Variance And Standard Deviation

Problem: Can you tell which of the following data has more spread ?

A) 1, 1, 0, 0, 5, 9, 9, 10, 10, 5

B) 4, 3, 5, 6, 7, 3, 5, 6, 4, 5

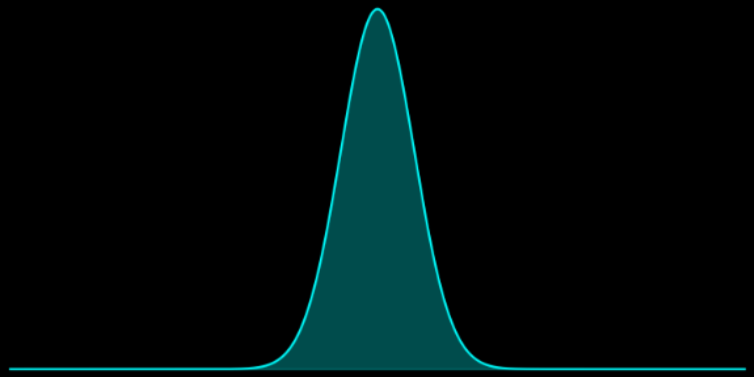
C) 5, 5, 5, 5, 5, 5, 5, 5, 5, 5

Variance and Standard deviation of population

Variance is a **measure of how much dispersed (or spread or scattered) your data is from its mean value**.

For a dataset $x_1, x_2, \ldots, x_n$ with mean μ:

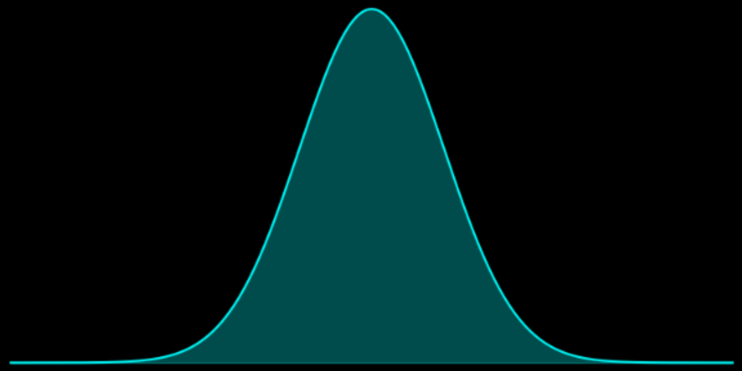
$$\text{Variance } (\sigma^2) = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$



Variance is **difficult to interpret** because of squared units.
For example, if $x_1, x_2, .. , x_n$ are in kg, then variance is going to be in kg^2

Standard Deviation σ is the square root of variance.

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$



Std. Dev. **easy to interpret** because it is in same unit as data points.
For example, if $x_1, x_2, .. , x_n$ are in kg, then standard dev. is going to be in kg.

Example1: Large spread around mean

Suppose monthly income of population in a city is 1, 1, 0, 0, 5, 9, 9, 10, 10, 5.

Calculate the variance and standard deviation for this population.

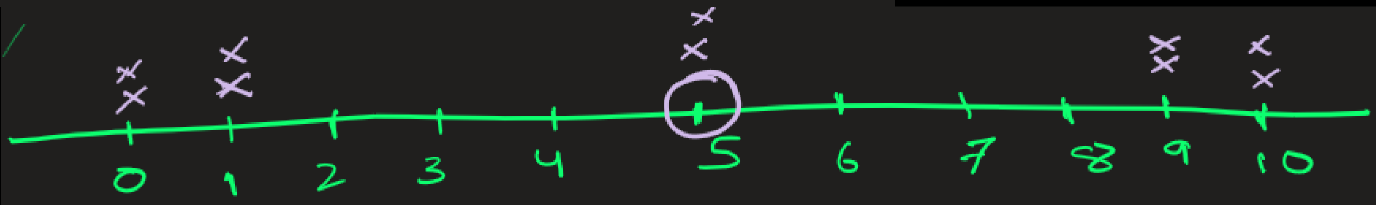
Ans: Here size n = 10, and **mean** = (1 + 1 + 0 + 0 + 5 + 9 + 9 + 10 + 10 + 5)/ 10 = 5

