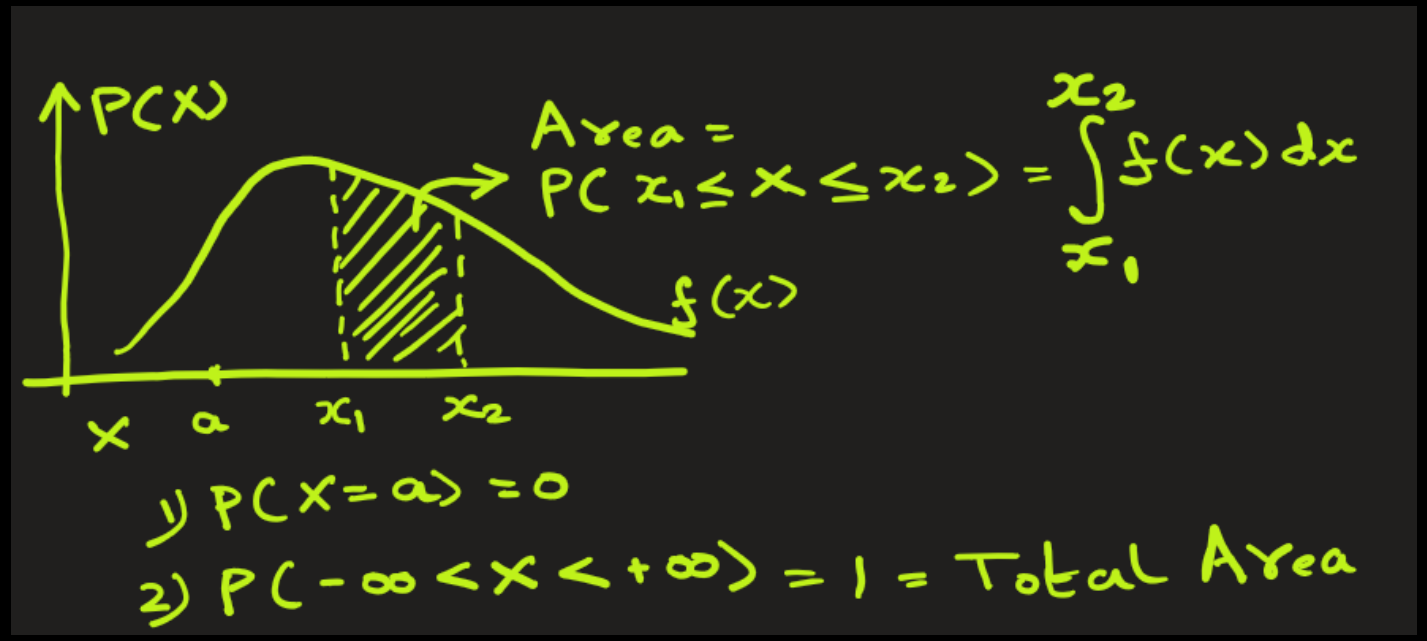
2 Types of Probability Distribution Functions: Discrete and Continuous

2. Continuous Probability Distribution

It applies when outcomes are uncountable, i.e., can take **any** value in a range. For examples height of people, weight of boxes, temperature, time to finish a task, etc. : Temperature could be 34, 32.3, -5.2, etc.

To calculate PDF for continuous variables, we calculate probability over an **interval**. The area under the curve represents probability for that interval.
Unlike discrete case, the probability of a single value is technically 0.



