Hypothesis Testing
(σ is known)
p-value

P-value

Problem: A medical researcher wants to find out whether a new medication will have any **undesirable side effects** on patients' pulse rates. The researcher is concerned with whether the pulse rate **increases, decreases, or stays the same** after taking the medication.

The researcher knows that the

- **mean pulse rate μ** for the population under study is **82 beats per minute (bpm)**, and
- **population standard deviation σ** is **5 bpm**.

The researcher selects a **random sample of 60 patients** who take the medication and finds that their **mean pulse rate** is **80.7 bpm**.

How would you test the claim that medication **does affect** the mean pulse rate, at significance level **α = 0.01 ?**

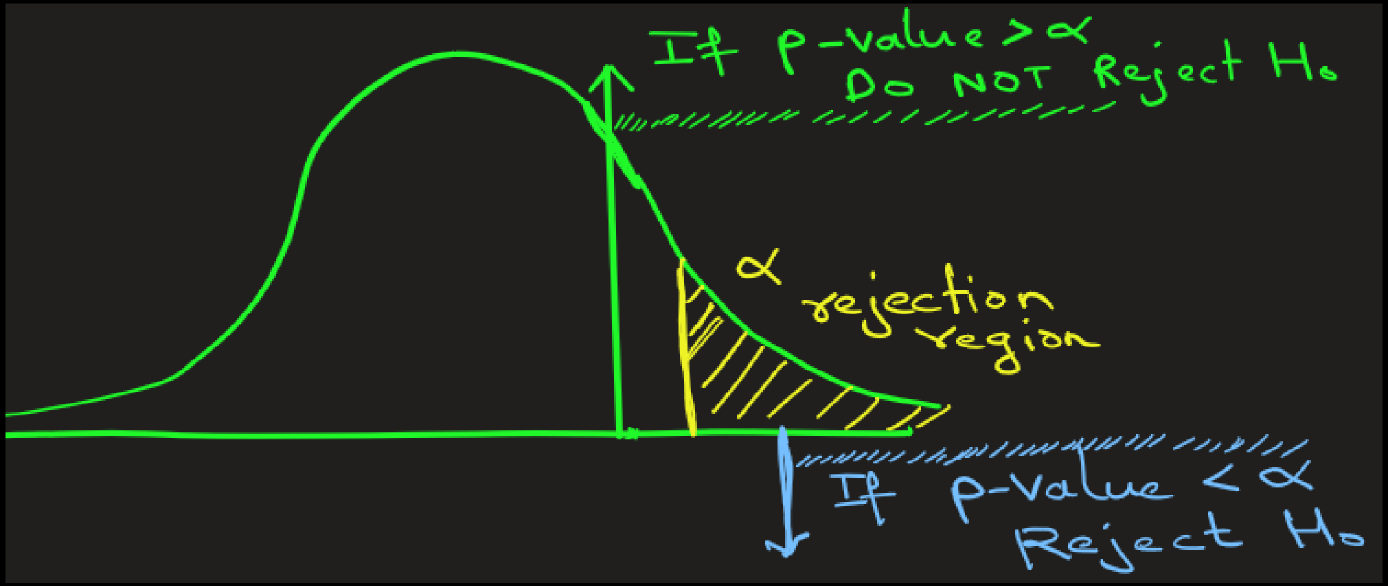
P-value

A **P-value** in hypothesis testing is the **probability of obtaining results as extreme as, or more extreme than, the observed data, assuming the null hypothesis is true**.

It measures the strength of evidence against the null hypothesis;

- a **small P-value** (typically < 0.05) suggests the results are statistically significant.
  Evidence is against null hypothesis → **Null hypothesis can be rejected**.

- a **large P-value** (typically > 0.05) suggests the observed results are likely due to random chance.
  Evidence is not against null hypothesis → **Null hypothesis cannot be rejected**.