$$\text{Variance}(\sigma^2) = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}$$

$$= \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}$$

$$= \frac{(1-5)^2 + (1-5)^2 + (0-5)^2 + (0-5)^2 + (5-5)^2 + (9-5)^2 + (9-5)^2 + (10-5)^2 + (10-5)^2 + (5-5)^2}{10}$$

$$= \frac{16 + 16 + 25 + 25 + 0 + 16 + 16 + 25 + 25 + 0}{10} = 16.8$$



Standard deviation σ = $\sqrt{16.8}$ = 4.1

There is some dispersion / spread in data

Example2: Few spread around mean

Consider the package weights (in kilograms) produced on a production line:

4, 3, 5, 6, 7, 3, 5, 6, 4, 5.

Calculate the variance and standard deviation for this population.

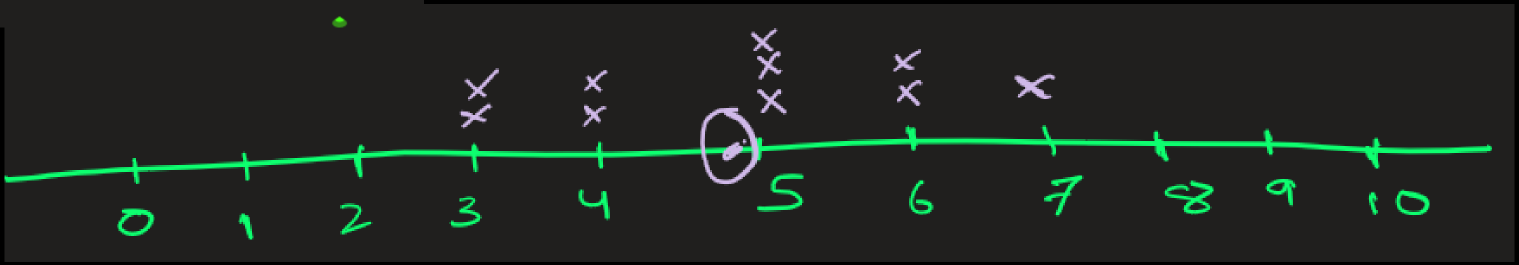
Ans: Here size n = 10, and **mean** = (4 + 3 + 5 + 6 + 7 + 3 + 5 + 6 + 4 + 5)/ 10 = 4.8

$$\text{Variance}(\sigma^2) = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}$$

$$= \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}$$

$$= \frac{(4-4.8)^2 + (3-4.8)^2 + (5-4.8)^2 + \cdots + (5-4.8)^2}{10}$$

$$= \frac{0.64 + 3.24 + 3.24 + \cdots + 3.24}{10} = 1.56$$



Standard deviation σ = $\sqrt{1.56}$ = 1.3 **Kg**

Here data is less dispersed.

Example3: Time intervals (in seconds) between each operation in an assembly line (high precision)
Consider the time intervals (in seconds) between operations on an assembly line for all 10 machines:
5, 5, 5, 5, 5, 5, 5, 5, 5, 5.
Calculate the variance and standard deviation for this population.

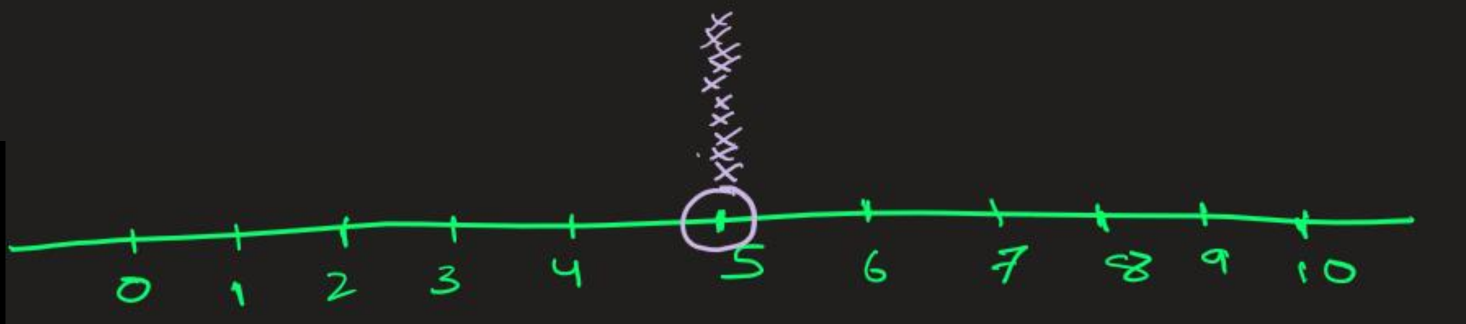
Ans: Here size n = 10, and **mean** = (5 + 5 + 5 + … + 5)/ 10 = 5

