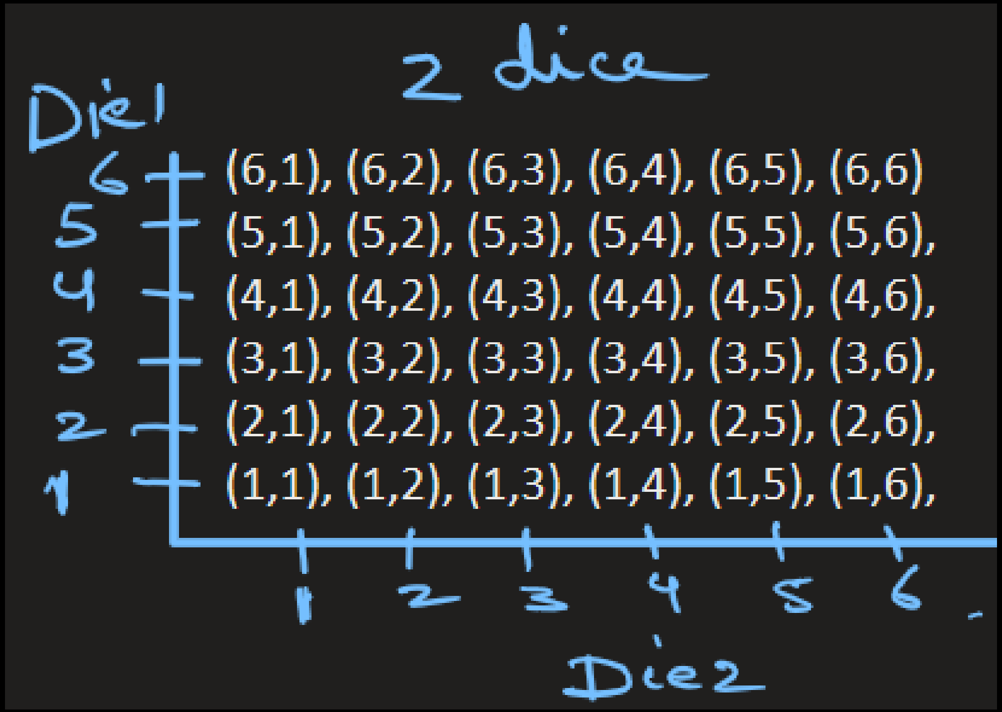
Addition Rule

Problem: Suppose we roll a pair of dice.

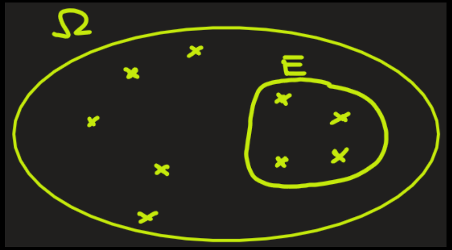


What is the probability that either first number is even **<u>or</u>** second number is odd?

Probability: Definition (Review)

Probability of an event E:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$



Or,

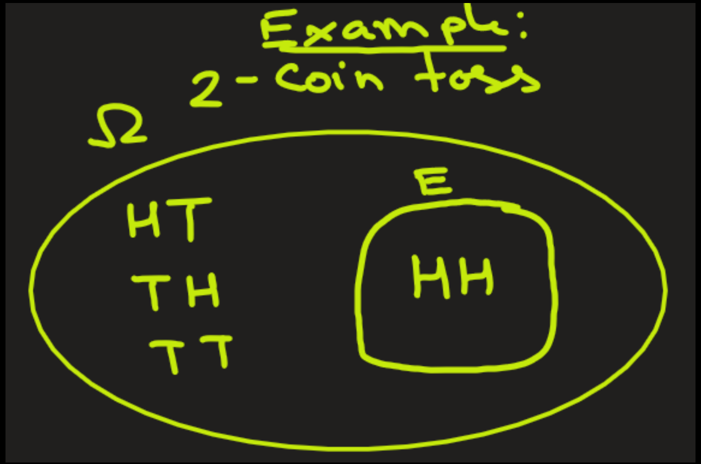
$$P(E) = \frac{|E|}{|\Omega|}$$

Here,

| E | = number (size) of elements in event E,

| Ω |= total number (size) of elements in sample space

coin1 coin2



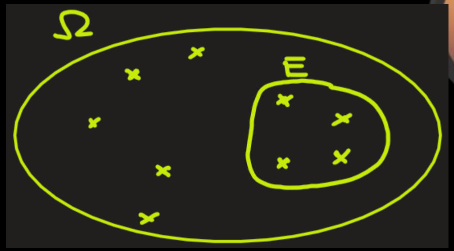
Example: In 2-coin toss, what is the probability of getting **both heads**?

Ans: Here sample space is Ω = {HH, HT, TH, TT}. Size is |Ω| = 4

The event E is both toss are heads: E = {HH}. Size is |E| = 1

P(E) = 1/4.

