# Law Of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

**Problem**: Suppose we have following 3 bags containing red and yellow balls

Bag 1: 2 red balls and 3 yellow balls

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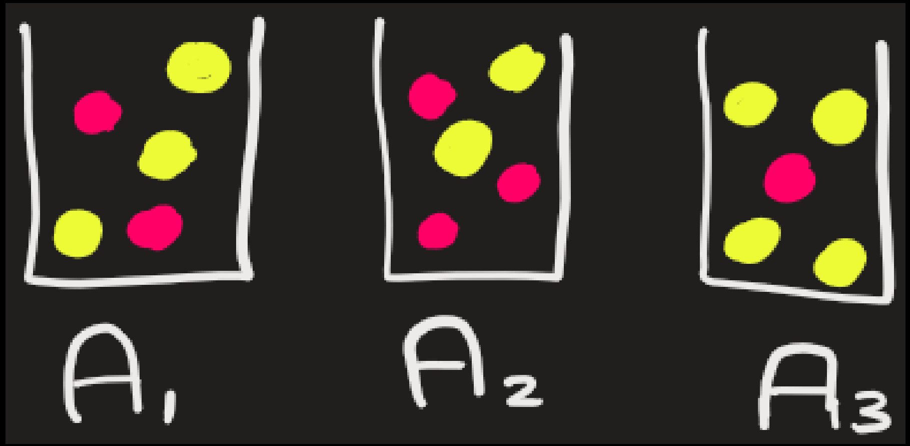
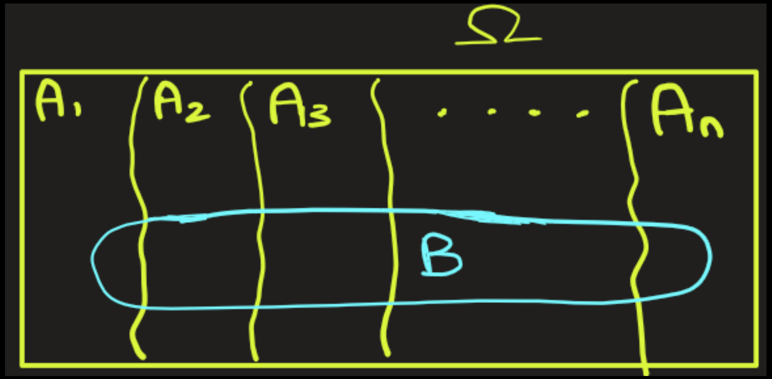



If a ball is randomly selected from a random bag, what is the probability that the ball is red ?

# Law Of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

Suppose we have set of events $A_1, A_2, \ldots, A_n$ that forms a **partition** of the sample space (i.e., they are mutually exclusive and cover all possibilities) and $B$ is any event in the sample space.




Then,

$$B \;=\; (A_1 \cap B) \;\cup\; (A_2 \cap B) \;\cup\; (A_3 \cap B) \;\cup \ldots \cup\; (A_n \cap B)$$

$$\Rightarrow \mathrm{P}(B) = \mathrm{P}(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) + \ldots + P(A_n \cap B)$$ (now apply conditional probability)

$$\Rightarrow P(B) = P(B|A_1) \times P(A_1) + P(B|A_2) \times P(A_2) + P(B|A_3) \times P(A_3) + \ldots + P(B|A_n) \times P(A_n)$$

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

**Why do we need Law of Total Probability ?**

Answer: Because sometimes we are given conditional probability, P(B|A), and P(A), and we have to find P(B).

# Law Of Total Probability

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Problem:

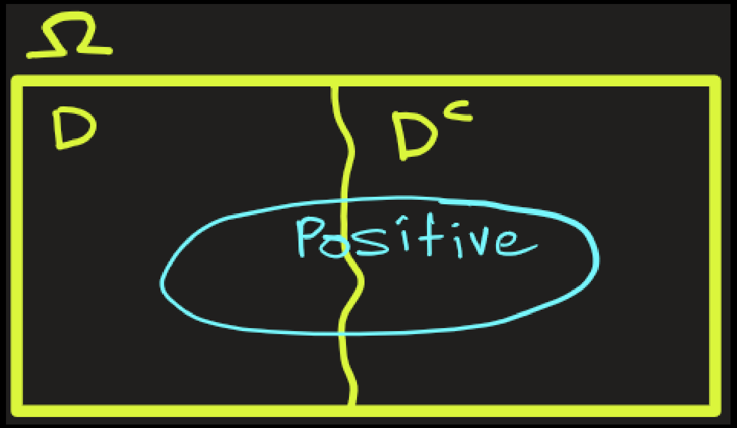
Suppose:

1% of people have a certain disease → $P(D) = 0.01$

A test detects the disease correctly 99% of the time → $P(Positive \mid D) = 0.99$

The test gives a false positive 2% of the time → $P(Positive \mid D^C) = 0.02$

What is the probability that a given test result is positive.