$$\text{Variance}(\sigma^2) = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n}$$

$$= \frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots + (x_n-\mu)^2}{n}$$

$$= \frac{(5-5)^2 + (5-5)^2 + (5-5)^2 + \cdots + (5-5)^2}{10}$$

$$= 0$$



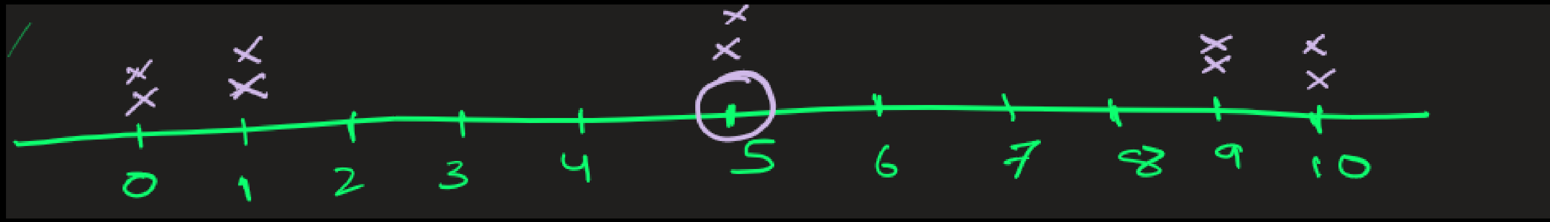
Standard deviation σ = $\sqrt{0} = 0$

The data has no dispersion.

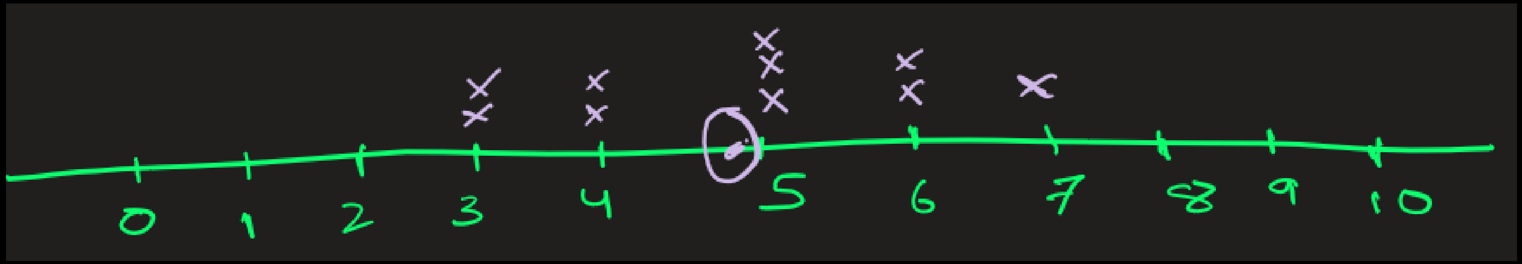
Conclusion:

- If data points are **far from the mean** -> Variance is **large**. Data is more dispersed or more spread
- If all data points are **close to the mean** -> Variance is **small**. Data is less dispersed or less spread