P-value

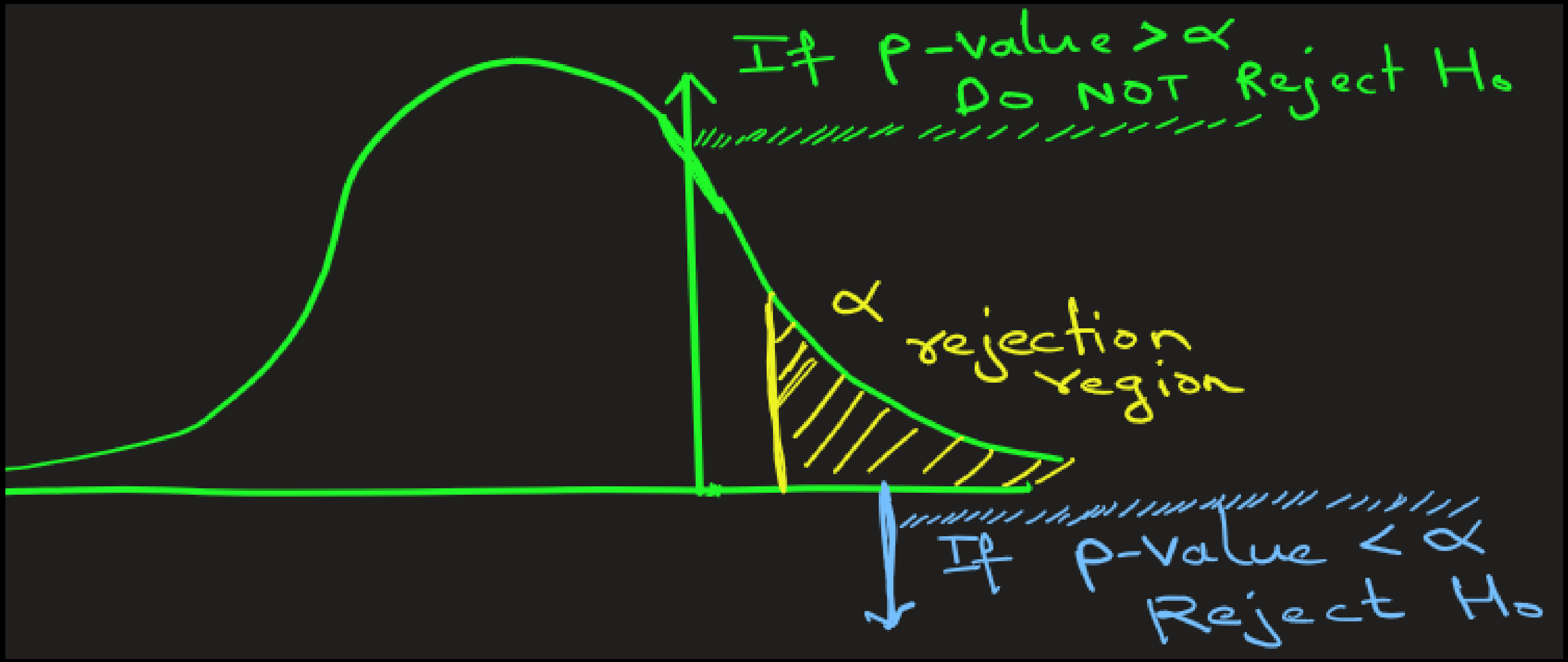
A simple example to understand p-value:

Let Null hypothesis = "The medicine doesn't work."

Then, p-value = "How surprising are my results *if* the medicine truly doesn't work?"

Let' say we choose a significance level, **α = 0.05 (5%)**, which sets our threshold for surprising result.

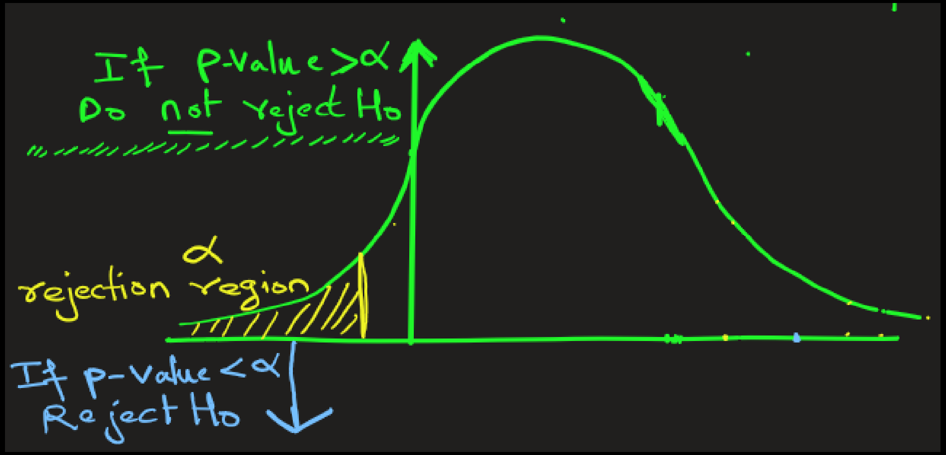
If p = 0.001 = 0.1% < 5%, the results are **very surprising** → strong evidence the medicine *might actually work*.
If p = 0.6 = 60% > 5% , the results are **not surprising** → no evidence that the medicine helps.



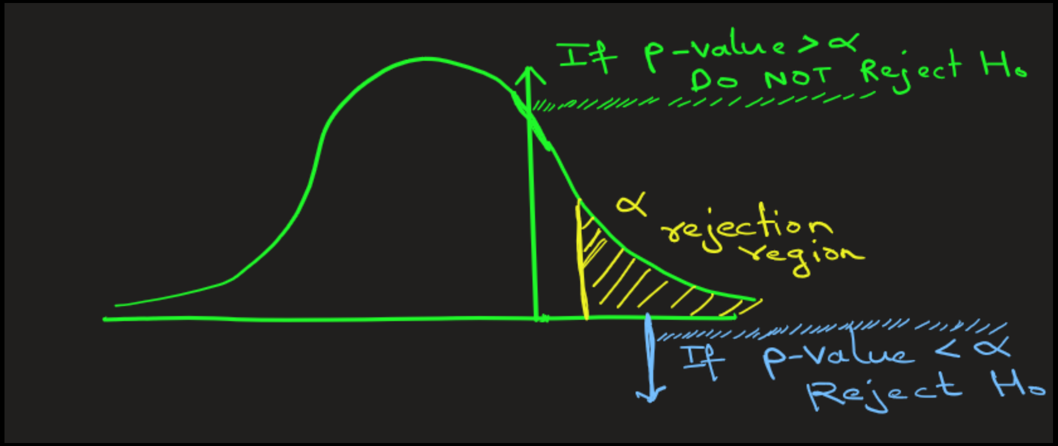
P-value

Hypothesis problems fall under 3 categories:

