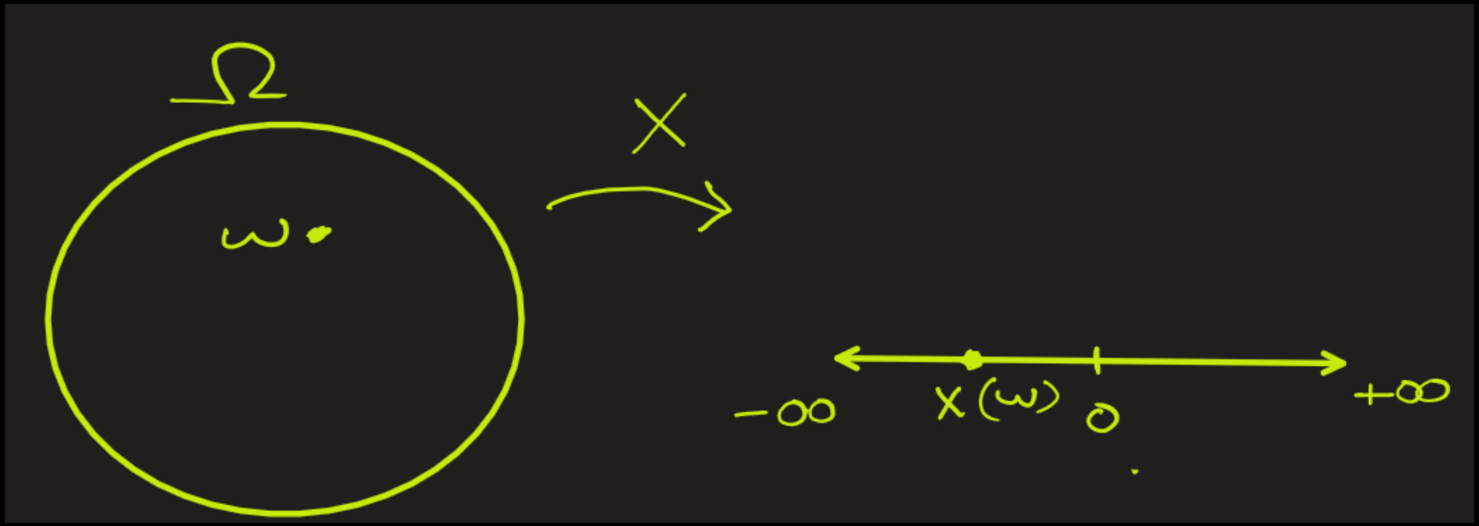
Random Variables: Formal Definition

A **random variable (RV)** is a function that assigns a **numerical value** to each possible outcome of a **random experiment**.

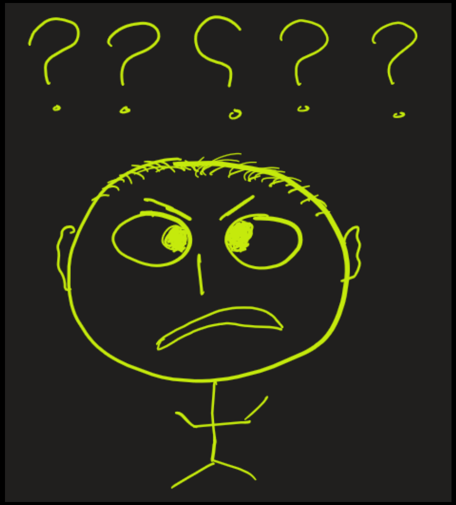
$$X : \Omega \to \mathbb{R}$$



Here,

Ω is the **sample space** (all possible outcomes),

X(ω) is the **numerical value** assigned to an outcome ω.



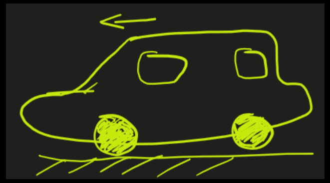
Random Variables: Plain English

Random variable is a **numeric variable** that represents the **outcome of a random experiment** — its value is not fixed but depends on chance.
These variable values may change randomly from one experiment to another.

They are often represented by a capital letter like **X**

For example,

- You roll a die, the outcome X could be 1, 2, 3, 4, 5, or 6
- You count number of cars passing through a toll in 1 minute, the outcome X could be 0, 1,2,3, …
- Measure the temperature of a room: Here outcome X could 10.3, 4, 5.34, etc.



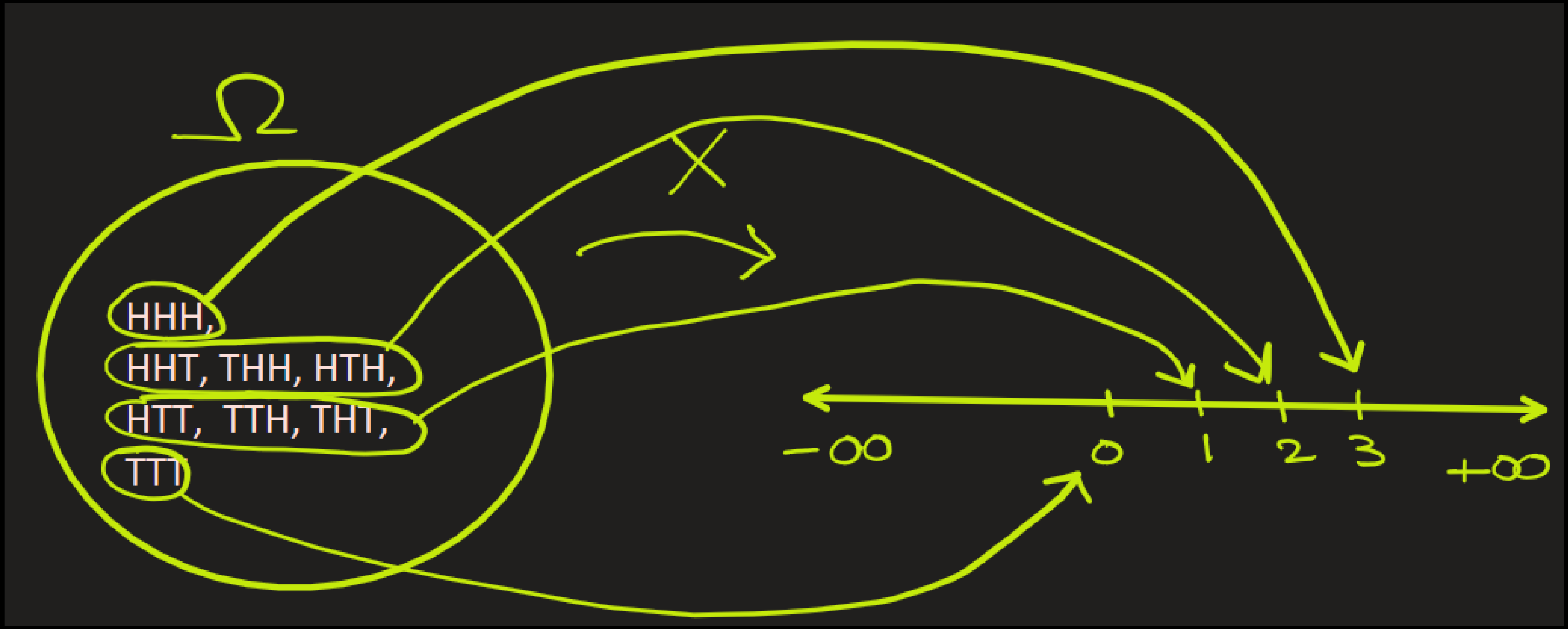
2 types of random variables

- Discrete RV
- Continuous RV

Discrete Random Variables

Definition:
A variable that can only take on a **finite or countably infinite number of specific values**.

