deep neural networks to understand and generate human-like text unlocking a whole host of new applications.

2.2.1 What are LLMs?

LLMs are deep neural networks trained on vast amounts of text data. They learn to predict the next word in a sentence, understand context, generate text, and perform various NLP tasks with minimal task-specific adjustments. The strength of LLMs lies in their ability to capture intricate patterns in language through extensive pre-training.

2.2.2 Architecture of LLMs

The architecture of most LLMs is based on the Transformer model, introduced by Vaswani et al., 2017, which relies on self-attention mechanisms to weigh the significance of different words in a sentence. The Transformer architecture is composed of two main components: an encoder and a decoder. The encoder takes the input text and produces a sequence of hidden states, which represent the meaning of the text. The decoder then takes the encoder's hidden states and generates the output text, one word at a time.

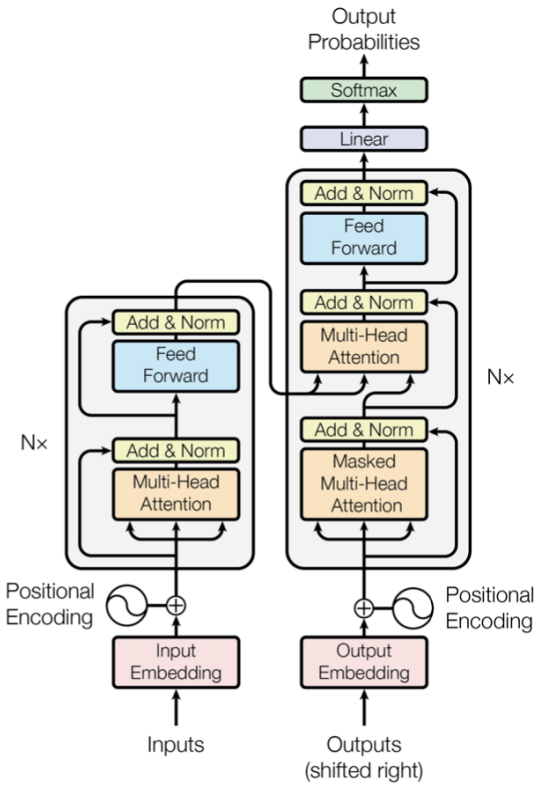


Figure 2.1: Transformer model architecture - Taken from "Attention Is All You Need"

At a high level, the goal of the Transformer is to take an input text and predict the next word in the sequence. The input text is broken into tokens, which are typically words or parts of words. Each token is then represented as a high-dimensional vector, known as its embedding. Initially, these embeddings encode only the individual meaning of the tokens without any contextual information.

Encoder

The encoder consists of a stack of N identical layers, each containing two sub-layers: a multi-head self-attention mechanism and a feed-forward neural network. Furtherly, a residual connection is employed around each of the two sub-layers, followed by layer normalization. So the output of each sub-layer is $\text{LayerNorm}(x + \text{SubLayer}(x))$, where $\text{SubLayer}(x)$ represents the function implemented by the sub-layer.

The self-attention mechanism allows the model to weigh the importance of different words in the input sequence when generating the output. The feed-forward neural network processes the output of the self-attention mechanism to produce the final hidden states of the encoder. A key feature of the encoder is that it processes all words in the input sequence in parallel, which contributes to the efficiency of the Transformer model. The output of the encoder is a sequence of vectors, each representing an input word in a high-dimensional space.

Decoder

The decoder, on the other hand, also consists of a stack of N identical layers, but with an additional third sublayer in each decoder layer, which performs multi-head attention over the encoder's output. This allows the decoder to focus on different parts of the encoder's output for each word in the output sequence. In the first sublayer of the decoder, self-attention is used, but with a constraint (masking) to prevent positions from attending to subsequent positions. This ensures that the predictions for position i can depend only on the known outputs at positions less than i. The purpose of the decoder is to generate an output sequence one word at a time, using the encoder's output and what it has produced so far as inputs.

Attention

The attention mechanism is a cornerstone of the Transformer model. Its primary function is to dynamically adjust the importance of each word in the input sequence based on its context, thereby refining the embeddings of these words to capture richer meanings.

The attention mechanism allows each token to attend to every other token in the sequence, effectively allowing the model to focus on the most relevant parts of the input when making predictions. This process is mathematically described as follows:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \tag{2.6}$$

Here, Q, K, and V are the query, key, and value matrices, respectively. These matrices are derived from the input embeddings by multiplying them with learned weight matrices. The dimension d_k is the size of the key vectors and serves to scale the dot products to prevent them from growing too large.

To understand how this works, consider a simple example: The phrases

- "After dinner, they enjoyed a sticky *date* pudding."
- "She circled the *date* of the concert on her calender."
- "They went on their first *date* to the zoo."

The word "date" has different meanings in each phrase, but its initial embedding would be the same. The attention mechanism helps to refine the meaning of "date" based on its context by allowing the embeddings of surrounding words to influence it.

Steps of the Attention Mechanism:

1. **Compute Query, Key, and Value Vectors:** For each embedded token $\overrightarrow{E_i}$, we compute query ($\overrightarrow{Q_i}$), key ($\overrightarrow{K_i}$), and value ($\overrightarrow{V_i}$) vectors by multiplying the token's embedding with learned matrices W^Q, W^K, and W^V:
$$\overrightarrow{Q_i} = W^Q \overrightarrow{E_i}, \quad \overrightarrow{K_i} = W^K \overrightarrow{E_i}, \quad \overrightarrow{V_i} = W^V \overrightarrow{E_i}$$

2. **Compute Attention Scores:** Calculate the dot products of the query vectors with all key vectors to measure how much focus each word should have on every other word, and divide by the square root of the key dimension for numerical stability:
$$\text{Scores} = \frac{QK^T}{\sqrt{d_k}}$$

3. **Apply Softmax Activation:** Normalize these scores using the softmax function to obtain attention weights:
$$\text{Attention Weights} = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)$$

4. **Compute Weighted Sum of Values:** Multiply the attention weights by the value vectors to get the final output for each token:
$$\text{Output} = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

which finally leaves us with Eq. 2.6.

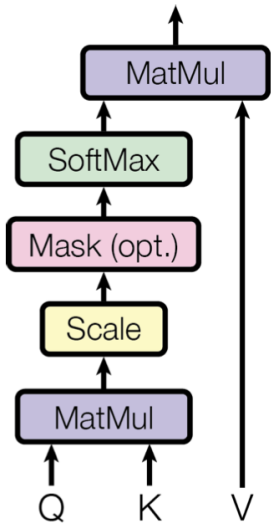


Figure 2.2: Illustration of the Attention Mechanism - Taken from "Attention Is All You Need"

This mechanism allows the model to dynamically adjust which parts of the input sequence to focus on, thereby capturing the contextual meaning of each word. This whole process is described as a single 'head' of attention, and multiple heads can be used in parallel to capture different aspects of the input sequence.

Multi-head Attention

Multi-head attention extends the single-head attention mechanism by allowing the model to attend to different parts of the input sequence simultaneously, capturing a variety of relationships and patterns. Instead of performing a single attention function, multi-head attention projects the queries, keys, and values into multiple subspaces and performs the attention function in each subspace independently.

This process is mathematically described as follows:

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \ldots, \text{head}_h)W^O \tag{2.7}$$

$$\textbf{where } \text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V) \tag{2.8}$$

Here the projections are the learned parameter matrices W_i^Q, W_i^K, and W_i^V for the i-th head, and W^O is the output matrix.

Steps of Multi-head Attention:

1. **Linear Projections**: Project the input embeddings into h different subspaces using learned weight matrices to create multiple sets of queries, keys, and values:
$$Q_i = QW_i^Q, \quad K_i = KW_i^K, \quad V_i = VW_i^V$$
2. **Parallel Attention Heads**: Apply the attention mechanism to each set of projections in parallel, producing multiple attention outputs:
$$\text{head}_i = \text{Attention}(Q_i, K_i, V_i)$$
3. **Concatenate and Project**: Concatenate the outputs of all attention heads and project them back to the original embedding space using a final learned matrix W^O:
$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, \ldots, \text{head}_h)W^O$$

Multi-head attention allows the model to capture different aspects of the input sequence in parallel, which enhances its ability to understand complex patterns and dependencies in the data.

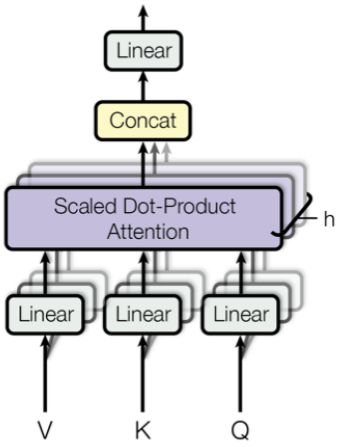


Figure 2.3: Illustration of Multi-head Attention - Taken from "Attention Is All You Need"

In summary, the attention and multi-head attention mechanisms enable the Transformer model to focus on relevant parts of the input sequence dynamically, improving its ability to capture context and generate accurate predictions. These mechanisms are crucial to the model's success in various natural language processing tasks.

Why Transformers?

The Transformer model has several advantages over traditional RNNs and LSTMs, including the ability to capture long-range dependencies, parallelize computation, and scale to larger datasets. The self-attention mechanism allows the model to focus on relevant parts of the input sequence, enabling it to learn complex patterns in the data. The multi-head attention mechanism further enhances the model's ability to capture different aspects of the input sequence in parallel, improving its performance on a wide range of NLP tasks such as translation, summarization, and image captioning. This is why Transformers have become the architecture of choice for many state-of-the-art NLP models, including GPT and BERT.

2.2.3 Training and Fine-Tuning

The foundation of an LLM's understanding and generation of human language lies in its pre-training phase. During this stage, the model is exposed to a large corpus of text data, often encompassing a wide range of topics, genres, and styles. The primary objective of pre-training is to enable the model to learn a generalized representation of language.

The pre-training is typically conducted using unsupervised learning techniques, where the model is trained on tasks like Masked Language Modeling (MLM) or Next Sentence Prediction (NSP). In MLM, for example, a percentage of the input tokens are randomly masked, and the model's objective is to predict the original tokens at these masked positions. The MLM objective can be formally represented as:

$$\mathcal{L}_{\text{MLM}} = -\sum_{i \in \mathcal{M}} \log p(x_i | x_{\setminus \mathcal{M}}) \tag{2.9}$$

where $\mathcal{M}$ is the set of masked positions, x_i is the original token at position i, and $x_{\setminus \mathcal{M}}$ represents the input with masked tokens.

Following pre-training, LLMs undergo a fine-tuning phase, wherein the model is specialized to perform specific NLP tasks. This phase involves training the pre-trained model on a smaller, task-specific dataset, allowing the model to adjust its weights to better perform the target task.

The fine-tuning process can be represented as a continuation of the training process, optimizing the following objective:

$$\mathcal{L}_{\text{fine-tune}} = -\sum_{(x,y) \in \mathcal{D}_{\text{task}}} \log p(y | x; \theta_{\text{pre-train}} + \Delta\theta) \tag{2.10}$$

where $\mathcal{D}_{\text{task}}$ is the task-specific dataset, (x, y) are the input-output pairs, $\theta_{\text{pre-train}}$ are the parameters learned during pre-training, and $\Delta\theta$ represents the parameter updates during fine-tuning.