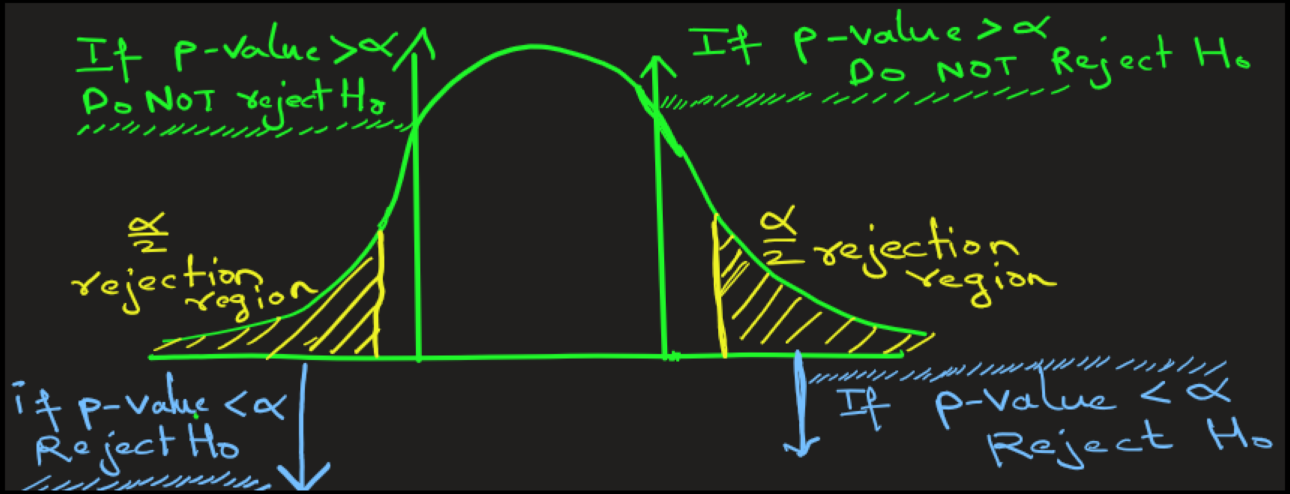
Left-tailed test



Right-tailed test



2-tailed test



P-value

Steps In Solving Hypothesis-testing Problems (P-Value method)

Step 1) State the hypothesis and identify the claim.

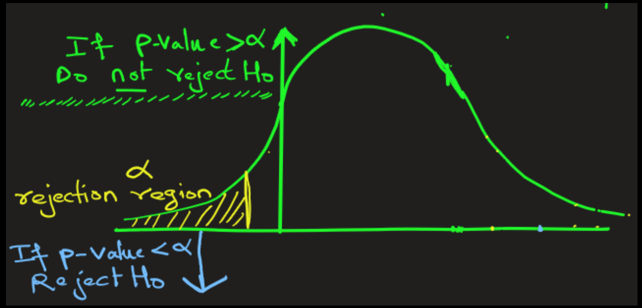
Step 2) Compute the z test value (or t test value)

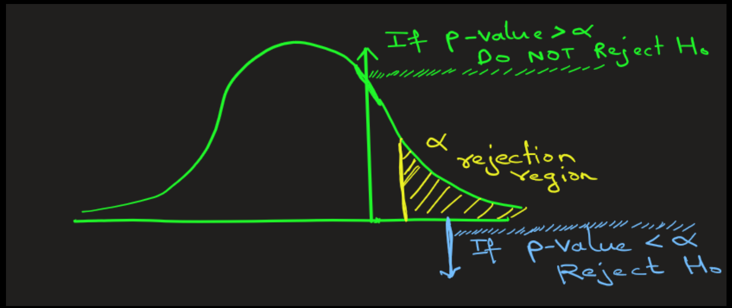
Step 3) Find the P-value for the above test value.

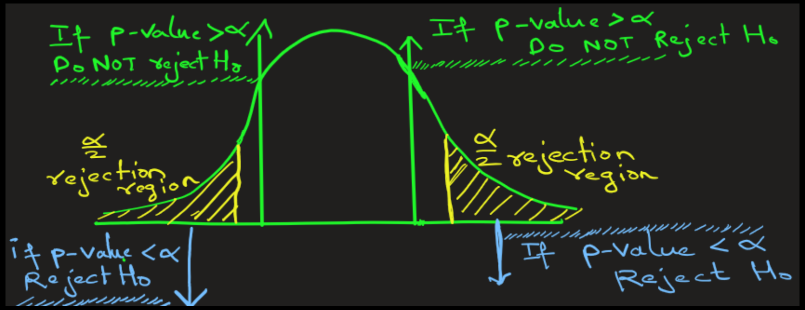
Step 4) Compare P-value with significance level α and make the decision:

- **If P-value <= α, reject the null hypothesis.**
- **If P-value > α, do not reject the null hypothesis.**

Step 5) Summarize the results.







