

Sample Size



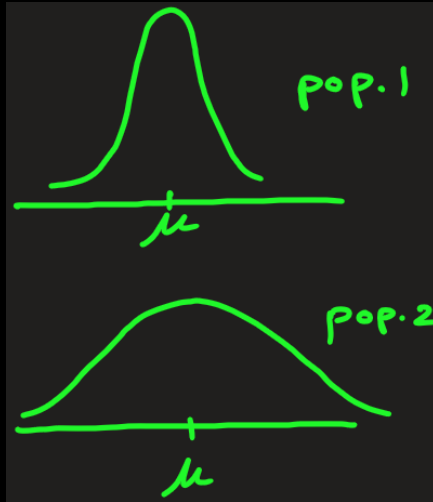
Problem: I have 2 sets of population with 15 data points in each:

Population1 = [3, 4, 2, 5, 2, 6, 3, 4, 3, 3, 2, 1, 3, 2, 2]

Population1 has standard deviation of 1.33.

Population2 = [-4.35, -4.35, -4.35, -4.35, -4.35, 10.35, 10.35, 10.35, 10.35, 10.35, 3.00, 3.00, 3.00, 3.00, 3.00]

Population2 has standard deviation of 6.



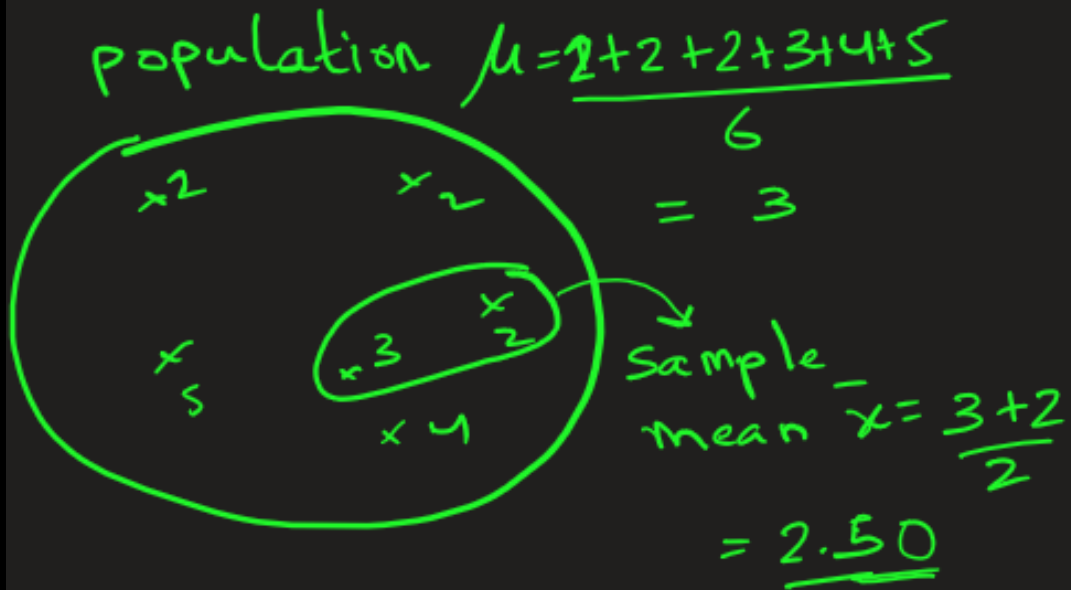
Both population have same mean. We have to determine that mean value.

If I am allowed to take only 10 sample points to determine mean value, which population should I use to get better estimate of the mean?

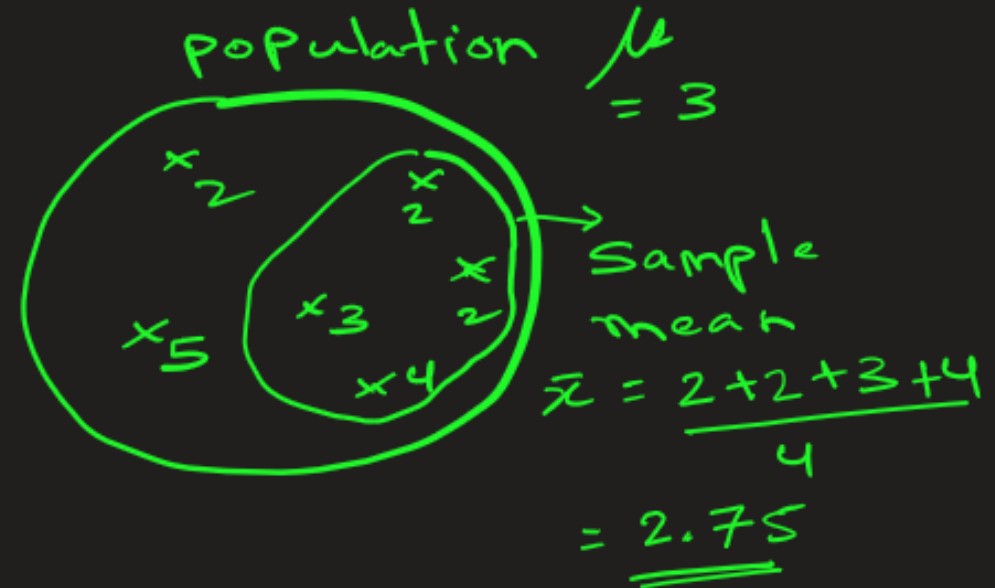
Factors that Effect the Accuracy of Estimate