Probability



Important Properties: (Die example)

1) $0 \leq P(E) \leq 1$

Probability of an event is always between 0 and 1

E.g.: P(even number) , P({5}), P({9}) are all between 0 and 1

2) $P(\Omega) = 1$

Sum of probabilities of sample space is 1

E.g.: $P(\ \Omega = \{1,2,3,4,5,6\}\ ) = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 1$

3) $P(\emptyset) = 0$

Probability of an event that is **not in** sample space is 0

E.g.: Probability of getting 8 = P({8}) = 0

4) $P(\bar{E}) = 1 - P(E)$ **Complementary Rule**

E.g.: P(not getting 4) = 1 – P(getting 4) = 1 – 1/6 = 5/6

Addition Rule: Disjoint Events

Two events are **mutually exclusive events or disjoint events** if they cannot occur at the same time (i.e., they have no outcomes in common)

If A and B are 2 disjoint events, then we denote them as

A and B = ∅ (null set or empty set)

=> $A \cap B = \emptyset$

Examples of disjoint event:

A: Rolling a die and getting an odd number {1, 3, 5}

B: Rolling a die and getting an even number : {2, 4, 6}

$A \cap B = \emptyset$

A: Randomly selecting a person with type A blood

B: Randomly selecting a person with type O blood

$A \cap B = \emptyset$

Examples of non-disjoint event

A: Rolling a die and getting an odd number {**1**, 3, 5}

B: Rolling a die and getting a number less than 3 : {**1**, 2}

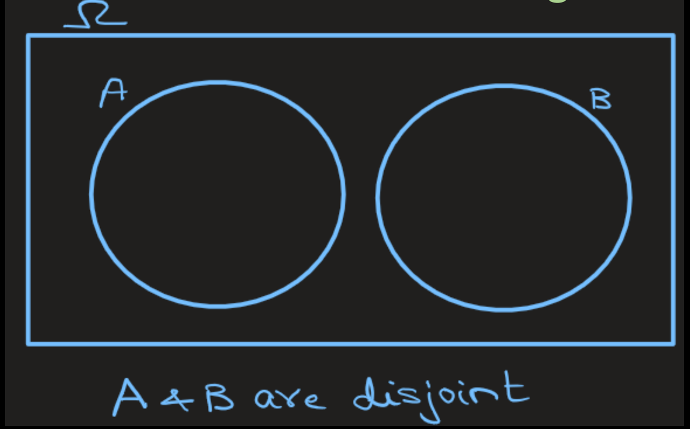
$A \cap B = \{1\}$

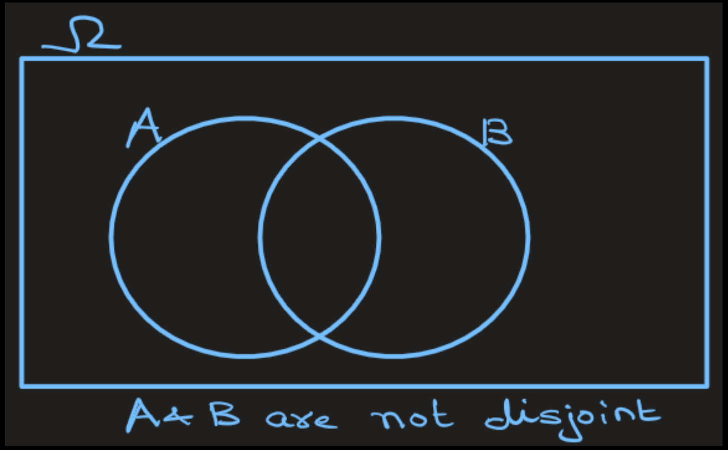
A: Randomly selecting a person with type A blood

