

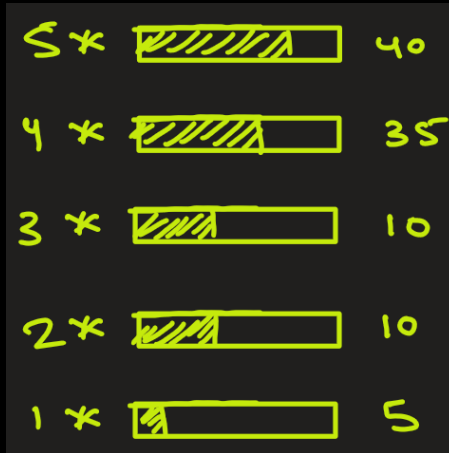


Probability



Problem: Suppose a product has been rated by customers with ratings given below:

5-star → 40 times
4-star → 35 times
3-star → 10 times
2-star → 10 times
1-star → 5 times



If a rating is randomly chosen, what is the probability that it is 4-star or 5-star?



What is probability in plain English ?



Think of probability as a **measure of uncertainty**

Whenever you're unsure what will happen — rolling a die, predicting rain, or getting a job offer — you're thinking in terms of probability.

Probability: Few key terms

Sample space (Ω): All possible outcomes. Let's see few examples.

- Toss 1 coin $\rightarrow \Omega = \{ H, T \}$. Size of Ω is 2



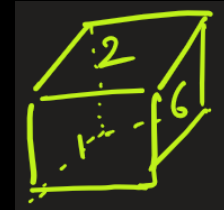
- Toss 2 coins $\rightarrow \Omega = \{ HH, HT, TH, TT \}$. Size of Ω is 4



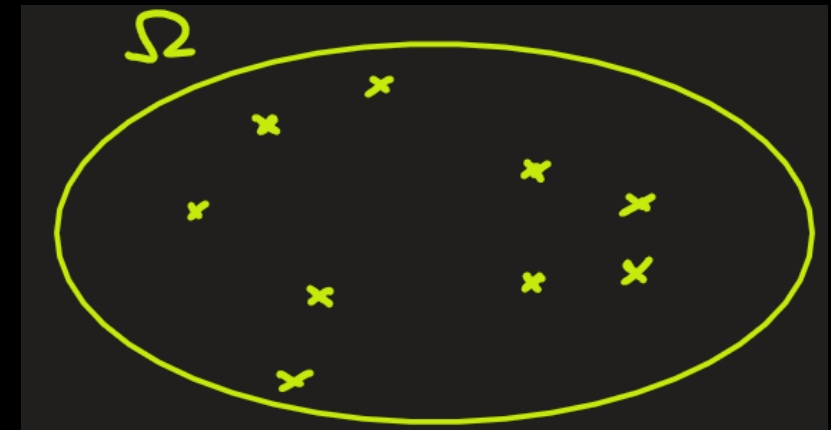
- Toss 3 coins $\rightarrow \Omega = \{ HHH, HHT, HTT, HTH, THH, TTH, THT, TTT \}$. Size of Ω is 8



