

Table 1: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 1-5

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
1	1,000	z1	18	20	0.036761	7.25E-09	16	17	0.022648	4.54E-09	20	22	0.039549	3.91E-09
		z2	18	20	0.021058	4.69E-09	17	19	0.015737	3.34E-09	19	21	0.019494	6.52E-09
		z3	23	25	0.01702	5.41E-09	19	21	0.014624	3.95E-09	19	21	0.013438	7.03E-09
		z4	20	22	0.015559	5.71E-09	16	18	0.013967	7.39E-09	19	21	0.014209	6.84E-09
		z5	16	18	0.014585	9.67E-09	16	18	0.013136	3.47E-09	18	20	0.01365	7.62E-09
		z6	17	19	0.015243	6.3E-09	17	19	0.015253	3.3E-09	18	20	0.014263	6.51E-09
	10,000	z1	19	21	0.068664	4.21E-09	37	39	0.150035	6.87E-09	20	22	0.065675	4.37E-09
		z2	18	20	0.075766	8.03E-09	17	19	0.075087	7.52E-09	20	22	0.069855	8.94E-09
		z3	19	21	0.077525	6.09E-09	18	20	0.073019	3.83E-09	19	21	0.061777	4.81E-09
		z4	19	21	0.075728	4.45E-09	18	20	0.084838	3.14E-09	18	20	0.063899	5.85E-09
		z5	17	19	0.069937	6.03E-09	17	19	0.058019	2.91E-09	19	21	0.06157	8.46E-09
		z6	18	20	0.069688	3.7E-09	17	19	0.074412	7.76E-09	19	21	0.061944	5.54E-09
2	1,000	z1	6	8	0.006587	6.52E-10	6	8	0.006603	3.26E-09	21	23	0.018055	3.64E-09
		z2	5	7	0.007581	6.03E-09	6	8	0.007437	2.11E-10	20	22	0.017478	4.16E-09
		z3	5	7	0.005959	3E-09	6	8	0.006788	2.98E-10	19	21	0.017544	8.36E-09
		z4	5	7	0.006457	1.36E-09	6	8	0.008042	2.31E-10	19	21	0.018713	5.47E-09
		z5	4	6	0.005274	2.69E-09	4	6	0.00505	2.52E-09	16	18	0.013618	8.71E-09
		z6	5	7	0.006165	2.98E-10	8	10	0.008471	7.99E-10	19	21	0.018228	5.48E-09
	10,000	z1	6	8	0.028325	1.41E-09	-	-	-	-	-	-	-	-
		z2	6	8	0.027008	1.37E-10	-	-	-	-	20	22	0.103719	6.03E-09
		z3	5	7	0.02477	6.68E-09	-	-	-	-	19	21	0.094654	4.66E-09
		z4	5	7	0.023423	3E-09	6	8	0.031557	4.76E-10	20	22	0.100849	5.66E-09
		z5	4	6	0.02045	6.03E-09	5	7	0.022135	1.9E-09	17	19	0.086231	9.2E-09
		z6	6	8	0.030225	4.88E-09	10	12	0.040806	2.5E-10	20	22	0.094794	5.32E-09
3	1,000	z1	6	8	0.005942	2.4E-10	7	9	0.00622	4.13E-09	20	22	0.017711	5.97E-09
		z2	5	7	0.005086	2.51E-10	7	9	0.006095	3.08E-09	20	22	0.015857	5.67E-09
		z3	5	7	0.005473	2.91E-10	7	9	0.007453	3.35E-09	20	22	0.016378	4.95E-09
		z4	5	7	0.006134	2.98E-10	7	9	0.006467	3.04E-09	20	22	0.014669	4.09E-09
		z5	5	7	0.005361	1.08E-10	5	7	0.005651	5.1E-09	18	20	0.015148	8.68E-09
		z6	12	14	0.01285	6.55E-09	21	23	0.01828	3.66E-09	18	20	0.018488	3.29E-09
	10,000	z1	6	8	0.024162	7.58E-10	7	9	0.031274	4.18E-09	21	23	0.082867	5.67E-09
		z2	5	7	0.021552	7.93E-10	7	9	0.028923	6.38E-09	20	22	0.079572	6.89E-09
		z3	5	7	0.02279	9.2E-10	7	9	0.02858	8.99E-10	21	23	0.097106	6.31E-09
		z4	5	7	0.020435	9.42E-10	7	9	0.028707	8.73E-09	20	22	0.080728	7.88E-09
		z5	5	7	0.021897	3.42E-10	6	8	0.02198	6.43E-09	19	21	0.09699	9.65E-09
		z6	13	15	0.066552	3.82E-09	22	24	0.083569	4.03E-09	18	20	0.098144	9.47E-09
4	1,000	z1	6	8	0.005619	2.87E-09	7	9	0.006823	1.19E-10	20	22	0.021271	4.63E-09
		z2	5	7	0.00595	9.89E-09	6	8	0.005469	5.21E-10	20	22	0.018736	5.87E-09
		z3	5	7	0.005407	3.92E-09	6	8	0.005786	3.94E-10	20	22	0.021209	3.86E-09
		z4	5	7	0.005703	1.46E-09	6	8	0.005812	2.31E-10	19	21	0.017197	6.7E-09
		z5	4	6	0.005901	1.98E-09	4	6	0.00612	1.85E-09	16	18	0.016049	8.84E-09
		z6	6	8	0.00682	1.52E-09	10	12	0.008768	4.15E-09	19	21	0.019414	5.55E-09
	10,000	z1	6	8	0.025622	9.09E-09	8	10	0.033583	2.22E-10	21	23	0.089595	8.04E-09
		z2	6	8	0.022932	3.13E-10	6	8	0.024428	2.93E-09	21	23	0.084526	6.5E-09
		z3	6	8	0.023912	1.24E-10	6	8	0.024945	2.8E-09	20	22	0.085129	6.81E-09
		z4	5	7	0.020152	4.63E-09	6	8	0.024826	5E-09	17	19	0.071388	4.43E-09
		z5	4	6	0.017415	6.25E-09	5	7	0.02025	1.95E-09	17	19	0.07388	9.76E-09
		z6	6	8	0.025838	4.79E-09	11	13	0.037641	1.13E-10	20	22	0.080899	5.62E-09
5	1,000	z1	7	9	0.00676	1.54E-09	11	13	0.007697	6.09E-09	20	22	0.015367	3.91E-09
		z2	6	8	0.006472	1.5E-09	10	12	0.008317	5.67E-09	19	21	0.015108	6.52E-09
		z3	6	8	0.006201	1.57E-09	10	12	0.008339	3.42E-09	19	21	0.014058	7.03E-09
		z4	6	8	0.005654	1.58E-09	10	12	0.007987	4.6E-09	19	21	0.015544	6.84E-09
		z5	5	7	0.006229	1.57E-09	9	11	0.007522	4.95E-09	18	20	0.014218	7.62E-09
		z6	5	7	0.005704	2.27E-10	5	7	0.005422	5.77E-09	18	20	0.012952	6.51E-09
	10,000	z1	7	9	0.027813	4.86E-09	12	14	0.039537	1.7E-10	20	22	0.069167	4.37E-09
		z2	6	8	0.022781	4.74E-09	11	13	0.037809	1.57E-10	20	22	0.064303	8.94E-09
		z3	6	8	0.021258	4.97E-09	11	13	0.033413	1.3E-10	19	21	0.062573	4.81E-09
		z4	6	8	0.02195	4.99E-09	11	13	0.034558	1.27E-10	18	20	0.059762	5.85E-09
		z5	5	7	0.018217	4.98E-09	10	12	0.030144	1.53E-10	19	21	0.06161	8.46E-09
		z6	5	7	0.017739	7.19E-10	6	8	0.020381	1.12E-10	19	21	0.061574	5.54E-09