Fine-tuning LLMs presents challenges such as catastrophic forgetting and overfitting, particularly when the task-specific dataset is small. Various strategies, including careful learning rate selection, regularization techniques, and the use of adapters, are employed to mitigate these issues.

Chapter 3

Literature Review

3.1 Overview of Large Language Models

The evolution of LLMs can be traced back to earlier models of machine learning that attempted to process and understand language. However, it was the introduction of models like Google's BERT (Bidirectional Encoder Representations from Transformers) and OpenAI's GPT (Generative Pre-trained Transformer) series that marked a significant leap in the capabilities of language models. Each iteration of these models has brought about improvements in understanding context, generating text, and general language comprehension, culminating in state-of-the-art models that are capable of writing essays, composing poetry, and even generating code.

3.2 Previous Studies on LLM Compression

Over the past few year, the increasing demand for Large Language Models (LLMs) across a vast spectrum of applications, from natural language processing to content generation, has underscored the critical need for optimization strategies tailored to these complex systems. Researchers have delved into a variety of optimization techniques aimed at refining LLMs, with a keen focus on enhancing model efficiency and reducing the computational burden without compromising the specialized performance that these models are known for. A key area of investigation has been model compression, which involves reducing the size and complexity of LLMs while maintaining their functionality and performance.

Low Rank Approximation

In the realm of large language models (LLMs), low-rank approximation has gained substantial traction as a method to fine-tune and streamline models. It has been utilized in various implementations, notably in Hu et al., 2021 and its subsequent adaptations (Valipour et al., 2023; Zhang et al., 2023; Chavan et al., 2024). These adaptations focus on leveraging low-rank structures to optimize the performance of LLMs without extensive resource demands.

A novel application of this technique is seen in TensorGPT (Xu, Xu, and Mandic, 2023), which employs a low-rank tensor format to manage large embeddings effi-

ciently. By adopting the Tensor-Train Decomposition (TTD), TensorGPT not only reduces the spatial complexity of LLMs but also potentially boosts their deployment on edge devices. This method treats each token embedding as a Matrix Product State (MPS), enabling the compression of the embedding layer by up to a factor of 38.40. Remarkably, this compression is achieved without sacrificing, and in some cases even enhancing, the performance of the model compared to its original configuration.

This focus on low-rank approximation underscores its critical role in advancing the field of LLMs by facilitating more efficient model architectures that maintain high performance while being computationally less demanding.

3.3 Introduction to BART

BART: Denoising Sequence-to-Sequence Pre-training for Natural Language Generation, Translation, and Comprehension by (Lewis et al., 2019), introduces BART as a versatile pretraining approach for natural language processing tasks. BART combines the benefits of both autoregressive (like GPT) and autoencoding (like BERT) paradigms into a unified model.

3.3.1 Architecture and Training

BART employs a Transformer-based sequence-to-sequence architecture from

(Vaswani et al., 2017), utilizing a bidirectional encoder (similar to BERT) and a left-to-right decoder (similar to GPT) to handle a wide array of tasks from generation to comprehension. For the base model (BART-base) and the large model (BART-large), the encoder and decoder consist of 6 and 12 layers, respectively (Appendix D). It is trained through a novel denoising objective, where the model reconstructs the original text from corrupted versions. This involves techniques such as token masking (Devlin et al., 2019) and text infilling, enhancing the model's ability to understand and generate contextually rich language.

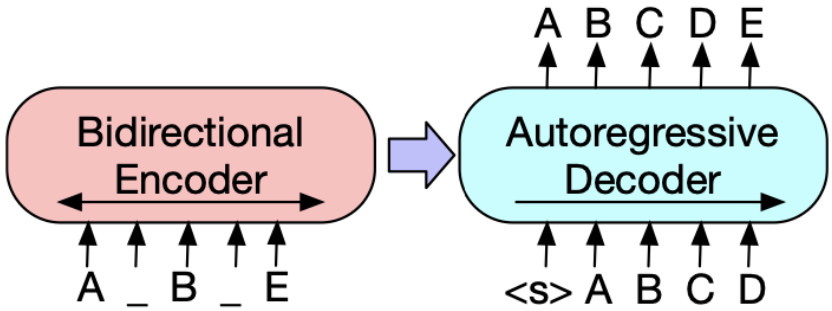


Figure 3.1: BART model's processing mechanism - Taken from Lewis et al., 2019

Figure 3.1 illustrates the BART model's processing mechanism. The original text sequence [ABCDE] is transformed into a masked version [A[MASK]B[MASK]E]. BART's encoder learns bidirectional representations from this altered input, allowing it to handle inputs that are not aligned with decoder outputs. The decoder

then attempts to reconstruct the original sequence autoregressively by optimizing the negative log-likelihood. For fine-tuning, the model processes an uncorrupted document to enhance accuracy and adaptability.

3.3.2 Efficacy and Applications

The versatility of BART is demonstrated across various NLP tasks including text generation, comprehension, and translation. Notably, BART outperforms previous work on benchmarks like ROUGE, showcasing substantial improvements particularly in abstractive summarization tasks. The model's ability to fine-tune to specific tasks using pre-trained weights allows it to excel in both generation and comprehension roles, making it a powerful tool for a broad range of applications.

3.4 Evaluating Summarization with ROUGE

The evaluation of automated summarization models is crucial for assessing their efficiency and effectiveness in capturing the essence of text data. ROUGE (Lin, 2004), which stands for Recall-Oriented Understudy for Gisting Evaluation, provides a set of metrics that are indispensable for this purpose. Introduced by Lin, 2004, ROUGE measures compare the overlap between computer-generated summaries and a set of reference summaries typically created by humans.

ROUGE Metrics ROUGE includes several specific metrics, such as ROUGE-N (n-gram overlap) and ROUGE-L (longest common subsequence), each suited for different aspects of summarization. For instance, ROUGE-N focuses on the exactness of content at various n-gram levels, providing insights into the precision of the summarization model in replicating key information. In contrast, ROUGE-L assesses the fluency and structure of the generated summaries by measuring sequence similarity, which is crucial for evaluating the narrative flow of the text.

Application and Relevance The application of ROUGE in evaluation has been extensively validated across various tasks, including the Document Understanding Conferences (DUC), where it has been used to measure the performance of summarization systems in a competitive environment. These metrics have proven to be reliable indicators of human judgment, making them a standard against which the summarization capabilities of LLMs are benchmarked. The relevance of ROUGE scores lies in their ability to provide quantifiable measures that correlate strongly with human evaluations, thus facilitating the improvement and development of more efficient summarization models.

Significance in LLM Research In the context of Large Language Models, understanding the effectiveness of different compression and optimization techniques often relies on the ability to evaluate how well the reduced models can summarize content. ROUGE metrics serve this purpose by quantifying the trade-offs between model complexity and performance retention, helping researchers and developers optimize LLMs without significant loss in functionality.

Chapter 4

Methodology

4.1 RAG Chatbot: Study Buddy

4.1.1 Motivation

As a teaching assistant, I have observed that students frequently resort to using chatbots like ChatGPT for quick answers to their questions. However, these chatbots often provide generic responses that may not be sufficiently helpful for academic purposes, as they lack the specificity and depth required for effective studying. This observation underscores a gap in the effectiveness of current chatbot solutions in educational contexts.

The Retrieval-Augmented Generation (RAG) model addresses this gap by combining a retriever and a generator, thereby enabling the delivery of more detailed and context-specific information.

To gain practical experience with large language models (LLMs) and their application in real-world language processing tasks, I developed a chatbot inspired by a project at BSS - Aarhus University (Appendix C). This chatbot, utilizing the RAG model Lewis et al., 2020, serves as a 'Study Buddy' to assist students in understanding complex topics, providing quick and relevant answers, and enhancing their learning experience for challenging subjects.

4.1.2 Development and Environment Tools

The implementation of the RAG chatbot was accomplished using the following tools and technologies:

- **OpenAI API:** Utilized to access the GPT-4 model for the chatbot's generation capabilities.
- **Astra DB:** Employed for storing and managing the data used by the retriever component.
- **DataStax RAGstack:** A curated stack of leading open-source software designed to facilitate the implementation of the RAG pattern in production-ready applications using Astra Vector DB as a vector store.
- **Langchain:** An open-source framework used for developing applications powered by LLMs.

- **Streamlit:** An open-source Python framework used for building and deploying data applications with minimal code.

4.1.3 Implementation Steps

The development of the RAG chatbot involves the following steps:

1. **Chatbot Interface:** Designing and creating a user-friendly interface for users to interact with the chatbot.
2. **Generator Component:** Implementing the generator component to provide responses based on the GPT-4 model.
3. **Retriever Component:** Developing the retriever component to search for relevant information based on user queries.
4. **Integration:** Integrating the retriever and generator components to create a functional RAG chatbot.
5. **PDF Uploader:** Implementing a feature that allows users to upload their own materials, enabling more meaningful and contextual responses.
6. **Deployment:** Deploying the chatbot on a cloud platform to make it publicly accessible.

The theoretical workflow of the RAG chatbot is illustrated in Figure 4.1. The process begins with the user inputting a query, which is then processed by the retriever to find relevant information. This information provides context based on the user's query. Subsequently, the generator creates a response using an embedded prompt template, the retrieved information, and the user's query, thereby providing the user with a detailed and specific answer.

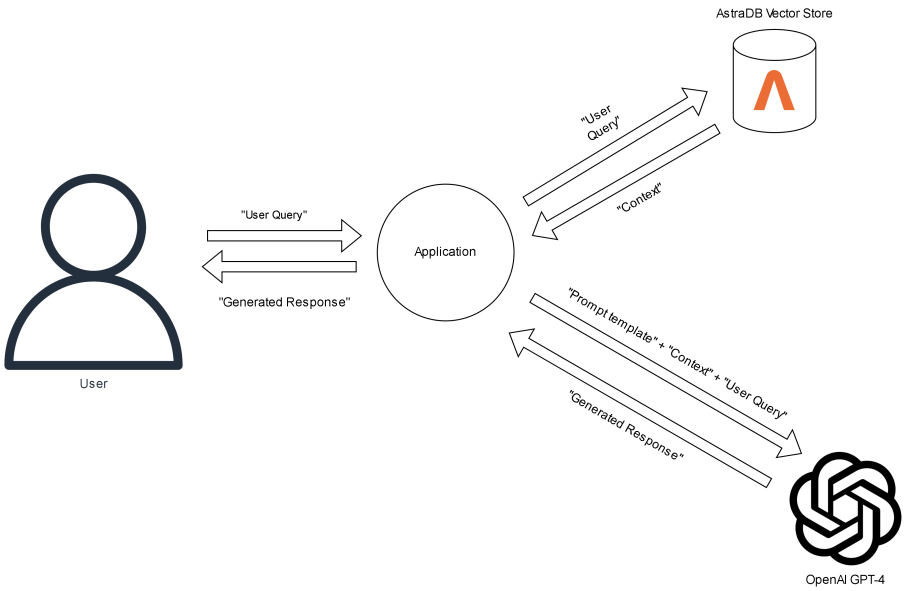


Figure 4.1: Diagram of Query Process

4.2 LLM Optimization

Optimizing Large Language Models (LLMs) for efficiency and performance without compromising their effectiveness is a critical area of research in the field of artificial intelligence. Various techniques have been developed to address this challenge, each employing unique strategies to reduce computational resources, decrease model size, and maintain, if not improve, the model's performance. This section explores the general methodologies applied in the optimization of LLMs, focusing particularly on model compression techniques. Among these, Low-Rank Approximation stands out as a highly effective approach. This technique leverages the mathematical properties of matrices to approximate the original model parameters with fewer components, thereby possibly reducing the model's computations and resource requirements.

