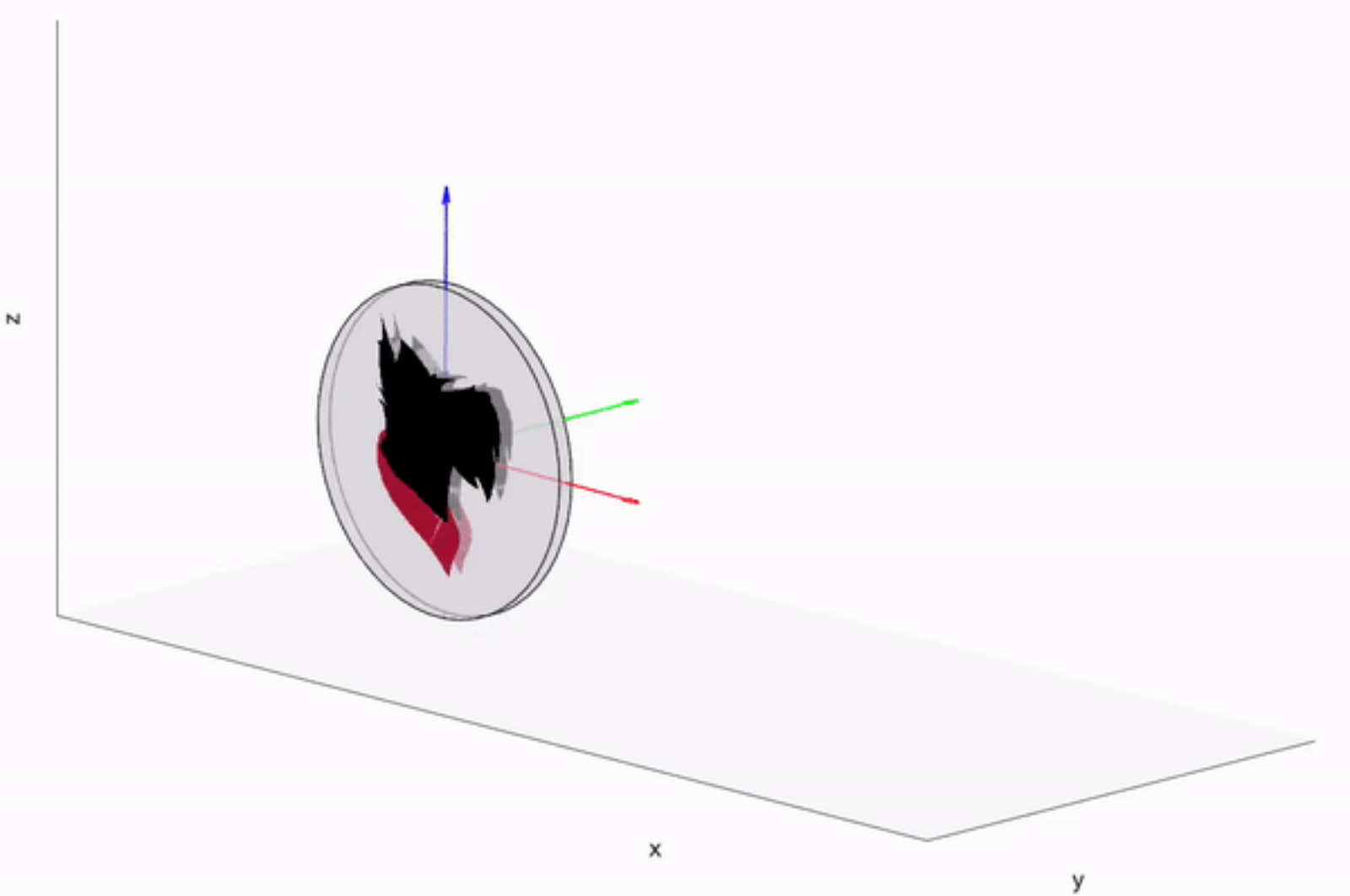
Velocity Constraints

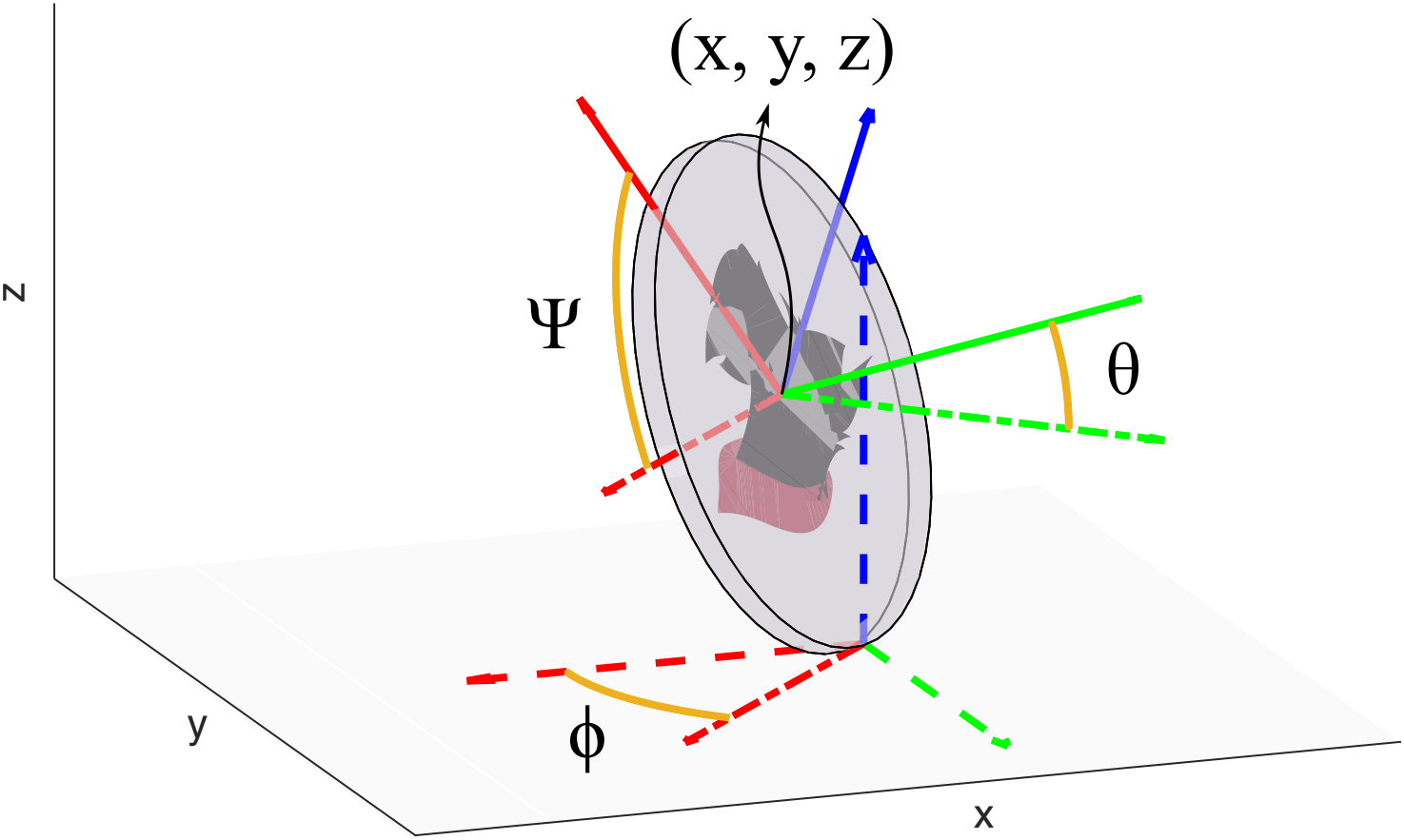
- Suppose our system has constraints