ECN 514: Detection & Estimation Theory
Assignment 2

Due date: Sunday, 14th April 2024 (by midnight)

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Maximum Marks: 100

1. Implement a scalar state-scalar observation Kalman filter by write a Matlab/Python code to compute the variables in the Kalman filter equations for $n = 0, 1, ..., 100$. The parameters associated with the state and observation equation are:

   - $a = 0.98$
   - $\sigma_u^2 = 1$
   - $\mu_s = 5$
   - $\sigma_s^2 = 1$

   (a) Write the code to generate and plot $s[n]$ and $x[n]$ (in the same plot) for $n = 0, 1, ..., 100$ for $\sigma_n^2 = \{0.9, 1, 1.2\}$ (Marks: $2 \times 3 = 6$)

   (b) Write the code to compute and plot Kalman prediction, minimum prediction MSE, Kalman gain, correction, and updated minimum MSE for $n = 0, 1, ..., 100$ for $\sigma_n^2 = \{0.9, 1, 1.2\}$ (Marks: $5 \times 3 = 15$)

   (c) Write the code to compute the innovation sequence, its auto-correlation function and the PSD for $n = 0, 1, ..., 100$ for $\sigma_n^2 = \{0.9, 1, 1.2\}$ (Marks: $3 \times 3 = 9$)

   (d) Comment on the whiteness of the innovation sequence based on its auto-correlation function and the PSD. (Marks: 3)

2. For the scalar state-scalar observation Kalman filter find an expression relating $M[n|n]$ to $M[n-1|n-1]$. Let $a = 0.9$, $\sigma_u^2 = 1$, and $\sigma_n^2 = n+1$. If $M[-1|-1] = 1$, write a code to compute and plot $M[n|n]$ for $n = 0, 1, ..., 100$. Explain your results. (Marks: 10 + 10 + 2)

3. Consider the problem of detecting change in the variance of a Gaussian statistic (Slide 32 in Lecture 12 - Hypothesis testing). Let $N = 2$, i.e., two sample measurements $x[0]$ and $x[1]$ are available. Find the relationship between P_{FA} and P_D for this problem. (Marks: 15)

4. A radar signal $s[n] = Acos(2\pi f_0 n)$ for $n = 0, 1, ..., N-1$ is received embedded in WGN with variance $\sigma^2 = 1$. A detector is to be designed that maintains $P_{FA} = 10^{-8}$. Find and plot the probability of detection versus A for following values of f_0 and N. (Marks: $3 \times 5 = 15$)

   (a) $f_0 = 0.2$ and $N = 50$

   (b) $f_0 = 0.25$ and $N = 25$

   (c) $f_0 = 0.4$ and $N = 10$

   (d) $f_0 = 0.5$ and $N = 30$

5. Consider a random signal $s[n]$ with zero mean and covariance matrix $C_s = diag(\sigma^2_{s_0}, \sigma^2_{s_1}, ..., \sigma^2_{s_{N-1}})$ embedded in WGN with variance σ^2. Let N measurements: $x[n]$ for $n = 0, 1, ..., N-1$ are taken. Find the NP detector for this problem. (Marks: 15)