4.2.1 Optimization Techniques for Large Language Models

Optimization strategies for Large Language Models (LLMs) are crucial for improving computational efficiency and model efficacy. These strategies are generally divided into two primary types: model pruning and parameter sharing.

Model Pruning

Model pruning aims to reduce the model's complexity and size effectively without substantial loss in performance. It involves systematically eliminating parameters or connections within the model that are least consequential to the output, thereby enhancing operational efficiency and making the model more adaptable for use in environments with limited resources.

Parameter Sharing

Conversely, parameter sharing utilizes the model's existing parameters across various parts of the model or different tasks. This method optimizes the use of the model's capacity, enabling multifunctional performance without an increase in parameter count.

Focus on Low-Rank Approximation

This thesis will specifically focus on model pruning techniques, with a particular emphasis on low-rank approximation. This approach approximates large matrices or tensors with ones of a lower rank, reducing the number of components and computational demands while preserving the model's critical information processing capabilities.

4.3 Case Study: Low-Rank Approximation

To assess the effectiveness of low-rank approximation in compressing LLMs, we consider Facebook's the BART-Base model (Lewis et al., 2019) as a case study.

BART is a transformer-based LLM that has been widely used for various natural language processing tasks such as summarization.

This thesis focuses on applying low-rank approximation specifically to the attention matrices. This choice stems from their pivotal role in the transformer architecture. These matrices, which help the model assess the relevance of different words within the input data, tend to be large and often encapsulate redundant information (Aghajanyan, Zettlemoyer, and Gupta, 2020).

By applying low-rank approximation to the attention weight matrices of the BART-base model, we aim to explore the possibility in reducing computational complexity and storage requirements while preserving its performance on a summarization task.

4.3.1 Implementation Steps

The implementation of low-rank approximation for BART involves the following steps:

1. **Singular Value Decomposition (SVD):** The attention weight matrices (Key, Query, Value, and Output) of the model are decomposed using SVD to obtain the low-rank approximation.

$$\begin{array}{lcl} Q & & U_q\ \Sigma_q\ V_q^T \\ V & \xrightarrow{\text{SVD}} & U_v\ \Sigma_v\ V_v^T \\ K & & U_k\ \Sigma_k\ V_k^T \\ O & & U_o\ \Sigma_o\ V_o^T \end{array}$$

2. **Custom Layer Implementation:** A custom layer is implemented to replace the original attention layers with the low-rank approximation.

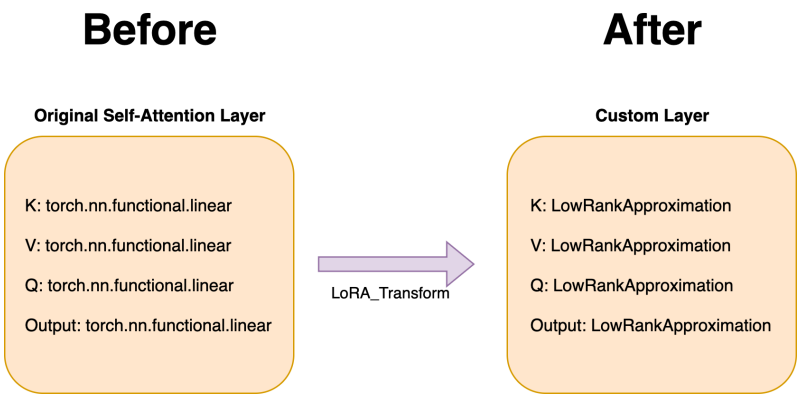


Figure 4.2: Components in attention layers replaced with their custom low-rank approximation

4.3.2 Evaluation Metrics

The approximated model is evaluated based on:

1. ROUGE scores (Lin, 2004): How well does the approximated model perform in comparison to the original BART-Base model on the summarization task? From which rank does the model start to lose performance?

2. Computational efficiency: How does the approximated model compare to the original BART-Base model in terms of computational resources required?

3. Cosine similarity: How well does the approximated model maintain the semantic properties of the original model?

4. Comparing some of the summaries generated by the compressed model with the original BART-Base model and the reference summaries using the Samsum dataset (Gliwa et al., 2019).

Dialogue

Hannah: Hey, do you have Betty's number?
Amanda: Lemme check
Hannah: <file_gif>
Amanda: Sorry, can't find it.
Amanda: Ask Larry
Amanda: He called her last time we were at the park together
Hannah: I don't know him well
Hannah: <file_gif>
Amanda: Don't be shy, he's very nice
Hannah: If you say so..
Hannah: I'd rather you texted him
Amanda: Just text him
Hannah: Urgh.. Alright
Hannah: Bye
Amanda: Bye bye

Summary

Hannah needs Betty's number but Amanda doesn't have it.
She needs to contact Larry.

Figure 4.3: SamSum Dataset `test[0]` dialogue and reference summary

4.3.3 Appropriate Rank Selection

In Section 2.1.4, we established the criterion for selecting an appropriate rank k to achieve computational efficiency and storage reduction. The criterion is given by:

$$k < \frac{\sqrt{4mn + (m+n)^2} - m - n}{2}$$

where m and n are the dimensions of the original matrix we wish to approximate with a low-rank representation.

BART-base Model:
For the BART-base model, the attention matrices have dimensions $m = n = 768$.

We know this by inspecting the model's architecture (Appendix D).
Therefore, the rank k for the approximation should satisfy:

$$k < \frac{\sqrt{4 \times 768 \times 768 + (768 + 768)^2} - 768 - 768}{2} \approx 318$$

to achieve a reduction in computational complexity and storage requirements.

BART-large Model:
Similarly, for the BART-large model, the attention matrices have dimensions $m = n = 1024$. Therefore, the rank k for the approximation should satisfy:

$$k < \frac{\sqrt{4 \times 1024 \times 1024 + (1024 + 1024)^2} - 1024 - 1024}{2} \approx 424$$

To determine the feasibility of low-rank approximation for the two BART models, we will evaluate the approximated model at various ranks and observe the impact on the ROUGE scores and the other metrics mentioned in section 4.3.2. This evaluation will help us understand the trade-offs between rank reduction and model performance.

Chapter 5

Implementation

5.1 RAG Chatbot

5.1.1 Chatbot Interface

The chatbot interface was developed using the `Streamlit` library, which provides a simple and interactive way to create web applications. The interface allows users to interact with the RAG chatbot by entering a question in the input field and receiving the corresponding answer from the model. The chatbot interface was designed to be user-friendly and intuitive, enabling users to easily communicate with the chatbot and obtain relevant information.

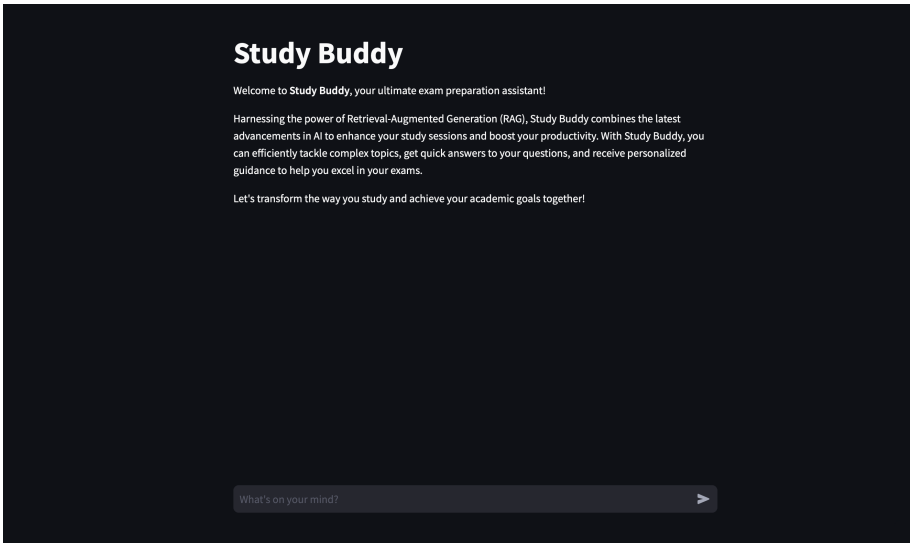


Figure 5.1: RAG Chatbot Interface

The chatbot interface displays a title, description and input field for user queries using the `st.title`, `st.markdown` and `st.chat_input` functions respectively. The title introduces the chatbot as "Study Buddy," while the description provides an overview of the chatbot's capabilities and features. By using the `Streamlit` library, the chatbot interface can be created by just a few lines of code.

5.1.2 Generator Component

The generator component of Study Buddy was implemented using the `Langchain` library, which provides ready-to-use methods to call OpenAI's GPT-4 model. Streamlit reruns the code in the `app.py` file every time a user interacts with the chatbot. The following code snippets demonstrate how the generator calls the GPT-4 model and generates responses to user queries.

```
from langchain_openai import ChatOpenAI

# Cache OpenAI Chat Model for reuse
@st.cache_resource()
def get_chat_model():
    return ChatOpenAI(
        temperature=0.3,
        model='gpt-4',
        streaming=True,
        verbose=True
    )
chat_model_instance = get_chat_model()
```

Listing 1: Caching the OpenAI Chat Model

The above code snippet details the implementation of the generator component for the Study Buddy chatbot, leveraging the `Langchain` library to interface with OpenAI's GPT-4 model. The generator component is responsible for producing the responses to user queries.

Key Elements of the Code:

- `ChatOpenAI`: This class from the `langchain_openai` module facilitates interaction with OpenAI's GPT-4 model.
- `@st.cache_resource`: This Streamlit decorator caches the chat model instance, ensuring efficient reuse across multiple interactions without the need to reinitialize it each time.
- `temperature`: A parameter that controls the randomness of the model's responses, with a value of 0.3 indicating a balance between creativity and determinism.
- `model='gpt-4'`: Specifies the use of the GPT-4 model for generating responses.
- `streaming=True`: Enables streaming of the model's output, allowing for real-time response generation.
- `verbose=True`: Provides detailed logs of the model's operations for debugging and transparency.

Explanation of Functionality:

1. **Caching the Chat Model:** The `get_chat_model()` function initializes and caches an instance of the GPT-4 model with specific parameters. By caching this instance, the system ensures that the model does not need to be reloaded each time a user interacts with the chatbot, thereby improving efficiency and response times.

2. **Model Parameters:** The temperature parameter is set to 0.3, which strikes a balance between generating creative responses and maintaining consistency. The `streaming=True` parameter enables the model to stream responses in real-time, providing a more interactive and immediate user experience. The `verbose=True` parameter allows for detailed logging, which is useful for monitoring the chatbot's operations and debugging issues.

This setup ensures that the generator component efficiently produces detailed and contextually relevant responses, enhancing the chatbot's effectiveness as an exam preparation assistant.

