Incorporating constraints with constraint forces

- Model as unconstrained system with additional forces (λ) that enforce constraints

- Q that generate constraint forces (λ) can be found by work balance

$$Q^T \dot{q} = \lambda^T \left(A(q) \dot{q} \right)$$

- Thus the constrained dynamics are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = A(q)^T \lambda + F_{\text{non-conservative}}$$

- Constrains on accelerations are additionally imposed

$$\frac{dA(q)}{dt}\dot{q} + A(q)\ddot{q} = 0$$

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The rolling disk example