- Roll a dice $\rightarrow \Omega = \{ 1, 2, 3, 4, 5, 6 \}$ Size of Ω is 6



- In drawing a card $\rightarrow \Omega = 52$ outcomes or elements



Probability: Few key terms

Events (E): It is a subset of the sample space Ω

e.g.1 In a 1-coin toss trial, event could be **getting H** : {H}. Size of E = 1

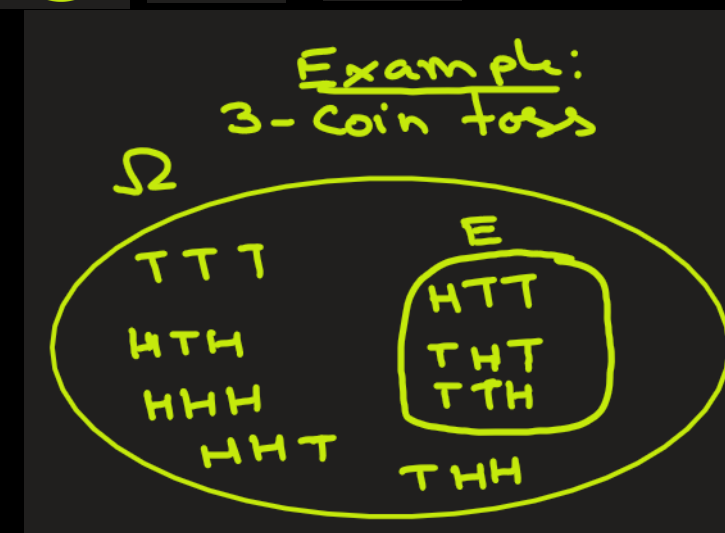
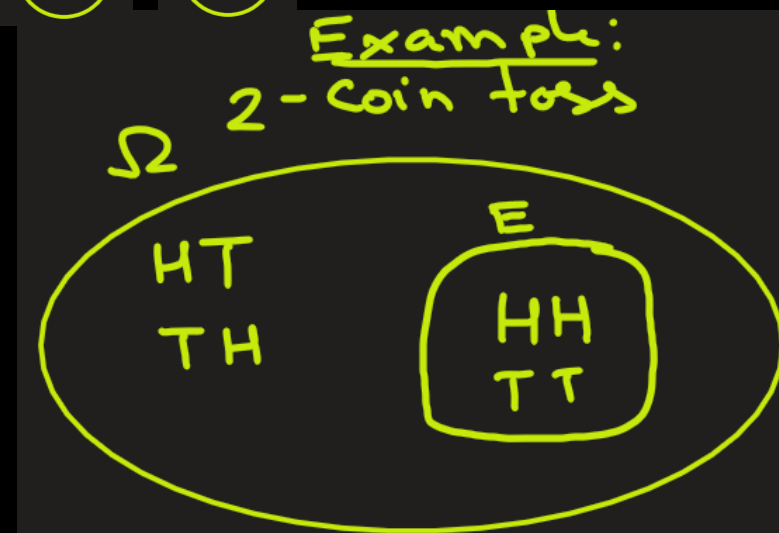
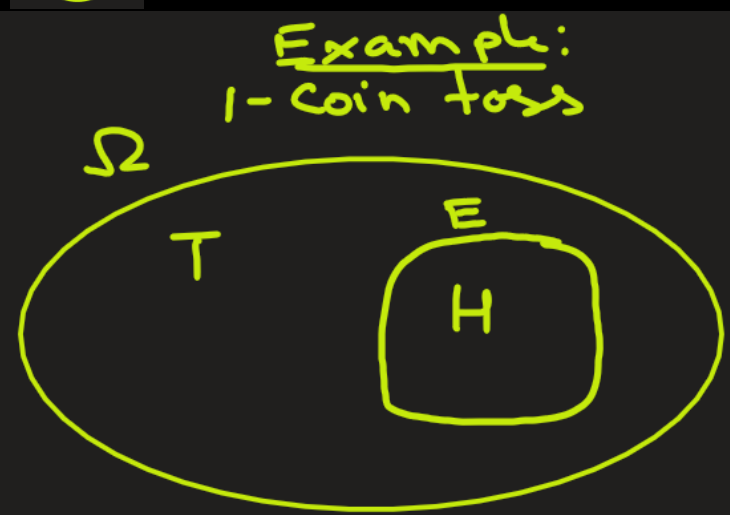
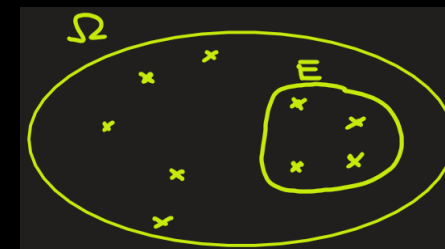
e.g.2 In 2-coin toss trial, event could be **same outcome** in both toss, {HH, TT}. Size of E = 2

e.g.3 In 3-coin toss trial, event could be getting **exactly 1 head** : {HTT, THT, TTH}. Size of E = 3

e.g.4 In dice roll, event could be the **even number** on dice { 2, 4, 6}. Size of E = 3

e.g.5 In drawing a card, event could be that the **card is ace**:

{Ace of heart, Ace of diamond, Ace of clubs, Ace of Spades}. Size of E = 4



Probability: Definition

Probability of an event E :

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Or,

$$P(E) = \frac{|E|}{|\Omega|}$$

Here,

$|E|$ = number (size) of elements in event E ,

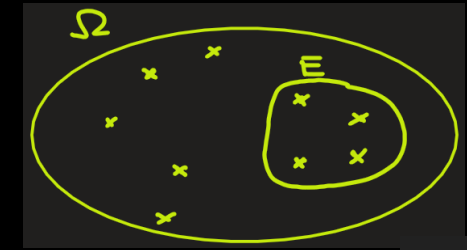
$|\Omega|$ = total number (size) of elements in sample space

Example: In 2-coin toss, what is the probability of getting **both heads**?