of the form $A(q)\dot{q} = 0$

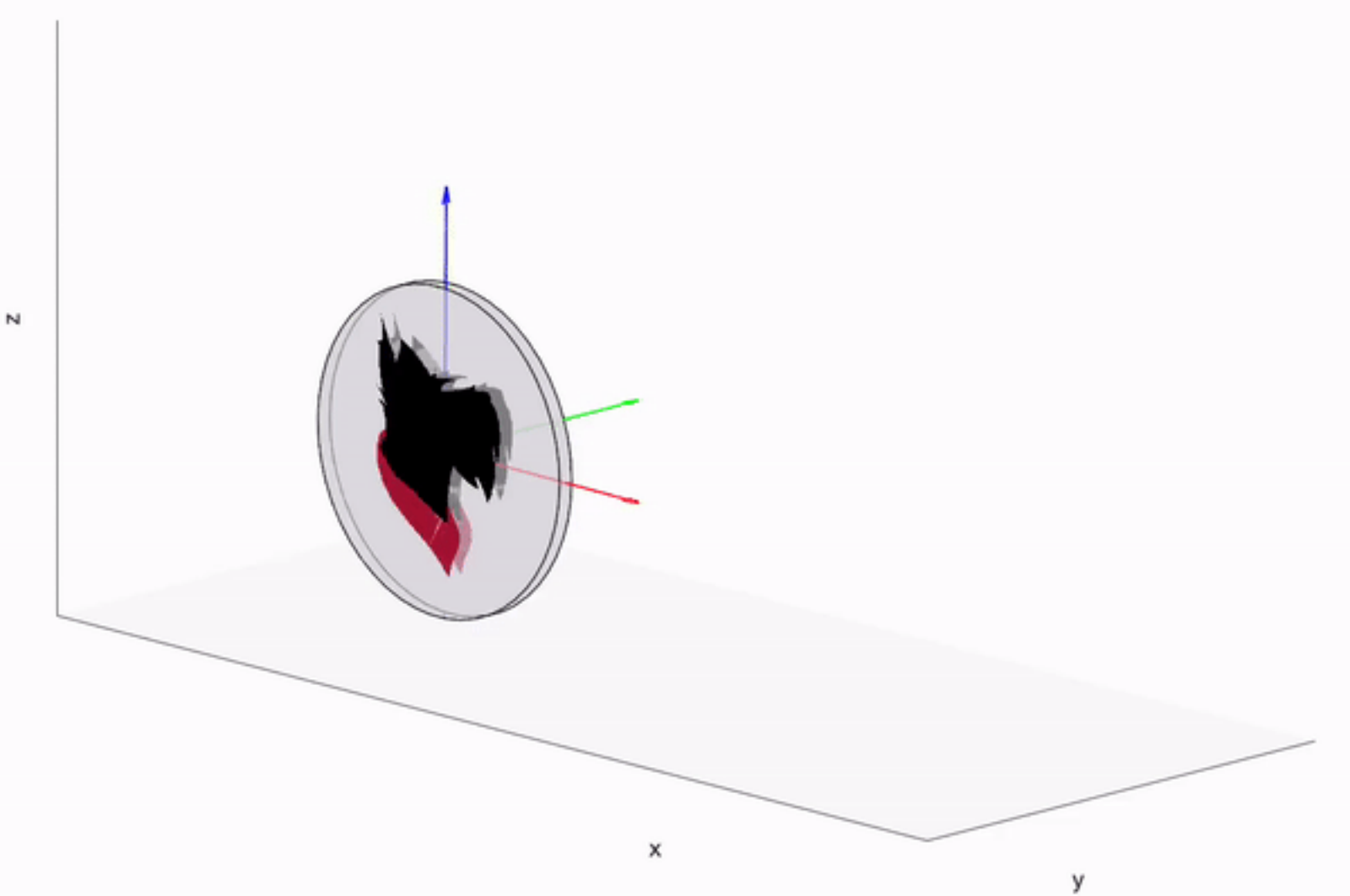
- Example : Rolling disk ($q = [x, y, z, \phi, \theta, \psi]$)