Example of finite values: The number of heads when you flip a coin three times.
Here outcomes are **Ω =** { HHH, HHT, HTT, HTH, THH, TTH, THT, TTT } .
Number of heads can be represented as X whose possible values are 0, 1, 2, or 3.
So, X = { 0, 1, 2, 3 }

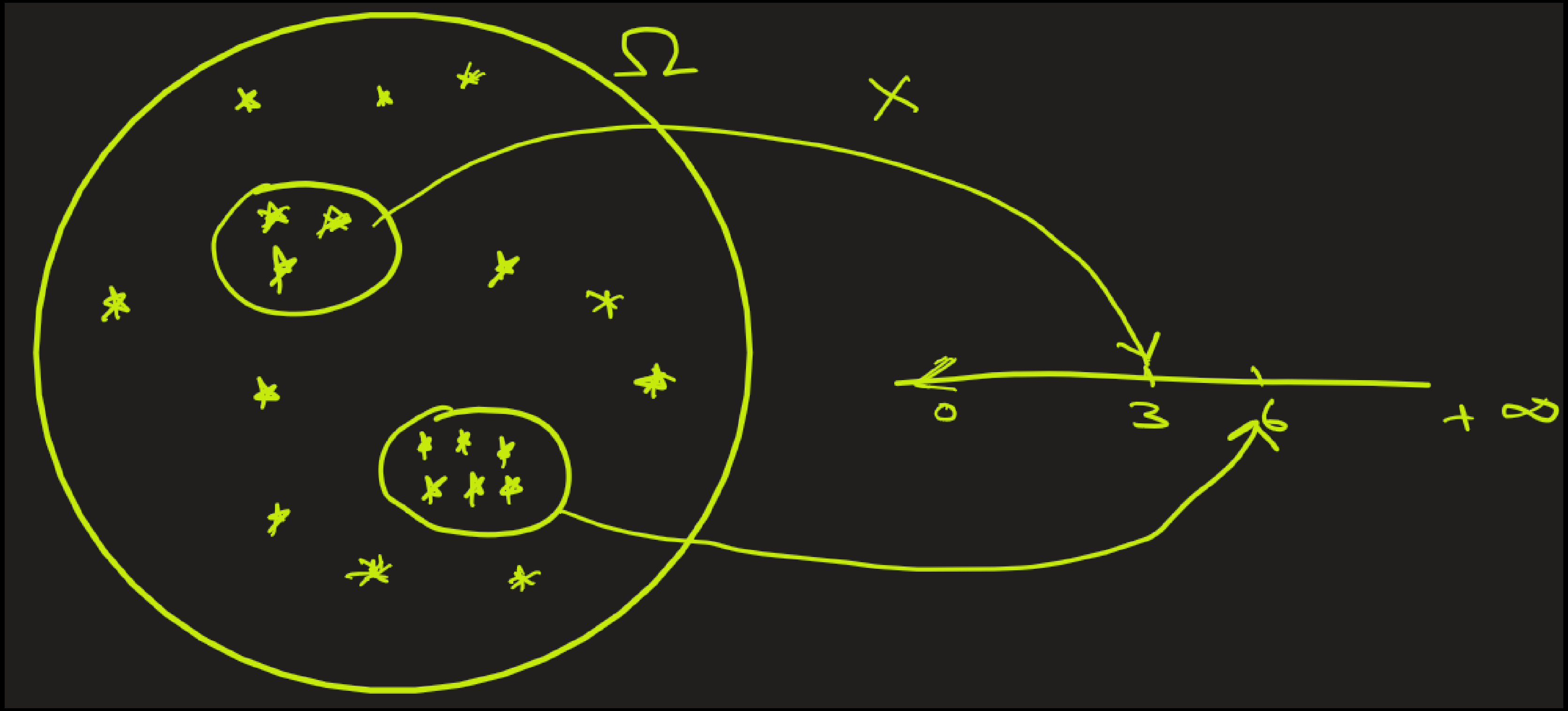


Discrete Random Variables

Example of infinite discrete values: Counting stars in a region of sky
Here outcomes are possible numbers of stars in a region, **Ω =** { 0, 1, 2, 3, … } .
X can be represented as number of stars in a selected region. Its values can 0, 1, 2, 3, ….

