Table 1: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 1-5

			TTHZP				IHZPM				EHZPM			
Problems	Dimension	IP	Iter	Funeval	Time(s)	$\|\Phi_k\|$	Iter	Funeval	Time(s)	$\|\Phi_k\|$	Iter	Funeval	Time(s)	$\|\Phi_k\|$
1	1,000	z1	18	20	0.036761	7.25E-09	16	17	0.022648	4.54E-09	20	22	0.039549	3.91E-09
		z2	18	20	0.021058	4.69E-09	17	19	0.015737	3.34E-09	19	21	0.019494	6.52E-09
		z3	23	25	0.01702	5.41E-09	19	21	0.014624	3.95E-09	19	21	0.013438	7.03E-09
		z4	20	22	0.015559	5.71E-09	16	18	0.013967	7.39E-09	19	21	0.014209	6.84E-09
		z5	16	18	0.014585	9.67E-09	16	18	0.013136	3.47E-09	18	20	0.01365	7.62E-09
		z6	17	19	0.015243	6.3E-09	17	19	0.015253	3.3E-09	18	20	0.014263	6.51E-09
	10,000	z1	19	21	0.068664	4.21E-09	37	39	0.150035	6.87E-09	20	22	0.065675	4.37E-09
		z2	18	20	0.075766	8.03E-09	17	19	0.075087	7.52E-09	20	22	0.069855	8.94E-09
		z3	19	21	0.077525	6.09E-09	18	20	0.073019	3.83E-09	19	21	0.061777	4.81E-09
		z4	19	21	0.075728	4.45E-09	18	20	0.084838	3.14E-09	18	20	0.063899	5.85E-09
		z5	17	19	0.069937	6.03E-09	17	19	0.058019	2.91E-09	19	21	0.06157	8.46E-09
		z6	18	20	0.069688	3.7E-09	17	19	0.074412	7.76E-09	19	21	0.061944	5.54E-09
2	1,000	z1	6	8	0.006587	6.52E-10	6	8	0.006603	3.26E-09	21	23	0.018055	3.64E-09
		z2	5	7	0.007581	6.03E-09	6	8	0.007437	2.11E-10	20	22	0.017478	4.16E-09
		z3	5	7	0.005959	3E-09	6	8	0.006788	2.98E-10	19	21	0.017544	8.36E-09
		z4	5	7	0.006457	1.36E-09	6	8	0.008042	2.31E-10	19	21	0.018713	5.47E-09
		z5	4	6	0.005274	2.69E-09	4	6	0.00505	2.52E-09	16	18	0.013618	8.71E-09
		z6	5	7	0.006165	2.98E-10	8	10	0.008471	7.99E-10	19	21	0.018228	5.48E-09
	10,000	z1	6	8	0.028325	1.41E-09	5	6	0.0802929	NaN	6	7	0.1170113	NaN
		z2	6	8	0.027008	1.37E-10	6	7	0.0865156	NaN	20	22	0.103719	6.03E-09
		z3	5	7	0.02477	6.68E-09	6	7	0.0841261	NaN	19	21	0.094654	4.66E-09
		z4	5	7	0.023423	3E-09	6	8	0.031557	4.76E-10	20	22	0.100849	5.66E-09
		z5	4	6	0.02045	6.03E-09	5	7	0.022135	1.9E-09	17	19	0.086231	9.2E-09
		z6	6	8	0.030225	4.88E-09	10	12	0.040806	2.5E-10	20	22	0.094794	5.32E-09
3	1,000	z1	6	8	0.005942	2.4E-10	7	9	0.00622	4.13E-09	20	22	0.017711	5.97E-09
		z2	5	7	0.005086	2.51E-10	7	9	0.006095	3.08E-09	20	22	0.015857	5.67E-09
		z3	5	7	0.005473	2.91E-10	7	9	0.007453	3.35E-09	20	22	0.016378	4.95E-09
		z4	5	7	0.006134	2.98E-10	7	9	0.006467	3.04E-09	20	22	0.014669	4.09E-09