B: Randomly selecting a person with gender as male

$A \cap B$ = Set of all males who have type A blood

Venn Diagram



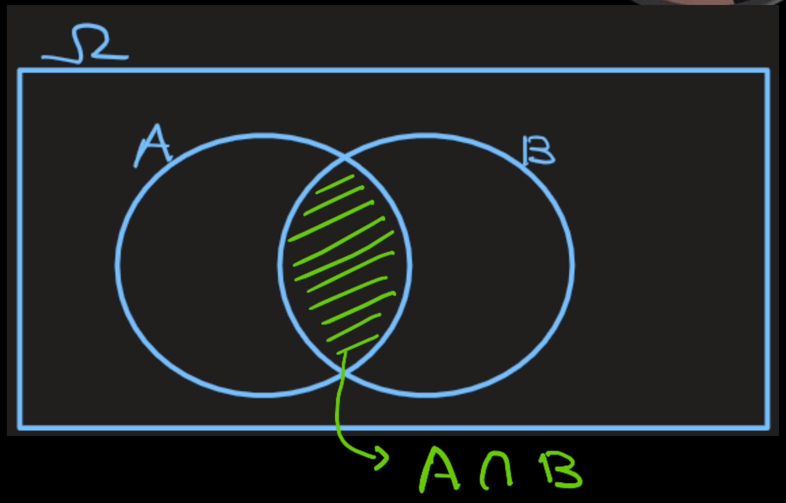


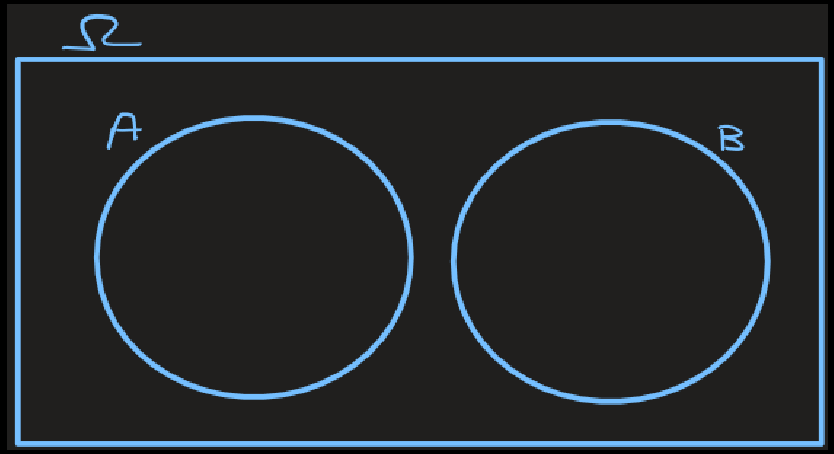
Addition Rule

General addition rule:

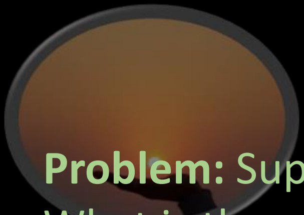
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$





Special case: If A and B are disjoint, then $P(A \cap B) = 0$.
In this case,
$P(A \cup B) = P(A) + P(B)$



Addition Rule

P(A U B) = P(A) + P(B) – P(A ∩ B)

Problem: Suppose you randomly select a whole number between 1 and 13.
What is the probability that is it multiple of 2 **or** multiple of 3 ?

Ans: The sample space is Ω **=** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}

Let

A: Randomly selecting whole number between 1 and 13 that is multiple of 2. A = { 2, 4, 6, 8, 10, 12 }

B: Randomly selecting whole number between 1 and 13 that is multiple of 3. B = { 3, 6, 9, 12 }

Here I have to find P(A U B).

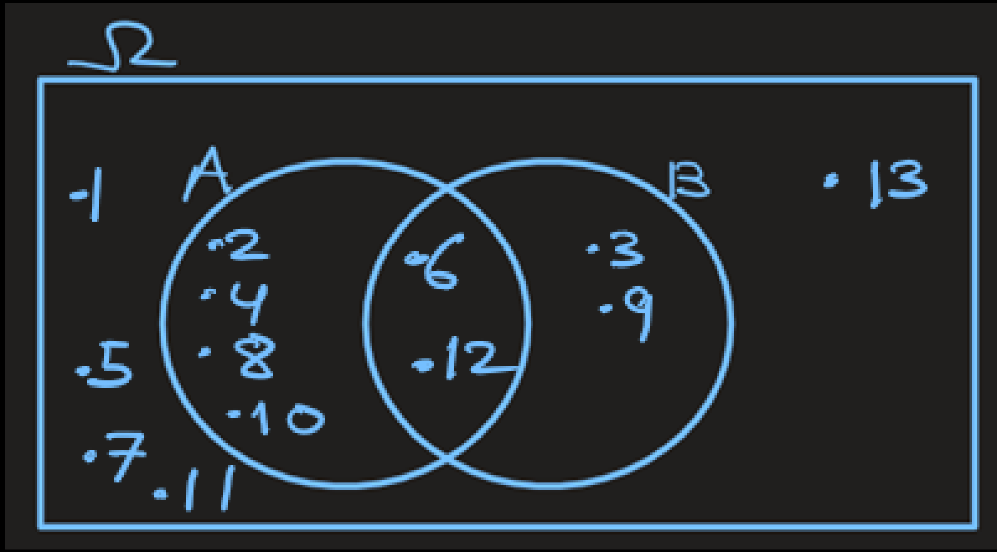
A ∩ B = numbers that are multiple of 2 **and** 3 = **{6, 12}.**

Here P(A) = 6/13 ; P(B) = 4/13; P(A ∩ B) = 2/13

So, P(A U B) = P(A) + P(B) – P(A ∩ B)