5.1.3 Retriever Component

The retriever component of Study Buddy was implemented using the `Langchain` library to retrieve the $k = 5$ most relevant documents from the Astra DB Vector Store.

```
from langchain_community.vectorstores import AstraDB
from langchain_openai import OpenAIEmbeddings

# Cache the Astra DB Vector Store for reuse
@st.cache_resource(show_spinner='Connecting to AstraDB')
def get_vector_store():
    vector_store_instance = AstraDB(
        embedding=OpenAIEmbeddings(),
        collection_name="datastax",
        api_endpoint=st.secrets['ASTRA_API_ENDPOINT'],
        token=st.secrets['ASTRA_TOKEN']
    )
    return vector_store_instance
vector_store_instance = get_vector_store()

# Cache the Retriever for reuse
@st.cache_resource(show_spinner='Initializing retriever')
def get_retriever():
    retriever_instance = vector_store_instance.as_retriever(
        search_kwargs={"k": 5}
    )
    return retriever_instance
retriever_instance = get_retriever()
```

Listing 2: Caching the Astra DB Vector Store and Retriever

The above code snippet demonstrates the implementation of the retriever component for the Study Buddy chatbot. The `Langchain` library is utilized to connect to the Astra DB Vector Store, which is a database optimized for vector searches. The

retriever component is responsible for identifying and retrieving the most relevant documents based on the user's query.

Key Elements of the Code:

- `AstraDB`: This class from the `langchain_community.vectorstores` module is used to create an instance of the vector store. It requires an embedding model, collection name, API endpoint, and token for authentication.
- `OpenAIEmbeddings`: This class from the `langchain_openai` module provides the embedding model used to convert text into numerical vectors that the vector store can handle.
- `@st.cache_resource`: This Streamlit decorator caches the resources, ensuring that the vector store and retriever instances are reused efficiently across different runs of the application. This caching mechanism helps to optimize performance by avoiding redundant connections and initializations.
- `get_vector_store()`: This function initializes and returns an instance of the vector store by connecting to Astra DB using the provided API endpoint and token.
- `get_retriever()`: This function initializes and returns an instance of the retriever, configured to fetch the top $k = 5$ most relevant documents based on the user's query.

Explanation of Functionality:

1. **Connecting to Astra DB:** The `get_vector_store()` function creates a connection to the Astra DB Vector Store using the OpenAI embeddings for document vectorization. The connection details, such as the API endpoint and token, are securely retrieved from the Streamlit secrets configuration.
2. **Initializing the Retriever:** The `get_retriever()` function sets up the retriever to query the vector store. The retriever is configured to return the top 5 relevant documents, which helps in providing contextually accurate responses to user queries.

This setup ensures that the retriever component can efficiently access and retrieve relevant information, enhancing the chatbot's ability to deliver detailed and context-specific answers.

5.1.4 Integration of Generator and Retriever

The generator and retriever components were integrated to provide a comprehensive response to user queries. The following code snippet demonstrates how the generator and retriever components work together to generate responses based on the user's input.

```
from langchain.schema.runnable import RunnableMap

input_data = RunnableMap({
        'context': lambda x: retriever_instance.get_relevant_documents(x['question']),
        'question': lambda x: x['question']
    })
response_chain = input_data | chat_prompt | chat_model_instance
```

Listing 3: Integration of Generator and Retriever

The above code snippet illustrates the integration of the generator and retriever components for the Study Buddy chatbot. This integration ensures that user queries are processed using both components to provide contextually accurate and comprehensive responses.

Key Elements of the Code:

- `RunnableMap`: This class is used to map the inputs (context and question) to the necessary functions for processing.
- `retriever_instance.get_relevant_documents`: This function retrieves the most relevant documents from the vector store based on the user's query.
- `chat_prompt`: A predefined template that formats the retrieved context and user question for the generator.
- `chat_model_instance`: The cached instance of the GPT-4 model used for generating responses.

Explanation of Functionality:

1. **Input Data Mapping:** The `RunnableMap` is used to create a mapping of inputs, where the 'context' key is assigned a lambda function that retrieves relevant documents based on the user's question, and the 'question' key simply maps the user's question.
2. **Response Chain Creation:** The `response_chain` is constructed by chaining the input data through the chat prompt and the chat model instance. This chain ensures that the user's question is enriched with relevant context before generating the final response.
3. **Generating Responses:** When invoked, the `response_chain` processes the inputs by first retrieving the relevant documents, then formatting them using the chat prompt, and finally generating a comprehensive response using the GPT-4 model.

This integration of the generator and retriever components ensures that the Study Buddy chatbot can deliver detailed and contextually relevant answers to user queries, enhancing its utility as an exam preparation assistant.

5.1.5 PDF Uploader

The PDF uploader component allows users to upload course materials, lecture notes, or other relevant documents to the Study Buddy chatbot. The uploaded documents are stored in the Astra DB Vector Store, enabling the chatbot to retrieve and reference them when responding to user queries.

```
# Sidebar for document upload
with st.sidebar:
    with st.form('upload_form'):
        uploaded_file = st.file_uploader('Upload a document for enhanced context', type=['pdf'])
        submit_button = st.form_submit_button('Save to AstraDB')
        if submit_button:
            process_and_vectorize_document(uploaded_file, vector_store_instance)
```

Listing 4: PDF Uploader Interface

The above code snippet creates an interface within the Streamlit sidebar for uploading PDF documents. This interface allows users to upload course materials, lecture notes, or other relevant documents, which are then processed and stored in the Astra DB Vector Store.

```
# Function to process and vectorize uploaded documents into Astra DB
def process_and_vectorize_document(uploaded_file, vector_db):
    if uploaded_file is not None:

        # Create a temporary file to store the uploaded document
        temp_dir = tempfile.TemporaryDirectory()
        temp_file_path = os.path.join(temp_dir.name, uploaded_file.name)
        with open(temp_file_path, 'wb') as temp_file:
            temp_file.write(uploaded_file.getvalue())

        # Load the PDF file
        document_pages = []
        pdf_loader = PyPDFLoader(temp_file_path)
        document_pages.extend(pdf_loader.load())

        # Initialize text splitter
        splitter = RecursiveCharacterTextSplitter(
            chunk_size=1500,
            chunk_overlap=100
        )

        # Split the document and add it to the vector store
        split_pages = splitter.split_documents(document_pages)
        vector_db.add_documents(split_pages)
        st.info(f"{len(split_pages)} pages have been loaded into the database.")
```

Listing 5: Processing and Vectorizing Uploaded Documents

The above code snippet defines a function that processes and vectorizes uploaded PDF documents, storing them in the Astra DB Vector Store. This function is crucial for enabling the chatbot to retrieve and reference user-uploaded materials when generating responses.

Key Elements of the Code:

- `tempfile.TemporaryDirectory()`: Creates a temporary directory to store the uploaded file.
- `open(temp_file_path, 'wb')`: Writes the uploaded file to the temporary directory.
- `PyPDFLoader`: Loads the PDF document into memory.
- `RecursiveCharacterTextSplitter`: Splits the document into smaller chunks for vectorization.
- `vector_db.add_documents`: Adds the vectorized document chunks to the Astra DB Vector Store.
- `st.info`: Displays an information message indicating the number of pages processed and loaded into the database.

Explanation of Functionality:

1. **Temporary File Storage:** The uploaded PDF document is stored temporarily to facilitate processing.
2. **Document Loading:** The PDF document is loaded into memory using the `PyPDFLoader` class.
3. **Text Splitting:** The loaded document is split into smaller chunks using the `RecursiveCharacterTextSplitter` class. This step is necessary to manage large documents and prepare them for vectorization.
4. **Vectorization and Storage:** The split document chunks are vectorized and added to the Astra DB Vector Store. This enables efficient retrieval and reference by the chatbot when responding to user queries.
5. **User Feedback:** A message is displayed to inform the user about the number of pages successfully processed and loaded into the database.

This comprehensive approach ensures that the uploaded documents are effectively processed and made available for contextual responses, significantly enhancing the chatbot's utility and relevance.

5.1.6 Deployment

The Study Buddy chatbot was deployed using the Streamlit sharing platform, which provides a simple and efficient way to host and share Streamlit applications. The deployment process involved uploading the application code to the Streamlit sharing platform and configuring the necessary settings to make the chatbot publicly accessible.

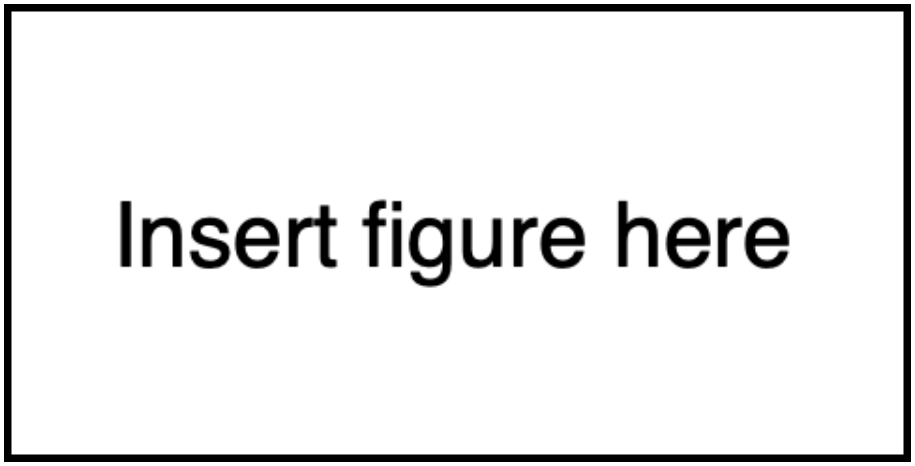


Figure 5.2: Study Buddy Chatbot Deployment

5.2 Case Study

5.2.1 Importing the Model

The BART-base model was imported from the Hugging Face platform using the `transformers` library.

```
model_checkpoint = "facebook/bart-base"
model = AutoModelForSeq2SeqLM.from_pretrained(model_checkpoint)
```

Listing 6: Importing the BART-base model

5.2.2 Dataset Preprocessing

The SamSum dataset was imported and preprocessed to facilitate the training and evaluation of the BART-base model. The dataset comprises conversations crafted and documented by linguists proficient in English, which were subsequently annotated with summaries. The preprocessing steps included tokenization, truncation, and padding to ensure uniform input sizes for the model.

Importing the Dataset

First, we imported the necessary libraries and loaded the SamSum dataset using the `datasets` library:

```
from datasets import load_dataset

raw_datasets = load_dataset("samsum")
```

Listing 7: Loading the SamSum dataset

The `load_dataset` function fetches the SamSum dataset, which contains dialogues and their corresponding summaries.

Setting Maximum Lengths

To ensure that the input and target sequences fit within the model's constraints, we set the maximum lengths for input and target sequences:

```
max_input_length = 512
max_target_length = 128
```

Listing 8: Setting maximum lengths for inputs and targets

Here, `max_input_length` is set to 512 tokens and `max_target_length` is set to 128 tokens, which are reasonable lengths for dialogues and summaries respectively.

Preprocessing Function

Next, we defined a preprocessing function to tokenize the inputs and targets. This function truncates and pads the sequences to the specified maximum lengths:

```
def preprocess_function(examples):
    inputs = [doc for doc in examples["dialogue"]]
    model_inputs = tokenizer(inputs, max_length=max_input_length, truncation=True)

    # Setup the tokenizer for targets
    with tokenizer.as_target_tokenizer():
        labels = tokenizer(examples["summary"], max_length=max_target_length, truncation=True)

    model_inputs["labels"] = labels["input_ids"]
    return model_inputs
```

Listing 9: Defining the preprocessing function

This function performs the following steps:

- Tokenizes the dialogue texts.
- Sets up the tokenizer for the target summaries.
- Tokenizes the summaries and adds them to the `model_inputs` dictionary as labels.

