Indian Institute of Technology Ropar
Department of Mathematics
MAL424/MA205/MA204 - Numerical Analysis
Second Semester of Academic Year 2020-2021

Lab Tutorial sheet 5 March 12, 2021

1. Solve the following system of equations using Gauss Seidel iterative method:
   (a) $4x + y + z + 0s + t = 6, -x - 3y + z + s = 6, 2x + y + 5z - s - t = 6,$
   (b) $-x - y - z + 4s = 6, 2y - z + s + 4t = 6$
   (c) $3x - y + z = 1, 3x + 6y + 2z = 0, 3x + 3y + 7z = 4$

2. For Solve the following system of equations using SOR method with optimal choice of ω, take initial approximation as zero
   (a) $-10x - y = 9, -x + 10y - 2z = 7, -2y + 10z = 6$
   (b) $10x + 5y = 6, 5x + 10y - 4z = 25, -4y + 8z - s = -11, -z + 5s = -11$

3. For the following system, compare the iterations for Gauss siedel and SOR with ω=1.25, taking initial approximation as (1,1,1) for both the methods

$$\begin{aligned} 4x + 3y &= 24 \\ 3x + 4y - z &= 30 \\ -y + 4z &= -24 \end{aligned}$$

4. Using the Gerschgorin's theorem, find bounds for the eigenvalues λ of the real $n \times n$ matrix $\mathbf{A}(n \geq 3)$

$$\mathbf{A} = \begin{bmatrix} a & b & & & \\ c & a & b & 0 & \\ & c & a & b & \\ & 0 & & \ddots & \\ & & & c & a \end{bmatrix}$$

   Show that the components x_i of the eigenvector x obey a linear difference equation and find all the eigenvalues and eigenvectors.

5. Find all the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

   using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.0005.

6. Find the first three iterations obtained by the Power method applied to the following matrices.
   a. $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$; Use $x^0 = (1, -1, 2)^t$.
   c. $\begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$; Use $x^{(0)} = (-1, 2, 1)^t$.

Algorithm

Gauss - Seidel

STEP 1: Input the matrix A, vector b and initial condition x.

STEP 2: if A is a square matrix
do STEP 3- STEP5
else Error, A must be a square matrix.

STEP 3: Decompose the matrix A as $L + D + U$, where
L is lower triangular matrix such that $L(i, j) = A(i, j)$, $i > j$
m D is diagonal matrix such that $D(i, i) = A(i, i)$ for $i = 1, 2, ..., n$.
U is upper triangular matrix with zero diagonal entries and $U(i, j) = A(i, j)$ for $i < j$.

STEP 4: Define error = norm(b-A*x)
Define epsilon= Tolerance

STEP 5: Let X and x be two successive approximations to the root.
While error > epsilon
do $X=$ $(L + D)^{-1} * (b - U * x)$
$x = X$
error = norm(b-A*x)
end while

Successive Over Relaxation (SOR)

Here the iterative formula is

$$X = (D + L\omega)^{-1} * (b\omega - (U\omega - (1 - \omega)D) * x)$$

where $A = L + D + U$, X is the new approximation and x is the old approximation. ω is the relaxation parameter.

********* End *********