= 6/13 + 4/13 – 2/13

= 8/13 = 0.615

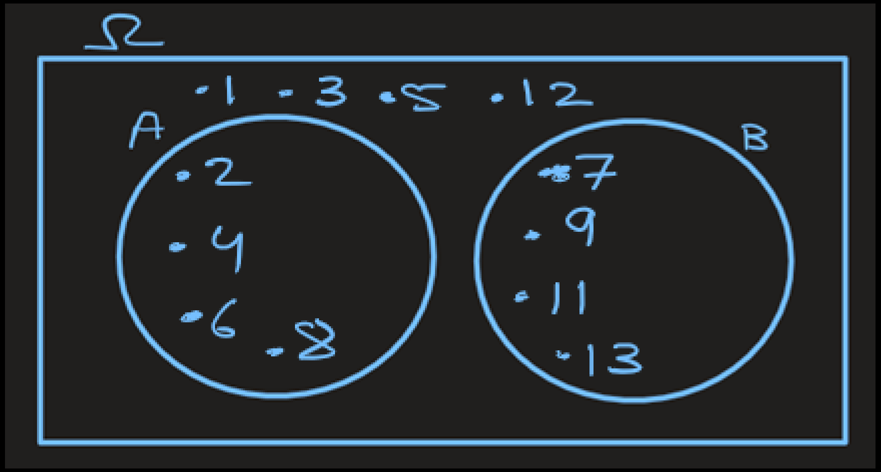


Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Problem: Suppose you randomly select a whole number between 1 and 13.
What is the probability that it is an even number less than 9 **or** an odd number more than 6 ?

Ans: The sample space is Ω **=** {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}
Let
A: Randomly selecting an even number less than 9. A = { 2, 4, 6, 8 }
B: Randomly selecting an odd number more than 6. B = { 7, 9, 11, 13 }



Here I have to find P(A U B).

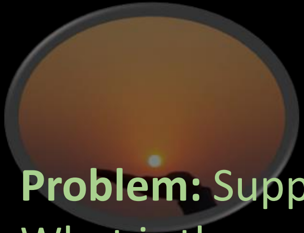
We see that **A ∩ B =** $\emptyset$ (an empty set AKA null set)

Here, P(A) = 4/13; P(B) = 4/13; P(A ∩ B) = 0

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 4/13 + 4/13 - 0$

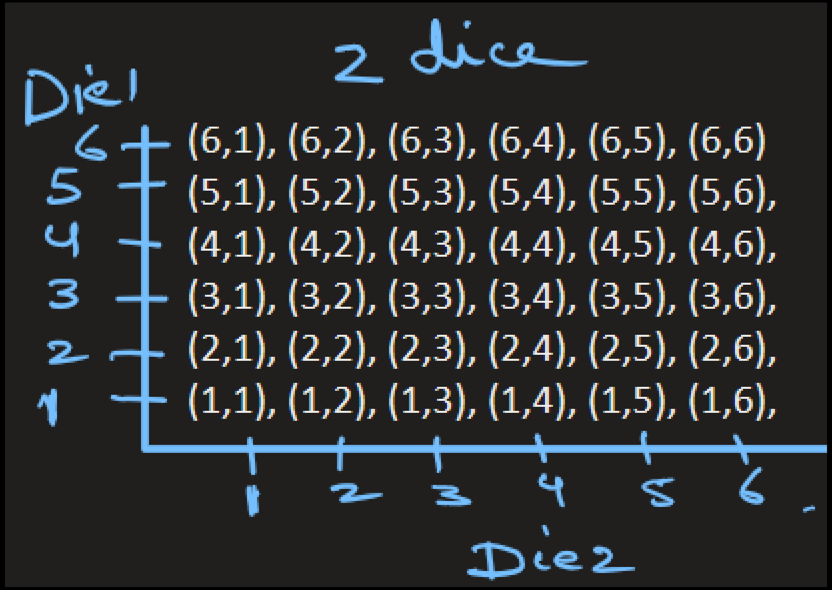
$= 8/13$



Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Problem: Suppose we roll a pair of dice.
What is the probability the sum of the two dice is 7 or number on the first die is 4?



Ans: The sample space is Ω is shown on figure.
Let
A: Sum of 2 dice is 7: $A = \{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)\}$
B: Number of 1st die is 4 : $B = \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

Here I have to find $P(A \cup B)$.

$A \cap B$ = sum of the two dice is 7 **and** number on the first die is 4 = $\{(4,3)\}$.

Here $P(A) = 6/36$; $P(B) = 6/36$; $P(A \cap B) = 1/36$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