So, X = { 0, 1, 2, 3, ….. }

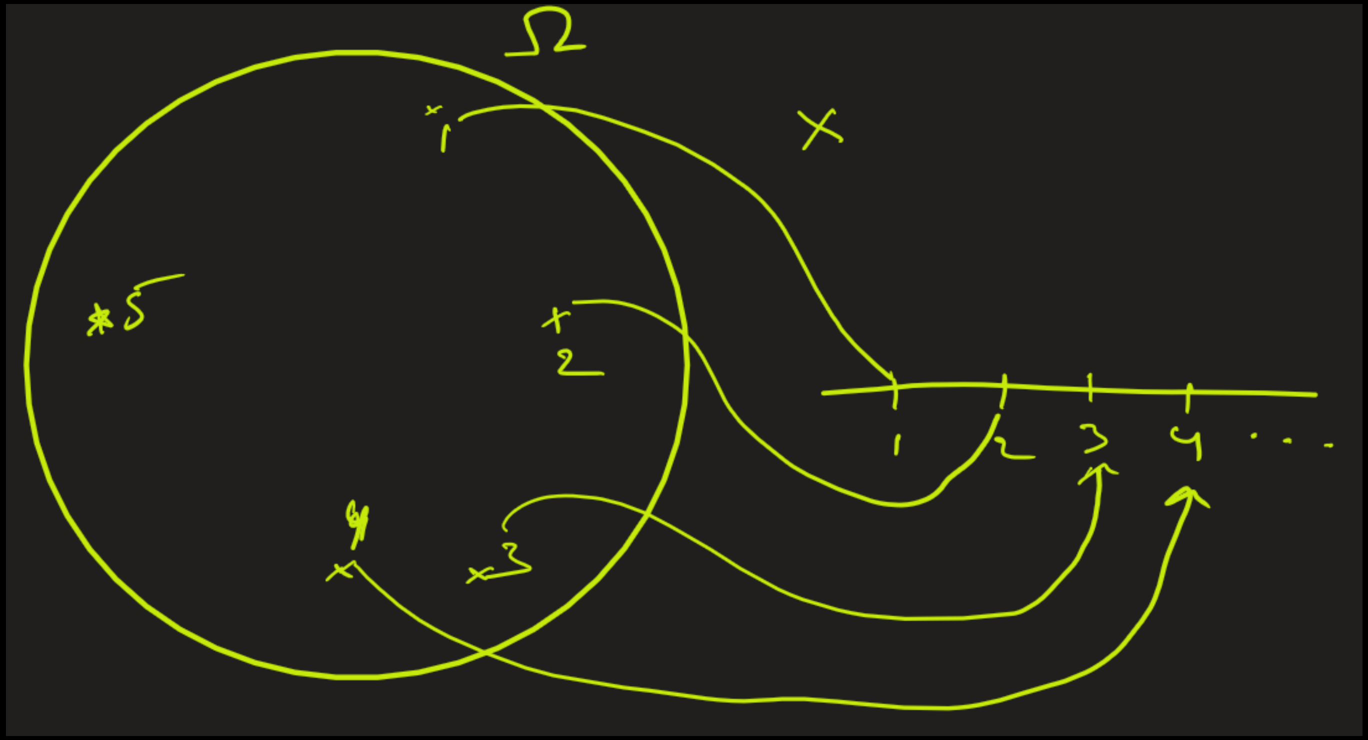
Here X = **Ω**



Discrete Random Variables

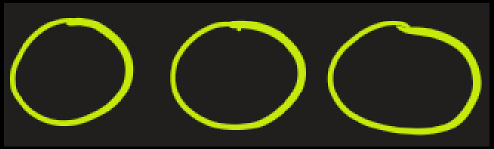
Example of infinite discrete values: Number of coin tosses until the first head appears

Each outcome represents the **trial number** on which the **first head** occurs:
So, the sample space Ω = {1, 2, 3, 4, 5, …}
Let **X** = "the number of tosses needed to get the first head."
Then X can take any positive integer value: X = {1, 2, 3, 4, 5, …}

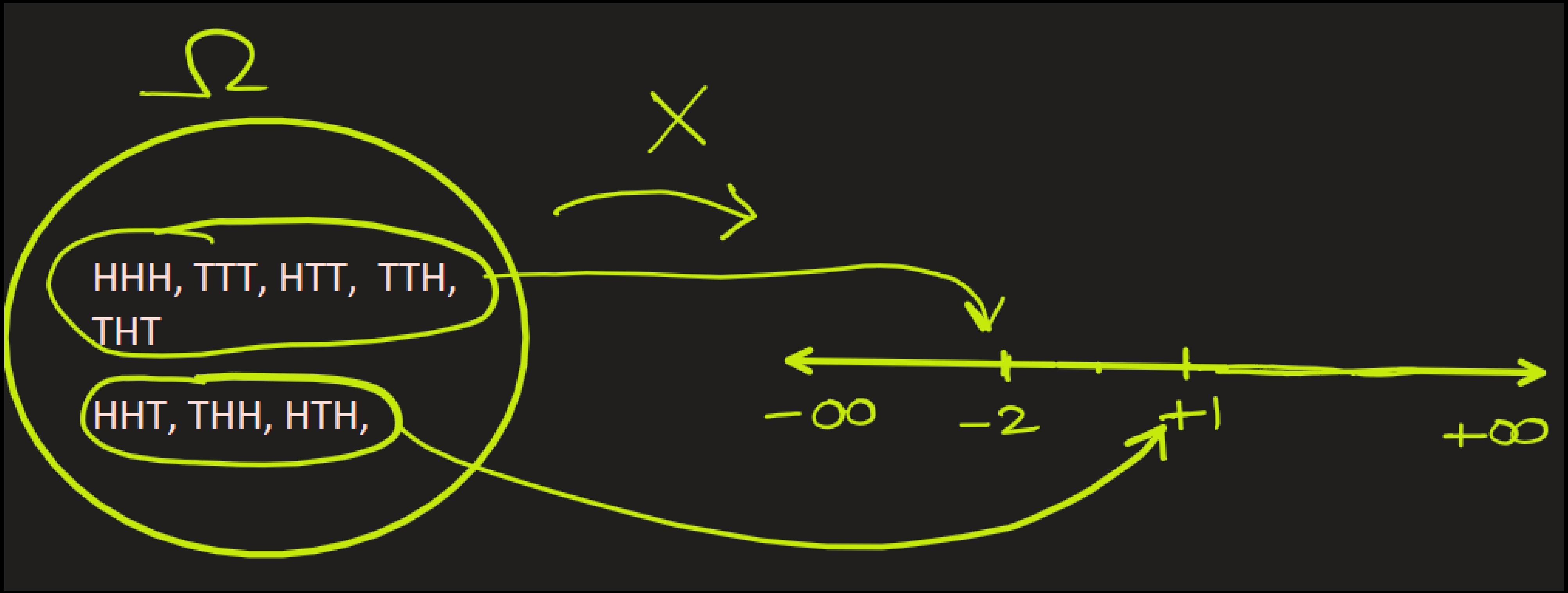


Discrete Random Variables

Example of negative random values: Gamble on coin toss



You flip a coin three times. if you get 2 heads you get \$1 else you lose \$2
Here outcomes are **Ω =** { HHH, HHT, HTT, HTH, THH, TTH, THT, TTT } .
X can be represented as win amount: +1 or -2
So, X = { -2, 1}