Applying the Preprocessing Function

Finally, we applied the preprocessing function to the entire dataset using the `map` method, which processes the dataset in batches:

```
tokenized_datasets = raw_datasets.map(preprocess_function, batched=True)
```

Listing 10: Applying the preprocessing function to the dataset

The `map` method applies the `preprocess_function` to each example in the dataset, ensuring that all dialogues and summaries are properly tokenized, truncated, and padded. This results in a tokenized dataset ready for training and evaluation with the BART-base model.

5.2.3 Finetuning the Model

The model was then fine-tuned on the SamSum dataset for dialogue summarization tasks.

Epoch	Training Loss	Validation Loss	Rouge1	Rouge2	Rougel	Rougelsum	Gen Len
0	1.81	1.54	47.39	24.45	40.03	43.77	18.21
2	1.48	1.49	47.98	24.7740	40.66	44.21	18.06

Table 5.1: Training and validation results across epochs.

5.2.4 Custom Layer Implementation

A custom low-rank layer was implemented to replace the Q, K, V and output matrices typically found in the attention mechanisms of transformers. This implementation utilizes the Singular Value Decomposition (SVD) from the PyTorch library to decompose and subsequently truncate the weight matrices, preserving only the k most significant components.

```
class LowRankLayer(nn.Module):
    """given a linear layer find low rank decomposition"""
    def __init__(self, rank, full_rank_layer):
        super().__init__()
        self.rank = rank
        self.bias = full_rank_layer.bias
        U, S, Vh = torch.linalg.svd(full_rank_layer.weight, driver = 'gesvd')
        S_diag = torch.diag(S)
        self.U = U[:, :self.rank]
        self.S = S_diag[:self.rank, :self.rank]
        self.Vh = Vh[:self.rank, :]
        self.weight = full_rank_layer.weight

    """forward pass through the low-rank layer"""
    def forward(self, x):
        output_t1 = F.linear(x, self.Vh)
        output_t2 = F.linear(output_t1, self.S)
        output = F.linear(output_t2, self.U, self.bias)
        return output
```

Listing 11: Custom Low-Rank Layer Implementation

The class `LowRankLayer` inherits from `nn.Module`, indicating it is a PyTorch neural network layer. The class contains two main components: the `init` method, which initializes the layer, and the `forward` method, which defines the forward pass.

Initialization In the `init` method, the low-rank decomposition is performed:

- The rank k and the full-rank layer are passed as parameters.
- The bias from the original layer is retained.

- Singular Value Decomposition (SVD) is performed on the weight matrix of the full-rank layer using `torch.linalg.svd`. This decomposes the weight matrix into U, S, and Vh.
- The diagonal matrix S is converted into a diagonal tensor using `torch.diag`.
- The top k singular vectors and values are retained:
  - `self.U` contains the first k columns of U.
  - `self.S` is the top $k \times k$ diagonal part of S.
  - `self.Vh` contains the first k rows of Vh.

Forward Pass The `forward` method performs the forward pass through the low-rank layer:

- The input x is first multiplied by Vh using `F.linear`.
- The result is then multiplied by the diagonal matrix S.
- Finally, the intermediate result is multiplied by U and the bias is added.

This approach maintains the key structural properties of the original layer. By focusing on the most significant singular values and vectors, the low-rank approximation retains the most important features of the weight matrices, thus aiming to preserve the performance of the model to a large extent.

5.2.5 Traversing the Model and Applying Low-Rank Approximation

The BART-base model was traversed to identify the attention layers that could be replaced with the low-rank approximation. The model's architecture was examined to determine the layers that could benefit from the low-rank decomposition.

```python
@dataclass
class LowRankConfig:
    rank:int
    target_modules: list[str]

# find the module that ends target suffix
def get_submodules(model, key):
    parent = model.get_submodule(".".join(key.split(".")[:-1]))
    target_name = key.split(".")[-1]
    target = model.get_submodule(key)
    return parent, target, target_name

# this function replaces a target layer with low rank layer
def recursive_setattr(obj, attr, value):
    attr = attr.split('.', 1)
    if len(attr) == 1:
        setattr(obj, attr[0], value)
    else:
        recursive_setattr(getattr(obj, attr[0]), attr[1], value)

# Traversing and modifying the BART model
def loRA_Transform(model, config):
  for key, module in model.named_modules():
      target_module_found = (
        any(key.endswith("." + target_key) for target_key in config.target_modules)
      )
      if target_module_found:
          low_rank_layer = LowRankLayer(config.rank, module)
          #replace target layer with low rank layer
          recursive_setattr(model, key, low_rank_layer)
```

Listing 12: Traversing the BART-base model for low-rank approximation

The `loRA_Transform` function traverses the BART-base model and replaces the target layers with low-rank layers. The function iterates over the model's modules and identifies the layers that match the target layer names specified in the configuration. For each target layer found, a low-rank layer is created using the `LowRankLayer` class and replaces the original layer in the model.

Chapter 6

Evaluation and Results

6.1 RAG Chatbot

The primary goal of developing the Study Buddy RAG chatbot was to gain hands-on experience with large language models (LLMs) and their application in real-world language processing tasks.

The evaluation of the Study Buddy chatbot involved querying the chatbot both before and after inserting relevant documents. The aim was to observe how the inclusion of additional context impacts the chatbot's performance in terms of accuracy, relevance, and response quality. This internal testing was done to verify that the chatbot was ready for potential future testing by users.

6.1.1 Functionality Testing

The functionality of the chatbot was tested by ensuring that all components (retriever, generator, and PDF uploader) worked seamlessly together. This involved verifying the following:

- **PDF Uploader:** Successfully uploading PDF documents and storing them in the Astra DB Vector Store.
- **Retriever Component:** Accurately retrieving relevant documents based on user queries.
- **Generator Component:** Generating coherent and contextually relevant responses using the GPT-4 model.
- **Integration:** Ensuring that the retriever and generator components integrate smoothly to provide comprehensive responses.

6.1.2 Case Study: Impact of Document Insertion on Response Quality

A series of specific queries were made to the Study Buddy chatbot before and after inserting a set of relevant documents. The goal was to observe changes in the quality of responses. The queries were related to complex topics that required detailed and specific information.

Example Query 1: The first query was related to the topic of "orthogonal matrices" and involved asking the chatbot to explain the properties of orthogonal matrices.

- **Before Document Insertion:** The chatbot provided a general and correct overview of the topic.

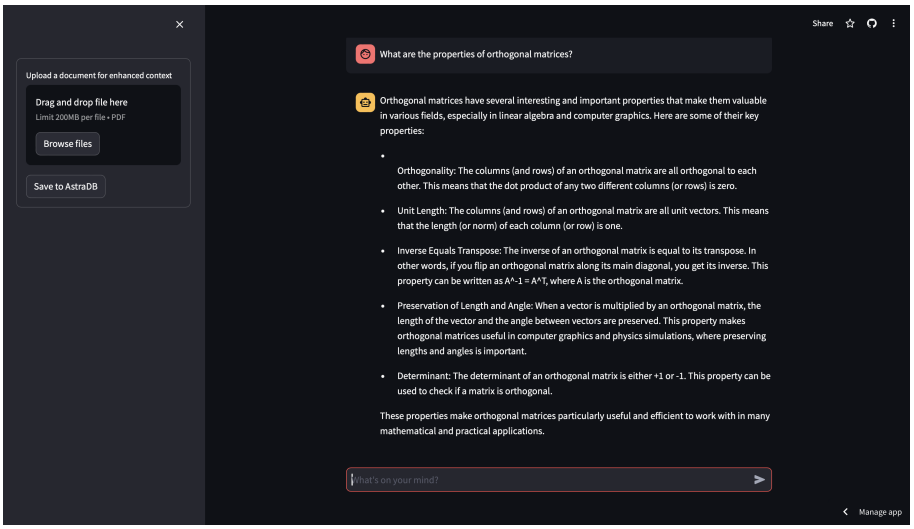


Figure 6.1: Response before insertion of lecture slides on orthogonal matrices

- **After Document Insertion:** The chatbot referenced specific information from the uploaded documents, while providing an accurate response.

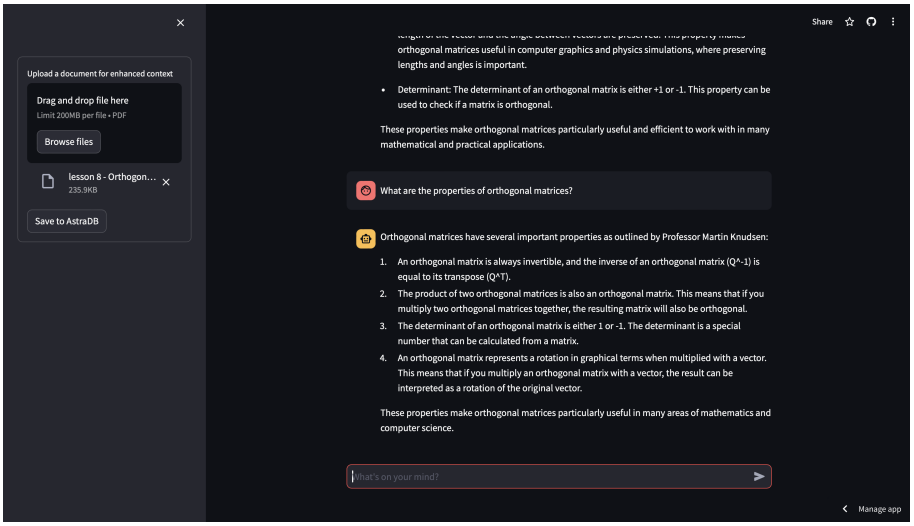


Figure 6.2: Response after insertion of lecture slides on orthogonal matrices

In this case we see that the professor who created the slides was referenced in the response, which was not present in the response before the document insertion. Furthermore, the response is almost identical in both structure and wording to one of the slides in the uploaded document.

Example Query 2: The second query was related to an uncommon topic, e.g. a project I did earlier in my studies, and involved asking the chatbot to explain the purpose of the system developed during the project. This query was chosen to test the chatbot's ability to provide information on topics that OpenAI's GPT-4 model is not familiar with.

- **Before Document Insertion:** A response was produced but lacked detailed information and context about the project.

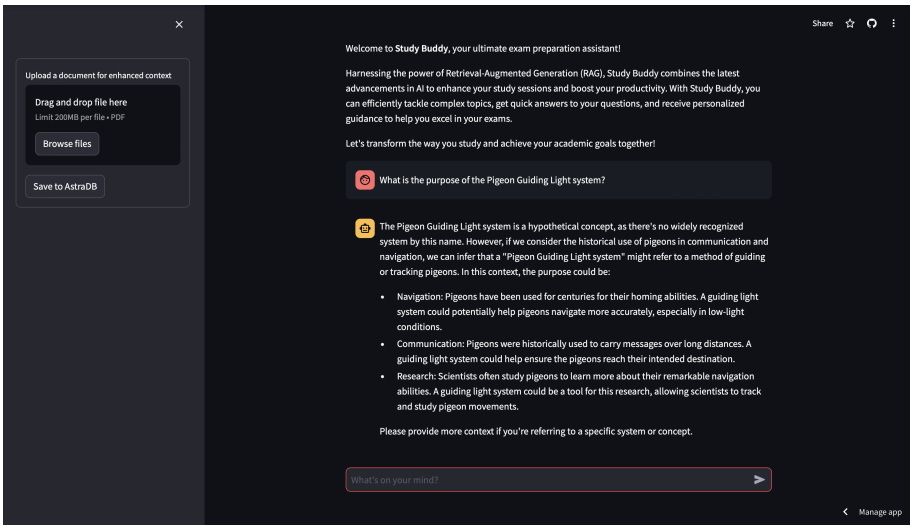


Figure 6.3: Response before insertion of project documentation

- **After Document Insertion:** The response included a detailed explanation in bullet points of the project, providing fast insight to a project that is not accesible for ChatGPT-4.

