Dividing Rectangles Attack Multi-Objective Optimization

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Motivation

- Multi-objective Optimization Problems (MOPs) involve a set of **conflicting** objectives that are to be optimized **simultaneously**.
- It is common that derivatives of the objectives $\mathbf{f}$ are neither **symbolically** nor **numerically** available.
- Evaluating $\mathbf{f}$ is typically **expensive** requiring some computational resources (e.g., a computer code or a laboratory experiment).
- Solve using a **finite budget** of function evaluations.

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- **Decomposition**-based evolutionary algorithms have been applied with success to MOPs.
- MOPs are broken into several subproblems, and solutions for the original problem are recognized in a coordinated manner.
- Approach the principle of decomposition-based methods, **divide-and-conquer**, from a **mathematical programming** view.
- We present `MO-DIRECT`: a multi-objective algorithmic instance of the sampling method, `DIRECT` (Jones et al., JOPT, 1993).

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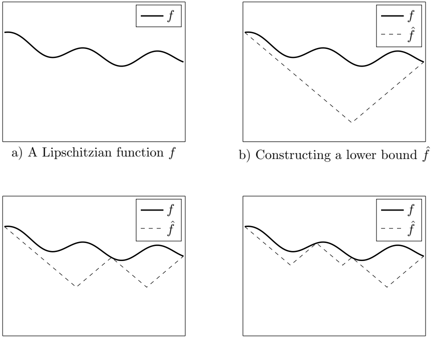
- We present `MO-DIRECT`: a multi-objective algorithmic instance of the sampling method, `DIRECT` (Jones et al., JOPT, 1993).

Background

- `Lipschitzian Optimization` (Piyavskii, USSR, 1972) and (Shubert, SIAM, 1972): piecewise linear bound from the Lipschitz condition.



a) A Lipschitzian function f
b) Constructing a lower bound $\hat{f}$
c) $\hat{f}$ after one sample
d) $\hat{f}$ after two samples

Figure: Working principle of `Lipschitzian Optimization` in one-dimensional search space.

Background

- Lipschitzian Optimization's limitation:
  1. **Lack of Knowledge** about the Lipschitz constant.
  2. **Complexity**: sampling vertices, bound computation.
- Motivated DIRECT (Jones et al., JOPT, 1993).
  1. **partitions** the search space into a set of hyperrectangles
  2. the objective is **evaluated** at the hyperrectangles centers.
  3. Based on the hyperrectangles **size** and the objective **value** at its center, a subset of the hyperrectangles are further partitioned.
  4. partition procedure ensures **good solutions** stay within **larger rectangles**.

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Background - DIRECT

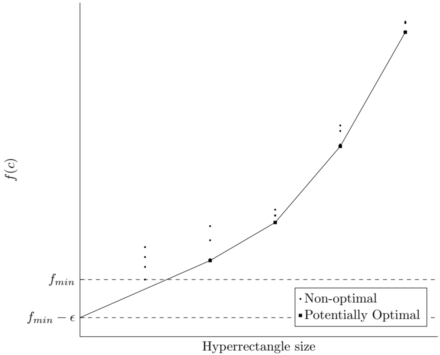


Figure: DIRECT projects rectangles into a **2-D space (size, function value)** and identifies *potentially-optimal* rectangles: those at the **bottom left of the convex hull**.

MO-DIRECT: From Single- to Multi-Objective (1)

- **Potentially-optimal rectangles**: given a set of hyperrectangles $\mathcal{H}$, the set of potentially-optimal hyperrectangles $\mathcal{I}$ can be defined as:

$$\mathcal{I} = \mathtt{ND}(\{(\mathbf{f}(\mathbf{c}_i), \sigma_i) : i \in \mathcal{H} , \sigma_i \geq \sigma_t\}) , \tag{1}$$

where

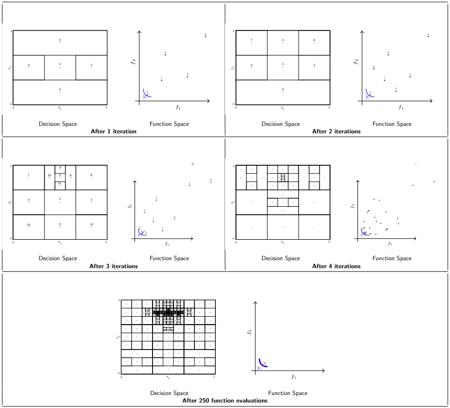
- $(\mathbf{f}(\mathbf{c}_i), \sigma_i)$ is the $(m+1)$-dimensional vector of hyperrectangle $i \in \mathcal{H}$ of center $\mathbf{c}_i$ and size σ_i,
- $\mathtt{ND}(\cdot)$ is an operator on a set of vectors A such that $\mathtt{ND}(A)$ is the set of non-dominated vectors in A,
- σ_t is the minimum size a hyperrectangle can have to be considered for potential optimality.