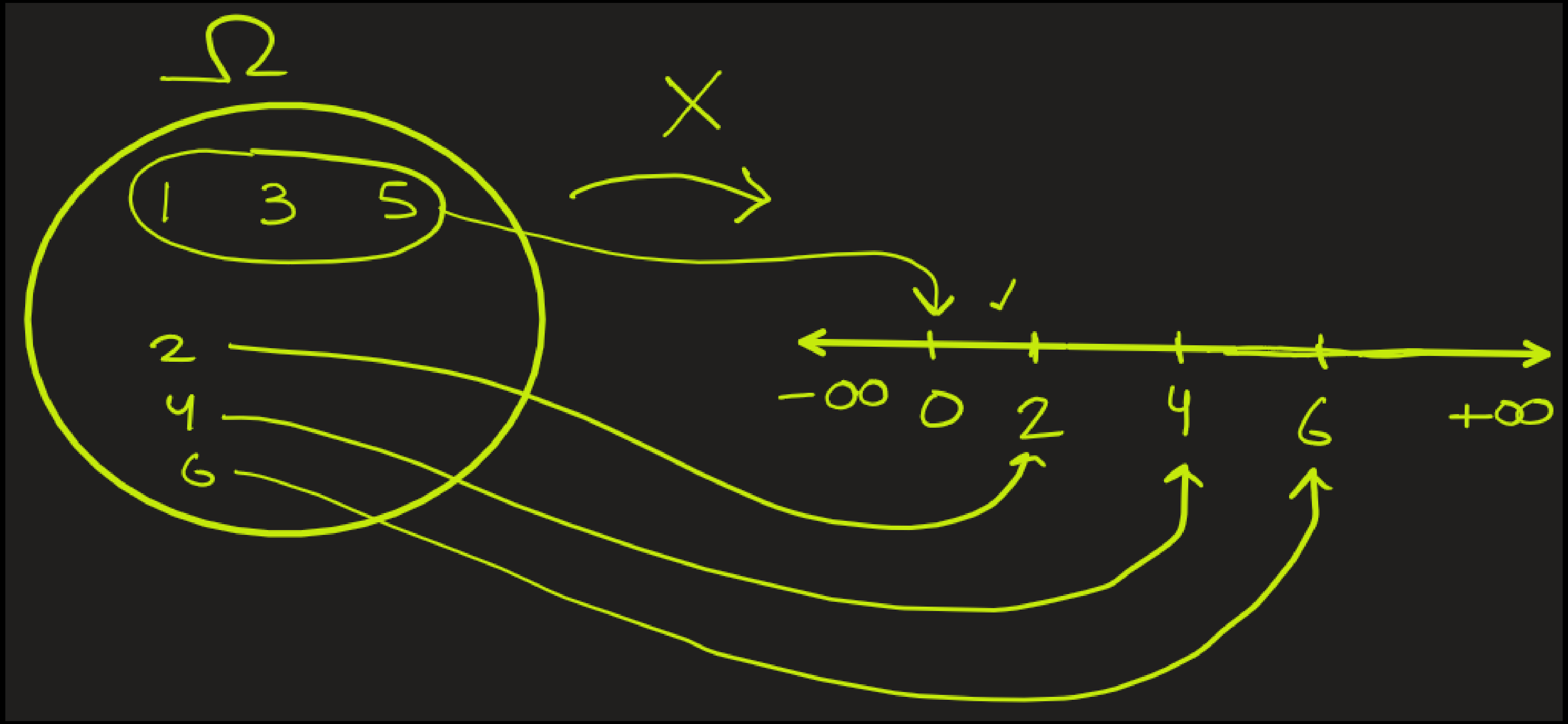


Discrete Random Variables

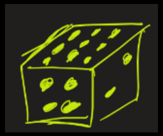
Example: Rolling a fair die once.

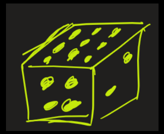
The sample space of all possible outcomes is $\Omega = \{1, 2, 3, 4, 5, 6\}$.
If we define the random variable **X** as "the number shown on the die when it is even else 0" then the possible values of **X** are {0, 2, 4, 6}.

X = { 0, 2, 4, 6 }



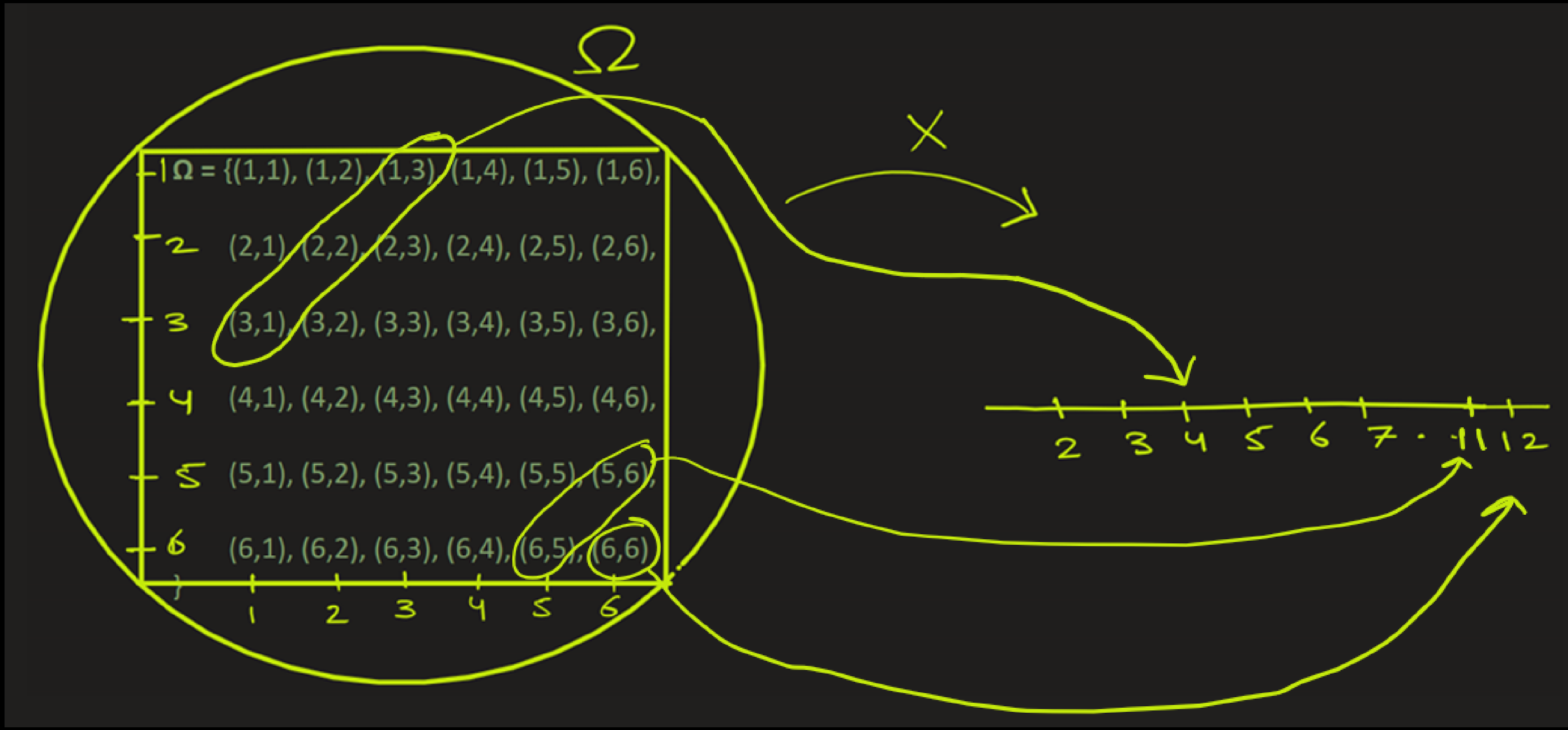
Discrete Random Variables





Example: Rolling 2 dice and adding the numbers on the pair of dice,
Here sample space of all possible outcomes is **Ω =**
{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }
If X is the sum of the numbers on the pair of dice, then
X = { 2, 3, 4, .., 12 }

Mapping here would be
(1,1) -> 2
(1,2) , (2,1) -> 3
(1,3), (2,2), (3,1) -> 4
And so on…

