Report on JHEP_096P_0320
Date: April 2, 2020

Author(s): Madhurima Pandey, Debasish Majumdar, Amit Dutta Banik, Ashadul Halder

Title: The Violation of Equivalence Principle and Four Neutrino Oscillations fo

Received: 2020-03-11 14:17:04.0

Referee report

Dear Editor,

I think this paper needs a large revision and I am still not sure about the relevance of this paper.

The idea of neutrino factories gets attention 10 yrs due to the very well know spectrum of neutrino sources and due to the possibility to switch polarity and to do studies of CP violation. Now it seems an unlikely possibility.

The paper shows probabilities and numbers of events and using some specific cases of the parameters it is said that it can relevant. This is more or less half of the way to write a paper. If you propose a new scenario, test of this scenario should be considered in a quantitive way. This was not done in the article.

Another point is that the authors put together two new sources of neutrino oscillations, a lighter sterile neutrino and a violation of the equivalence principle (VEP). It is not clear which effect is relevant to them. The authors should compare with the standard three-neutrino without sterile neutrino and VEP to have a clear idea of what is the source of the change of the probability.

Another point is that the number of events of the author is exaggerated. I checked some references, that I include below and these numbers are off by a factor of 100.

1. As a general suggestion, the authors can include a LATEX package that enumerates the lines. It is simpler for the referred quote in this way than the lines that he is talking about.

2. Page 3, the authors write ... *the neutrino oscillation probabilities are significant even if the* Δf_{ij} *takes a value as small as* $\sim 10^{-24}$

   • If the authors compare different effects that are working, this it is not surprising. For example, the relevant parameters are $\frac{\Delta m^2_{41}}{4E_\nu}, \frac{\Delta m^2_{31}}{4E_\nu}, V_{CC}, 2E_\nu \Delta f_{41/31}$. The authors disregard, $\frac{\Delta m^2_{21}}{4E_\nu}$. Putting the numbers

$$
\begin{aligned}
\frac{\Delta m^2_{41}}{4E_\nu} &= 150 \times 10^{-15}\mathrm{eV} \left(\frac{\frac{\Delta m^2_{41}}{3\times 10^{-3}\mathrm{eV}^2}}{\frac{E_\nu}{5\mathrm{GeV}}} \right) \\
\frac{\Delta m^2_{31}}{4E_\nu} &= 130 \times 10^{-15}\mathrm{eV} \left(\frac{\frac{\Delta m^2_{31}}{2.5\times 10^{-3}\mathrm{eV}^2}}{\frac{E_\nu}{5\mathrm{GeV}}} \right) \\
\frac{\Delta m^2_{21}}{4E_\nu} &= 3.5 \times 10^{-15}\mathrm{eV} \left(\frac{\frac{\Delta m^2_{31}}{7.5\times 10^{-5}\mathrm{eV}^2}}{\frac{E_\nu}{5\mathrm{GeV}}} \right) \\
2E_\nu \Delta f_{41/31} &= 10 \times 10^{-15}\mathrm{eV} \left(\frac{E_\nu}{5\mathrm{GeV}} \right) \left(\frac{\Delta f_{41/31}}{10^{-24}} \right) \\
\sqrt{2} G_f N_e &= 623 \times 10^{-15}\mathrm{eV} \left(\frac{N_e}{4.15\mathrm{g/cm}^3} \right)
\end{aligned} \tag{1}
$$

   From these numbers, we can conclude that the value of $\frac{\Delta f_{41/31}}{10^{-24}} = 1$ which is not small. It is about 10% of the value due the Δm^2_{31} scale, and it is bigger than the value due to Δm^2_{21} scale.

3. Page 3, the authors quote that they are going to use the distance of 7000 km and a consrant density of 4.15 g/cc. For this distance the neutrino should cross mantle and core then by sure teh density it is not constant. It changes by a factor of two between mantle and core. Then this assumption of a constant matter density it is not correct.

