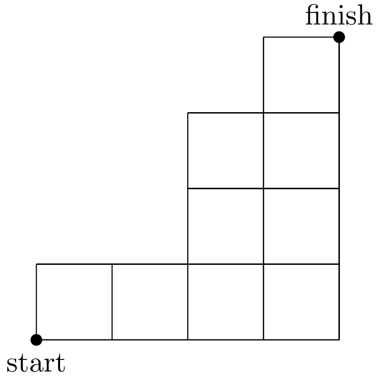
Creative Thinking

IMSA *Mu Alpha Theta*

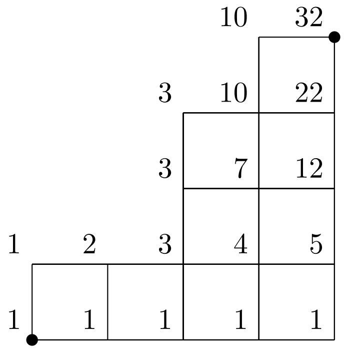
March 6, 2024

1 Taxi to the Store!

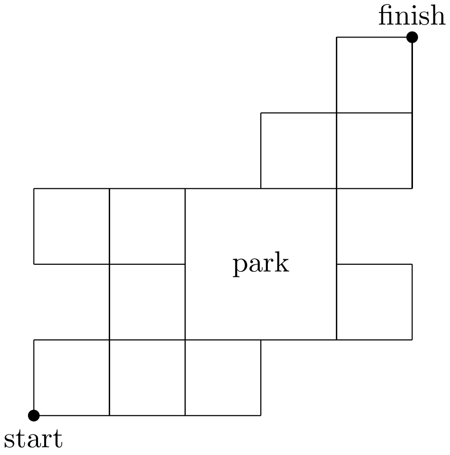
Roy lives in a city whose map is shown below. He lives at the far southwest corner (marked "start") and wishes to trvel to a store at the far northeast corner (marked "finish"). He is going to take a taxi that charges him more the farther he travels, so he only wants to travel north or east on the roads of the city.



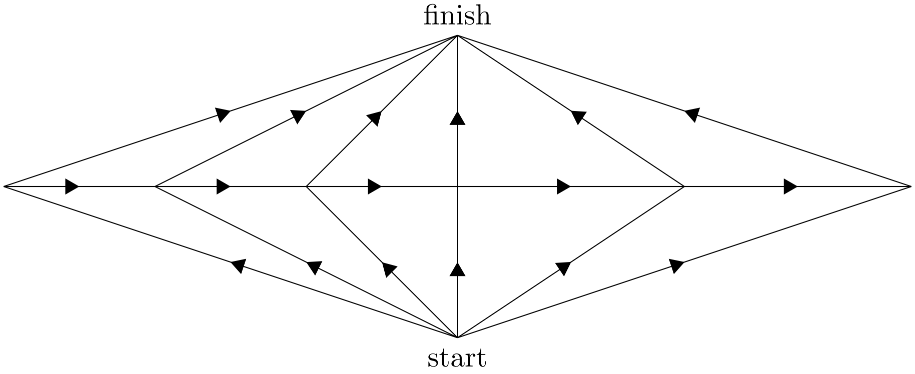
In the map below, each corner is labeled with the number different routes the taxi could take to arrive at that corner, following the rule of only traveling north or east.



a) Joy lives in a city with a different map, as seen below (top of next page), but she wants to follow the same rules of only traveling north or east to have the smallest cost of the taxi ride. Notice that in the center of the city is a park that you can't drive through. How many different paths could Joy take from start to finish?


finish
park
start

b) Troy lives in the crazy city shown below. All the streets are one-way streets that travel northward. The main east-west avenue is also only one way, going east. How many ways are there for Troy to get from start to finish?


finish
start

2 Aashima's Airports

Aashima has a helicopter that can land anywhere it wants, but can only go very specific distances in very specific directions for each flight. When she takes off, she can either go 1 mile east and 4 miles north or else she can go 2 miles east and 9 miles north before having to land.

a) If Aashima starts at airport A and wants to get to airport B, which is 9 miles east and 40 miles north of airport A, how many total flights must she take to get there?

b) Aashima adds an upgrade to her copter, and she now has the additional option of flying 3 miles west and 13 miles south. She wants to take off exactly 11 times in her trip from A to B. What is the farthest east (in miles) that she could go and still make it to B?

3 Skyelar's Magic Squares

Skyelar is playing a game with Roy and Drew, where she introduces the idea of a *magic square*. A magic square is a square array of numbers, in which each row, each column, and the two diagonals add up to the same result. For example

8	3	4
1	5	9
6	7	2

4	17	18	7
15	10	9	12
11	14	13	8
16	5	6	19

are magic squares. The 3×3 magic square's rows, columns, and diagonals each add up to 15, while the 4×4 square's rows, columns, and diagonals add up to 46. Notice that each square only uses consecutive numbers; 1–9 for the 3×3 square and 4–19 for the 4×4 square.

Skyelar now challenges her friends. If she gives them a set of rules, they need to construct a magic square that obeys those rules to win a prize.

a) If Roy was asked to construct a 3×3 magic square out of the numbers 10–18 where 10 is the entry in the bottom of the second column what would the top number in the second column be?

b) If Drew was asked to make a 4×4 magic square whose columns, diagonals, and rows added up to 150, using 16 consecutive integers, what would the smallest entry be?

4 Split Nim

Vidyoot and Arjun have been playing the game Nim with two piles of beans. Arjun always goes first. In the game of Nim, there are piles of beans, and two players take turns; when it is your turn you can take away any number (including all) of the beans from any one pile. The person who takes the last bean wins.

Playing with two piles, Vidyoot and Arjun quickly become bored. This is because if the piles start out equal in size, Arjun always loses because no matter what Arjun does, Vidyoot just does the same to the other pile, leaving two smaller equal piles. So when Arjun takes the last bean from one pile, Vidyoot takes the last bean from the other pile and wins. But if the piles start out uneven, then Arjun always wins by making his first move to take enough beans from the larger pile to make the piles equal, which forces Vidyoot to lose as in the previous scenario.

So they decide to make the game more interesting. Instead of the normal first move of taking some beans from a pile, Arjun instead takes one of the piles and splits it into two piles (they don't have to be the same size!) and then the game continues from there.

Now, for example, if they start with two piles with 6 beans each, Arjun will win by splitting one of them into a pile of 2 and a pile of 4 (convince yourself that Vidyoot will lose no matter what he does now!). But faced with a starting position with 4 beans in each pile, everything he does will allow Vidyoot to win (hint: if he splits one of the piles into two equal parts, Vidyoot will just remove all the beans from the other pile, leaving Arjun with two equal piles and a loss; if Arjun starts by splitting the pile into one pile of 3 and one pile of 1, then Vidyoot will win by taking two beans from the other pile, leaving Arjun with piles of 1, 2, and 3 beans, which you should be able to convince yourself is a losing position).

We will say that Arjun's winning move from the 6-6 start is 6-4-2. Always list the piles in decreasing order of size.

What is Arjun's winning move starting from:

a) 8-6?

b) 17-9?