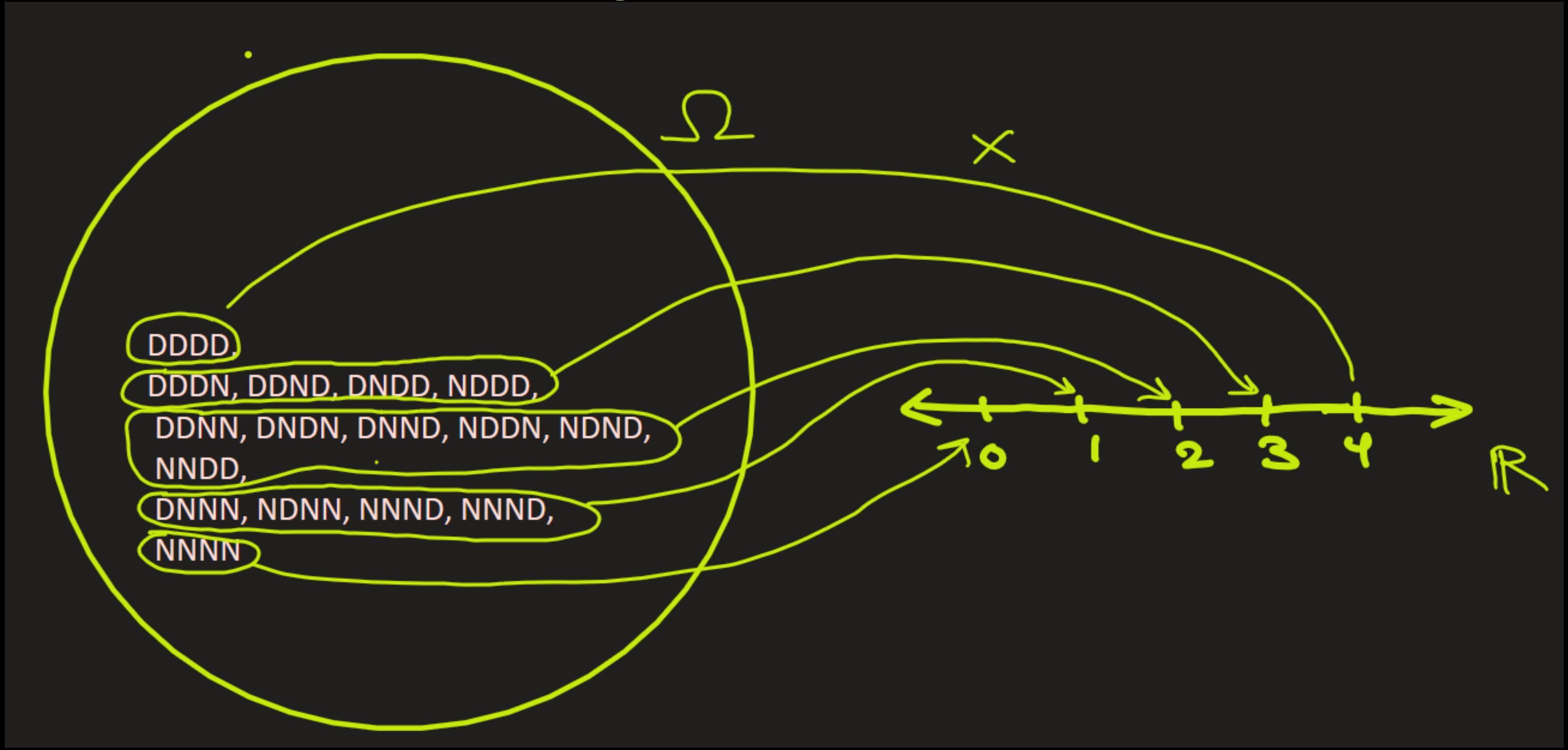


Discrete Random Variables

Example: Counting defective items in a batch of 4

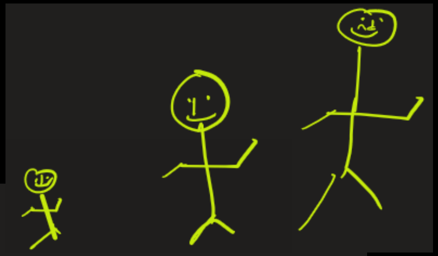
Here sample space is Ω
= { DDDD, DDDN, DDND, DNDD, NDDD, DDNN, DNDN, DNND, NDDN, NDND, NNDD, DNNN, NDNN, NNND, NNND, NNNN }
D is defective and N is Non-defective. There are $2^4 = 16$ total outcomes.

Let X represent the number of defective items in a batch of 4. Its values range from 0 to 4.
So, X = { 0, 1, 2, 3, 4 }



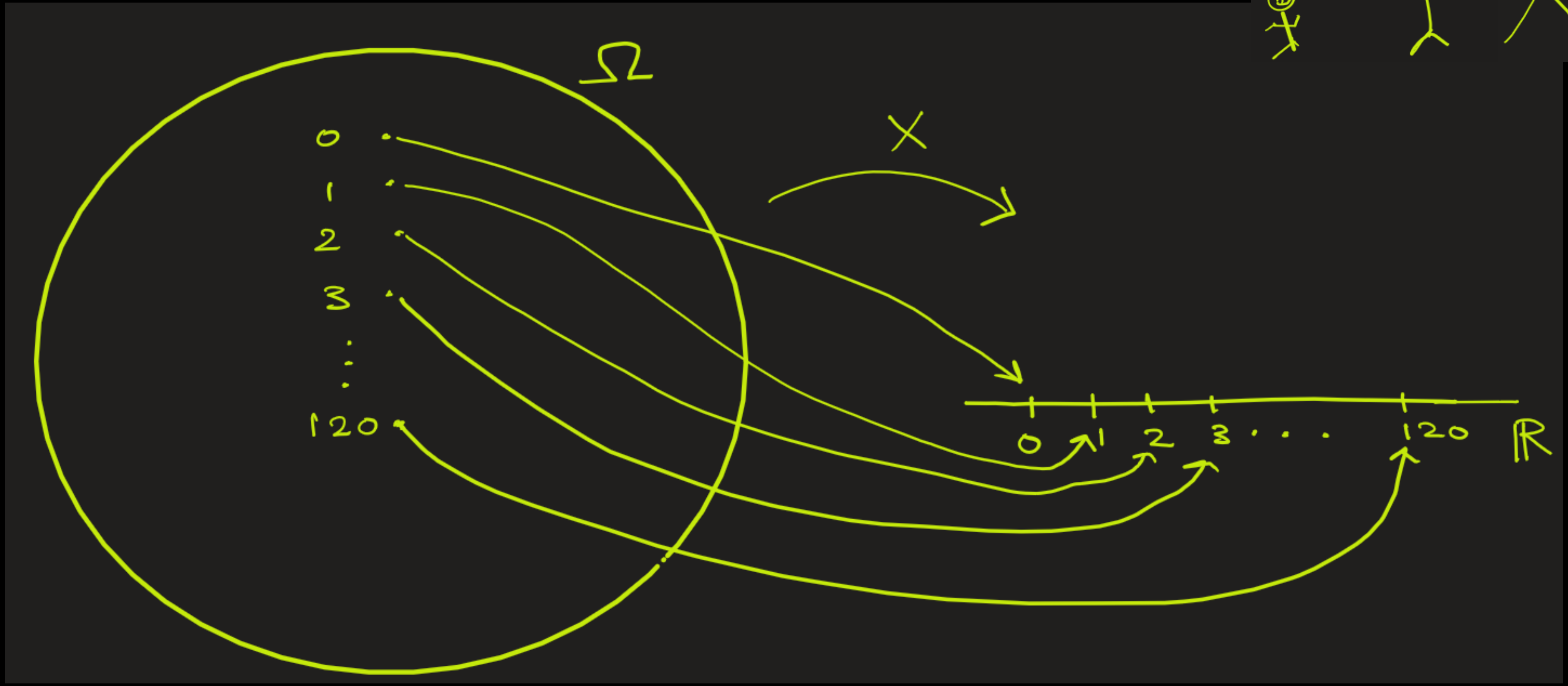
Discrete Random Variables

Example: Picking a person's age expressed in years.
In everyday use we generally count age in years as whole number. 0 year could be a 2-month-old baby.
Here sample space is Ω = { 0, 1, 2 , 3,…, 120}



Representing X as age, we see it could be any discrete number between 0 years to 120 years.
X = { 0, 1 ,2, 3, …., 120 }

Here X = Ω



CONTINUOUS R.V.

