№2

$\rho\sigma^2 + (1-\rho)\frac{\sigma^2}{M}$ – дисп. среди.

Δ Пусть a_i – о.р.с.в с дисп σ^2

$$a(x) = \frac{1}{k}\sum_{i=1}^{k} a_i(x)$$

$$E_{X,Y,X,y}(a(x)) = \frac{1}{k} E_{X,Y,X,y}(a_i(x)) = E_{X,Y,X,y}(a_1(x))$$

$$Var_{X,Y,X,y}(a(x)^2) = \frac{1}{k^2} Var_{X,Y,X,y}\left(\sum_{i,j} a_i(x)a_j(x)\right) = \frac{1}{k} Var_{X,Y,X}\, a_1(x) +$$

$$+ \frac{1}{k^2}\sum_{i\neq j} cov(a_i(x), a_j(x)) .$$

$$\frac{1}{k} Var_{X,Y,X}\, a_1(x) + \frac{1}{k^2}\sum_{i\neq j} cov(a_i(x), a_j(x)) = \frac{1}{k} Var_{X,Y,X}\, a_1(x) +$$

$$+ \frac{1}{k^2}\sum_{i\neq j} \underset{\text{коэф. кор.}}{\underbrace{r}} Var_{X,Y,X}\, a_1(x) = \left(\frac{1}{k} + \frac{r(k-1)}{k}\right) Var_{X,Y,X}\, a_1(x)$$ ⊠

№3. Пусть $\{y_i\}_{i=1}^N$ – о.р. сл. вел.

$$D(\bar{y}) = D\left(\frac{1}{N}\sum_{i=1}^N y_i\right) = \frac{1}{N^2} E\left(\sum_{i=1}^N y_i - E\sum_{i=1}^N x_i\right)^2 = \frac{1}{N^2} E\left(\sum_{i=1}^N (y_i - Ey_i)\right)^2$$

$$= \frac{1}{N^2}\left(\sum_{i=j}^N E(y_i - Ey_i)^2 + \sum_{i\neq j}^N E(y_i - Ey_i)(y_j - Ey_j)\right) = \frac{1}{N^2}\cdot$$

$$\cdot\left(\sum_{i=1}^N D x_1 + \sum_{i\neq j}^N cov(y_j, y_i)\right) = \frac{\sigma^2 N}{N^2} + \frac{N(N-1)}{N^2}\rho\sigma^2 =$$

$$= \frac{\sigma^2}{N} + \left(1 - \frac{1}{N}\right)\rho\sigma^2 = \rho\sigma^2 + (1-\rho)\frac{\sigma^2}{N} \quad \boxtimes$$

№1.

$$E_{x,y} E_{X^\ell} (y - a_{X^\ell}(x))^2 = \underbrace{E_{x,y}(y - E(y|x))^2}_{\text{шум}} + \underbrace{E_{x,y}(E(y|x) - E_{X^\ell} a_{X^\ell}(x))^2}_{\text{смещение}} + \underbrace{E_{x,y} E_{X^\ell}(a_{X^\ell}(x) - E_{X^\ell} a_{X^\ell}(x))^2}_{\text{разброс}}$$

Δ $y = f(x) + \varepsilon$,

$E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma^2$

$$E(a(x) - y)^2 = E(E(a(x) - y)^2 | x)) = E(E((a(x) - f(x) - \varepsilon)^2 | x)) =$$

$$= E(E((a(x) - f(x))^2 - 2\varepsilon(a(x) - f(x)) + \varepsilon^2 | x) = E(E((a(x) - f(x))^2 | x) -$$

$$- 2E(\varepsilon | x) E(a(x) - f(x) | x) + E(\varepsilon^2 | x)) = E(E((a(x) - f(x))^2 | x) -$$

$$- 2E(\varepsilon) E(a(x) - f(x) | x) + E(\varepsilon^2)) = E(E((a(x) - f(x))^2 | x) + \sigma^2) =$$

$$= E(E((a(x) - E(a(x)) + E(a(x)) - f(x))^2 | x) + \sigma^2) = \Big| \bar{a}(x) = E(a(x)) \Big|$$

$$E(E((a(x) - \bar{a}(x) + \bar{a}(x) - f(x))^2 | x) + \sigma^2) = E(E((a(x) - \bar{a}(x)))^2 | x) +$$

$$+ E((\bar{a}(x) - f(x))^2 | x) - 2E((a(x) - \bar{a}(x)))(\bar{a}(x) - f(x)) | x) + \sigma^2) =$$

$$= E(E((a(x) - \bar{a}(x)))^2 | x) + E((\bar{a}(x) - f(x))^2 | x) - 2(\bar{a}(x) - f(x)) E$$

$$E((a(x) - \bar{a}(x))) | x) + \sigma^2) = Variance(x) + Bias(x)^2 + \sigma^2$$

$$Var(x) = E((a(x) - \bar{a}(x)))^2 | x) = E((a(x) - \bar{a}(x)))^2)$$

$$Bias(x) = \bar{a}(x) - f(x) = E(a(x) - f(y))$$

$$E_{x,y}(a(x) - y)^2 = E_x(Variance(x) + Bias(x)^2) + \sigma^2$$