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Q. Given three integers  $n, a$  and  $b$ ; return  $n$ th magical no.  
 Since the ans may be very large no. so return  
 $\text{mod } 10^9 + 7$ .

A magical no. — if no. is divide by  $a$  or  $b$ .  
 $n=1, a=2, b=3$   
 output = 2.

No. divisible by:	Magical
2 — div by 2	✓
3 — 3	✓
4 — 2	✓
5 ×	
6 — 2 and 3	✓

Ans. magical no. (div by  $a$  or  $b$ )

- multiple of  $a$
- multiple of  $b$

but count only once

$$\text{Count}(x) = \left( \frac{x}{a} + \frac{x}{b} - \frac{x}{\text{lcm}(a,b)} \right)$$

Approach:

1. compute  $\text{lcm}(a, b)$
2. The answer lies between  ~~$\min(a, b)$~~ <sup>1</sup> and  ~~$\max(a, b)$~~   
 $n \times \min(a, b)$
3. Use binary search to find  $n$ th magical no.
4. Return result %  $(10^9 + 7)$

$n=5$

$a=2$

$b=3$

		Numbers	Divisible by 2	Div by 3	Magical
2	<del>2</del>	1	×	×	×
3	<del>3</del>	2	✓	×	✓
4	<del>4</del>	3	×	✓	✓
5	<del>5</del>	4	✓	×	✓
6	<del>6</del>	5	×	×	×
7	<del>7</del>	6	✓	✓	✓
8	<del>8</del>	7	×	×	×
9	<del>9</del>	8	✓	×	✓

for  $n=5$  output = 8

## # Brute force code

```
#include <bits/stdc++.h>
using namespace std;
```

```
int nthMagical (int n, int a, int b) {
    int count = 0;
    int num = 1;
    while (true) {
        if (num % a == 0 || num % b == 0) {
            count++;
            if (count == n) {
                return num;
            }
        }
        num++;
    }
    return -1;
}
```

## # Optimal code in c++

$$// \text{count}(x) = \underbrace{\frac{x}{a} + \frac{x}{b}}_{\text{Inclusion}} - \underbrace{\left(\frac{x}{\text{lcm}(a,b)}\right)}_{\text{Exclude to prevent counting same no. twice as using count to return nth magical.}}$$

```
#include <bits/stdc++.h>
using namespace std;
```

```
long long gcd (long long a, long long b) {
    while (b != 0) {
        long long t = a % b;
        a = b;
        b = t;
    }
    return a;
}
```

```
long long lcm (long long a, long long b) {
    return (a / gcd(a, b)) * b;
}
```

```
int main() {
    int n, a, b;
    cin >> n >> a >> b;
```

const int MOD = 1e9 + 7;

long long low = 1;

long long high = (long long) n \* min(a, b);

long long L = lcm(a, b);

while (low < high) {

long long mid = low + (high - low) / 2;

long long count = mid / a + mid / b - mid / L;

if (count < n)

low = mid + 1;

else

high = mid;

}

count < low % MOD;

return 0;

}