Continuous Random Variables

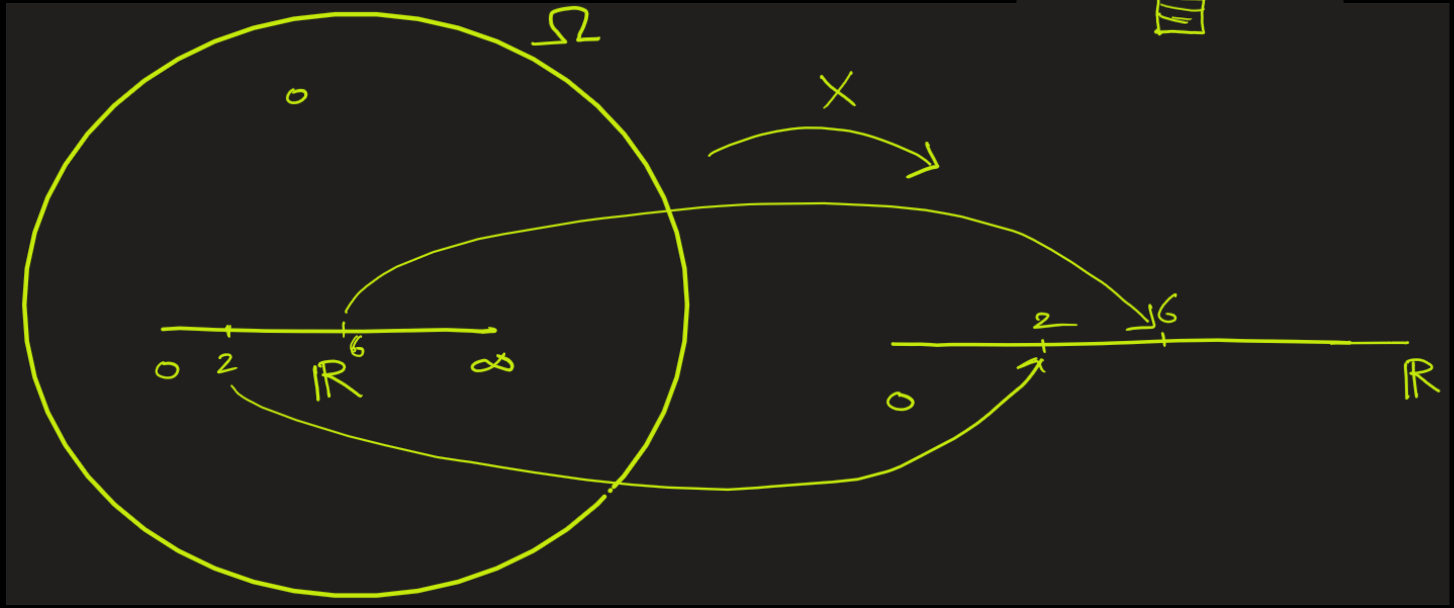
Definition:
A variable that can take **any value within a given interval**.
That is, takes **infinitely many values** (real numbers in an interval).

Example: You test a single light bulb and record how long it lasts before burning out.

Each outcome corresponds to a specific **lifetime** (in hours).
The lifetime could be **any positive real number**, so
Ω = [0, ∞)
Here 0 hours → burns out immediately

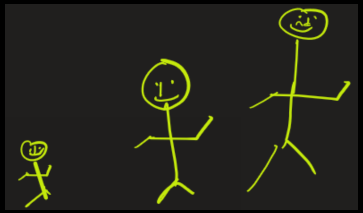
Here X represents the life-time of the bulb that you bought. It can take **any real value** in the interval [0, ∞)

Here X = **Ω =** [0, ∞)



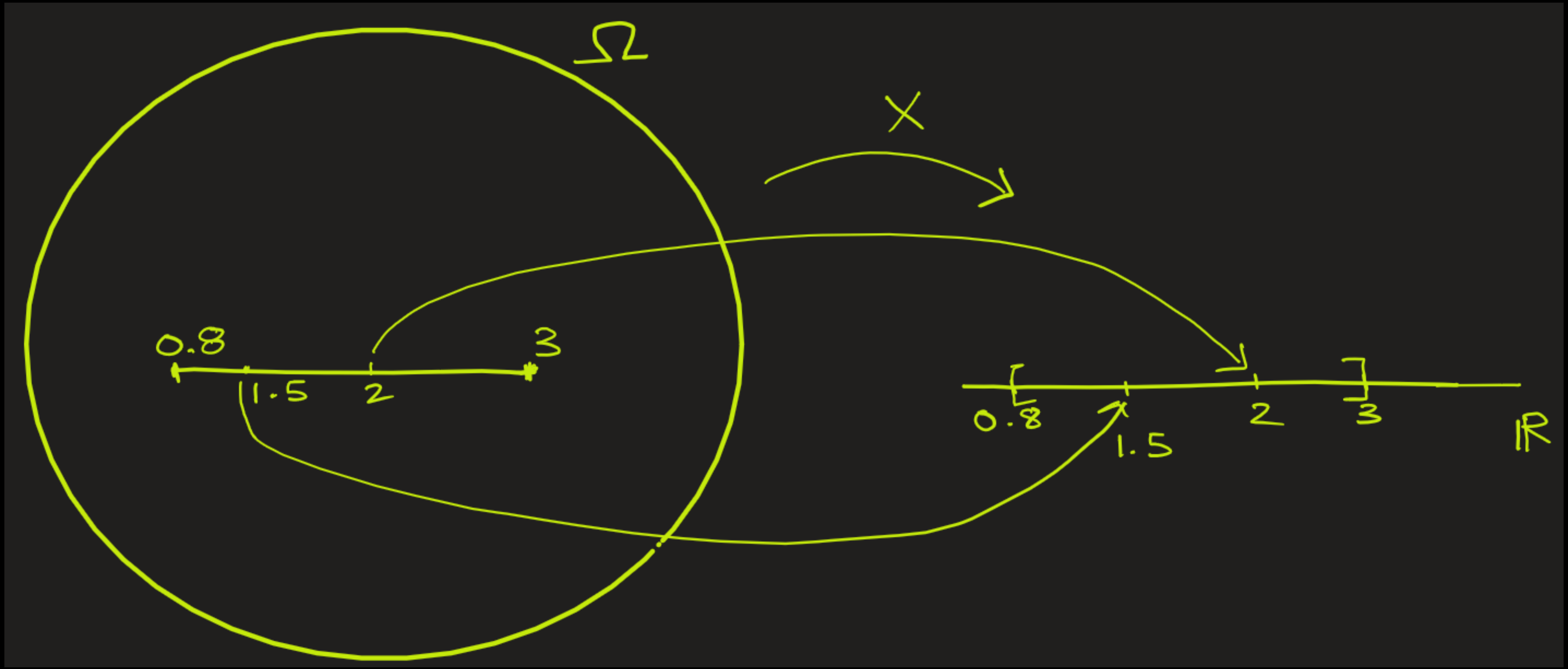
Continuous Random Variables

Simple Example: The height of a person in a crowd.