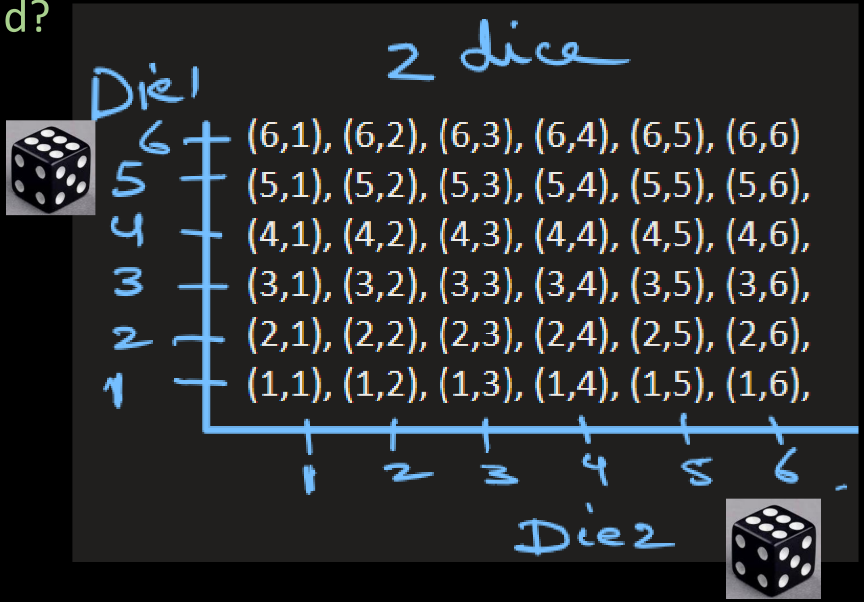
$= 6/36 + 6/36 - 1/36$

$= 11/36$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Addition Rule

Problem: Suppose we roll a pair of dice.
What is the probability that either first number is even **o**r second number is odd?



Ans:

1) Let

A: First number is even

A will have 18 elements

B: Second number is odd

B will have 18 elements

We have to find $P(A \cup B)$.

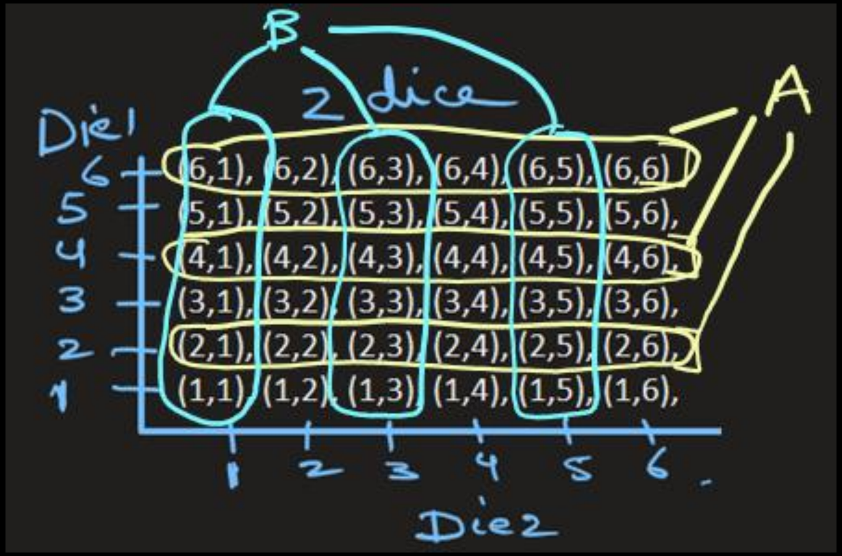
$|A \cap B|$ = |first number is even **<u>and</u>** second number is odd| = 9

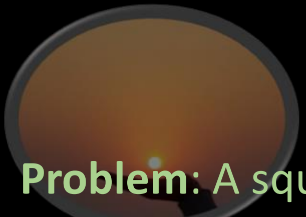
Here $P(A) = 18/36$; $P(B) = 18/36$; $P(A \cap B) = 9/36$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 18/36 + 18/36 - 9/36$

$= 27/36 = 3/4$





Addition Rule

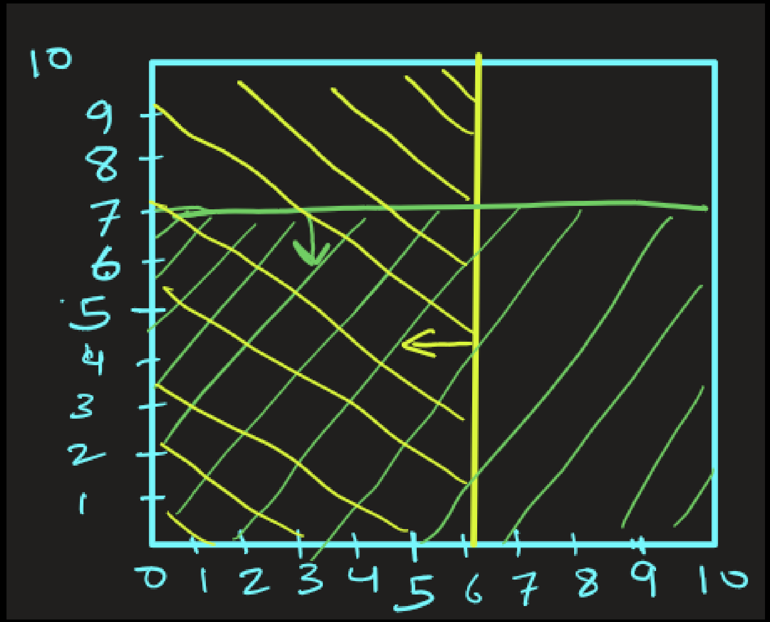
P(A U B) = P(A) + P(B) – P(A ∩ B)