4. Page 5, The Eq.(1) it is not correct. It should be $U^*_{\alpha i}$ for neutrino states.

5. Page 6, I dont think it is necessary to type the rotation matrices $R(\theta_{ij})$. the name says by itself what it is the mathematical formulae. The Eq.(4) it is not necessay.

   • The same applies to Eq.(5) and (6). Why the authors decomposed the matrix separating the 4×4 new matrix $R(\theta_{i4})$? The authors can say the same " we use rotation matrices.

6. Pages 7 to 9: The authors after Eq.(12) and Eq.(20) discuss the case when the gravitational basis $\nu_{\mathrm{G_i}}$ is not equal to the mass basis ν_i. But at the end of the day, they assume that they are the same bases. This is a waste of space. I suggest to delete whole this text and equations.

   - Beside this there are inconsistencies in the text. For example,

   (a) page 7, the Eq.(13) shows that the new mixing matrix $U'_{4\times4}$ which relates the gravitational basis to flavor basis is inconsistent with Eq.(15). In the text after this equation, ν_α should be on the right side of the equation.

   (b) In the same place, the authors use the notation of α_{ij}. Later the authors use the notation Δf_{ij} for the same parameters. Why two different notations?

   (c) page 8, Eq.(17) it is wrong. If you have a Hamiltonian in the gravitational basis H_G. When you change to the mass basis, by a U' matrix, then you have the H_G in the mass basis. In the gravitational basis, the U' evolution is due H_G, that does not commute with Hamiltonian in mass basis H_d. Then in the mass basis the new Hamiltonian $U'H_GU'^\dagger$, H_G is constant, but now in the mass basis, the U' quantity did not commute with H_d. It becomes a time-dependent Hamiltonian.

   (d) But this discussion is vain since in the end the authors only use the Eq.(20) in the paper. I suggest to remove all this text and focus on Eq.(20).

7. Page 9, assumptions of the paper: In the text between the Eq.(20) and (21) the authors assume that $\Delta m^2_{21} \to 0$. From the typical numbers quoted in Note 2, mentioned above, we have

$$\begin{aligned}
\frac{\Delta m^2_{21}}{4E_\nu} &= 5.9\times10^{-15}\mathrm{eV}\left(\frac{\frac{\Delta m^2_{31}}{7.5\times10^{-5}\mathrm{eV}^2}}{\frac{E_\nu}{3\mathrm{GeV}}}\right)\\
2E_\nu\Delta f_{41/31} &= 6\times10^{-15}\mathrm{eV}\left(\frac{E_\nu}{3\mathrm{GeV}}\right)\left(\frac{\Delta f_{41/31}}{10^{-24}}\right)
\end{aligned}$$

   This mean that for neutrino energies below $E_\nu < 3$ GeV, the mass scale at Δm^2_{21} it is too small. Then the assumption is not correct for energy range that they choose.

8. Page 9, the value of Δm^2_{31} quoted in Eq.(23) is wrong.

9. Page 11, I dont think there is the need of Eq.(27). It is exactly the Eq.(25) but the sum is explicitly written.

10. Page 11, in Eq.(26) the authors use the variable for the vaccum energies and for the eigenvalues of H''. It is confunsing notation.

11. Page 13, The authors quote that they use Eqs(17-27). but in reality they use Eqs.(20-27).

    (a) In the same parahtaph they say that in computations they use constant density for L=7000 km. The density it is not constant for this distance. Neutrinos have mantle, core, mantle in the path. The authors should compute the probabilities, using the cnange of the density.

12. Page 15, the Figs1(a–e) consider the effect of both the sterile neutrino and the VEP contribution. The comparison with the standard case, with three neutrinos without sterile and without VEP is essential. All plots are without explanation and it is not clear if the main effect is due the sterile neutrino or the VEP.