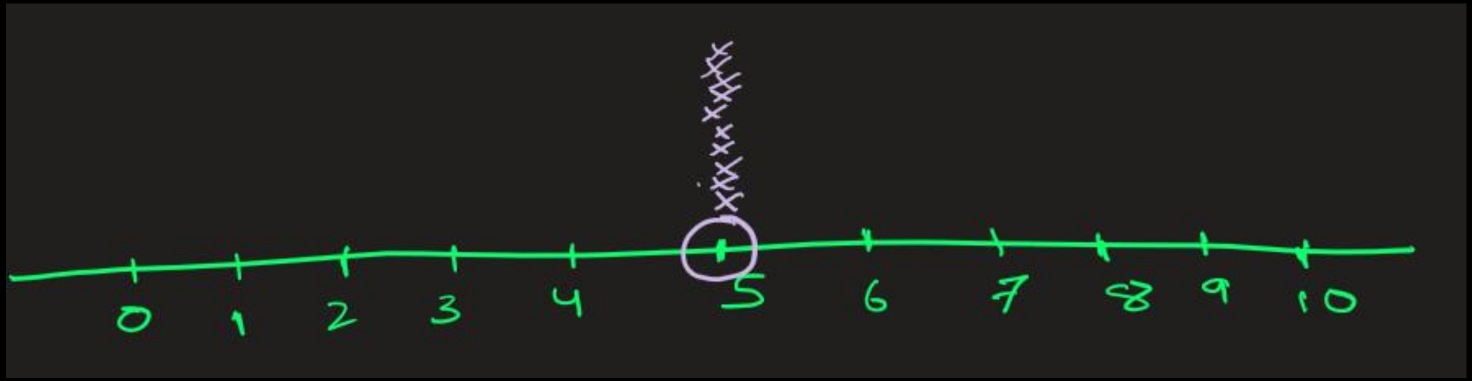
Case A: 1, 1, 0, 0, 5, 9, 9, 10, 10, 5
variance = 16.8



Case B: 4, 3, 5, 6, 7, 3, 5, 6, 4, 5
variance = 1.56



Case C: 5, 5, 5, 5, 5, 5, 5, 5, 5, 5
variance = 0

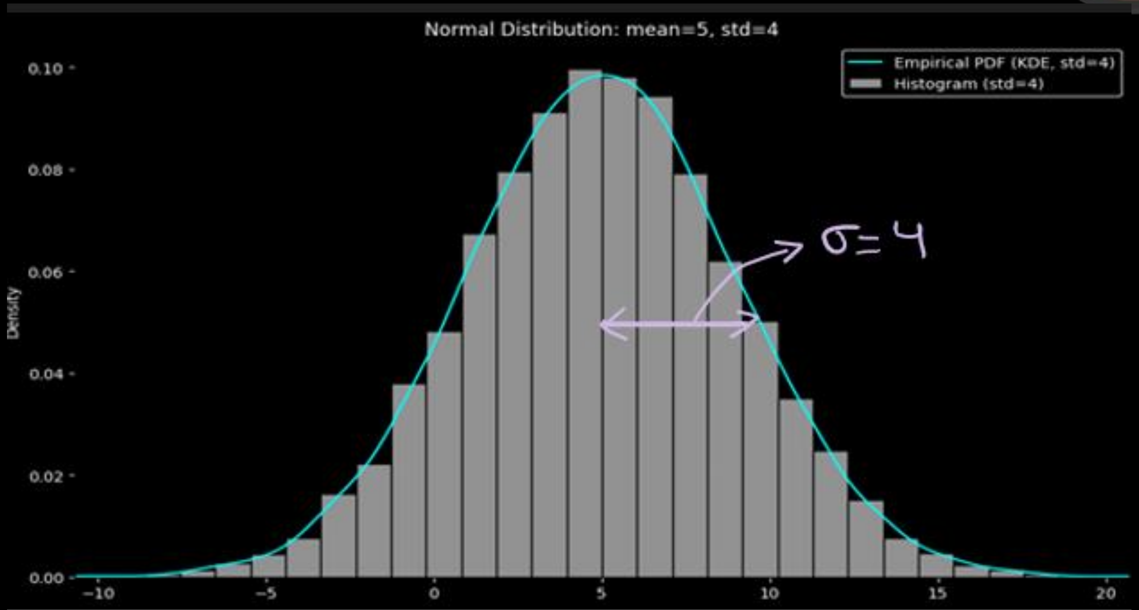


How standard deviation controls dispersion/spread of distribution curve:

1) Following is 100 samples taken from population with 1050 data points with high std dev. (This could be temperature.)

