7^{th} Grade Team Contest

IMSA *Mu Alpha Theta*

March 6, 2024

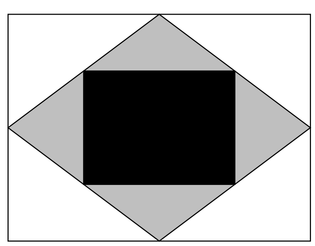
1. A parallelogram has consecutive angles $(9x + 2)^\circ$ and $(8x + 8)^\circ$. Determine the value of x.

2. Define two operations on numbers: $x \blacklozenge y = x^y$, and $x \lozenge y = x + 2y + 3$. Compute $2 \blacklozenge (1 \lozenge (3 \blacklozenge 2) \lozenge -10)$.

3. Given that x is positive and $x = \dfrac{5x}{x+1}$, compute the value of x.

4. Each of the squares on an 8×8 chessboard has a tile on it that is white on one side and either red or black on the other side. Initially all the tiles are white-side up. 10 of the tiles are red on the other side; the rest are black on the other side. If a tile is flipped over at random, what is the probability that it is black on the other side?

5. A flag is in the shape of a rectangle. The background of the flag is white. On top of the background is a blue quadrilateral (shown as gray below) whose vertices are the midpoints of the four sides of the rectangle. On top of this is a red quadrilateral (shown as black below) whose vertices are the midpoints of the four sides of the blue quadrilateral. Compute the quotient $\dfrac{\text{Area of white portion of flag}}{\text{Area of blue portion of flag}}$.



6. A *diagonal* of a polygon is a line segment that connects two non-neighboring vertices. How many diagonals does an octagon have?

7. What percentage of numbers from 1 to 100 are divisible by either 2, 3, or 5?

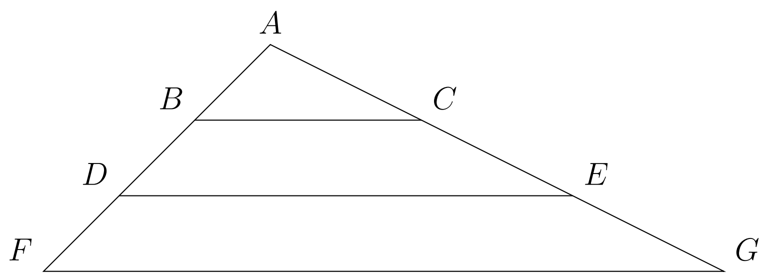
8. Fifty students took a test on which it was possible to earn integer scores anywhere from 1 to 10. Thirty-five students passed by earning 7 or higher, while the other students did not pass the test. Compute the highest possible average of the students' scores.

9. Sasha has written down a number using just the digits 1, 6, 7, and 8. No digit is used more than once, and some of the digits may not have been used at all. The number is a multiple of 3. What is the largest possible number that Sasha could have written down?

10. The sum of two positive integers x and y is 100. Their product is a multiple of 37. Compute the minimum possible value of $x - y$.

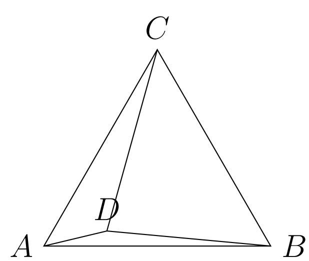
11. Champernowne's constant, C, is the number $0.12345678910111213\ldots$ which is simply writing the numbers in order after the decimal point. The 11^{th} digit to the right of the decimal point is a 0. Determine the 2024^{th} digit to the right of the decimal point.

12. In triangler AFG points B and D lie on side $\overline{AF}$ while C and E lie on side $\overline{AG}$. Segments $\overline{BC}$, $\overline{DE}$ and $\overline{FG}$ are parallel, and $AB = BD = DF$. Compute the ratio of the area of quadrilateral $BDEC$ to the area of quadrilateral $DFGE$. Write your answer as a common fraction.



13. Let $n = 2^4 3^6 5^6$. How many of the factors of n are less than $\sqrt{n}$?

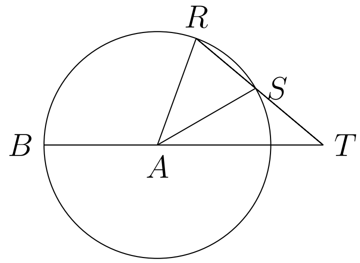
14. Sam the slug is sliding along the edges tetrahedron $ABCD$. Each time he reaches a vertex, he slides along an edge chosen at random to a different vertex, but not to the vertex he just came from. If Sam starts at vertex A, what is the probability that he reaches all the other vertices and then slides back to A within 4 moves?



15. There are six cards lying on a table. Each card has a letter on one side and a number on the other side. The visible sides of the cards show the characters A, B, E, 3, 4, and 5. What is the minimum number of cards that must be turned over to verify that the following statement is true: If a card has a vowel on one side, then it has a prime number on the other side.

16. Given that x and y are integers with $x < y$ and $2xy + 68 = 12x + 12y$, compute all possible values for x.

17. A is the center of a circle. B, R, and S are points on the circle, and the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{RS}$ meet at point T. The measure of both angles $\angle RAS$ and $\angle RTB$ are $40°$. What is the measure of minor arc $\overparen{BR}$, in degrees?



18. Alice has a collection of marbles. When she divides the marbles into three equal piles, there is one marble left over. When she divides the marbles into four equal piles, there are two left over. Given that Alice has fewer than 100 marbles, what is the largest possible number of marbles she could have?

19. Given that $\dfrac{2^{\sqrt{49-x^2}}}{2^{\sqrt{25-x^2}}} = 16$, compute the value of $2^{\sqrt{49-x^2}} \cdot 2^{\sqrt{25-x^2}}$.

20. Three tangent circles are inscribed in a larger circle as shown below. What is the ratio of the total area of the smaller circles to the area of the larger circle? Express your answer in simplest radical form.