7^{th} Grade Individual Contest

IMSA *Mu Alpha Theta*

March 6, 2024

1. What is 50% of 19.5?
2. Compute $(4 + 11)/3 \cdot 7 - 5$.
3. Let a, b, c, and d be whole numbers. $2^a \cdot 11^b \cdot 23^c = 2024$. Find $2a - 11b + 23c$.
4. Azamon is experimenting with a new drone to deliver packages. The drone is capable of flying at 90 mph (miles per hour). If the course for its test flight is 7 miles long, how long does it take for the drone to complete the test flight? Answer as a fraction or mixed number in minutes.
5. Alice is 8 times as old as her niece, Kate. If the sum of their ages is 63, how old is Alice?
6. Banana and Blueberry are competing tech companies that make cheap phones that are both sold for \$80 at first. Banana phones become very popular, and the price of its phone rises by 50%. But then it loses its popularity and the price falls by 50%. Then Blueberry becomes even more popular, and its phone price rises by 60% and when it loses popularity the price then falls by 60%. In dollars and cents, compute the difference between the prices of the phones after all these changes occur.
7. Xavier buys 2 pens and 4 pencils for \$4 at a store. Yolanda buys 7 pens and 2 pencils for \$8 at the same store. Compute the cost of 1 pen and 1 pencil at this store.
8. Esmerelda was born on January 1, 2000. Her mother was born on January 1, 1976, and her father was born on January 1. 1975. In what year was the sum of their ages equal to 100?
9. The perimeter of a square is eight times its area. Find the length of a side of this square.
10. The flow rate of a bathtub faucet is 4.5 gallons per minute, but the plug is leaky and the water drains at a rate of 2 gallons per minute. If the tub can hold 60 gallons of water and is empty when the faucet is turned on, how many minutes will it take for the tub to become $\frac{3}{4}$ full?
11. Larry has invented a machine that can figure out prime numbers. When you put in a positive integer, n, the machine outputs the nth prime number. However, the machine occasionally malfunctions and outputs the square of the nth prime number instead. For example, if you put in "7" the machine should output 17, the 7^{th} prime. But it might accidentally output $17^2 = 289$ instead. If the machine outputs 529, what was the value of n?

12. Let N be a positive integer such that the sum of the digits of N is equal to 22. If N is divisible by 4, what is the largest possible value of N that is less than 1000?

13. The units digit of a number is the rightmost digit of a number. For example, 3 is the units digit of 73. What is the units digit of $4^{4^{4^4}}$?

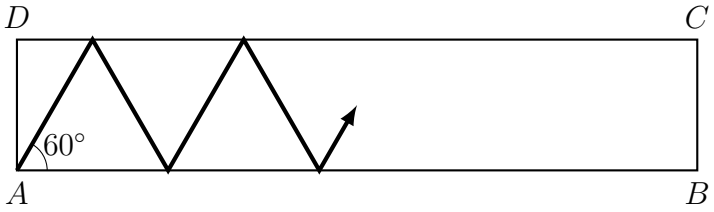
14. 10 people enter a room one at a time, 5 of whom are right-handed and 5 are left-handed. When a person first enters the room, if they are right-handed, they will shake hands with everyone who is already in the room, but if they are left-handed, they will only shake hands with the other left-handed people. The first person who enters the room is left-handed, and then the next person is right-handed, and it continues to alternate left-right-left-right until all 10 people have come into the room. Once everyone is inside, how many handshakes have occurred?

15. In a school, there are 400 students. Each student takes at least one of the following subjects: Math, Science, or English. It is known that 240 students take Math, 260 students take Science, and 280 students take English. Furthermore, 120 students take both Math and Science, 180 students take both Math and English, and 140 students take both Science and English. There are 60 students enrolled in all three. How many students take only one of those three subjects?

16. The ! operation, called the *factorial*, multiplies a given integer by all the positive integers less than it. For example, 5! is equal to $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, or 120. For 100!, starting at the units digit and moving left, how many zeros appear before the first non-zero digit is reached?

17. Given that $x^4 - y^4 = 56$, $x^2 + 2xy + y^2 = 8$, and $x^2 - 2xy + y^2 = 6$, determine the value of $x^2 - y^2$.

18. A rectangular hallway, indicated by rectangle $ABCD$ below, is lined with mirrors. A laser beam is fired from corner A at an angle of 60° and bounces down the hallway until it hits the far wall, $\overline{CD}$. Given that the length of the hallway is $AB = 17$ meters, how far did the laser beam travel?



19. In a survey, 54% of the respondents' favorite desert was pie, 32% reported that cake was their favorite, and 35 people responded that ice cream was the best desert. There were no other answers given. How many people responded that cake was their favorite desert?

20. Alice and Betty take turns rolling a fair 4-sided die. Three sides of the die are painted red and one is painted blue, and whoever gets the blue side first wins the game. If Alice rolls first, what is the probability that she will win?