Small sample size gives more uncertainty i.e. less confident
+ less accurate

Large sample size gives less uncertainty, i.e. more confident
+ more accurate



Small sample
(\bar{x} is less closer to μ)

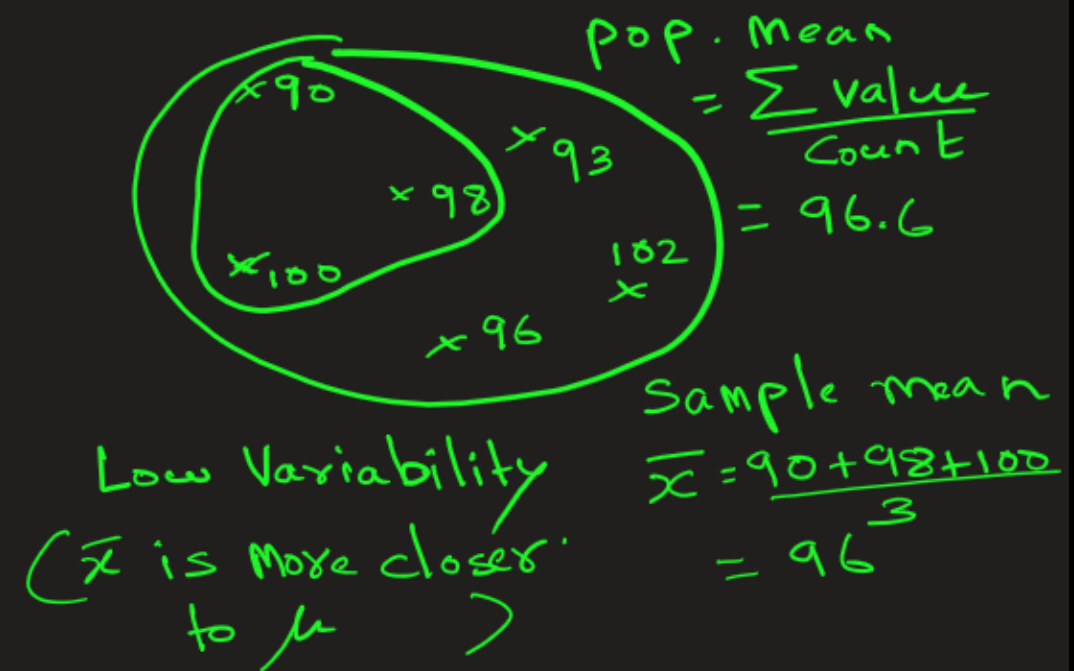
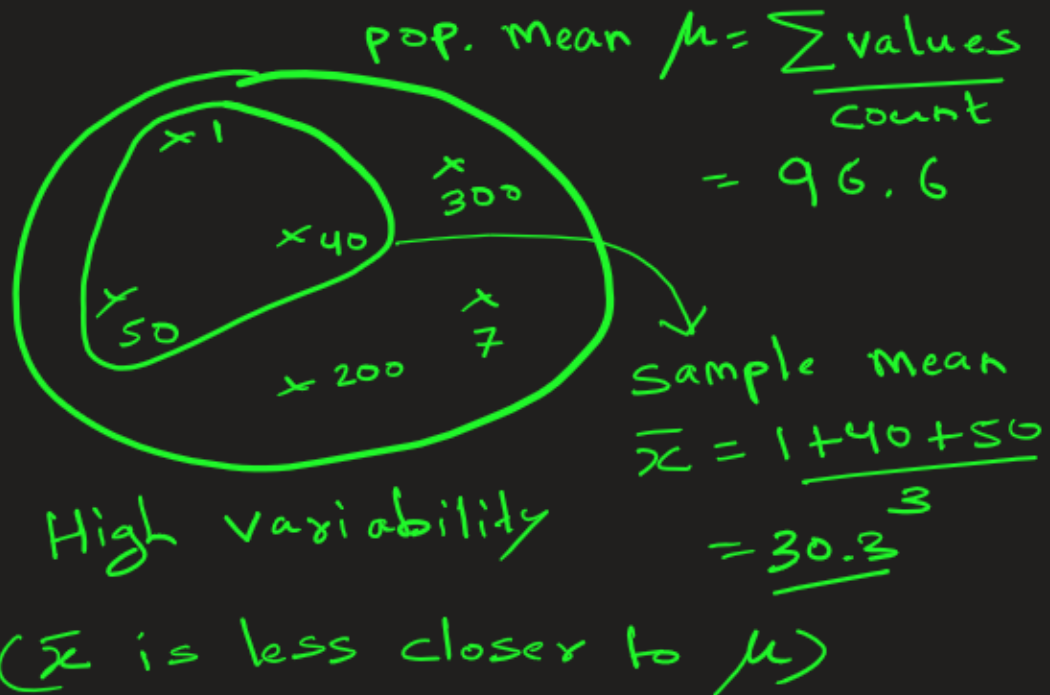


Large Sample
(\bar{x} is more closer to μ)

Factors that Effect the Accuracy of Estimate

High variability (i.e. high standard deviation σ) in the population gives more uncertainty i.e. less confident + less accurate

Low variability (i.e. low standard deviation σ) in the population gives less uncertainty, i.e. more confident + more accurate





How large a sample is necessary to make an accurate estimate ?