P-value: Applications

Problem : In a population of metal components, the average lifetime **μ without** a protective paint coating is 101 days. The lifetime is normally distributed, and the population standard deviation σ is **15** days. An engineer claims that applying the **new** protective paint coating **increases** the mean lifetime of the component beyond 101 days.
She tests this claim by selecting a random sample of **50 components** with the coating and observes following lifetime:
[114 107 115 124 106 106 125 116 104 114 104 104 111 90 92 103 99 112 100 95 123 107 109 95 103 110 97 113 103 106 103 127 109 98 117 97 111 89 96 111 116 111 108 106 94 102 104 119 112 91]
The mean of above 50 samples is 107 days approx.
Test the engineer's claim at significance level **α = 0.05.** Use the P-value method.

Ans:
Step 1) State the hypothesis and identify the claim.

H0: μ = 101 (The mean lifetime is 101 days; the coating has no effect)
H1: μ > 101 (claim: The mean lifetime > 101 days; the coating increases the lifetime) This is right-tailed test

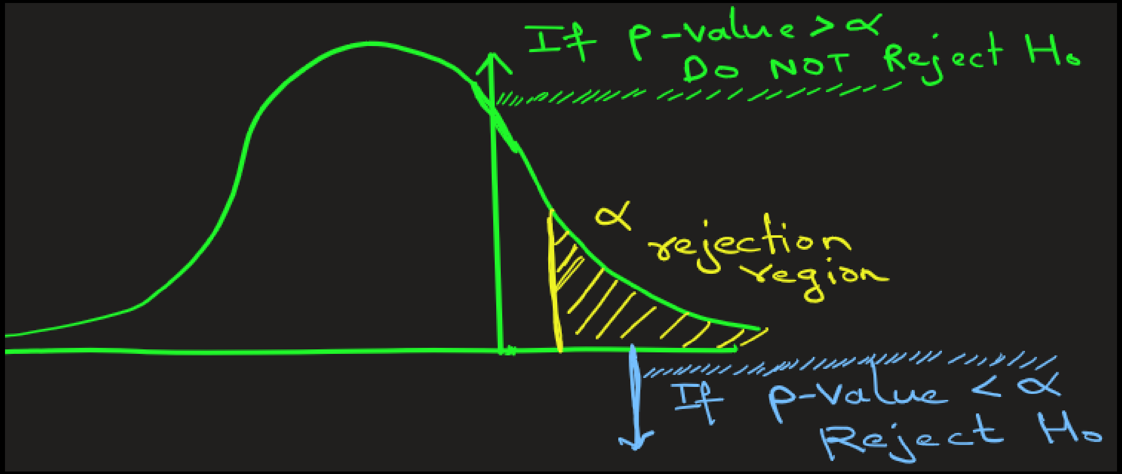
Step 2) Here σ is known, so compute the z test value for μ = 101:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{107-101}{15/\sqrt{50}} = 2.828 \approx 2.83$$

Step 3) Find the p-value
Look up the Z value in the Standard Normal table for Z=2.83

Now, P(Z>2.83)
= 1 − P(Z<2.83) (here look up in SN table)
= 1 – 0.9977 = 0.0023
So, the p-value is 0.0023



Step 4)
Since the P-value 0.0023 is less than α value of 0.05, we **reject the null hypothesis** at 5% significance level

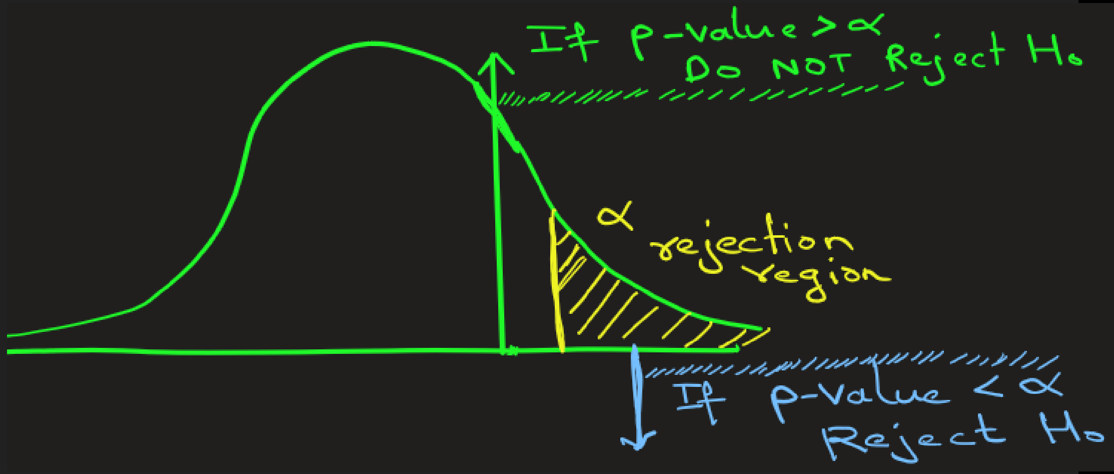
Step 5) There **is enough evidence at the 5% significance level** to conclude that the new protective coating has no effect.