- Constraints :


z
x
y

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -r\cos(\phi)\sin(\theta) & -r\sin(\phi)\cos(\theta) & -r\cos(\phi) \\ -r\sin(\phi)\sin(\theta) & r\cos(\phi)\cos(\theta) & -r\sin(\phi) \\ r\sin(\theta) & 0 & \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = 0$$


(x, y, z)
Ψ
θ
ϕ
z
y
x


z
x
y

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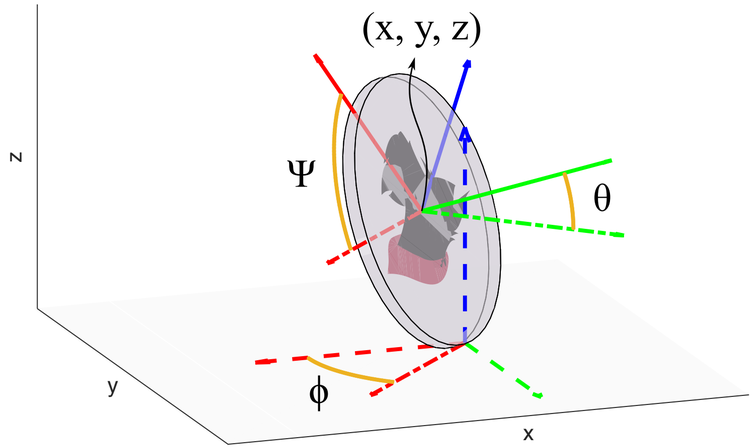

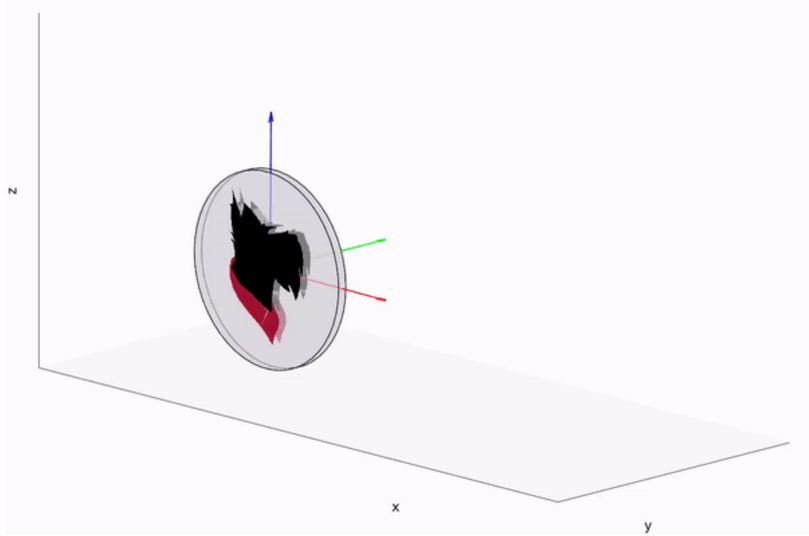

- Constraints : $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -r\cos(\phi)\sin(\theta) & -r\sin(\phi)\cos(\theta) & -r\cos(\phi) \\ -r\sin(\phi)\sin(\theta) & r\cos(\phi)\cos(\theta) & -r\sin(\phi) \\ r\sin(\theta) & 0 & \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = 0$

Incorporating constraints with constraint forces