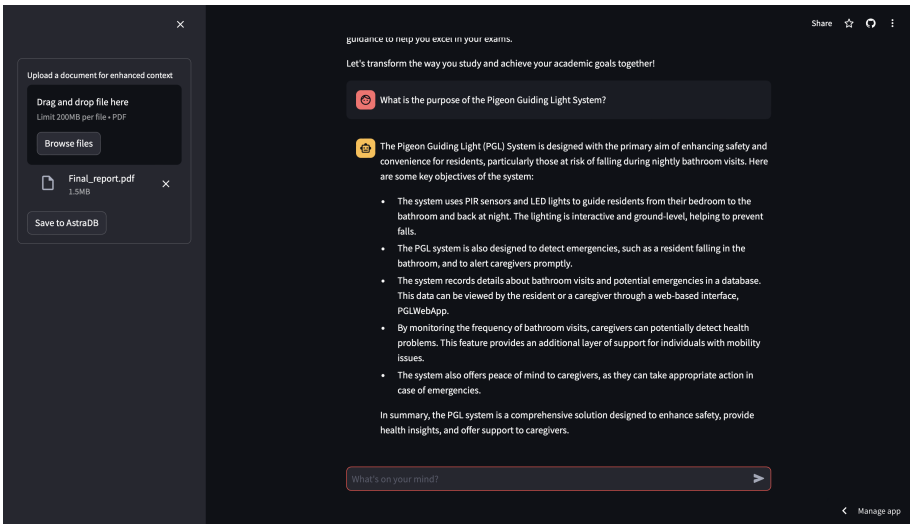


Figure 6.4: Response after insertion of project documentation

6.1.3 Impact of Document Insertion on Unrelated Query Response

- **Document Insertion of a Single Document:** After inserting only a document on "orthogonal matrices," a query was made on a different topic, e.g. "eigenvectors". The chatbot was not able to provide a relevant response, indicating that the document insertion did impact the chatbot's ability to respond to unrelated queries.

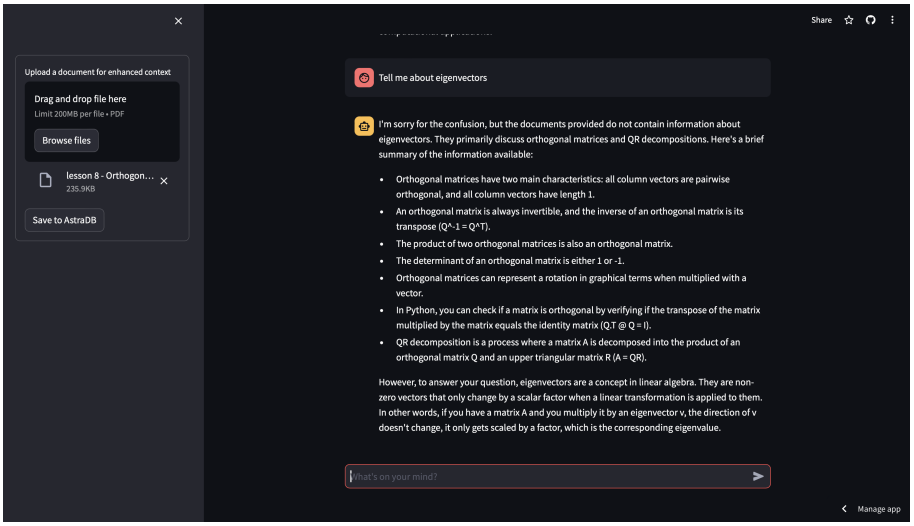


Figure 6.5: Response to an unrelated query after insertion of orthogonal matrices slides

- **Document Insertion of Multiple Documents:** After inserting another document on "eigenvectors," the chatbot was able to provide a relevant response to the same query on "eigenvectors." This demonstrates that the chatbot's ability to respond to queries on different topics can be improved by inserting the relevant documents.

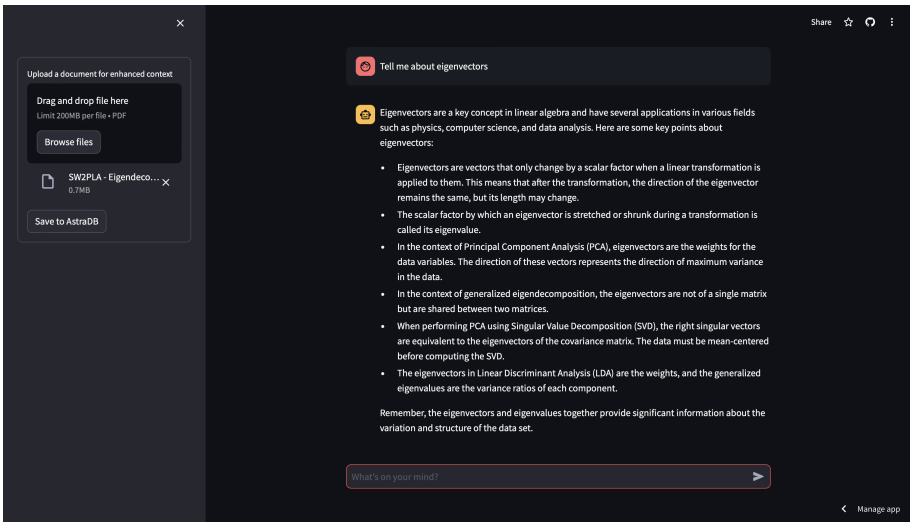


Figure 6.6: Response to query after insertion of eigenvectors slides

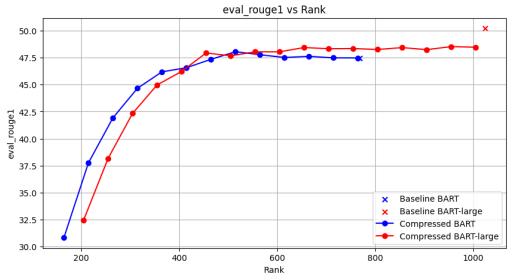
It should be noticed that the response is tailored after the inserted document, such that it also covers PCA and SVD, which are specifically stated as related topics in the slides.

6.2 Case Study

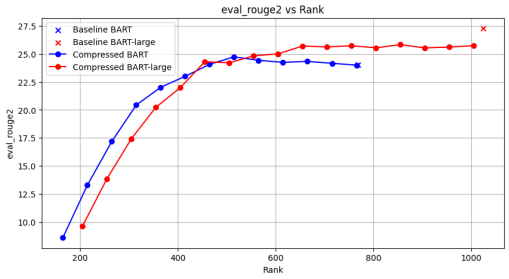
The evaluation of compressing the BART-base and BART-large model using low-rank approximation reveals the following insights:

6.2.1 ROUGE scores

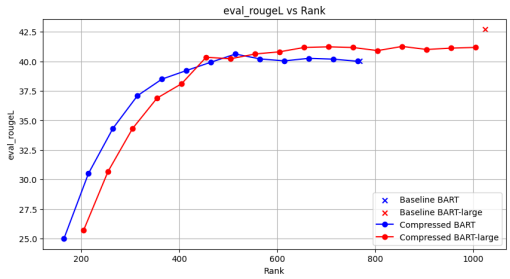
The low-rank approximation technique achieves comparable ROUGE scores until rank $r = 510$, after which the scores begin to decline. This indicates that the model's performance is preserved up to a certain rank, beyond which the approximation starts to impact the summarization quality. The following figures illustrate the ROUGE scores for various ranks:



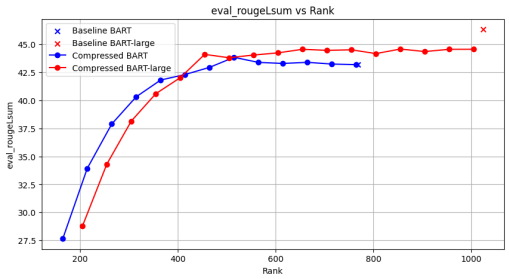
(a) ROUGE-1 scores.



(b) ROUGE-2 scores.



(c) ROUGE-L scores.



(d) ROUGE-Lsum scores.

Figure 6.7: ROUGE scores for different ranks.

As depicted in Figure 6.7, the ROUGE-1, ROUGE-2, ROUGE-L, and ROUGE-Lsum scores maintain a high level of performance up to a rank of $r = 510$. However, it is important to note that for the BART-large model, there is a slight decline in performance even when the model is low-rank approximated at full rank. Beyond the rank of $r = 510$, a noticeable degradation in scores is observed, indicating the limitations of the low-rank approximation in preserving the model's quality.

6.2.2 Comparing Summaries

The quality of the generated summaries demonstrates a noticeable decline as the rank of the low-rank approximation decreases. Initially, at higher ranks close to full rank, the summaries remain somewhat coherent and consistent, closely matching the original summary at full rank. However, as the rank decreases to $r = 510$, the summaries start to lose details and exhibit increased errors and inconsistencies.

At ranks 765 through 730, the summary remains consistent:

> *Hannah is looking for Betty's number. Amanda can't find it. Larry called her last time they were at the park together.*

However, a shift is noticed at rank 725 through 720:

> *Hannah doesn't know Betty's number. She texted Larry last time they were at the park together.*

This slight change introduces minor variations in the narrative.

The original phrasing returns from rank 715 through 655:

> *Hannah is looking for Betty's number. Amanda can't find it. Larry called her last time they were at the park together.*

By rank 650 through 490, a critical detail is lost at times, but the original phrasing returns.

> *Hannah is looking for Betty's number. Larry called her last time they were at the park together.*

As the rank decreases further, the summary begins to diverge more substantially:

> Rank 485: *Amanda can't find Betty's number. Hannah doesn't know Larry.*

> Rank 470: *Betty's number is not in Hannah's phone.*

By the time the rank reaches around 450, the summaries are severely affected:

> Rank 435: *Hannah has Betty's number. She texted Larry last time they were at the park together.*

Further reduction in rank leads to highly inconsistent and erroneous summaries:

> Rank 325: *Amanda can't find Betty's number. Hannah and Amanda don't know each other.*

> Rank 310: *Amanda and Hannah don't know each other.*

At the lowest ranks, the summaries become almost nonsensical:

Rank 205: *Amanda, Hannah and Amanda are going to meet up with Larry.*

Rank 175: *Amanda and Amanda don't know each other person in the park.*

Rank 150: *Amanda and Amanda are going to the park.*

These observations further indicates that the low-rank approximation method can maintain summarization quality up to a certain rank, beyond which the coherence and accuracy of the summaries degrade significantly. This highlights the trade-off between computational efficiency and model performance in the context of low-rank approximation.

Chapter 7

Discussion

7.1 Interpretation of Results

7.1.1 RAG Chatbot

The Study Buddy RAG chatbot successfully demonstrated the potential of combining retrieval-augmented generation (RAG) models with large language models (LLMs) to enhance the learning experience for students. The integration of the retriever and generator components, along with the ability to upload and reference specific documents, significantly improved the chatbot's response quality and relevance. However, the specialization of the chatbot's responses due to the document upload feature can restrict its ability to handle unrelated queries effectively. This trade-off highlights a critical aspect of Study Buddy's current implementation. While it succeeds in providing detailed responses within the scope of the uploaded documents, its performance on broader, unrelated topics may diminish as it leans heavily on the specific context provided by those documents. Furthermore, The quality of the data retrieved by a RAG implementation is directly dependent on the quality of the data it has access to. If the information in the underlying source systems—such as databases, online file storage, or other data repositories—is outdated, incomplete, or biased, the RAG implementation cannot identify or correct these flaws. Consequently, it will retrieve and pass along this flawed information to the language model responsible for generating the final output.