Computing p-value programmatically

In a population of metal components, the average lifetime without a protective paint coating is 101 months. The lifetime is normally distributed, and the population standard deviation is 15 months. An engineer claims that applying the new protective paint coating increases the mean lifetime of the component beyond 101 months. She tests this claim by selecting a random sample of 50 components with the coating and finds that the sample mean lifetime is 107 months. Test the engineer's claim at α = 0.05.

[ ]:

```python
import scipy.stats as stats
import numpy as np

# Given values
mu_0 = 101         # null hypothesis mean
sample_mean = 107
sigma = 15         # population std dev
n = 50
alpha = 0.05

# Compute Z-statistic
z_stat = (sample_mean - mu_0) / (sigma / np.sqrt(n))

# One-tailed p-value (right tail)
p_value = 1 - stats.norm.cdf(z_stat)

# Decision
reject_null = p_value < alpha

# Output
print(f"Z-statistic: {z_stat:.4f}")
print(f"P-value: {p_value:.4f}")
print("Reject Null Hypothesis?" , "Yes" if reject_null else "No")
```




```
Z-statistic: 2.8284
P-value: 0.0023
Reject Null Hypothesis? Yes
```

P-value: Applications

Problem: A water treatment plant claims their new filtration system reduces contaminant levels to **below 5 ppm which is the population mean μ.** Environmental engineers want to verify the claim.
A random sample of **60 water samples** is taken after treatment. The sample shows a mean contaminant level of **4.9 ppm**, with a known population standard deviation σ of **0.8 ppm**. Test the water treatment plant claim at significance level **α = 0.10**.

Ans:
Step 1) State the hypothesis and identify the claim.

H0: μ = 5 ppm (No improvement)
H1: μ < 5 ppm (Claim: System reduces mean contaminant level)
This is left-tailed test

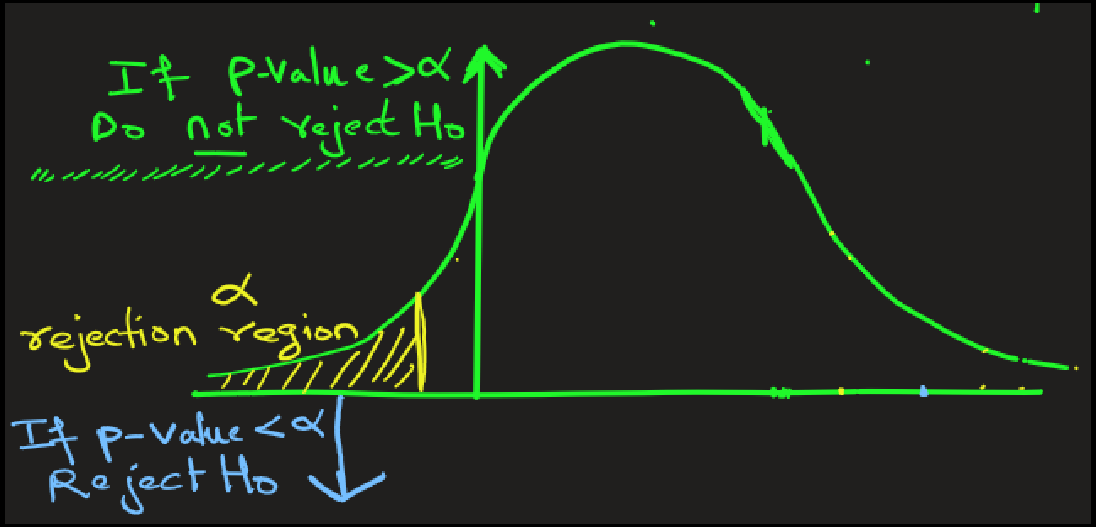
Step 2) Here σ is known, so compute the z test value .

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{4.9-5}{0.8/\sqrt{60}} = -0.968 \approx -0.97$$

Step 3) Find the p-value
Look up the Z value in the Standard Normal table for Z = -0.97