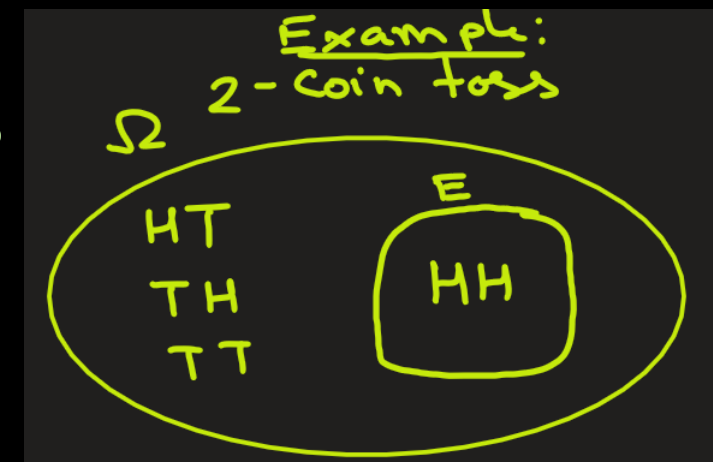
Ans: Here sample space is $\Omega = \{HH, HT, TH, TT\}$. Size = 4

The event E is both toss are heads: $E = \{HH\}$. Size of $E = 1$

$$P(E) = 1/4.$$



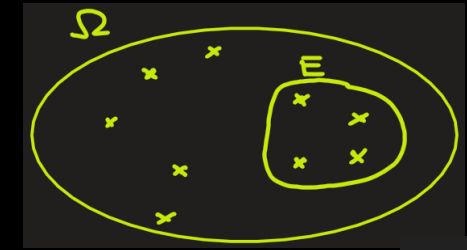
$$P(E) = \frac{|E|}{|\Omega|}$$



Probability: Application

Problem: In 3-coin toss, what is the probability of getting

- a) exactly two heads?
- b) at least one head ?
- c) all tails ?



$$P(E) = \frac{|E|}{|\Omega|}$$

Ans: Here sample space is $\Omega = \{ HHH, HHT, HTT, HTH, THH, TTH, THT, TTT \}$. Size of Ω is $|\Omega| = 8$

a) The event E is exactly 2 heads: $E = \{ HHT, HTH, THH \}$. Size of E is $|E| = 3$

$$P(E) = 3/8.$$

b) The event E is at least 1 head: $\{ HHH, HHT, HTT, HTH, THH, TTH, THT \}$. Size of E is $|E| = 7$

$$P(E) = 7/8.$$

c) The event E is of all tails: $\{ TTT \}$. Size of E is $|E| = 1$

$$P(E) = 1/8.$$

Probability: Application

Problem: Suppose your inbox contains the following 7 emails:

{ spam, spam, non-spam, spam, spam, non-spam, non-spam }

If you randomly open one email from your inbox, what is the probability that the email is a spam message?

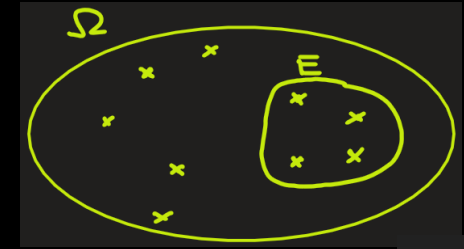
Ans:

Here sample space is $\Omega = \{ \text{spam, spam, non-spam, spam, spam, non-spam, non-spam} \}$.

Size is $|\Omega| = 7$

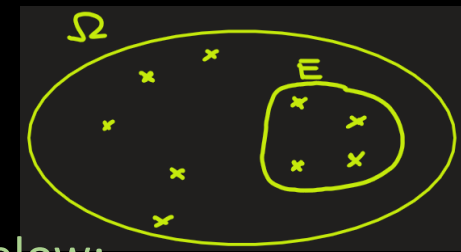
The event E is mail opened is spam: $E = \{ \text{spam, spam, spam, spam} \}$. Size of E is $|E| = 4$

$$P(E) = 4/7$$



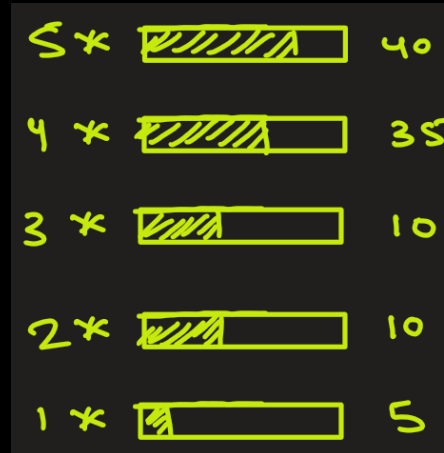
$$P(E) = \frac{|E|}{|\Omega|}$$

Probability: Application



Problem: Suppose a product has been rated by customers with ratings given below:

5-star \rightarrow 40 times
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$$P(E) = \frac{|E|}{|\Omega|}$$

If a rating is randomly chosen, what is the probability that it is 4-star or higher?

Solution:

Here sample space Ω = all ratings. The size of sample space is $|\Omega| = 40 + 35 + 10 + 10 + 5 = 100$

Here event E is randomly chosen rating is 4-star or 5-star. Size of E is $|E| = 35 + 40 = 75$

$$P(E) = 75/100 = 0.75$$