6.987, 4.447, 7.591, 11.092, 4.063, 4.063, 11.317, 8.070, 3.122, 7.170, 3.146, 3.137, 5.968, -2.653, -1.900, 2.751, 0.949, 6.257, 1.368, -0.649, 10.863, 4.097, 5.270, -0.699, 2.822, 5.444, 0.396, 6.503, 2.597, 3.833, 2.593, 12.409, 4.946, 0.769, 8.290, 0.117, 5.835, -2.839, -0.313, 5.787, 7.954, 5.685, 4.537, 3.796, -0.914, 2.121, 3.157, 9.228, 6.374, -2.052, 6.296, 3.460, 2.292, 7.447, 9.124, 8.725, 1.643, 3.763, 6.325, 8.902, 3.083, 4.257, 0.575, 0.215, 8.250, 10.425, 4.712, 9.014, 6.447, 2.420, 6.446, 11.152, 4.857, 11.259, -5.479, 8.288, 5.348, 3.804, 5.367, -2.950, 4.121, 6.428, 10.912, 2.927, 1.766, 2.993, 8.662, 6.315, 2.881, 7.053, 5.388, 8.875, 2.192, 3.689, 3.432, -0.854, 6.184, 6.044, 5.020, 4.062

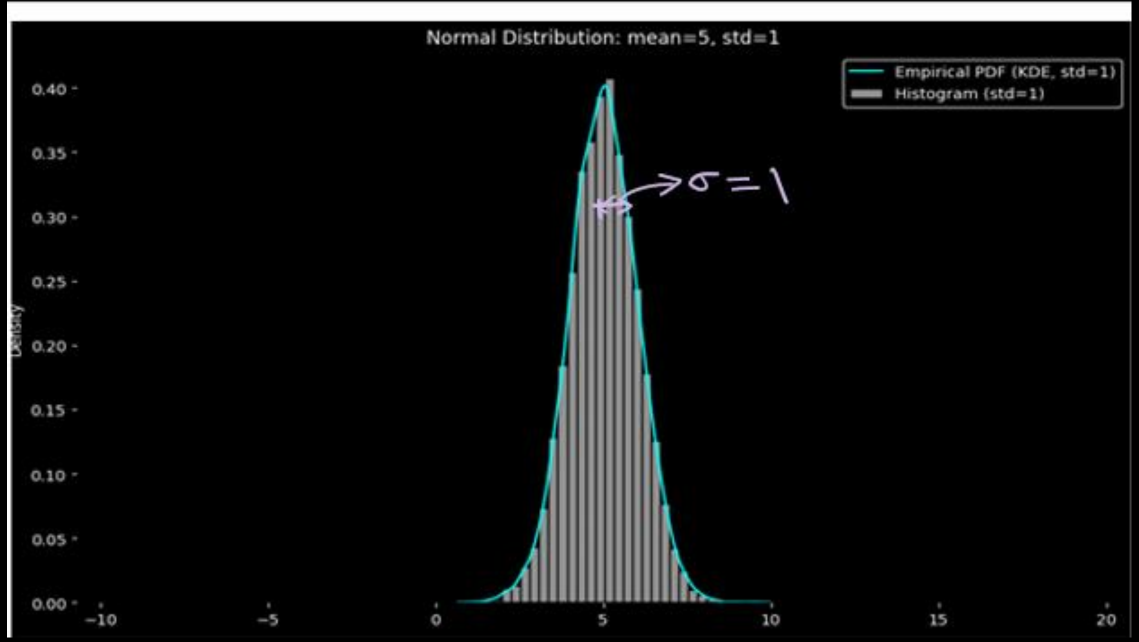
Sample mean: 5, Sample std dev: **4**



2) Following is 100 samples taken from population with 1050 data points with low std dev