Now P(Z < -0.97) = 0.1660
So, the p-value is 0.1660



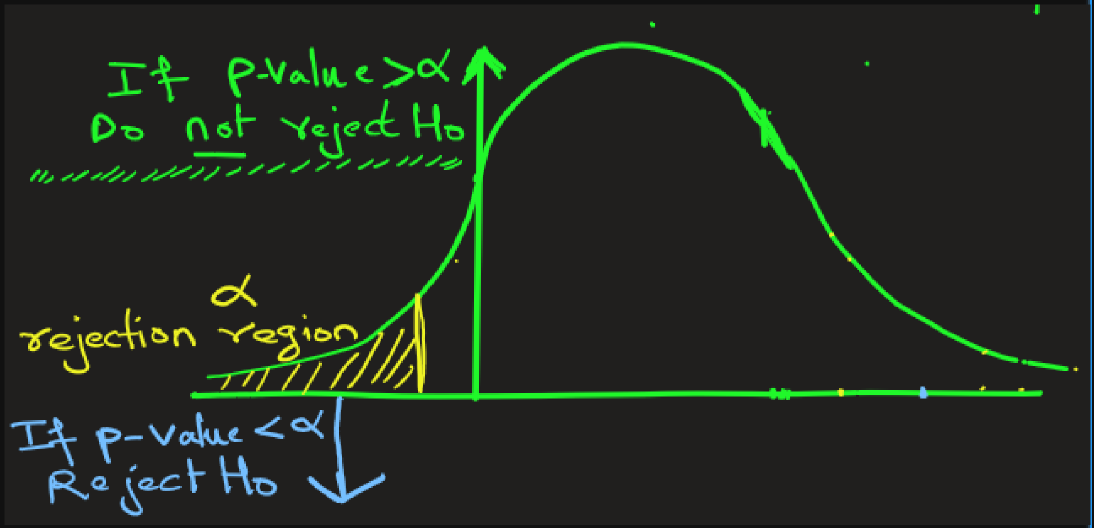
Step 4)
Since the P-value 0.1660 is greater than α value of 0.10, we do **not reject the null hypothesis** at 10% significance level

Step 5)
Although the sample mean (4.9 ppm) is lower than 5 ppm, there is **not enough evidence at the 10% level** to conclude that the new filtration system reduces contaminant level.

Computing p-value programmatically

A water treatment plant claims their new filtration system reduces contaminant levels to below 5 ppm which is the population mean. Environmental engineers want to verify that the system performs better than this standard. A random sample of 60 water samples is taken after treatment. The sample shows a mean contaminant level of 4.9 ppm, with a known population standard deviation of 0.8 ppm. Test the water treatment plant claim at α = 0.10.

[2]:
```
import scipy.stats as stats
import numpy as np

# Given values
mu_0 = 5          # null hypothesis mean
sample_mean = 4.9
sigma = 0.8       # population std dev
n = 60
alpha = 0.10

# Compute Z-statistic
z_stat = (sample_mean - mu_0) / (sigma / np.sqrt(n))

# One-tailed p-value (left tail)
p_value = stats.norm.cdf(z_stat)

# Decision
reject_null = p_value < alpha

# Output
print(f"Z-statistic: {z_stat:.4f}")
print(f"P-value: {p_value:.4f}")
print("Reject Null Hypothesis?" , "Yes" if reject_null else "No")
```




```
Z-statistic: -0.9682
P-value: 0.1665
Reject Null Hypothesis? No
```

P-value: Applications

Problem: A medical researcher is interested in finding out whether a new medication will have any **undesirable side effects** on patients' pulse rates. The researcher is concerned with whether the pulse rate **increases, decreases, or stays the same** after taking the medication.
The researcher knows that the **mean pulse rate μ** for the population under study is **82 beats per minute (bpm)**, with a **population standard deviation σ** of **5 bpm**. To investigate, the researcher selects a **random sample of 60 patients** who take the medication and finds that their **mean pulse rate** is **80.7 bpm**. Test the claim that medication **does affect** the mean pulse rate at significance level **α = 0.01**

Ans:
Step 1) State the hypothesis and identify the claim.

H0: μ = 82 BPM (The medication has **no effect** on mean pulse rate.)
H1: μ != 82 BPM (claim: The medication **does affect** the mean pulse rate (it could be higher or lower). This is 2-tailed test

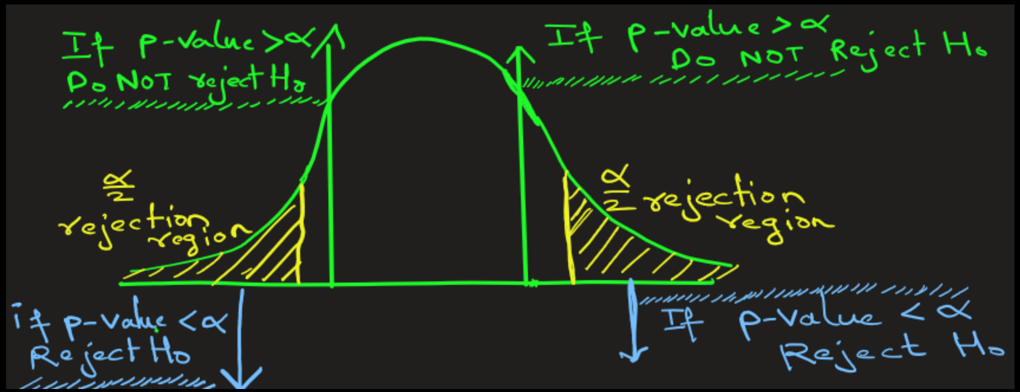
Step 2) Here σ is known, so compute the z test value .

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$Z = \frac{80.7 - 82}{5/\sqrt{60}} = -2.0137 \approx -2.014$$

Step 3) Find the P-value
Look up the Z value in the standard normal table for Z = -2.014

Here, P(|Z| > 2.014) = 2*P(Z > 2.014) because of symmetrical nature of standard distribution curve.
Now, P(Z > 2.014)
= 1 – P(Z < 2.014) (here look up in SN table)
= 1 – 0.9778 = 0.0222
So, the p-value is P(|Z| > 2.013) = 2(0.0222)= 0.0444



Step 4) Since the P-value 0.0444 is greater than α value of 0.01, we **do not reject the null hypothesis at 1% significance level**.