		z5	5	7	0.005361	1.08E-10	5	7	0.005651	5.1E-09	18	20	0.015148	8.68E-09
		z6	12	14	0.01285	6.55E-09	21	23	0.01828	3.66E-09	18	20	0.018488	3.29E-09
	10,000	z1	6	8	0.024162	7.58E-10	7	9	0.031274	4.18E-09	21	23	0.082867	5.67E-09
		z2	5	7	0.021552	7.93E-10	7	9	0.028923	6.38E-09	20	22	0.079572	6.89E-09
		z3	5	7	0.02279	9.2E-10	7	9	0.02858	8.99E-10	21	23	0.097106	6.31E-09
		z4	5	7	0.020435	9.42E-10	7	9	0.028707	8.73E-09	20	22	0.080728	7.88E-09
		z5	5	7	0.021897	3.42E-10	6	8	0.02198	6.43E-09	19	21	0.09699	9.65E-09
		z6	13	15	0.066552	3.82E-09	22	24	0.083569	4.03E-09	18	20	0.098144	9.47E-09
4	1,000	z1	6	8	0.005619	2.87E-09	7	9	0.006823	1.19E-10	20	22	0.021271	4.63E-09
		z2	5	7	0.00595	9.89E-09	6	8	0.005469	5.21E-10	20	22	0.018736	5.87E-09
		z3	5	7	0.005407	3.92E-09	6	8	0.005786	3.94E-10	20	22	0.021209	3.86E-09
		z4	5	7	0.005703	1.46E-09	6	8	0.005812	2.31E-10	19	21	0.017197	6.7E-09
		z5	4	6	0.005901	1.98E-09	4	6	0.00612	1.85E-09	16	18	0.016049	8.84E-09
		z6	6	8	0.00682	1.52E-09	10	12	0.008768	4.15E-09	19	21	0.019414	5.55E-09
	10,000	z1	6	8	0.025622	9.09E-09	8	10	0.033583	2.22E-10	21	23	0.089595	8.04E-09
		z2	6	8	0.022932	3.13E-10	6	8	0.024428	2.93E-09	21	23	0.084526	6.5E-09
		z3	6	8	0.023912	1.24E-10	6	8	0.024945	2.8E-09	20	22	0.085129	6.81E-09
		z4	5	7	0.020152	4.63E-09	6	8	0.024826	5E-09	17	19	0.071388	4.43E-09
		z5	4	6	0.017415	6.25E-09	5	7	0.02025	1.95E-09	17	19	0.07388	9.76E-09
		z6	6	8	0.025838	4.79E-09	11	13	0.037641	1.13E-10	20	22	0.080899	5.62E-09
5	1,000	z1	7	9	0.00676	1.54E-09	11	13	0.007697	6.09E-09	20	22	0.015367	3.91E-09
		z2	6	8	0.006472	1.5E-09	10	12	0.008317	5.67E-09	19	21	0.015108	6.52E-09
		z3	6	8	0.006201	1.57E-09	10	12	0.008339	3.42E-09	19	21	0.014058	7.03E-09
		z4	6	8	0.005654	1.58E-09	10	12	0.007987	4.6E-09	19	21	0.015544	6.84E-09
		z5	5	7	0.006229	1.57E-09	9	11	0.007522	4.95E-09	18	20	0.014218	7.62E-09
		z6	5	7	0.005704	2.27E-10	5	7	0.005422	5.77E-09	18	20	0.012952	6.51E-09
	10,000	z1	7	9	0.027813	4.86E-09	12	14	0.039537	1.7E-10	20	22	0.069167	4.37E-09
		z2	6	8	0.022781	4.74E-09	11	13	0.037809	1.57E-10	20	22	0.064303	8.94E-09
		z3	6	8	0.021258	4.97E-09	11	13	0.033413	1.3E-10	19	21	0.062573	4.81E-09
		z4	6	8	0.02195	4.99E-09	11	13	0.034558	1.27E-10	18	20	0.059762	5.85E-09
		z5	5	7	0.018217	4.98E-09	10	12	0.030144	1.53E-10	19	21	0.06161	8.46E-09
		z6	5	7	0.017739	7.19E-10	6	8	0.020381	1.12E-10	19	21	0.061574	5.54E-09