We can derive the formula for sample size n from margin of error E , population standard deviation σ and critical z -value as follows:

$$\begin{aligned} E &= Z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) \\ E\sqrt{n} &= Z_{\alpha/2} \cdot \sigma \\ n &= \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2 \end{aligned}$$

where,

E = desired **margin of error** (the maximum acceptable difference between sample mean and true mean)

$Z_{\alpha/2}$ = z -score corresponding to the confidence level (e.g. 1.645 for 90%, 1.96 for 95%, 2.58 for 99%)

The answer to above question depends on three factors:

- Margin of error (E): How much error are you willing to accept ? If you want less E , choose bigger sample size.
- Population standard deviation (σ): How much variation in the data exist ? If σ is high (high variability), then more samples are needed
- Degree of confidence (critical z -value) : What percentage of confidence level you want ? If you want high confidence, $Z_{\alpha/2}$, you need more samples.

How large a sample is necessary to make an accurate estimate ?

Problem: An aerospace engineer is estimating the average fuel consumption rate of an aircraft engine during cruise conditions. For flight planning and performance certification, the engineer needs 99% confidence that the estimate is within 2 gallons per hour of the true mean consumption rate. Flight test data shows a standard deviation of 4.3 gallons per hour. How many minimum samples he needs?

Ans:

Given: $z_{\alpha/2} = 2.58$ (for 99% confidence level) ,

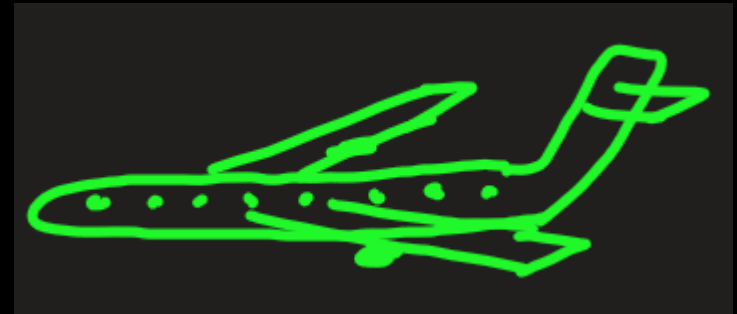
$\sigma = 4.3$,

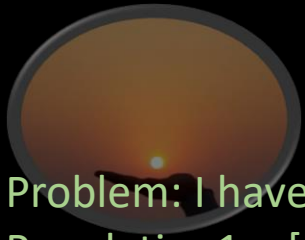
$E = 2$

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

$$n = \left(\frac{2.58 \times 4.3}{2} \right)^2 = 32$$

To be 99% confident that the estimate is within 2 units of the true mean, the engineer needs a sample of **at least 32 measurements.**





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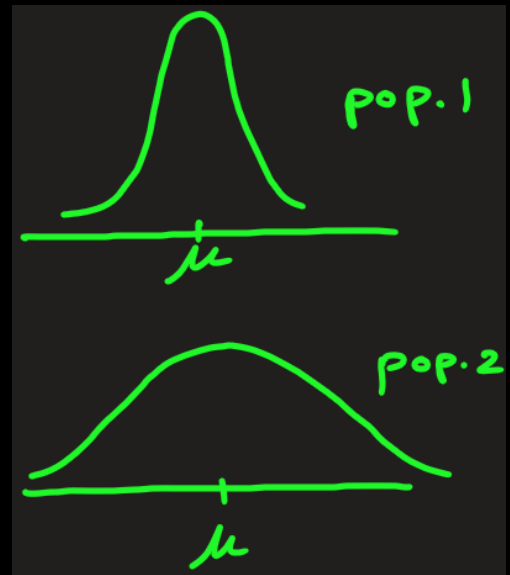
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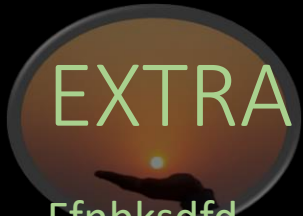
Ans: From the relation,

$$E = Z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right)$$

we see that for E, margin of error, to be minimum we should select population with low standard deviation σ .
Select population1.





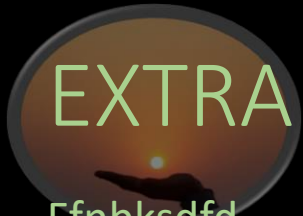


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