Here **sample space (Ω)** is the set of **all possible outcomes** — i.e., all possible heights that could occur in this experiment. We know from experience that height of any person is between 0.8 to 3 meters, i.e. such as 1.75 meters or 1.755 meters or 1.32 meters, etc.
So, **Ω** = [0.8, 3] . Here **Ω is** 1-dimensional.

Let X represent the height of person you selected. It can take any value between [0.8, 3]
Here, X = **Ω** = [0.8, 3]



Continuous Random Variables

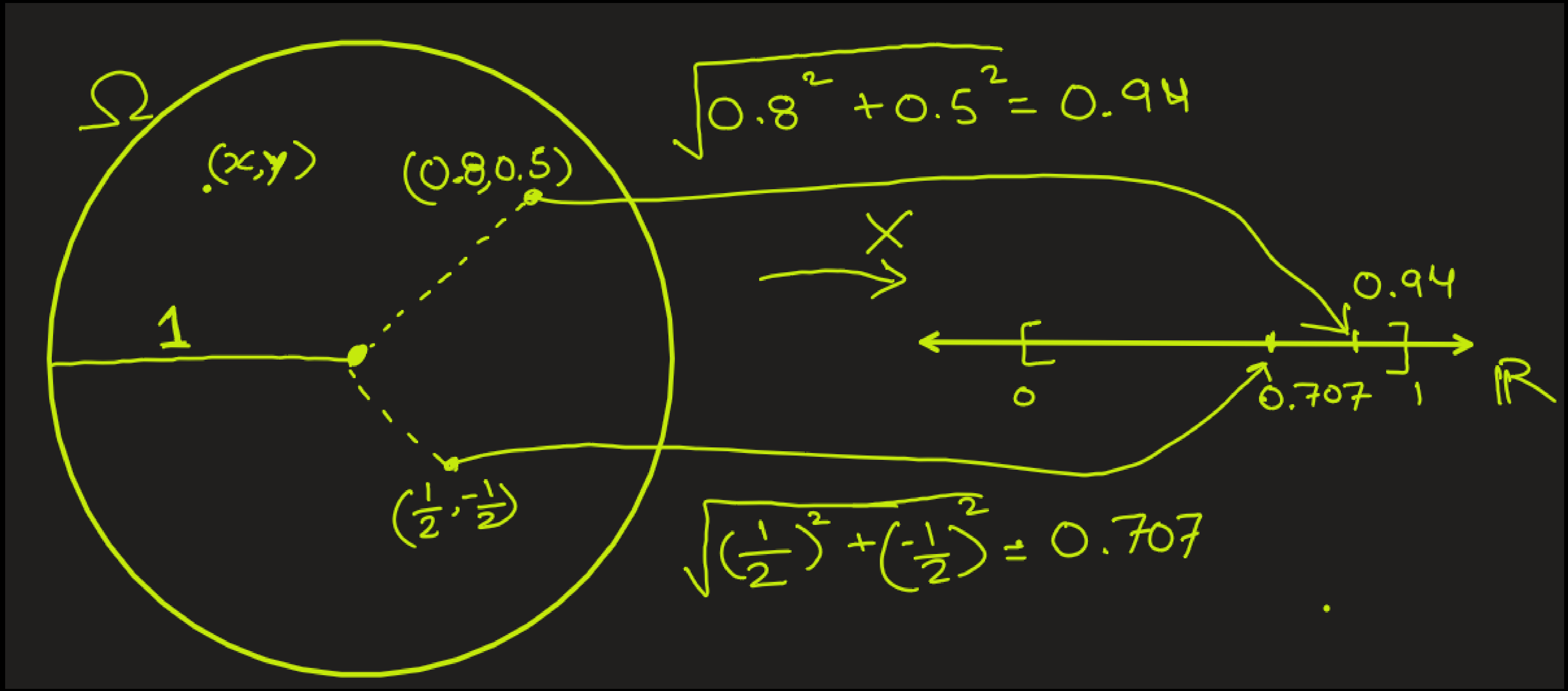
Example: Tossing a Dart at a Circular Board of radius 1 meter

You throw a dart at a **circular dartboard**, and record where it lands. Each outcome is a **point (x, y)** on the circular board.

$$\Omega = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$$

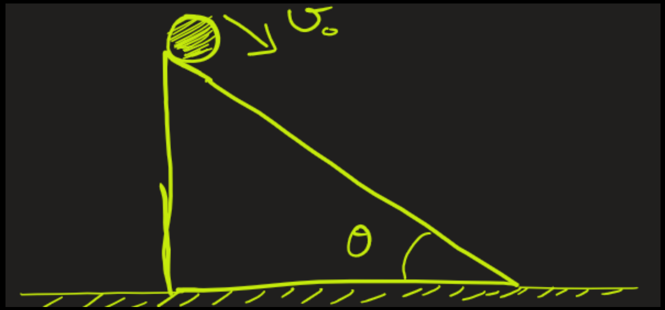
Here sample space (Ω) is a 2-dimensional — all the points within and on the circle.
Let **X** be the distance from the center of the board to the point on which the dart lands.

Here **X** can range from 0 (center) to 1 (edge of the board):
X = [0, 1]



Continuous Random Variables

Example: Rolling a Ball Down a Slope



You roll a smooth ball down a slope of length 1 meter and record the **time it takes to reach the bottom**.
Each possible outcome can depend on **many physical factors** — for example:

- initial velocity v_0 , say it ranges from 0 to 10 mt. per sec
- The angle of slope Ɵ . Say it ranges from 45 to 90 degrees

Each trial (roll) has a different combination of these factors. So, the **sample space** (Ω) can be represented as:

$$\Omega = \{(v_0, \theta),\ 0 \le v_0 \le 10,\ 45^\circ \le \theta \le 90^\circ\}$$

Here sample space (Ω) is a 2-dimensional

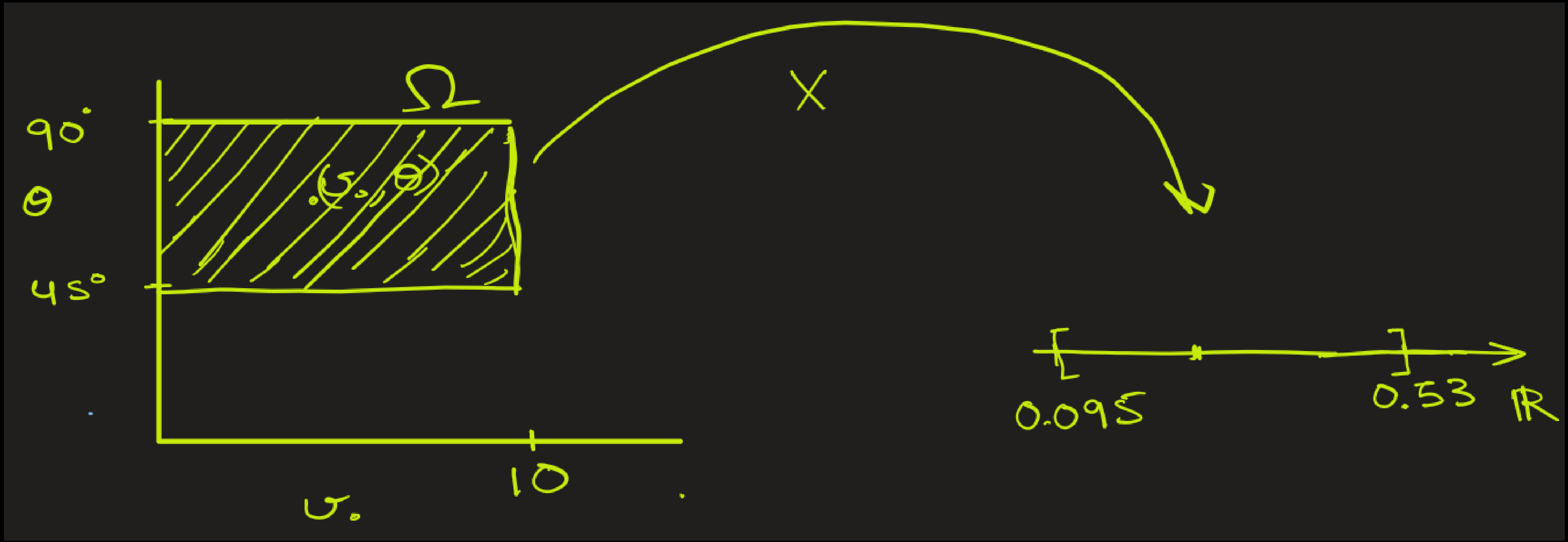
Let X be the time (in seconds) it takes for the ball to reach the bottom. Using principles of physics,