Table 2: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 6-10

			TTHZP				IHZPM				EHZPM			
Problems	Dimension	IP	Iter	Funeval	Time(s)	$\|\Phi_k\|$	Iter	Funeval	Time(s)	$\|\Phi_k\|$	Iter	Funeval	Time(s)	$\|\Phi_k\|$
6	1,000	z1	8	10	0.012043	9.74E-09	17	19	0.019679	8.73E-09	16	18	0.021584	4.96E-09
		z2	12	14	0.016164	4.6E-09	18	20	0.020792	9.61E-09	16	18	0.019559	5.33E-09
		z3	12	14	0.015438	2.09E-09	18	20	0.019869	8.83E-09	14	16	0.017468	3.33E-09
		z4	11	13	0.01453	3.88E-09	18	20	0.021325	3.8E-09	14	16	0.019084	3.83E-09
		z5	11	13	0.019421	2.14E-09	17	19	0.019787	9.85E-09	14	16	0.018743	4.62E-09
		z6	11	13	0.016568	8.52E-09	19	21	0.022795	2.91E-09	15	17	0.019811	2.41E-09
	10,000	z1	10	12	0.068632	5.02E-09	15	17	0.096181	5.21E-09	17	19	0.117131	6.27E-09
		z2	11	13	0.066346	2.46E-09	15	17	0.088532	3.35E-09	17	19	0.11579	9.41E-09
		z3	10	12	0.062165	6.81E-09	13	15	0.074248	3.21E-09	15	17	0.105101	9.16E-09
		z4	10	12	0.068928	2.04E-09	14	16	0.079345	2.98E-09	15	17	0.098544	7.04E-09
		z5	9	11	0.062373	6.68E-09	15	17	0.085929	5.87E-09	14	16	0.092003	9.78E-09
		z6	10	12	0.061319	4.53E-09	15	17	0.084891	6.17E-09	16	18	0.109527	8.97E-09
7	1,000	z1	8	10	0.010557	2.91E-09	9	11	0.01233	5.05E-10	21	23	0.025551	6.53E-09
		z2	8	10	0.010211	3.69E-09	10	12	0.014542	4.39E-09	19	21	0.027144	4.95E-09
		z3	8	10	0.009844	3.89E-09	11	13	0.01505	1.84E-10	21	23	0.023852	7.66E-09
		z4	8	10	0.011307	4.09E-09	11	13	0.013188	3.71E-10	21	23	0.029021	5.6E-09
		z5	8	10	0.009348	4.72E-09	12	14	0.015335	4.02E-10	21	23	0.025973	8.92E-09
		z6	8	10	0.008962	5.48E-09	12	14	0.016211	1.53E-10	20	22	0.027174	6.02E-09
	10,000	z1	7	9	0.040889	6.7E-10	6	7	0.1132785	NaN	21	23	0.127687	4.23E-09
		z2	7	9	0.040382	8.54E-10	6	7	0.1087542	NaN	20	22	0.128844	6.96E-09
		z3	7	9	0.038999	9.02E-10	6	7	0.1087542	NaN	22	24	0.150906	3.76E-09
		z4	7	9	0.038577	9.5E-10	5	6	0.107879	NaN	21	23	0.183199	5.79E-09
		z5	7	9	0.037446	1.1E-09	5	6	0.1037079	NaN	7	8	0.1411949	NaN
		z6	7	9	0.040569	1.28E-09	5	6	0.1046396	NaN	6	7	0.1133257	NaN
8	1,000	z1	14	16	0.013145	2.53E-09	19	21	0.015142	4.51E-09	16	18	0.015983	7.33E-09
		z2	13	15	0.011644	3.02E-09	18	20	0.015376	5.13E-09	15	17	0.014381	6.78E-09
		z3	11	13	0.010263	6.73E-09	16	18	0.013695	4.76E-09	13	15	0.01355	9.43E-09
		z4	13	15	0.011276	2.74E-09	18	20	0.015435	4.45E-09	15	17	0.012986	5.92E-09
		z5	14	16	0.012295	3.22E-09	19	21	0.014737	6.93E-09	16	18	0.015932	8.16E-09
		z6	14	16	0.013548	8.16E-09	16	18	0.012236	8.85E-09	14	16	0.014473	8.64E-09
	10,000	z1	14	16	0.058877	8.01E-09	17	19	0.071658	4.69E-09	17	19	0.085409	3.39E-09
		z2	13	15	0.053878	9.55E-09	19	21	0.068069	5.03E-09	16	18	0.086833	5.8E-09
		z3	12	14	0.055133	4.62E-09	17	19	0.064957	5.01E-09	14	16	0.075968	8.11E-09
		z4	13	15	0.05626	8.66E-09	19	21	0.073393	4.57E-09	16	18	0.087378	5.08E-09
		z5	15	17	0.068114	2.21E-09	20	22	0.073496	6.38E-09	17	19	0.09017	3.34E-09
		z6	15	17	0.066863	5.6E-09	19	21	0.073209	7.47E-09	15	17	0.078666	4.64E-09
9	1,000	z1	14	16	0.020469	3.38E-09	22	24	0.03124	5.1E-09	17	19	0.031742	5.11E-09
		z2	13	15	0.023402	3.18E-09	21	23	0.031896	4.34E-09	18	20	0.030163	8.28E-09
		z3	12	14	0.021335	7.92E-09	21	23	0.029192	3.78E-09	18	20	0.033684	5.24E-09
		z4	12	14	0.018372	1.94E-09	20	22	0.028301	7.45E-09	17	19	0.02955	9.93E-09
		z5	12	14	0.019083	3.64E-09	20	22	0.025039	4.94E-09	17	19	0.027181	5.31E-09
		z6	11	13	0.017712	5.97E-09	20	22	0.028376	6.04E-09	17	19	0.024195	6.41E-09
	10,000	z1	15	17	0.116284	1.97E-09	21	23	0.182182	9.75E-09	20	22	0.192527	4.8E-09
		z2	14	16	0.10961	1.86E-09	22	24	0.155455	7.99E-09	19	21	0.192352	6.74E-09
		z3	13	15	0.100289	4.63E-09	22	24	0.191745	5.27E-09	18	20	0.167451	5.77E-09
		z4	12	14	0.111041	6.15E-09	22	24	0.153829	3.69E-09	18	20	0.175169	9.66E-09
		z5	13	15	0.165186	2.12E-09	21	23	0.149497	5.47E-09	18	20	0.172095	5.03E-09
		z6	12	14	0.096057	3.48E-09	20	22	0.146935	3.96E-09	18	20	0.172834	4.93E-09
10	1,000	z2	14	16	0.014524	2.28E-09	21	23	0.023109	4.45E-09	18	20	0.021997	6.12E-09
		z3	13	15	0.015234	2.04E-09	22	24	0.022636	7.02E-09	18	20	0.021251	4.32E-09
		z4	14	16	0.014873	1.91E-09	20	22	0.019614	9.01E-09	19	21	0.020221	4.12E-09
		z5	13	15	0.013688	6.31E-09	21	23	0.02173	8.92E-09	19	21	0.024237	3.68E-09
		z6	13	15	0.014566	2.77E-09	19	21	0.02147	8.13E-09	17	19	0.020174	3.26E-09
	10,000	z2	14	16	0.113189	7.2E-09	5	6	0.0269789	NaN	17	19	0.1052	9.63E-09
		z3	13	15	0.072997	6.45E-09	6	7	0.0363439	NaN	18	20	0.107023	9.26E-09
		z4	14	16	0.085927	6.04E-09	6	7	0.0377671	NaN	19	21	0.118409	4.78E-09
		z5	14	16	0.060437	3.68E-09	22	24	0.151209	6.34E-09	19	21	0.118739	4.35E-09
		z6	13	15	0.063288	8.77E-09	20	22	0.093252	4.79E-09	17	19	0.106091	7.05E-09

