## №1.

$$E_{x,y}E_{X^\ell}(y-a_{X^\ell}(x))^2 = \underbrace{E_{x,y}(y-E(y|x))^2}_{\text{шум}} + \underbrace{E_{x,y}(E(y|x) - E_{X^\ell}a_{X^\ell}(x))^2}_{\text{смещение}} + \underbrace{E_{x,y}E_{X^\ell}(a_{X^\ell}(x) - E_{X^\ell}a_{X^\ell}(x))^2}_{\text{разброс}}$$

$\Delta$ $y = f(x) + \varepsilon$

$E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma^2$

$E(a(x)-y)^2 = E(E(a(x)-y)^2|x)) = E(E((a(x)-f(x)-\varepsilon)^2|x)) =$

$= E(E((a(x)-f(x))^2 - 2\varepsilon(a(x)-f(x)) + \varepsilon^2|x) = E(E((a(x)-f(x))^2|x) -$

$- 2E(\varepsilon|x)E(a(x)-f(x)|x) + E(\varepsilon^2|x)) = E(E((a(x)-f(x))^2|x) -$

$- 2E(\varepsilon)E(a(x)-f(x)|x) + E(\varepsilon^2)) = E(E((a(x)-f(x))^2|x) + \sigma^2) =$

$= E(E((a(x)-E(a(x)) + E(a(x)) - f(x))^2|x) + \sigma^2) = \Big|\, \bar{a}(x) = E(a(x))$

$E(E((a(x)-\bar{a}(x) + \bar{a}(x) - f(x))^2|x) + \sigma^2) = E(E((a(x)-\bar{a}(x)))^2|x) +$

$+ E((\bar{a}(x)-f(x))^2|x) - 2E((a(x)-\bar{a}(x)))(\bar{a}(x)-f(x))|x) + \sigma^2) =$

$= E(E((a(x)-\bar{a}(x)))^2|x) + E((\bar{a}(x)-f(x))^2|x) - 2(\bar{a}(x)-f(x))$

$E((a(x)-\bar{a}(x)))|x) + \sigma^2) = Variance(x) + Bias(x)^2 + \sigma^2$

$Var(x) = E((a(x)-\bar{a}(x)))^2|x) = E((a(x)-\bar{a}(x)))^2)$

$Bias(x) = \bar{a}(x) - f(x) = E(a(x) - f(y))$

$E_{X,Y,x,y}(a(x)-y)^2 = E_x(Variance(x) + Bias(x)^2) + \sigma^2$

## №3

$\rho\sigma^2 + (1-\rho)\frac{\sigma^2}{M}$ – дисп средн.

$\Delta$ Пусть $a_i$ – о.р.с.в с дисп $\sigma^2$

$a(x) = \frac{1}{k}\sum_{i=1}^{k} a_i(x)$

$E_{X,Y,x,y}(a(x)) = \frac{1}{k}E_{X,Y,x,y}(a_i(x)) = E_{X,Y,x,y}(a_1(x))$

$Var_{X,Y,x,y}(a(x)^2) = \frac{1}{k^2}Var_{X,Y,x,y}\left(\sum_{i,j} a_i(x)a_j(x)\right) = \frac{1}{k}Var_{X,Y,x}a_i(x) +$

$+ \frac{1}{k^2}\sum_{i\neq j} cov(a_i(x), a_j(x))$

$\frac{1}{k}Var_{X,Y,x}a_i(x) + \frac{1}{k^2}\sum_{i\neq j} cov(a_i(x), a_j(x)) = \frac{1}{k}Var_{X,Y,x}a_i(x) +$

$+ \frac{1}{k^2}\sum_{i\neq j} \underset{\text{козф. кор.}}{r}\, Var_{X,Y,x}a_i(x) = \left(\frac{1}{k} + \frac{r(k-1)}{k}\right)Var_{X,Y,x}a_i(x)$ ⊠