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Q. Given three integers m , a and b ; return m th magical no.
Since the ans may be very large no. so return
 $\text{mod } 10^9 + 7$.

A magical no. — If no. is divisible by a or b .
 $n=1, a=2, b=3$
output = 2.

No. divisible by:	Magical
2	X
3	X
4	X
5	X
6	2 and 3

Ans. magical no. (div by a or b)
→ multiple of a
→ multiple of b] best count only once

$$\text{Count}(x) = \left(\frac{x}{a} + \frac{x}{b} - \frac{x}{\text{lcm}(a,b)} \right)$$

Approach:
1. Compute $\text{lcm}(a,b)$
2. The answer lies between ~~min(a,b)~~ and ~~$n * \text{min}(a,b)$~~

- Use binary search to find m th magical no.
- Return result % $(10^9 + 7)$

$n=5$

$a=2$ $b=3$

Number	Divisible by 2	Div by 3	Magical
1	X	X	X
2	V	X	V
3	X	V	V
4	V	X	V
5	X	X	X
6	V	V	V
7	X	X	X
8	V	V	V

for $m=5$ output = 8

Brute force code

```
#include <bits/stdc++.h>
using namespace std;

int nthMagical (int n, int a, int b) {
    int count = 0;
    int num = 1;
    while (true) {
        if (num % a == 0 || num % b == 0) {
            count++;
            if (count == n) {
                return num;
            }
        }
        num++;
    }
    return -1;
}
```

Optimal code in c++

$$\text{# count}(x) = \frac{x}{a} + \frac{x}{b} - \left(\frac{x}{\text{lcm}(a,b)} \right)$$

Inclusion Exclusion

```
#include <bits/stdc++.h>
using namespace std;
```

```
long long gcd (long long a, long long b) {
    while (b != 0) {
        long long t = a % b;
        a = b;
        b = t;
    }
    return a;
}
```

```
long long lcm (long long a, long long b) {
    return (a / gcd(a,b)) * b;
}
```

```
int main() {
    int n, a, b;
    cin >> n >> a >> b;
```

```
const int MOD = 1e9 + 7;  
long long low = 1;  
long long high = (long long)n * min(a, b);  
long long L = lcm(a, b);  
while (low < high) {  
    long long mid = low + (high - low) / 2;  
    long long count = mid / a + mid / b - mid / L;  
    if (count < n)  
        low = mid + 1;  
    else  
        high = mid;  
}  
if (count < low % MOD)  
    return 0;
```