7.1.2 Case Study: Low Rank Approximation

Recall from Section 4.3.3 that we needed to low-rank approximate the self-attention matrices with at most rank $r = 318$ and rank $r = 424$ for BART-base and BART-large respectively to achieve a significant reduction in computational complexity and storage requirements. However, the results show that the summarization quality declines significantly before this threshold. The low-rank approximation of the BART-Base model does not present a viable approach to compressing large language models for summarization tasks. Although the technique can achieve significant reductions in computational complexity and storage requirements, the summarization quality declines significantly before reaching a practically useful rank. These findings highlight the limitations of low-rank approximations for preserving the performance of large language models in summarization tasks.

In summary, while low-rank approximation effectively reduces the computational burden and storage requirements of the BART-Base model, the associated decline in summarization quality limits its practical applicability. This work underscores the need for alternative approaches to optimize and deploy large language models in resource-constrained environments without compromising performance.

7.2 Limitations and Challenges

Chapter 8

Conclusion and Future Work

8.1 Summary of Key Findings

8.2 Contributions to the Field

8.3 Recommendations for Future Research

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Appendix A

Educational Synergy in Teaching and Research

Introduction

This appendix outlines the dual role I embraced during my semester as both a Teaching Assistant (TA) in the course "SW2PLA: Practical Linear Algebra for Software Engineers" and doing research for my Bachelor's thesis. My unique position allowed me to bridge theoretical understanding and practical application, fostering an educational environment where concepts in linear algebra, that are taught in the SW2PLA course, were directly linked to research in Large Language Models (LLMs).

Dual Role and Synergistic Learning

As a Teaching Assistant in the SW2PLA course, my responsibilities included assisting students with practical exercises, grading assignments, and engaging in discussions about linear algebra during lectures. These activities were aimed at deepening students' understanding of linear algebra's application in modern computational technologies, particularly within the realms of basic AI and and elementary machine learning. Simultaneously, my bachelor's thesis focused on integrating linear algebra within the development and optimization of LLMs, specifically assessing the effectiveness of low-rank approximation by SVD of Facebook's BART-model's attention weight matrices. Which gave me a new perspective on the practical applications of linear algebra in AI research and a further understanding of the linear algebra concepts taught in the SW2PLA course.

Project Overview in SW2PLA

The final project for the SW2PLA course required students to undertake a practical application of eigendecomposition or SVD. The project aimed to provide hands-on experience with linear algebra's potent applications, offering several avenues for exploration:

1. **Recommender Systems**: Using SVD to predict user preferences based on past interactions.
2. **PageRank Algorithm**: Implementing an eigenvector-based approach to rank web pages.
3. **Image Compression**: Applying SVD to reduce the dimensionality of image data without losing significant information.
4. **Fibonacci Algorithm** Using eigendecomposition to compute the Fibonnaci algorithm without recursion.
5. **Eigenportfolios**: Employing eigendecomposition for optimized stock portfolio selection.
6. **Facial Recognition**: Utilizing PCA (a form of eigendecomposition) to identify and classify facial features in images.
7. **Open Project**: Any project idea involving Eigendecomposition or SVD. Whether it's applying these techniques to optimize algorithms, enhance data processing, or explore new applications, approaches or innovative use of linear algebra.

Integration with Bachelor's Thesis

During the course, I also had the opportunity to present the methodologies and intermediate findings of my bachelor's thesis to the class. This presentation served as a practical demonstration of how linear algebra is utilised within the development and optimization of LLMs—particularly through techniques like low-rank approximations and matrix factorizations—to enhance computational efficiency and model scalability. This further provided students with real-world applications of their theoretical studies.

Student Projects and Feedback

The culmination of the course was the student presentations, where they demonstrated their projects' outcomes. This session provided an interactive platform for peer learning and feedback, where students could showcase their application of linear algebra in various projects. My presentation alongside the students' highlighted the reciprocal nature of teaching/learning and research.

External Presentation and Recognition

In addition to sharing my findings within the SW2PLA course, I had the opportunity to present the methodologies and intermediate results of my bachelor's thesis at a poster competition held during the Center for Language Generation and

AI (CLAI) workshop. This presentation allowed me to engage with a broader audience of experts and peers in the field, receiving valuable feedback that furthered my research endeavors.

The highlight of this event was the recognition of the work, as the 3rd place in the competition was secured. This achievement not only affirmed the quality and relevance of my research in the field of AI but also provided a platform to showcase the practical applications of linear algebra in optimizing Large Language Models. This external validation brought additional perspective to the educational content I was delivering in the SW2PLA course, enhancing the overall teaching and learning experience.

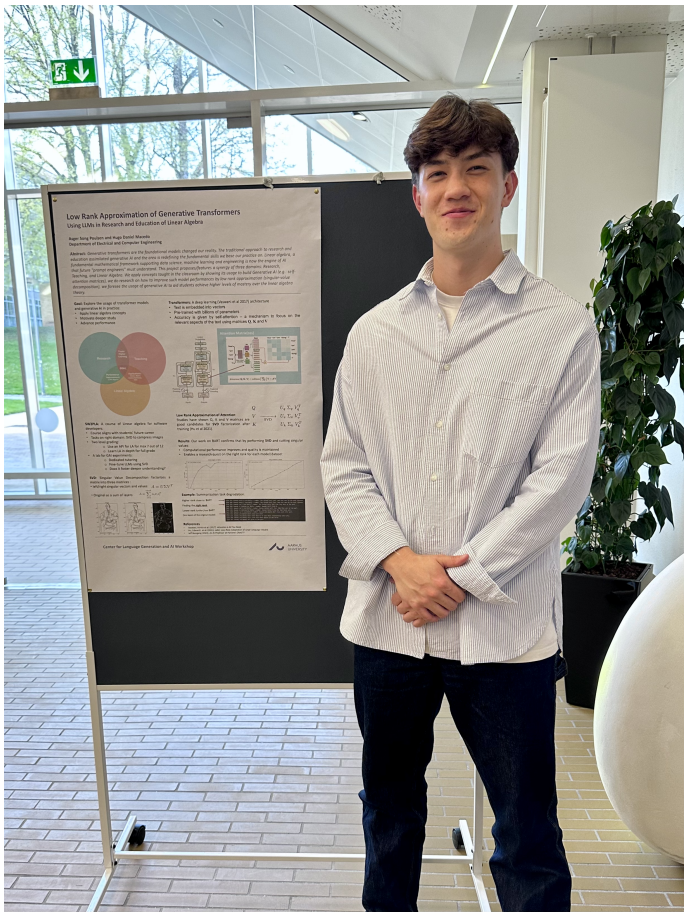


Figure A.1: 3rd place in the CLAI Poster Competition

It was an honor to represent the Department of Electrical and Computer Engineering (ECE) at such a forum. This accolade not only represents a personal achievement but also served as a good practice for me to present research findings to a broader audience, including students and faculty members from various disciplines.

Conclusion

My involvement in the SW2PLA course as a TA, while simultaneously conducting research for my bachelor's thesis, created a rich educational synergy. This experience

not only enhanced my understanding and teaching of linear algebra but also allowed me to directly apply and evaluate theoretical concepts in practical, research-based applications. It underscores the importance of an integrated approach in education, where teaching responsibilities and academic research complement and enrich each other, preparing students, and myself included, for future challenges in technology and engineering.

Appendix B

Poster

The following page contain the poster presented at the CLAI Workshop 2024.

Low Rank Approximation of Generative Transformers

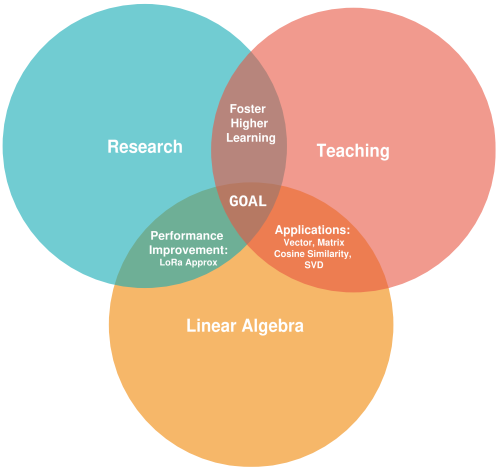
Using LLMs in Research and Education of Linear Algebra

Asger Song Poulsen and Hugo Daniel Macedo
Department of Electrical and Computer Engineering

Abstract: *Generative transformers are the foundational models changed our reality. The traditional approach to research and education assimilated generative AI and the area is redefining the fundamental skills we base our practice on. Linear algebra, a fundamental mathematical framework supporting data science, machine learning and engineering is now the engine of AI that future "prompt engineers" must understand. This project proposes/features a synergy of three domains: Research, Teaching, and Linear Algebra. We apply concepts taught in the classroom by showing its usage to build Generative AI (e.g.: self-attention matrices), we do research on how to improve such model performances by low rank approximation (singular-value decomposition), we foresee the usage of generative AI to aid students achieve higher levels of mastery over the linear algebra theory.*

Goal: Explore the usage of transformer models and generative AI in practice:

- Apply linear algebra concepts
- Motivate deeper study
- Advance performance

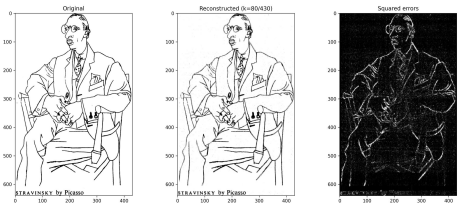


SW2PLA: A course of Linear algebra for software developers:

- Course aligns with students' future career
- Tasks on right domain: SVD to compress images
- Two level grading:
  - Use an API for LA for max 7 out of 12
  - Learn LA in depth for full grade
- A lab for GAI experiments:
  - Dedicated tutoring
  - Fine-tune LLMs using SVD
  - Does it foster deeper understanding?