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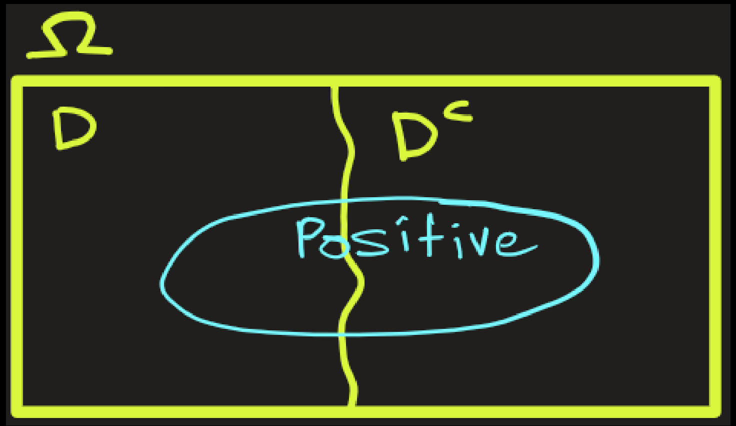


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What is the probability that a given test result is positive.

Ans: Here
D is an event that person has disease
$D^C$ is the **complementary** event that person does not have the disease

So, D and $D^C$ **partition** the sample space.
We can apply the Law of Total Probability

P(Positive) = P(Positive ∩ D) + P(Positive ∩ $D^C$)

= P(Positive | D) x P(D) + P(Positive | $D^C$) x P($D^C$)

= 0.99 x 0.01 + 0.02 x 0.99
= 0.0297

# Law Of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

Problem: Let

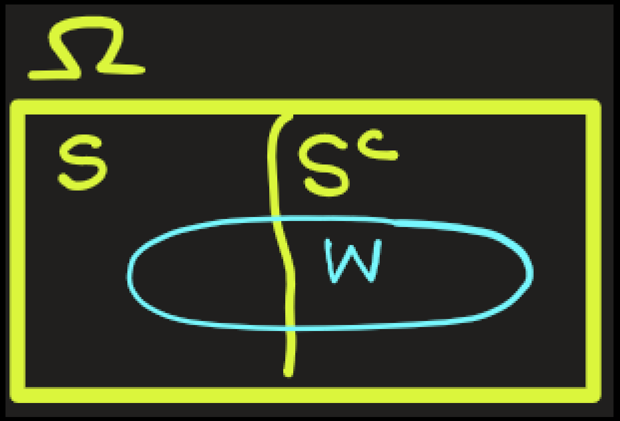
S be the event that email is spam and W be the event that email contains word “win” in it.

$P(S) = 0.3$ → 30% of emails are spam

$P(W \mid S) = 0.8$ → 80% of spam emails contain the word “win”

$P(W \mid S^C) = 0.1$ →10% of non-spam emails contain the word “win”

Find $P(W)$ :Probability that an email contains “win”.



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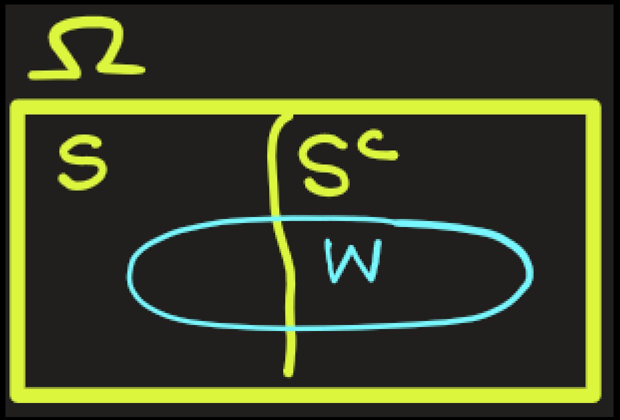



Ans:

Here S and $S^C$ **partition** the sample space.

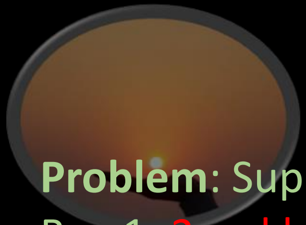
So, we can apply the Law of Total Probability

P(W) = P(W ∩ S) + P(W ∩ $S^C$)

= P(W | S) x P(S) + P(W | $S^C$) x P($S^C$)

= 0.8 x 0.3 + 0.1 x 0.7

= 0.31



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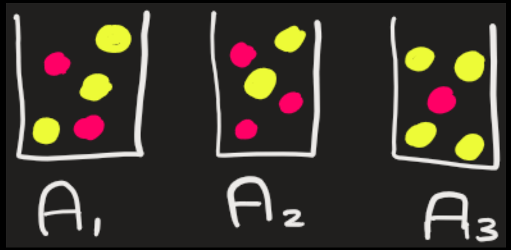