$$X = \frac{-v_0 + \sqrt{v_0^2 + 2g\sin\theta}}{g\sin\theta}$$

Continued

When v_0 = 10 and Θ = 90 -> X = 0.095 seconds, ball takes least time. i.e. X is minimum
When v_0 = 0 and Θ = 45 -> X = 0.53 seconds, ball takes longest time. i.e. X is maximum

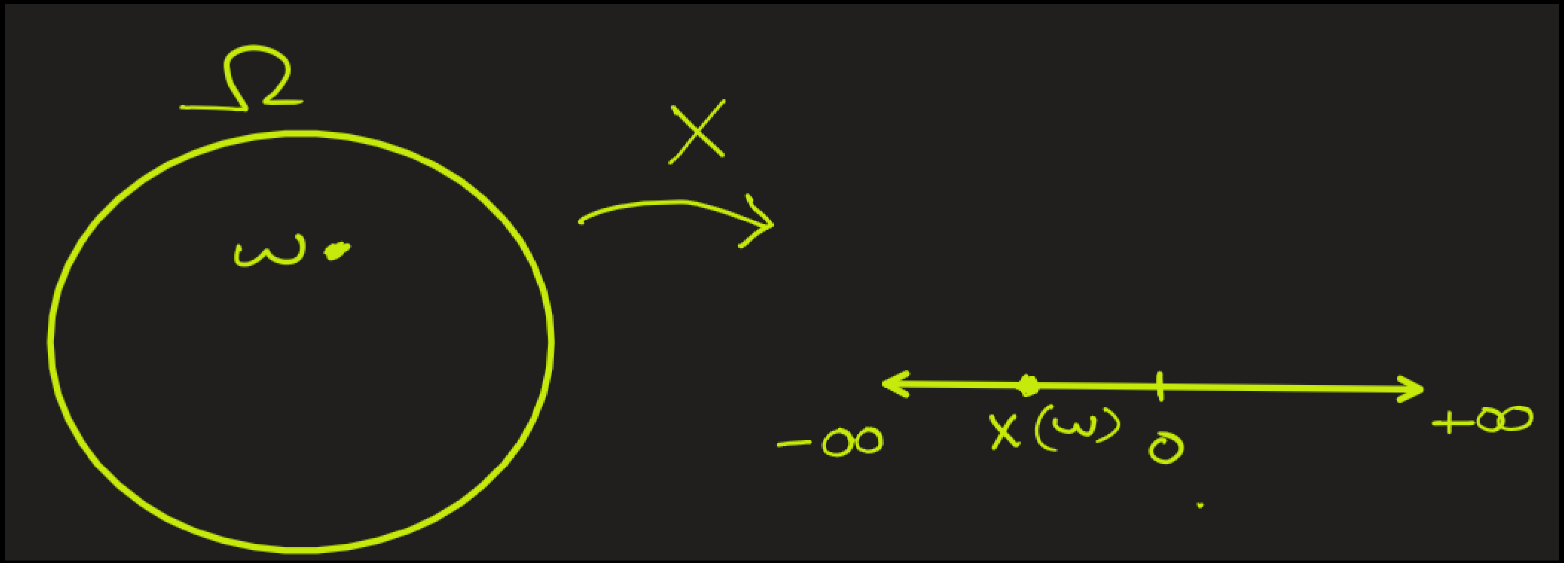
So, X = [0.095, 0.53) .



Random Variables: Formal Definition

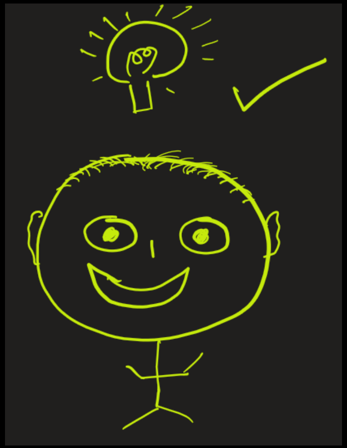
A **random variable (RV)** is a function that assigns a **numerical value** to each possible outcome of a **random experiment**.

$$X : \Omega \to \mathbb{R}$$



Here,
Ω is the **sample space** (all possible outcomes),

X(ω) is the **numerical value** assigned to an outcome ω.



EXTRA-TODO

Continuous Random Variables

Example 1: ADVANCED Rolling a Ball Down a Slope

You roll a smooth ball down a slope of length d and record the **time it takes to reach the bottom**.

Each possible outcome can depend on **many physical factors** — for example:

- initial velocity v_0 , say it ranges from 0 to 10 mt. per sec
- The angle of slope ϴ, say it ranges from 45 to 90 degrees
- friction coefficient μ, say it ranges from 0 to 1

Each trial (roll) has a different combination of these factors. So, the **sample space** (Ω) can be represented as:

$$\Omega = \{ (v_0, \mu, \theta), 0 \le v_0 \le 10, 0 \le \mu \le 1, 45^\circ \le \theta \le 90^\circ \}$$

Here sample space (Ω) is a 3-dimensional

Let X be the time (in seconds) it takes for the ball to reach the bottom. Using principles of physics,

$$X = \frac{-v_0 + \sqrt{v_0^2 + 2gd[\sin\theta - \mu\cos\theta]}}{g\sin\theta - \mu\cos\theta}$$

Depending on initial conditions, X can take range of values

Continuous Random Variables

Example: You **spin a wheel labeled from 0 to 1** — imagine a continuous circular scale starting at 0 and ending at 1.
When the wheel stops, the pointer lands at some value between **0 and 1** (like 0.37, 0.852, etc.).
Here sample space is Ω = { all numbers between 0 and 1 } = [0, 1]
Here X is the position where wheel stops. So, X = [0, 1]
In this case Ω = X

Example: Person’s age in precise form
A person's exact age could be 25 years, 6 months, 3 days, 12 hours, and 5 minutes, which could be measured with even greater precision if needed. Therefore, age ,X, could be any continuous number between 0 years to 120 years.
So, X = [0,120]