Problem: A square dartboard has corners at the points (0,0) , (0,10), (10,0), (10,10).
A dart is thrown randomly at the board such that every point on the board is equally likely to be hit.
Define the following events:
Event A: The dart lands to the **left** of the vertical line $x = 6$.
Event B: The dart lands **below** the horizontal line $y = 7$.
Using the **Addition Rule of Probability**, compute the probability that the dart lands **either** to the left of the line $x = 6$ **or** below the line $y = 7$. (Assume probability is proportional to area, i.e. uniform probability distribution over the board)

Ans: We have to find P(A U B) using addition rule. Here the probability is proportional to the area.

A: X < 6.

$$P(A) = \frac{6\text{x}10}{10\text{x}10} = 0.6$$



B: $Y < 7$

$$P(B) = \frac{7\text{x}10}{10\text{x}10} = 0.7$$

Overlap (A ∩ B): $X < 6$ and $Y < 7$

$$P(\text{A}\cap\text{B}) = \frac{6\text{x}7}{10\text{x}10} = 0.42$$

So, P(A U B) = P(A) + P(B) – P(A ∩ B)
= 0.6 + 0.7 – 0.42 = 0.88

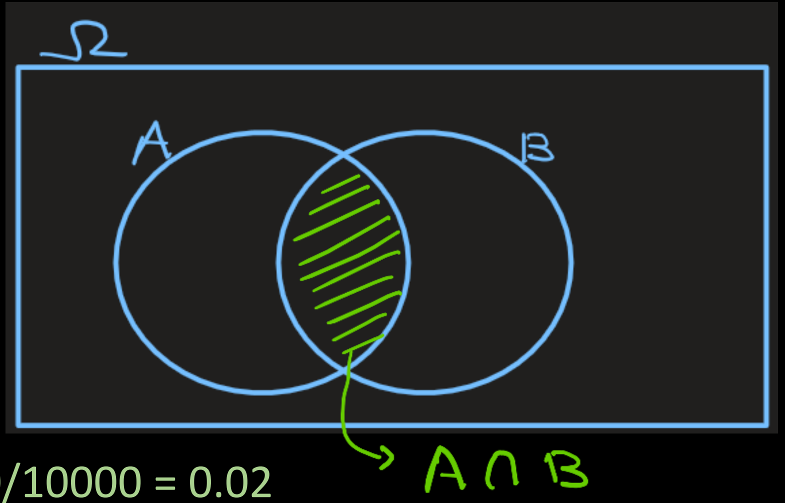
EXTRA

Extra questions.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Addition Rule

Problem Suppose you're designing a **fraud detection** system for online transactions.
Based on past historical data of 10,000 transactions, we have following empirical results.
1000 transaction were large (that is, > $5000)
500 transaction were from unusual locations
200 transactions were large **AND** from an unusual location
You receive some notification of some random transaction. What is the probability that transaction is either large **OR** from an unusual location?



Answer : Let
Event A: Transaction is large (over $5,000). So, P(A) = 1000/10000 = 0.10
Event B: Transaction is from an unusual location. So, P(B) = 500/10000 = 0.05

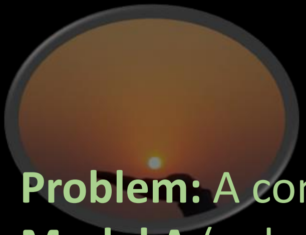
Here I have to find $P(A \cup B)$.

Event $A \cap B$: 200 transactions were large and from an unusual location → $P(A \cap B) = 200/10000 = 0.02$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.10 + 0.05 - 0.02$$

$$= 0.13$$



Addition Rule

P(A U B) = P(A) + P(B) – P(A ∩ B)

Problem: A company trains two spam detection models:
Model A (a deep neural network) and **Model B** (a random forest).

Probability that Model A flags an email as spam: $P(A) = 0.4$
Probability that Model B flags an email as spam: $P(B) = 0.3$
Probability that *both* models flag the same email as spam: $P(A \cap B) = 0.15$

What is the probability that **at least one** model flags the email as spam?

