INDIAN INSTITUTE OF TECHNOLOGY ROPAR
Department of Mathematics
MAL 424/204/205 - Numerical Analysis
Second Semester of Academic Year 2020 - 2021

Tutorial sheet 3

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1. Show that $\mathcal{O}\left(x^a\right)+\mathcal{O}\left(x^b\right)=\mathcal{O}\left(x^{min(a,b)}\right)$
   Hint: Around 0 we have $x^a=\mathcal{O}\left(x^b\right)$ for all $b \leq a$.

   Write a MATLAB program for each of the following.

2. Using Gauss elimination, solve the system $Ax=b$, where A and b are given as
   (a) $A=\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}, b=\begin{bmatrix}0\\3\\2\end{bmatrix}$
   (b) $A=\begin{bmatrix}4&3&2&1\\3&4&3&2\\2&3&4&3\\1&2&3&4\end{bmatrix}, b=\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix}$
   (c) $A=\begin{bmatrix}0&1\\2&3\end{bmatrix}, b=\begin{bmatrix}2\\8\end{bmatrix}$
   (d) $A=\begin{bmatrix}0.001&1\\2&3\end{bmatrix}, b=\begin{bmatrix}2\\8\end{bmatrix}$
   What do you observe in (c), (d)?

3. Consider the following system of equations:
$$\begin{aligned}0.729x+0.81y+0.9z&=0.6867\\x+y+z&=0.8338\\1.331x+1.21y+1.1z&=1\end{aligned}$$
   Solve using Gauss elimination method:
   (a) Without pivoting
   (b) With partial pivoting
   Compare your result with the exact solution: $x=0.2245, y=0.2814, z=0.3279$.

4. Using Guass - Jordan, solve the system $Ax=b$, where A and b are given as in 1.

Algorithm for Gauss Elimination method for solving $Ax=b$

Step 1: Enter the coefficient matrix A and the vector b.
Step 2: Form the augmented matrix C = $[A\ |b]$.
Step 3: Check for existence of solution by checking rank of matrix A and augmented

matrix C. If a unique solution exists, go to step 4 otherwise Gauss elimination is not possible.

Without pivoting

Step 4a: **Elimination:** Apply row operations on the augmented matrix C so as to reduce the matrix A into an upper triangular matrix.

Let row and col be the number of rows and columns in A. if C(i,i) $\neq 0$
for i = 1, 2, ..., col-1
for j = i+1, i+2, ..., row
$\lambda = C(j,i)/C(i,i)$
$R_j \rightarrow \; R_j - \lambda \; R_i$
end
end
else Stop, Gauss Elimination is not applicable.
end

Step 5a: Use **back substitution** to obtain the final solution.
$x_n = \frac{C(n,n+1)}{C(n,n)}$
% Here row = col = n(say)
for k = n-1, n-2, ..., 1
Define sum = 0
for j = k+1, k+2, ..., n
sum = sum + $C(k,j) * \; x_j$
end
$x_k = \dfrac{1}{C(k,k)}(C(k,n+1) - sum)$
end

With pivoting

Aim: To get the maximum element of the column i as the pivot element when working on column i

Step 4b: **Partial pivoting and elimination:**
for i = 1, 2, ..., col-1
for j = i+1, i+2, ..., row
if $|C(j,i)| \geq |C(i,i)|$
$R_j \leftrightarrow \; R_i$
end
if C(i,i) $\neq 0$
$\lambda = C(j,i)/C(i,i)$
$R_j \rightarrow \; R_j - \lambda \; R_i$
else Stop, Gauss elimination not applicable.
end
end
end

Step 5b: **Back substitution** Same as step 5a.

MATLAB commands needed

a) To add λ times row i to row j
$A(i,:) = A(j,:) + \lambda A(i,:)$
b) To interchange row i and row$j$

$A([i,j],:) = A([j,i],:)$
c) To interchange column i and column j
$A(:,[i,j]) = A(:,[j,i])$
or a three line code can be written
d) max(abs(A)), where A is a n $\times$n matrix.
e) return