    (a) In Figure 1a, the dips should be due to the Δm^2_{41} related MSW resonance and probably due to some interference with Δm^2_{31}. Probably some of the dips around $E_\nu \sim 4-6$ are due Δm^2_{31} and the other due Δm^2_{41}. This is because P_{es} has an enhancement for these energies.

    (b) In Fig. 1c, it seems that there is an enhancement of the probability due to the Δf_{31}.

    (c) Comparing Fig1a with Fig2a, you can notice that one of the dips it is really due to the sterile Δm^2_{41}.

    (d) Comparing Fig1a,b,c, with Fig2a,b,c one can notice that sensitivity to VEP disappears. This is due to suppression of the probably due to matter effect.

    (e) It will helpfull if the authors get a simpler analytical expression to see the relevance of the effects. The problem is that the authors choose Δm^2_{41} to be in the interval $(1-3)\times 10^{-3}$ eV^2 that is of the same order as Δm^2_{31}. Maybe one solution is to put the value of $\Delta m^2_{41} = 1\text{eV}^2$ for which thereis clear separation of Δm^2_{41} and Δm^2_{31}. Maybe, in this case, one can get a simple analytical explanation.

13. Page 19, The number of injected muons is not given.

    (a) The authors change the baseline to be 7150km. I suggest to keep the same distance, for the probabilities and for the number of events.

    (b) There is a problem: Following Refs [1], assuming the same muon energy one expects for the baseline 7000km 10^4 events as can be seen in Page 211 of that paper. The author's value is $\sim 2 \times 10^6$ events, which is a factor of 100 different. Also in Ref.[2] the numbers of events seem to be too high.

(b) Also there is a problem of internal inconsistency of the author's results. In Figure 2, for $\Delta m^2_{41} = 3 \times 10^{-3}\mathrm{eV}^2$, $\Delta f_{31} = 10^{-24}$ there are no changes in the $P(\nu_\mu \to \nu_\mu)$ and in $P(\nu_\mu \to \nu_e)$ probabilities. But in Page 21, Table 4, the same muon signal changes when Δf_{41} changes. This it is not correct. It has some mistakes here.

(c) Comparing Figure 1, for for $\Delta m^2_{41} = 1 \times 10^{-3}\mathrm{eV}^2$, $\Delta f_{31} = 10^{-24}$ with numbers in Table 2, one finds something strange. The probability increases when Δf_{41} increases but the number of events decreases. I think there is some mistake here.

(b) Also there is the background in wrong sign muon channel due τ decay which is sometimes much bigger than the signal itself. See e.g. the right side of Figure 82 on page 211 of Refs [1].

14. The final comment is that the authors made the computation of the probability and the number of events without checks of the quantitative bounds on VEP parameters.

15. Also the bounds on $\Delta f_{31} < 7 \times 10^{-27}$[3] makes assumptions of larger value for Δf_{31} pointless. These bounds assume 3 neutrinos, but for the energy range of Ref.[3] a lighter sterile will have no effect.

My recommendation **is to reject this paper**. The internal numbers of paper are not consistent in the Figures with probabilities and the numbers of events, the assumptions are not reasonable, and for the oscillation experimentf that authors consider computations should be done for variable density profile.

References

[1] A. Bandyopadhyay *et al.* [ISS Physics Working Group], Rept. Prog. Phys. **72** (2009), 106201 doi:10.1088/0034-4885/72/10/106201 [arXiv:0710.4947 [hep-ph]].

[2] A. Cervera, F. Dydak and J. Gomez Cadenas, Nucl. Instrum. Meth. A **451** (2000), 123-130 doi:10.1016/S0168-9002(00)00558-1

[3] A. Esmaili, D. R. Gratieri, M. M. Guzzo, P. C. de Holanda, O. L. G. Peres and G. A. Valdiviesso, Phys. Rev. D **89**, no. 11, 113003 (2014) doi:10.1103/PhysRevD.89.113003 [arXiv:1404.3608 [hep-ph]].