4.322, 4.695, 4.403, 5.110, 6.197, 4.229, 6.001, 4.218, 4.152, 5.819, 5.922, 5.851, 3.684, 4.534, 5.823, 5.042, 3.926, 5.458, 4.285, 6.795, 6.545, 5.604, 6.361, 5.065, 5.765, 6.478, 5.245, 4.745, 3.295, 4.917, 5.823, 5.946, 5.504, 4.459, 3.023, 4.505, 4.696, 4.687, 5.619, 6.986, 5.124, 4.783, 4.741, 5.124, 4.172, 5.120, 5.451, 5.210, 5.458, 5.434, 3.228, 5.637, 4.329, 3.904, 3.896, 5.434, 4.777, 3.318, 5.478, 3.560, 5.140, 5.238, 5.886, 6.782, 3.635, 4.948, 4.724, 5.434, 5.216, 4.647, 4.559, 3.884, 5.988, 5.459, 5.918, 4.690, 4.343, 3.922, 5.359, 3.960, 6.849, 3.692, 5.274, 6.567, 5.599, 5.176, 4.032, 3.618, 5.770, 6.506, 4.561, 3.861, 2.542, 4.019, 4.886, 6.599, 5.673, 5.305, 4.600, 4.417

Sample mean: 5, Sample std dev: **1**



Why the 2 measure: variance and std. dev.?

Variance is the average squared deviation from the mean. So, **if your data is in, say, cm, then variance is in square cm (cm^2).** Squared units aren't directly interpretable: nobody says "the spread of your height is 100 cm^2".

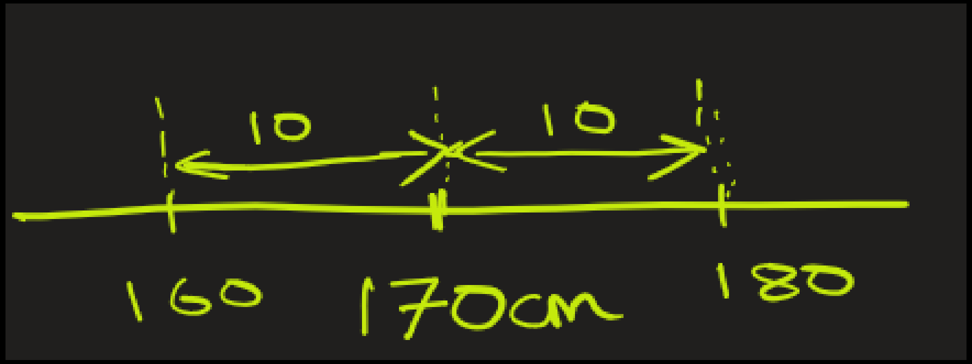
By taking the square root as in standard deviation, you bring the units back to the original scale: cm -> cm

If the mean height is 170 cm and variance is 100 cm^2, then standard deviation would 10 cm: that is , you can say **"Most data is within 10 cm above or below the mean."**

For a dataset $x_1, x_2, \ldots, x_n$ with mean μ:

$$\text{Variance } (\sigma^2) = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$



Population variance versus Sample variance

If you have a **population** of size N with values $x_1, x_2, \ldots, x_N$, the **population variance** is:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

x_i = each individual value
μ = population mean = $\frac{1}{N}\sum_{i=1}^{N} x_i$
N = size of the population

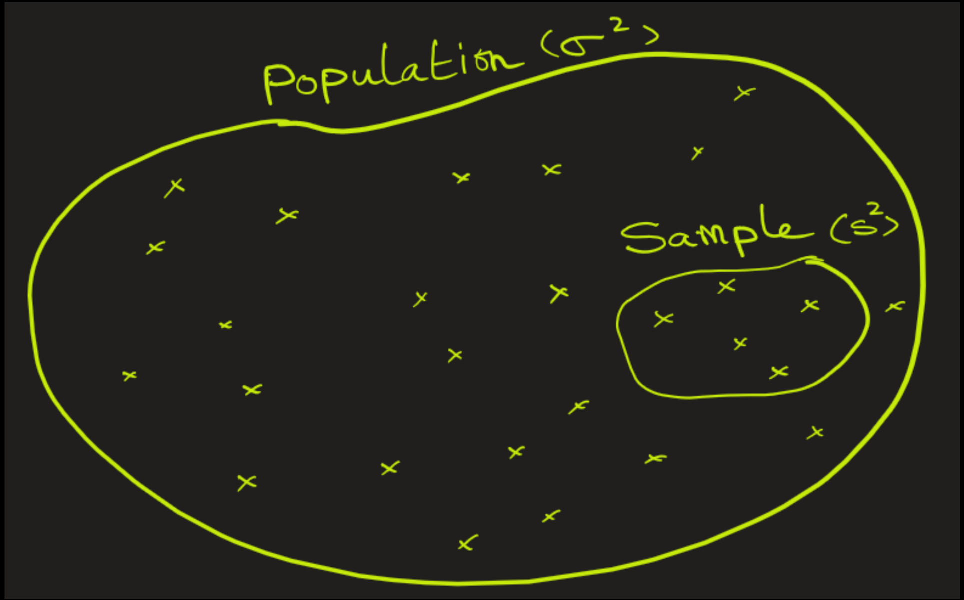
If you have a **sample** of size n drawn from a population, the **sample variance** is:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

