8^{th} Grade Team Contest

IMSA *Mu Alpha Theta*

March 6, 2024

1. A book that originally cost \$16 is on sale for \$10. By what percent has the price of the book been reduced?

2. Trish is rowing a boat up a river, against the current. If the river is flowing at a rate of 2 mph and Trish rows at a rate of 3 mph, she will reach her destination in 90 minutes. At those rates, how long will it take Trish to row back, with the current, to her starting place?

3. To the nearest whole week, how many weeks is 2024 hours?

4. The bulb in the light in your room burns out. You go to replace it and there are six spare bulbs in the closet. Five are good bulbs, but one of them is from a cheap manufacturer; 12% of the bulbs from the cheap manufacturer don't work. When you put the new bulb in your light, what is the probability that it will work?

5. The last two digits of 1234 are "34." What are the last two digits of 2024^{2024}?

6. A patient was given 1000 mg of a medication at 2:00 p.m., and was given another 1000 mg does at 4:00 p.m. Every two hours, 80 percent of the medication in the body is used up. How many milligrams of the medication remain in the patient's body at 6:00 p.m.?

7. Max has discs of radius 2 inches, 3 inches, and 5 inches, each with height equal to its radius. Max builds a tower by placing the largest disc on the table, the middle-size disc on top of that, and then the smallest disc on top of that. How much area (in square inches) of the tower is visible? Parts of the discs that are touching each other or the table are not visible. Answer exactly.

8. Solve for x: $\sqrt[3]{x \cdot \sqrt[3]{x \cdot \sqrt[3]{x \cdot \sqrt{x}}}} = 10$.

9. On a math test, 25% of the class gets a score of 70, 10% of the class gets a score of 80, 5% of the class gets a score of 85, 50% of the class gets a score of 90, and the rest of the class gets a score of 100. What is the difference between the mean and median in scores?

10. Forgetful Felix only remembers to bring his lunch to work one day out of every ten, and has to go out for lunch the other days. Clever Clarence remembers his lunch half the

time, and only has to go out for lunch half the time. For the 20 work days in a month, what is the smallest number of days that they might both have to go out to lunch?

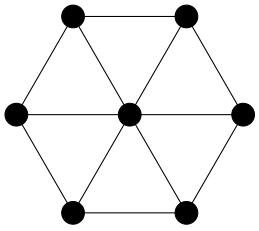
11. The graphs $|2x + 3y| = 11$ and $|3x - 2y| = 17$ taken together form the boundaries of a quadrilateral in the coordinate plane. Find the area of this quadrilateral.

12. A dairy farmer wants to build a rectangular pen for some of the cows. The farmer decides to build the pen so that one side is the wall of the barn to reduce the amount of fencing that needs to be purchased. The pen is to have an area of 200 m^2. What is the minimum amount of fencing, in meters, required to construct the pen?

13. Fievel picks a number, and then adds the five consecutive numbers starting with the number he picked. For instance, if Fievel picks 73, he would add $73+74+75+76+77 = 375$. What is the greatest number less than 1000 Fievel could pick so that his total would be divisible by 13?

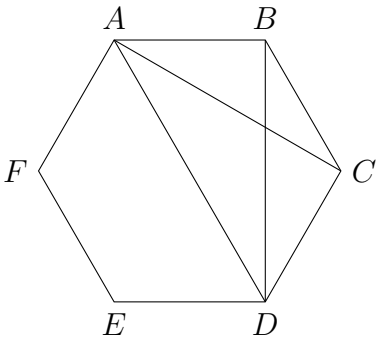
14. The perfect squares 1, 4, 9, 16, ... are put into a neverending string of digits that begins 1491625.... Determine the 23^{rd} number in this string.

15. Arjun the ant starts at the center of the figure below, which is a regular hexagon with its three long diagonals drawn through the center. Each second Arjun moves from the point he is currently at to one of the neighboring points. How many different paths are there that Arjun could travel over the first four seconds given that he ends up back at the center of the figure after four seconds?



16. Speedy Sarah and Slow Sam are painting the rooms in a building, which are all the same size. Sarah is able to paint a room in 90 minutes by herself. Working together, Sarah and Sam can paint a room in 70 minutes. How long does it take Sam to paint a room?

17. Given regular hexagon $ABCDEF$ with side length 2. Compute the difference between the area of $\triangle ABC$ and $\triangle ABD$



18. A circle of radius 4 has four circles of radius r surrounding it. The centers of these circles form a square and each of these circles is externally tangent to the original circle and to two of the other radius-r circles. Compute r.

19. Sally's sister swipes Sally's stuff. So Sally purchased a safe to keep her stuff away from her sister. Sally needs to choose a 4-digit combination for the safe. She knows her sister will try combinations with 1, 2, 0, and 7, because Sally's birthday is 12/07 and of course her sister knows that. Sally's favorite number is 9, and her sister doesn't know that, so Sally will choose a combination with at least one 9 and none of 1, 2, 0, or 7. How many different combinations could Sally use?

20. The equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ has exactly 15 ordered pairs of positive integers (x, y) of solutions. Compute the smallest possible value of z.