

3. Aufgabe 2

a)  $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$   $P_A = A(A^T A)^{-1} A^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \left[ \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$

$a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$   $a_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$   $A = QR$

$R = Q^T A = Q^T A = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$

$A = QR$

b)

$B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

$b_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$   $a_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$b_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$Q = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{3}}{3} \\ 0 & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{3} \end{pmatrix}$

$P_B = B(B^T B)^{-1} B^T = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \left[ \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \right]^{-1} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} =$

$= \frac{1}{6} \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}$

$R = Q^T B = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}$

$B = QR$