x_i = each sample value
$\bar{x}$ = sample mean = $\frac{1}{n}\sum_{i=1}^{n} x_i$
n = size of the sample
$n-1$ is used instead of n (Bessel's correction) to get an **unbiased estimate** of the population variance

- For **population** variance, divide by n
- For **sample** variance, divide by n-1

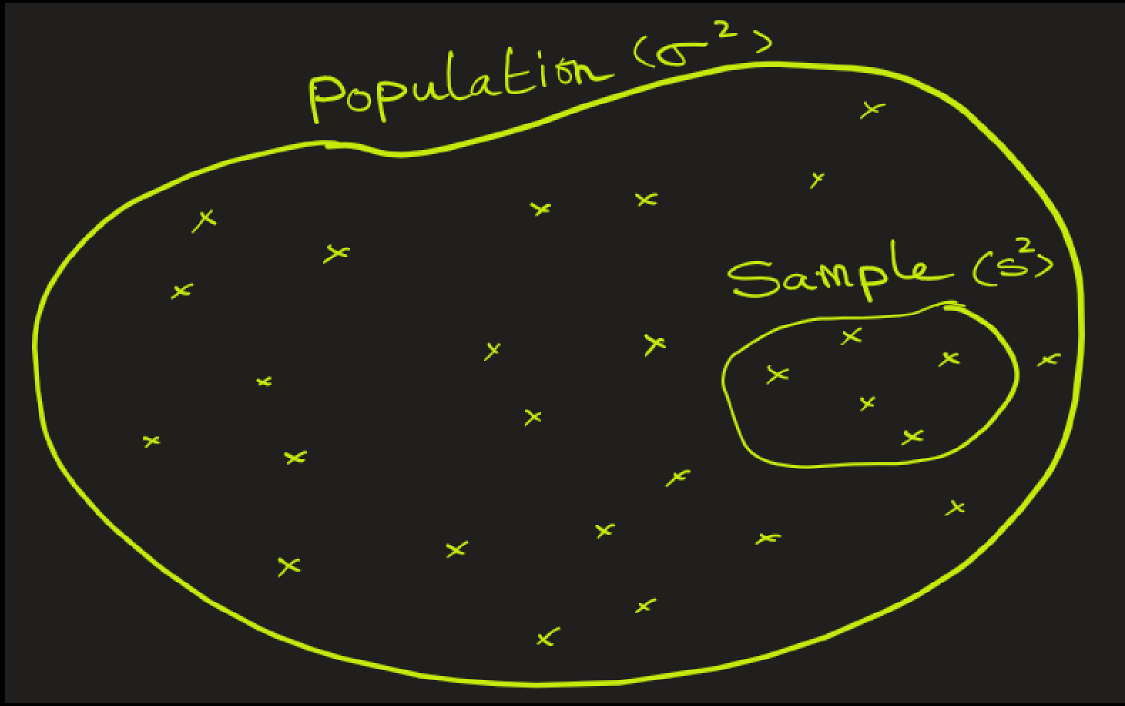
For large n, the result is almost same



How to decide which variance to use: population or sample?

- If you are analyzing **all the data from the group** you're interested in, then use **population**.
- If you are analyzing a **subset (a sample) of the full group** and trying to generalize, then use **sample**.

Situation	Population or Sample?	Formula to Use
You recorded heights of **all 100 students** in a school	Population	Divide by n
You took a **random sample of 10 students** to estimate the average height in the school	Sample	Divide by n - 1
You analyzed **every** product made in a factory on one day	Population	Divide by n
You tested **20 random products** to estimate quality	Sample	Divide by n - 1



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