MO-DIRECT: From Single- to Multi-Objective (2)

- **Partition procedure**: should increase the attractiveness of search near points with good function values
- by making the **biggest** rectangles contain the **best** solutions as bigger rectangles are more likely to be potentially-optimal, with everything else equal.
- Proposition: a hyperrectangle is divided such that the biggest produced hyperrectangles contain the distant solutions—in the objective space—from that of the hyperrectangle, increasing the likelihood of visiting unexplored regions of the function space.

MO-DIRECT: Demo

available at `https://goo.gl/VWiAc1`.



Convergence Analysis

- MO-DIRECT is **asymptotically optimal**.

Lemma

As the number of iterations t goes to ∞, the fewest number of divisions $r(t)$ undergone by any hyperrectangle $i \in \mathcal{H}$ created by MO-DIRECT approaches ∞.

Theorem

In the limit, MO-DIRECT converges to the Pareto front of a continuous MOP.

Empirical Analysis

- 20 MOPs from the literature.
- compared against decomposition-based algorithms: `MOEA/D` (Zhang et al., IEEE TEC, 2007) and `MO-HOO` (Van Moffaert et al., IJCNN, 2014).
- results in hypervolume-based data profiles.
- More details in the paper.

Performance Evaluation

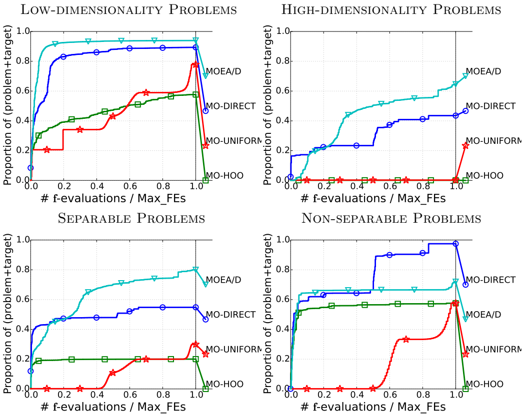


Figure: Hypervolume-based data profiles over different problems for MO-DIRECT, MOEA/D, MO-HOO, and MO-UNIFORM (Max_FEs = 20000 FEs).

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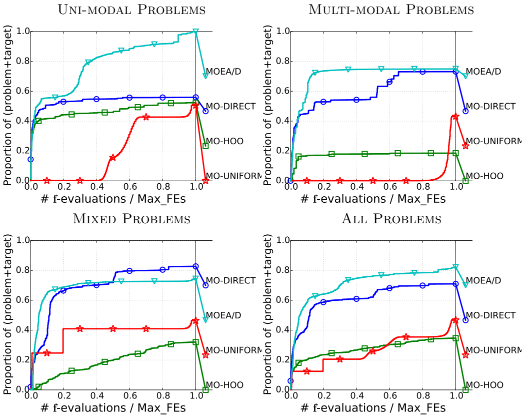

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Conclusion

- mathematical programming is emerging as a promising framework for black-box multi-objective optimization: `MULTMADS`, `DMS`, `MO-DIRECT`.
- `MO-DIRECT` is **asymptotically optimal**. Suitable for low-dimensional and/or non-separable problems.
- Can be used as an **initializer** for local-search multi-objective solvers.
- Limitations/Future work:
  1. **Scaling** `MO-DIRECT`: inefficient partition procedure along all the coordinates, exhaust $\mathcal{O}(2n)$ f-evals.
  2. The `ND` operator does not consider solutions diversity. **Indicator-based techniques**?

BMOBench

- Inspired by COCO, we built BMOBench
- a platform with **100** MOPs.
- accommodate **stochastic and deterministic** algorithms.
- data profiles generated in terms of 4 quality indicators.
- Special session at SSCI'2016, Greece.[1] (**Deadline: 15-August-2016**)
- We invite the multi-objective community to test their published/novel algorithms on these problems.

[1] http://ash-aldujaili.github.io/BMOBench/

Thank you