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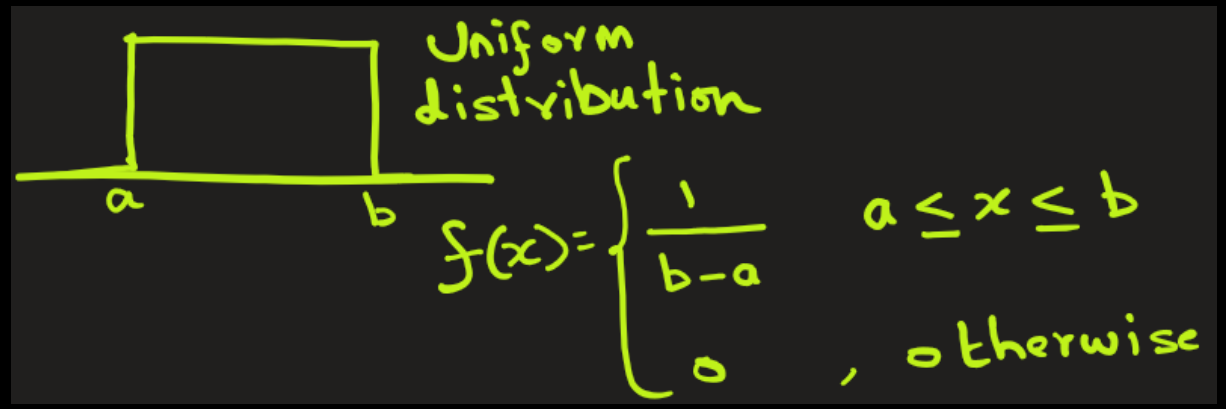
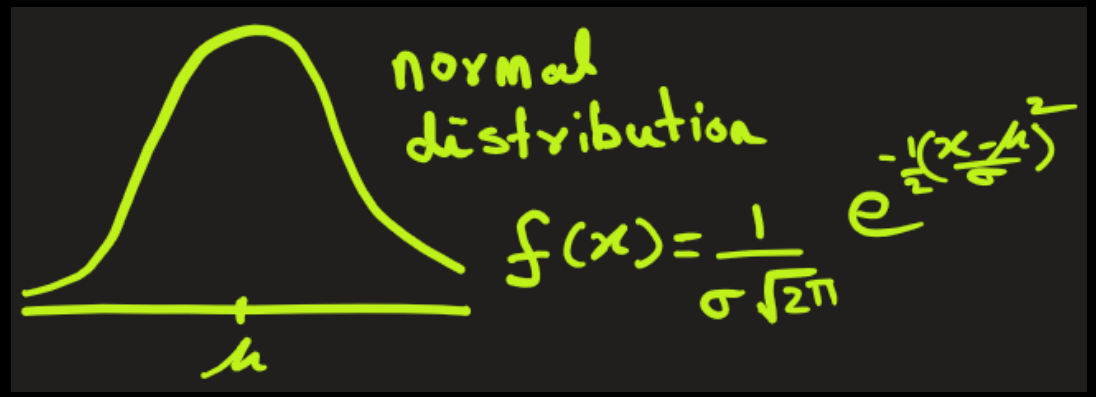
Example of continuous PDF:



Here we find area of rectangle using length X height



Here we find area under curve using z-table

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Example: Imagine a bus that runs every 10 minutes, and you arrive randomly at the bus stop. Your waiting time X follows a Uniform(0, 10) distribution, i.e. you could wait anywhere between 0 and 10 minutes with equal probability. After arriving at bus stop, what is the probability that you wait

a) Less than 4 minutes
b) More than 7 minutes
c) Between 3 and 8 minutes
d) Between 3 and 12 minutes
e) Between 11 and 12 minutes
f) Exactly 2 minutes

Ans:

a) P(0 < X < 4) = Area between 0 and 4 = 0.1 x 4 = 0.4
b) P(7 < X < 10) = Area between 7 and 10 = 0.1 x 3 = 0.3
c) P(3 < X < 8) = Area between 3 and 8 = 0.1 x 5 = 0.5
d) P(3 < X < 12) = Area between 3 and 12 = 0.1 x 7 = 0.7
c) P(11 < X < 12) = Area between 11 and 12 = 0 x 1 = 0
d) P(X = 2) = Area between 2 and 2 = 0.1 x 0 = 0