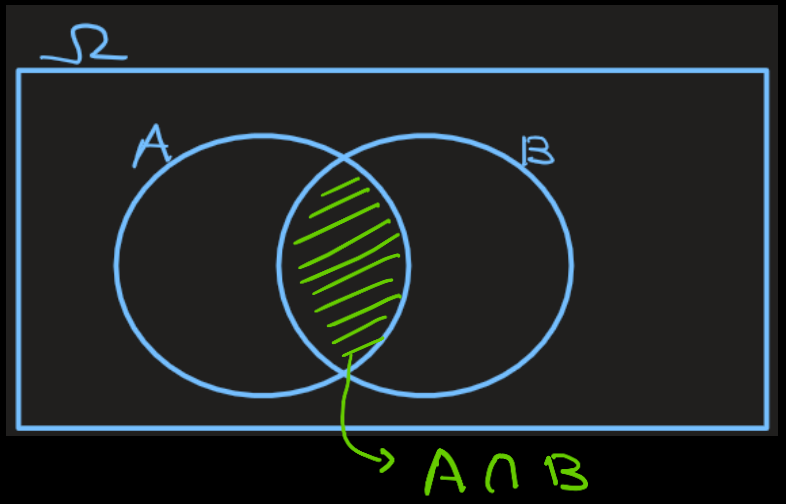
Ans:
P (**at least one** model flags the email as spam)

= P(A U B)

= P(A) + P(B) – P(A ∩ B)

= 0.4 + 0.3 – 0.15

= 0.55



P(A U B) = P(A) + P(B) – P(A ∩ B)

Addition Rule

Problem: An AI content moderation system flags content as
Hate speech (H): $P(H) = 0.2$
Misinformation (M): $P(M) = 0.25$
Both: $P(H \cap M) = 0.05$

What is the probability that a post is flagged for *either hate speech* ***or*** *misinformation*?

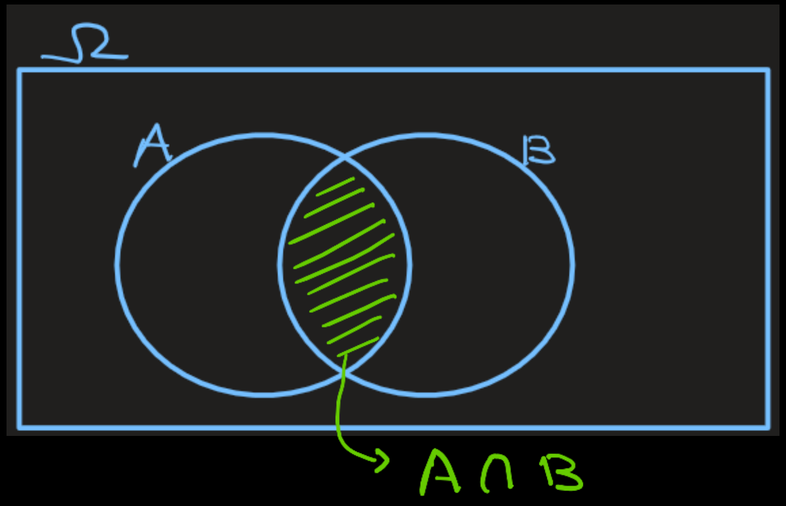
Ans:
P (a post is flagged for *either hate speech or misinformation*)

= P(H U M)

= P(H) + P(M) – P(H ∩ M)

= 0.2 + 0.25 – 0.05

= 0.4



Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Same problem as before but different scenario: Suppose you're designing a **recommendation** system
Based on past historical data of 10,000 user views, we have following empirical results.
1000 user liked sci-fi
500 user liked fantasy
200 user liked sci-fi **and** fantasy
You pick a random user from the database. What is the probability that this **user likes sci-fi OR fantasy**?

Answer : Let
Event A: user likes sci-fi. So, $P(A) = 1000/10000 = 0.10$
Event B: user liked fantasy . So, $P(B) = 500/10000 = 0.05$

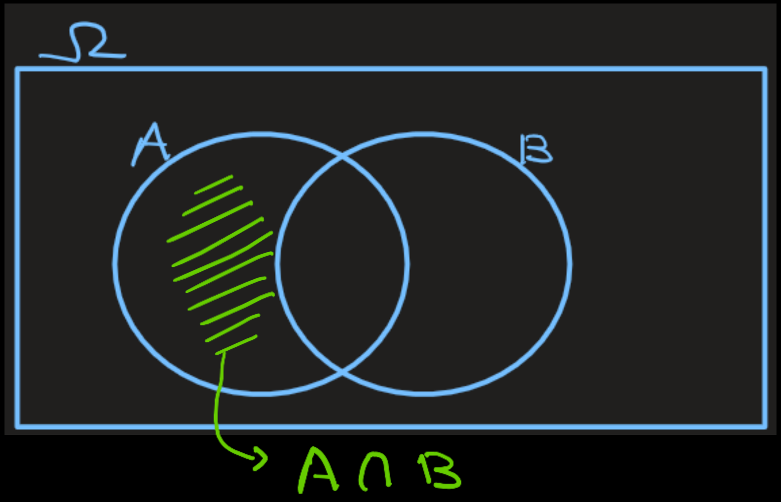
We have to find $P(A \cup B)$.