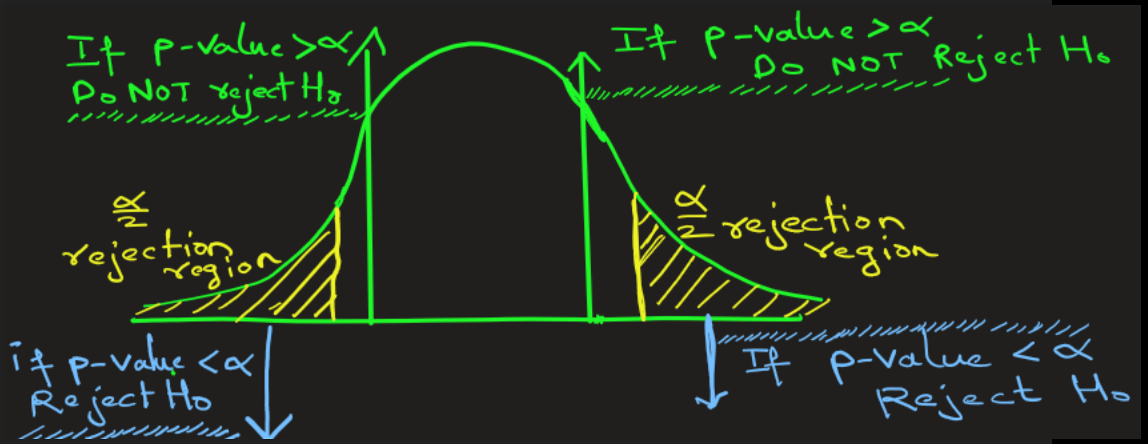
Step 5) Although the sample mean pulse rate after medication (80.7 bpm) is lower than the population mean 82.0 bpm, there is **not enough evidence at the 1% level** to conclude that the medication **has any effect** on mean pulse rate.

Computing p-value programmatically

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects on patients' pulse rates. The researcher is particularly concerned with whether the pulse rate increases, decreases, or stays the same after taking the medication. The researcher knows that the mean pulse rate for the population under study is 82 beats per minute (bpm), with a population standard deviation of 5 bpm. To investigate, the researcher selects a random sample of 60 patients who take the medication and finds that their mean pulse rate is 80.7 bpm. Test the claim that medication does affect the mean pulse rate at $\alpha = 0.01$

[3]:

```python
import scipy.stats as stats
import numpy as np

# Given values
mu_0 = 82             # null hypothesis mean
sample_mean = 80.7
sigma = 5             # population std dev
n = 60
alpha = 0.01

# Compute Z-statistic
z_stat = (sample_mean - mu_0) / (sigma / np.sqrt(n))

# Two-tailed p-value
p_value = 2 * (1 - stats.norm.cdf(abs(z_stat)))

# Decision
reject_null = p_value < alpha

# Output
print(f"Z-statistic: {z_stat:.4f}")
print(f"P-value: {p_value:.4f}")
print("Reject Null Hypothesis?" , "Yes" if reject_null else "No")
```



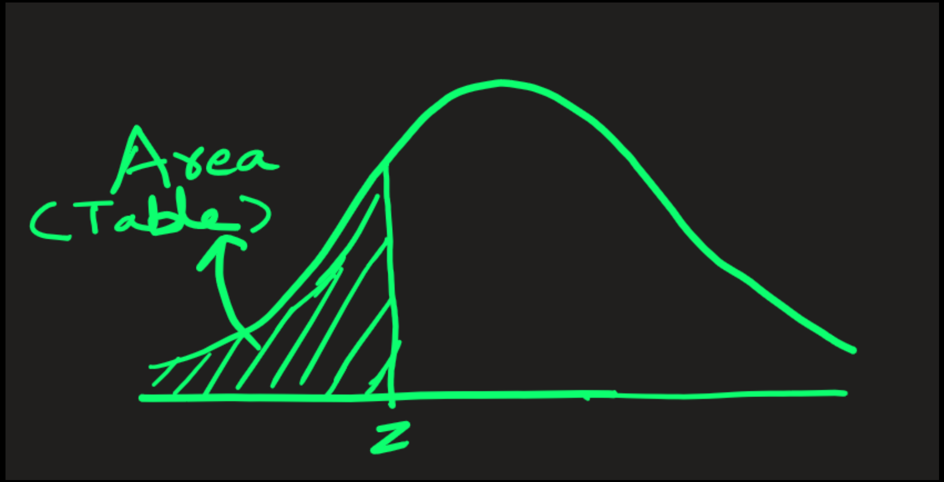

```
Z-statistic: -2.0140
P-value: 0.0440
Reject Null Hypothesis? No
```

EXTRA

Standard Normal Distribution:

Table values represent AREA to the LEFT of the z-score

This table shows area for
z-values between **-3.1 to -0.1**

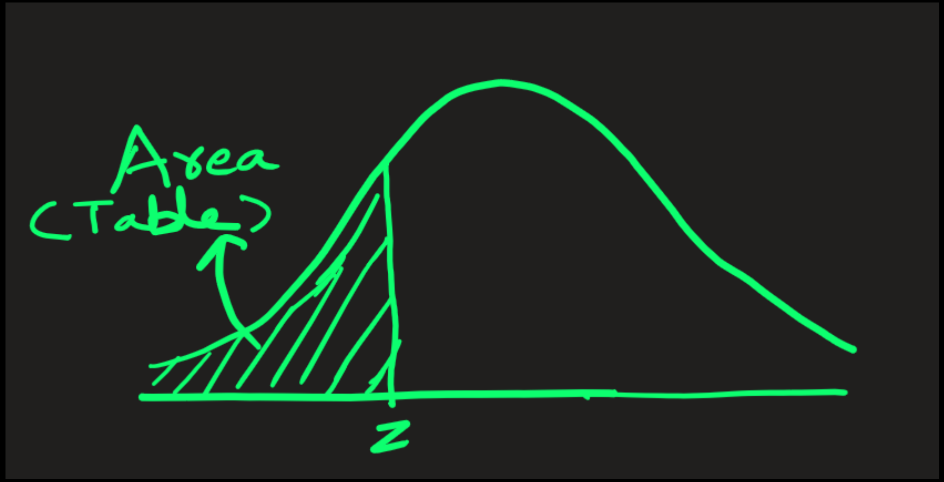


Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0076	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2644	0.2611	0.2579	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4091	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247

Standard Normal Distribution:

Table values represent AREA to the LEFT of the z-score

This table shows area for
z-values between **0.0 to +3.0**



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4841	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

A **statistical hypothesis** is a conjecture about a population parameter (e.g. mean, variance, etc.). This conjecture may or may not be true.

The **null hypothesis (H0):** is a statistical hypothesis that states that **there is no difference between a population parameter and the observed parameter.** That is, nothing is happening. Status quo

The **alternative hypothesis (H1):** is a statistical hypothesis that states that **there is a difference between population parameter and the observed parameter.** That is, something is happening.

Simple Example: Imagine you want to test if a new teaching method improves test scores.

Null hypothesis (H0): The new teaching method has **no effect** on test scores
(Students scores remained same as before. Nothing happened. **No significant** change)

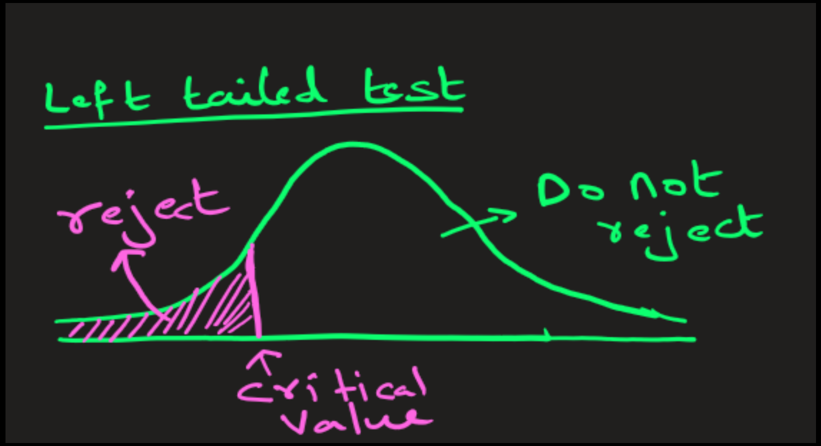
Alternative hypothesis(H1): The new teaching method has **positive effect** test scores.
(Students scores increased. Something happened. There **was significant** change)

Side Note: The null hypothesis isn’t always only about comparing “population mean vs observed mean” — it can also be about proportions, variances, correlations, regression coefficients, etc.

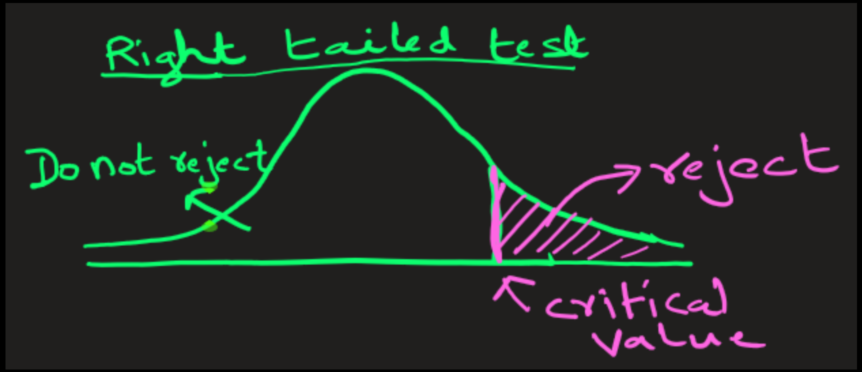
P-value

Hypothesis problems fall under following 3 categories:

Left-tailed test



Right-tailed test



2-tailed test