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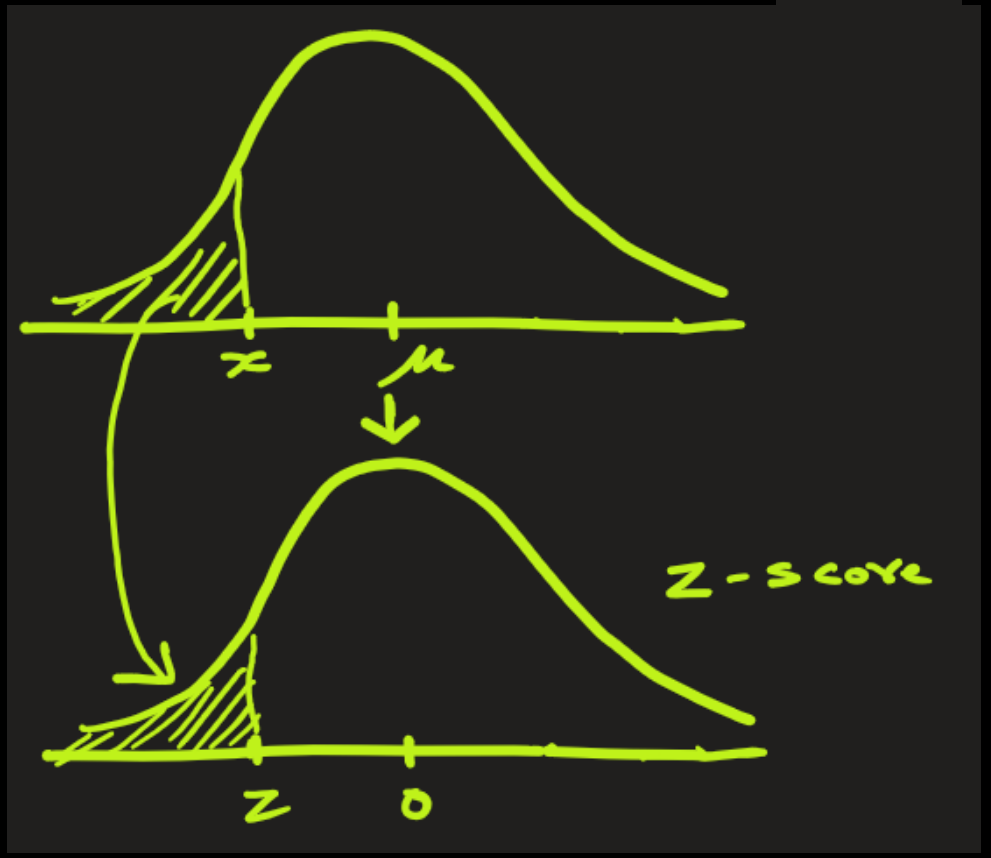
Example: Adult male heights, X, in a city are approximately **normally distributed** with mean $\mu = 170\ cm$ and standard deviation $\sigma = 10\ cm$. If you randomly pick an adult, then what is the probability that his

a) Height is less than 155 cm;
b) Height is greater than 190 cm;
c) Height is between 160 cm and 180 cm
d) Height is exactly 190 cm;



Ans: It's given that height is normally distributed. So, to answer above we convert above values into z-score and then we calculate their probability using z-table.

a) X = 155 cm -> z-score = (X - μ) / σ = (155 - 170) / 10 = -1.5

P(X < 155)
= P (z < -1.5) (this is area under the curve and now use z-table)
= 0.0668 = 6.68%