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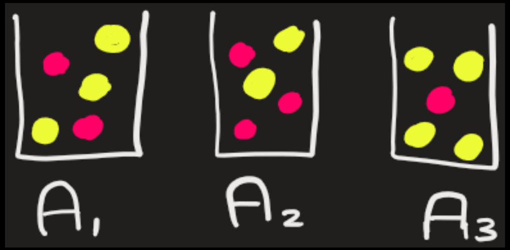



Ans: Let

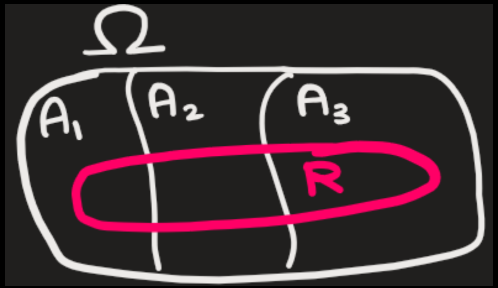
$A_1$ : Event that ball is selected from bag 1

$A_2$ : Event that ball is selected from bag 2

$A_3$ : Event that ball is selected from bag 3

R : Event that the ball selected is red

Since the red ball has to come from one these bags, so $A_1, A_2, A_3$ forms a **partition** of the sample space.




$$\mathrm{P}(R) = \mathrm{P}(A_1 \cap R) + P(A_2 \cap R) + P(A_3 \cap R)$$ ,now apply conditional probability

$$P(R) = P(R|A_1) \times P(A_1) + P(R|A_2) \times P(A_2) + P(R|A_3) \times P(A_3)$$

$$P(R) = \frac{2}{5} \times \frac{1}{3} + \frac{3}{5} \times \frac{1}{3} + \frac{1}{5} \times \frac{1}{3}$$

$$P(R) = \frac{6}{15}$$

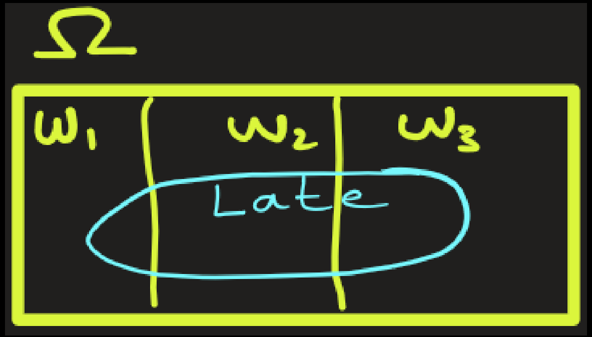
# Law Of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

Problem: A person can commute to work in three different ways, depending on the weather:

$W_1$ : **Sunny day** $W_2$ : **Rainy day** $W_3$ : **Snowy day**

Assume exactly one type of weather must occur each day.




| **Event** | **Probability** | **Probability of being late given the event** |
|---|---|---|
| $W_1$: Sunny | P($W_1$) = 0.6 | P(Late\|$W_1$) = 0.1 |
| $W_2$ : Rainy | P($W_2$) = 0.3 | P(Late\|$W_2$) = 0.4 |
| $W_3$: Snowy | P($W_3$) = 0.1 | P(Late\|$W_3$) = 0.7 |

Find the **total probability of being late** $P(Late)$

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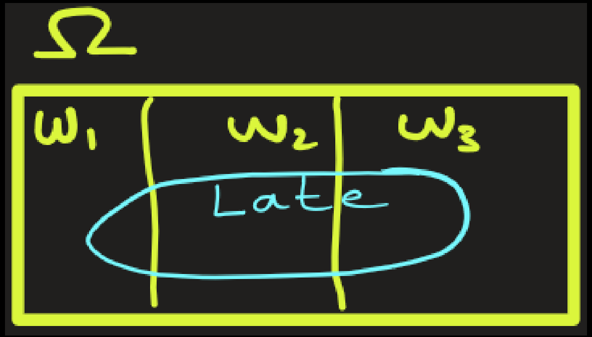


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| $W_3$: Snowy | P($W_3$) = 0.1 | P(Late\|$W_3$) = 0.7 |

Find the **total probability of being late** $P(Late)$

Ans: These three events $W_1, W_2, W_3$ form a **partition** of the sample space because exactly one type of weather must occur each day. So, we can apply the Law of Total Probability

$$P(\text{Late}) = P(\text{Late}|W_1)P(W_1) + P(\text{Late}|W_2)P(W_2) + P(\text{Late}|W_3)P(W_3)$$

Substitute values:

$$P(\text{Late}) = (0.1)(0.6) + (0.4)(0.3) + (0.7)(0.1)$$

$$P(\text{Late}) = 0.06 + 0.12 + 0.07 = 0.25$$

There’s a **25% chance** the person will be late overall, considering all weather conditions.

# Law Of Total Probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

# Law Of Total Probability

Jlkdfds

**Flsdf\p**

# Extra

Jlkdfds

**Flsdf\p**

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \times P(A_i)$$

$$P(B) = \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)$$