Table 3: TTHZP methods for problems 3, 7 & 8 with IP that are far from the solutions

TTHZP						
Problems	Dimension	IP	Iter	Funeval	Time(s)	Norm
3	1,000,000	1,000,000	10	12	5.39215	3.45E-09
		2,000,000	10	12	5.82631	3.36E-09
		3,000,000	10	12	6.03510	8.71E-09
		4,000,000	10	12	6.14975	9.48E-09
		5,000,000	10	12	6.12519	5.04E-09
		6,000,000	10	12	6.39293	9.19E-09
		7,000,000	10	12	6.39766	6.72E-09
		8,000,000	10	12	6.58383	9.47E-09
		9,000,000	11	13	6.85648	3.15E-09
		10,000,000	11	13	6.98976	2.77E-09
7	1,000,000	1,000,000	12	14	5.96538	9.24E-09
		2,000,000	13	15	6.45715	1.88E-10
		3,000,000	13	15	6.47056	6.07E-10
		4,000,000	13	15	6.40774	6.29E-10
		5,000,000	13	15	6.41550	7.80E-10
		6,000,000	13	15	6.52297	7.96E-10
		7,000,000	13	15	6.87300	7.35E-10
		8,000,000	14	16	7.08038	5.44E-10
		9,000,000	14	16	7.04773	5.25E-10
		10,000,000	14	16	7.01074	3.63E-10
8	1,000,000	1,000,000	10	12	5.84685	4.67E-10
		2,000,000	11	13	5.99018	5.28E-10
		3,000,000	11	13	6.12070	6.15E-10
		4,000,000	10	12	5.60422	6.23E-10
		5,000,000	10	12	5.53956	5.85E-10
		6,000,000	10	12	5.34310	6.26E-10
		7,000,000	10	12	5.71980	4.94E-10
		8,000,000	11	13	5.99898	5.26E-10
		9,000,000	11	13	5.91940	3.85E-10
		10,000,000	11	13	5.86425	4.90E-10

Tables 1 and 2 present detailed results of the experiments conducted by displaying initial starting point "IP", number of iterations "Iter", number of function evaluations "Funeval", processing time "Time (s)", and $\|\Phi_k\|$ is the norm of the residual at the termination point. The method is said to have failed when the machine reads the norm of the residual at the termination point as NaN (not a number). A careful analysis of Tables 1– 2 reveals that the TTHZP method outperforms the IHZPM and EHZPM methods since the former solved all of the problems in the experiments and the latter failed to solve some of the problems during the iteration process by clear indication from the tables. Furthermore, because it has fewer iterations, the TTHZP method converges to the solution faster than the IHZPM and EHZPM methods. Indeed, the proposed method outperforms the compared methods since it takes less processing time, iterations, and function evaluations.

In Table 3, we choose Problems 3, 7, and 8 to demonstrate the behavior of the TTHZP method. We take ten different initial points with the dimension $1,000,000$, which are far from the solutions and efficiently converge to the solutions of (1). However, more iterations are required when the starting point is further away from the solution. Therefore, the proposed method is competitive for solving large-scale problems.