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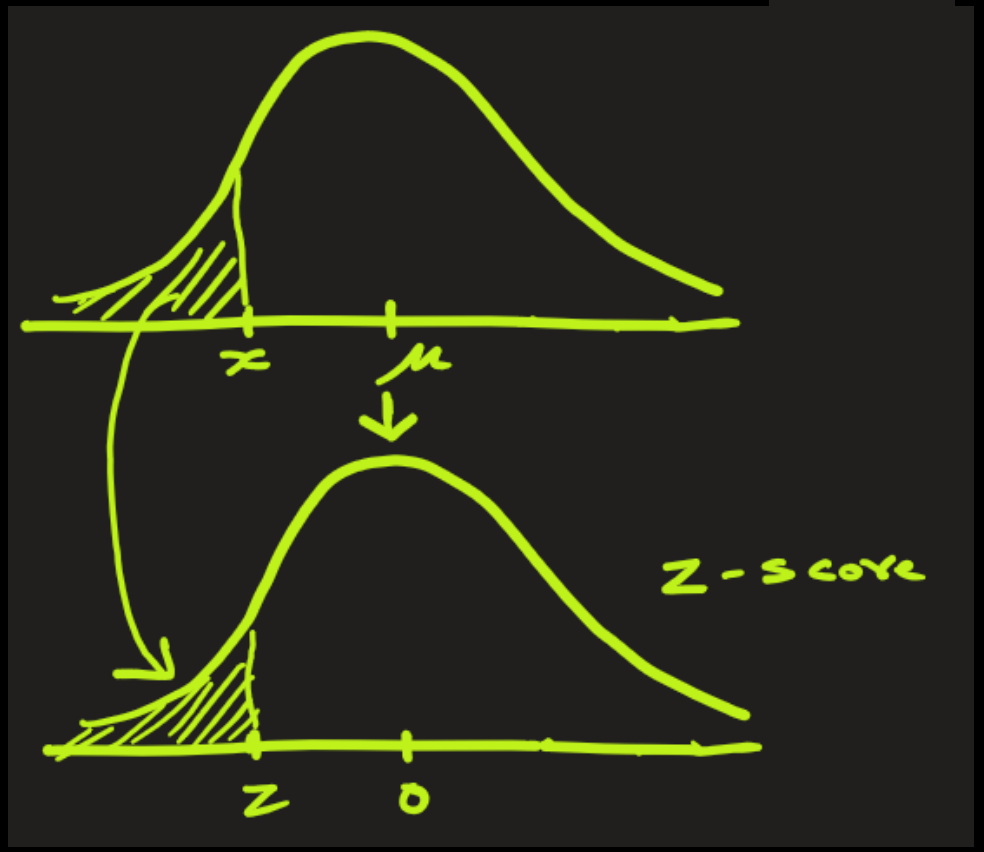
b) X = 190 cm -> z-score = (X - μ) / σ = (190 - 170) / 10 = 2.0

P(X > 190)
= P (z > 2.0) (this is area under the curve and now use z-table)
= 1 – 0.977 = 0.023 = 2.3%



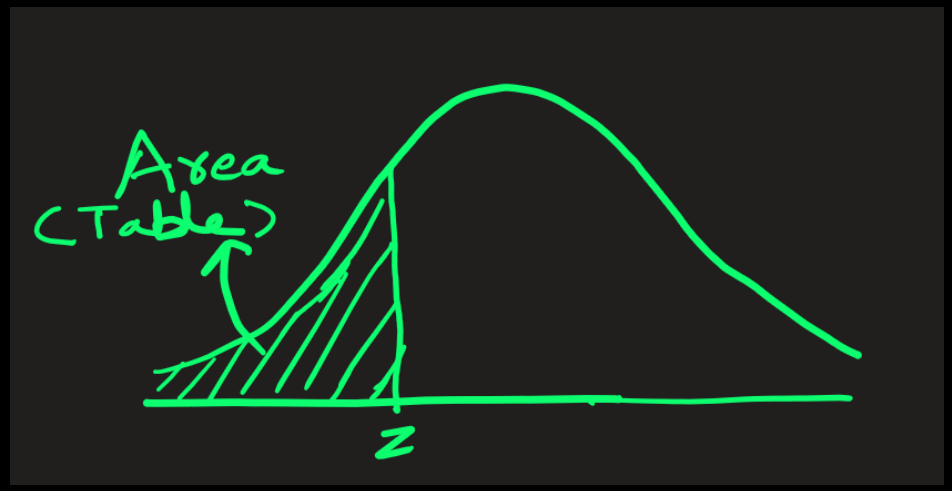
c) X = 160 cm -> z-score = (X - μ) / σ = (160 - 170) / 10 = - 1.0
X = 180 cm -> z-score = (X - μ) / σ = (180 - 170) / 10 = +1.0

P (160 < X < 180)
= P (-1.0 < z < +1.0) (this is area under the curve and now use z-table)
= 0.6827 = 68.27%

d) X = 190 cm.
We do not need to convert this into z-score because
P(X = 190 cm) = 0

Standard Normal Distribution: Table values represent AREA to the LEFT of the z-score

This table shows z-values from **-3.1 to -0.1**



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0076	0.0073	0.0071	0.0070	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2644	0.2611	0.2579	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4091	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247

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