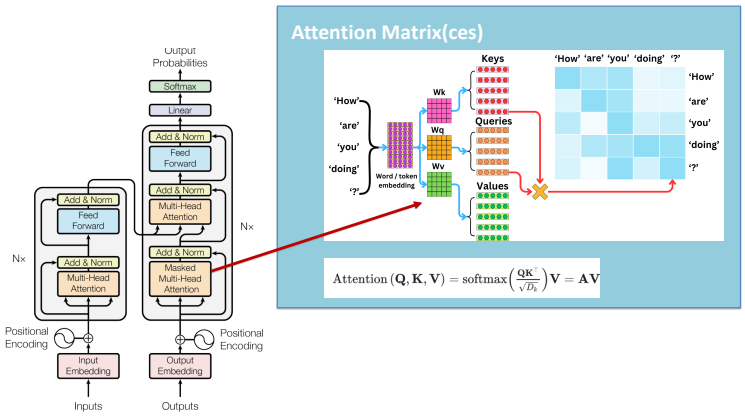
SVD: Singular Value Decomposition factorizes a matrix into three matrices :

- left/right singular vectors and values: $A = U\Sigma V^T$
- Original as a sum of layers: $A = \sum_{i=1}^{rank} u_i \sigma_i v_i^T$



Transformers: A deep learning (Vaswani et al 2017) architecture

- Text is embedded into vectors
- Pre-trained with billions of parameters
- Accuracy is given by self-attention – a mechanism to focus on the relevant aspects of the text using matrices $\mathbf{Q}$, $\mathbf{K}$ and $\mathbf{V}$

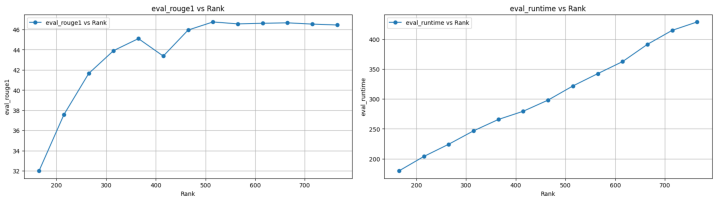


Low Rank Approximation of Attention: Studies have shown Q, K and V matrices are good candidates for **SVD** factorization after training (Hu et al 2021).

$$Q \longrightarrow U_q\ \Sigma_q\ V_q^T$$
$$V \xrightarrow{\text{SVD}} U_v\ \Sigma_v\ V_v^T$$
$$K \longrightarrow U_k\ \Sigma_k\ V_k^T$$

Results: Our work on BART confirms that by performing **SVD** and cutting singular values:

- Computational performance improves and quality is maintained
- Enables a research quest on the right rank for each model/dataset



Example: Summarization task degradation:

Higher rank *closer to* BART

Finding the **right rank**

Lower rank *further from* BART

(less layers of the original model)

```
Rank 765: Hannah doesn't know Betty's number. She texted Larry last time they were at the park together.
Rank 660: Hannah doesn't know Betty's number. She texted Larry last time they were at the park together.
Rank 510: Hannah can't find Betty's number. She texted Larry last time they were at the park.
Rank 410: Hannah is looking for Betty's number. Amanda can't find it.
Rank 370: Amanda can't find Betty's number. Hannah texted Larry last time they were at the park.
Rank 365: Amanda is looking for Betty's number. Hannah doesn't know her boyfriend Larry.
Rank 360: Amanda can't find Betty's number. Hannah and Amanda don't know each other well.
Rank 315: Amanda and Hannah don't know each other.
Rank 210: Amanda and Amanda don't know what to do with their relationship with Larry.
Rank 190: Amanda and Amanda don't know each other person in the world.
Rank 165: Amanda and Amanda are going to the park.
```

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Appendix C

Interview with Carsten Bergenholtz

Customized RAG Chatbot at Aarhus University

In the initial phase of the project, I engaged with faculty members at Aarhus University to gain insights into the current research and application of Large Language Models (LLMs) within the university. This collaboration aimed to understand both the potential and the challenges associated with the use of LLMs in academic settings.

Associate Professor Carsten Bergenholtz from the Department of Management contributed significantly by sharing his experiences with LLMs. His involvement provided crucial insights that shaped the direction of our project.

Interview Summary

During our discussions, concerns were raised about the use of chatbots like ChatGPT-4 in educational environments. While these systems can produce impressive outputs, they also pose risks due to potential inaccuracies and a lack of course-specific knowledge. However, solutions exist to mitigate these challenges. Professor Bergenholtz shared his experience with implementing a customized Retrieval-Augmented Generation (RAG) chatbot for his Philosophy of Science course, which had an enrollment of 550 students.

Chatbot Implementation

The customized chatbot was used approximately 20,000 times by the students, indicating strong engagement and utility. Professor Bergenholtz uploaded about 250 pages of course-relevant documents, from text to subtitles from his online lectures, to create a knowledge base for the chatbot. This setup, based on AU's Microsoft Azure platform, ensured that the chatbot was:

- GDPR compliant, adhering to data protection regulations.
- Based on the ChatGPT-4 model, ensuring advanced conversational capabilities.

- Freely accessible to all students, removing financial barriers.
- Equipped with an appealing user interface, enhancing user experience.
- Integrated within the existing university systems, ensuring seamless access.

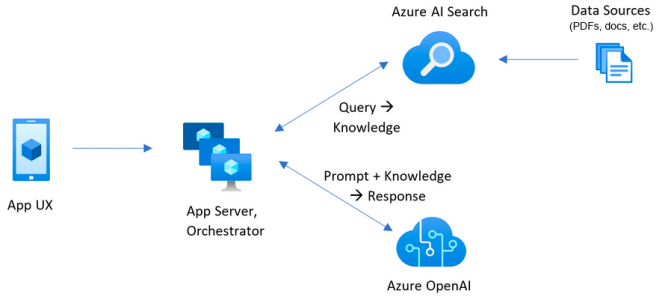

Figure C.1: Figure to put somewhere

The RAG chatbot was specifically designed to respond only to queries related to the course content. Questions outside the course scope received a 'cannot answer this' response to maintain the focus and academic integrity of the tool. Additionally, the chatbot provided a link to the source of its answers, enhancing transparency and trust.

Chatbot Evaluation and Costs

The quality of the chatbot's responses was satisfactory, with about 85% of interactions leading to useful answers, and only 2-4.5% of responses being flawed. Notably, the flawed responses were quickly identified by users through follow-up questions. Despite its imperfections, the chatbot was considered a significant improvement over traditional search methods or regular chatbots used by students. The total cost of the chatbot was approximately 600€, with actual running costs around 400€ for 20,000 interactions, showcasing its cost-effectiveness.

Student Feedback and Future Prospects

The chatbot was well-received based on survey responses, with students appreciating its ability to clarify complex concepts, compare texts, and summarize content. This tool proved particularly useful for large classes and when copyright for the necessary materials was held by the course instructor. Plans are in place to continue and enhance this service in future courses, focusing on guiding students to ask more effective questions.

Conclusion

The implementation of the RAG chatbot at Aarhus University exemplifies the practical application of LLMs in enhancing educational experiences. The project set a precedent for future educational tools that leverage AI to support learning and inquiry. This initiative highlights the synergy between innovative technology and

traditional educational practices, paving the way for more dynamic and interactive learning environments.

Appendix D

Architecture of BART

One can inspect the model architecture by using the `transformers` library by Hugging Face.

BART-Base

```
from transformers import AutoModelForSeq2SeqLM
model_checkpoint = "facebook/bart-base"
model = AutoModelForSeq2SeqLM.from_pretrained(model_checkpoint)
```

Listing 13: Loading pre-trained BART-base model from the Hugging Face model repository

This code snippet fetches the configuration, tokenizer, and weights of the BART model from the Hugging Face model repository. The model can be inspected by printing the model object, which will output the model architecture as shown in Listing 14.

```
BartForConditionalGeneration(
  (model): BartModel(
    (shared): Embedding(50265, 768, padding_idx=1)
    (encoder): BartEncoder(
      (embed_tokens): BartScaledWordEmbedding(50265, 768, padding_idx=1)
      (embed_positions): BartLearnedPositionalEmbedding(1026, 768)
      (layers): ModuleList(
        (0-5): 6 x BartEncoderLayer(
          (self_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=768, out_features=768, bias=True)
            (v_proj): Linear(in_features=768, out_features=768, bias=True)
            (q_proj): Linear(in_features=768, out_features=768, bias=True)
            (out_proj): Linear(in_features=768, out_features=768, bias=True)
          )
          (self_attn_layer_norm): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
          (activation_fn): GELUActivation()
          (fc1): Linear(in_features=768, out_features=3072, bias=True)
          (fc2): Linear(in_features=3072, out_features=768, bias=True)
          (final_layer_norm): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
        )
      )
      (layernorm_embedding): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
    )
    (decoder): BartDecoder(
      (embed_tokens): BartScaledWordEmbedding(50265, 768, padding_idx=1)
      (embed_positions): BartLearnedPositionalEmbedding(1026, 768)
      (layers): ModuleList(
        (0-5): 6 x BartDecoderLayer(
          (self_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=768, out_features=768, bias=True)
            (v_proj): Linear(in_features=768, out_features=768, bias=True)
            (q_proj): Linear(in_features=768, out_features=768, bias=True)
            (out_proj): Linear(in_features=768, out_features=768, bias=True)
          )
          (activation_fn): GELUActivation()
          (self_attn_layer_norm): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
          (encoder_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=768, out_features=768, bias=True)
            (v_proj): Linear(in_features=768, out_features=768, bias=True)
            (q_proj): Linear(in_features=768, out_features=768, bias=True)
            (out_proj): Linear(in_features=768, out_features=768, bias=True)
          )
          (encoder_attn_layer_norm): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
          (fc1): Linear(in_features=768, out_features=3072, bias=True)
          (fc2): Linear(in_features=3072, out_features=768, bias=True)
          (final_layer_norm): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
        )
      )
      (layernorm_embedding): LayerNorm((768,), eps=1e-05, elementwise_affine=True)
    )
  )
  (lm_head): Linear(in_features=768, out_features=50265, bias=False)
)
```

Listing 14: BART-base model architecture

The model has a total of 139 million parameters.

BART-Large

Similarly, for the BART-large model:

```
from transformers import AutoModelForSeq2SeqLM
model_checkpoint = "facebook/bart-large"
model = AutoModelForSeq2SeqLM.from_pretrained(model_checkpoint)
```

Listing 15: Loading pre-trained BART-large model from the Hugging Face model repository

The model architecture is shown in Listing 16.

```
BartForConditionalGeneration(
  (model): BartModel(
    (shared): Embedding(50265, 1024, padding_idx=1)
    (encoder): BartEncoder(
      (embed_tokens): BartScaledWordEmbedding(50265, 1024, padding_idx=1)
      (embed_positions): BartLearnedPositionalEmbedding(1026, 1024)
      (layers): ModuleList(
        (0-11): 12 x BartEncoderLayer(
          (self_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (v_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (q_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (out_proj): Linear(in_features=1024, out_features=1024, bias=True)
          )
          (self_attn_layer_norm): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
          (activation_fn): GELUActivation()
          (fc1): Linear(in_features=1024, out_features=4096, bias=True)
          (fc2): Linear(in_features=4096, out_features=1024, bias=True)
          (final_layer_norm): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
        )
      )
      (layernorm_embedding): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
    )
    (decoder): BartDecoder(
      (embed_tokens): BartScaledWordEmbedding(50265, 1024, padding_idx=1)
      (embed_positions): BartLearnedPositionalEmbedding(1026, 1024)
      (layers): ModuleList(
        (0-11): 12 x BartDecoderLayer(
          (self_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (v_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (q_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (out_proj): Linear(in_features=1024, out_features=1024, bias=True)
          )
          (activation_fn): GELUActivation()
          (self_attn_layer_norm): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
          (encoder_attn): BartSdpaAttention(
            (k_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (v_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (q_proj): Linear(in_features=1024, out_features=1024, bias=True)
            (out_proj): Linear(in_features=1024, out_features=1024, bias=True)
          )
          (encoder_attn_layer_norm): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
          (fc1): Linear(in_features=1024, out_features=4096, bias=True)
          (fc2): Linear(in_features=4096, out_features=1024, bias=True)
          (final_layer_norm): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
        )
      )
      (layernorm_embedding): LayerNorm((1024,), eps=1e-05, elementwise_affine=True)
    )
  )
  (lm_head): Linear(in_features=1024, out_features=50265, bias=False)
)
```

Listing 16: BART-large model architecture

The model has a total of 406 million parameters.