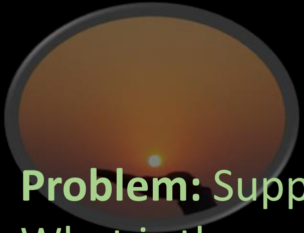
Event A ∩ B: 200 **user likes sci-fi and fantasy** → $P(A \cap B) = 200/10000 = 0.02$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.10 + 0.05 - 0.02$$

$$= 0.13$$



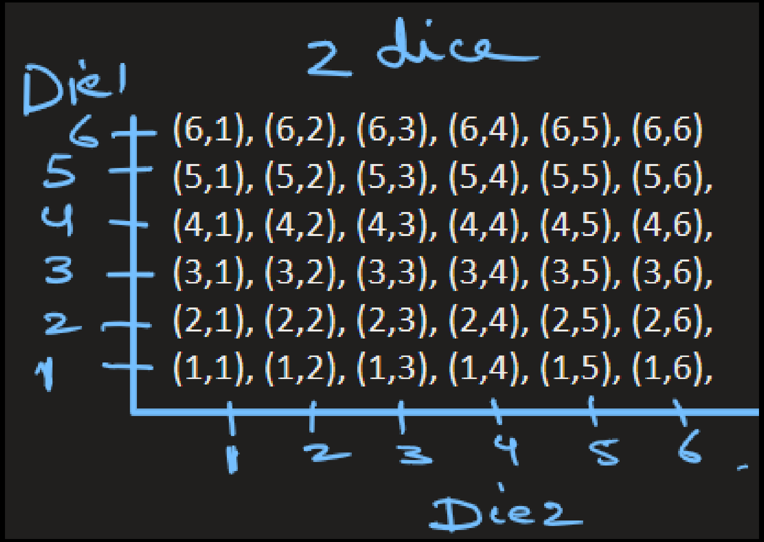


Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Problem: Suppose we roll a pair of dice.
What is the probability the sum of the two dice is $\geq$ 10 or number on first die is 3 ?

Ans:
Let
A: Sum of 2 dice number $\geq$ 10 : A = {(6,4), (5,5), (4,6), (6,5), (5,6), (6,6)}
B: Number of 1st die is 3 : B = {(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)}



Here I have to find $P(A \cup B)$.

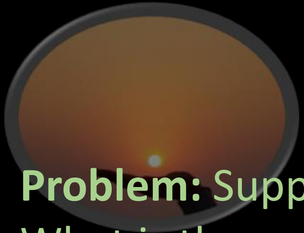
$A \cap B$ = Sum of 2 dice number $\geq$ 10 **<u>and</u>** number of 1st die is 3 = $\emptyset$

Here $P(A) = 6/36$; $P(B) = 6/36$; $P(A \cap B) = 0$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

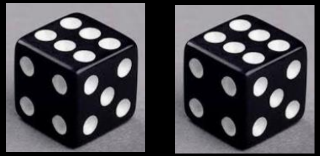
$= 6/36 + 6/36 - 0$

$= 12/36$
$= 1/4$

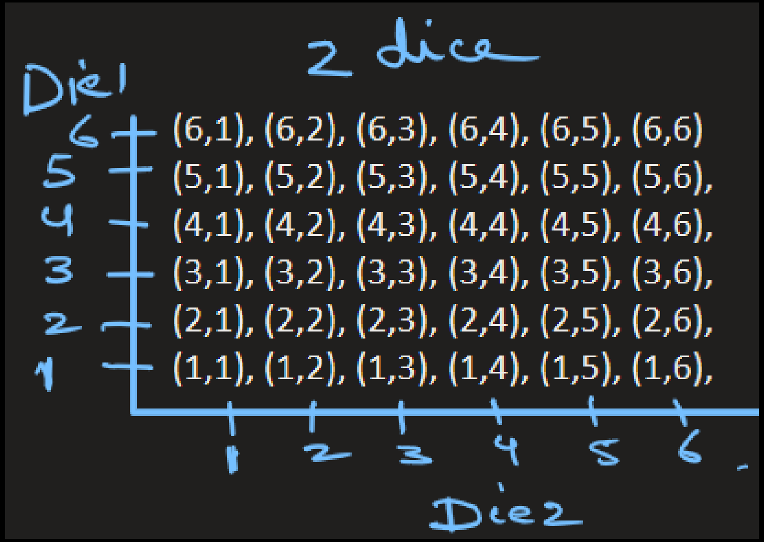


Addition Rule

P(A U B) = P(A) + P(B) – P(A ∩ B)



Problem: Suppose we roll a pair of dice.
What is the probability the sum of the two dice is ≥ 7 or both numbers are same?



Ans:

1) Let

A: sum of the two dice is ≥ 7

A = {(6,1), (6,2), (6,3), (6,4), and so on-upper diagonal } = 21 elements

B: both numbers are same

B = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

Then, **A ∩ B** = the sum of the two dice is ≥ 7 **<u>and</u>** both numbers are same = {(4,4), (5,5), (6,6)}.

Here P(A) = 21/36 ; P(B) = 6/36; P(A ∩ B) = 3/36

So, **P(A U B) = P(A) + P(B) – P(A ∩ B)**

= 21/36 + 6/36 **–** 3/36

= 24/36