Table 2: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 6-10

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
6	1,000	z1	8	10	0.012043	9.74E-09	17	19	0.019679	8.73E-09	16	18	0.021584	4.96E-09
		z2	12	14	0.016164	4.6E-09	18	20	0.020792	9.61E-09	16	18	0.019559	5.33E-09
		z3	12	14	0.015438	2.09E-09	18	20	0.019869	8.83E-09	14	16	0.017468	3.33E-09
		z4	11	13	0.01453	3.88E-09	18	20	0.021325	3.8E-09	14	16	0.019084	3.83E-09
		z5	11	13	0.019421	2.14E-09	17	19	0.019787	9.85E-09	14	16	0.018743	4.62E-09
		z6	11	13	0.016568	8.52E-09	19	21	0.022795	2.91E-09	15	17	0.019811	2.41E-09
	10,000	z1	10	12	0.068632	5.02E-09	15	17	0.096181	5.21E-09	17	19	0.117131	6.27E-09
		z2	11	13	0.066346	2.46E-09	15	17	0.088532	3.35E-09	17	19	0.11579	9.41E-09
		z3	10	12	0.062165	6.81E-09	13	15	0.074248	3.21E-09	15	17	0.105101	9.16E-09
		z4	10	12	0.068928	2.04E-09	14	16	0.079345	2.98E-09	15	17	0.098544	7.04E-09
		z5	9	11	0.062373	6.68E-09	15	17	0.085929	5.87E-09	14	16	0.092003	9.78E-09
		z6	10	12	0.061319	4.53E-09	15	17	0.084891	6.17E-09	16	18	0.109527	8.97E-09
7	1,000	z1	8	10	0.010557	2.91E-09	9	11	0.01233	5.05E-10	21	23	0.025551	6.53E-09
		z2	8	10	0.010211	3.69E-09	10	12	0.014542	4.39E-09	19	21	0.027144	4.95E-09
		z3	8	10	0.009844	3.89E-09	11	13	0.01505	1.84E-10	21	23	0.023852	7.66E-09
		z4	8	10	0.011307	4.09E-09	11	13	0.013188	3.71E-10	21	23	0.029021	5.6E-09
		z5	8	10	0.009348	4.72E-09	12	14	0.015335	4.02E-10	21	23	0.025973	8.92E-09
		z6	8	10	0.008962	5.48E-09	12	14	0.016211	1.53E-10	20	22	0.027174	6.02E-09
	10,000	z1	7	9	0.040889	6.7E-10	–	–	–	–	21	23	0.127687	4.23E-09
		z2	7	9	0.040382	8.54E-10	–	–	–	–	20	22	0.128844	6.96E-09
		z3	7	9	0.038999	9.02E-10	–	–	–	–	22	24	0.150906	3.76E-09
		z4	7	9	0.038577	9.5E-10	–	–	–	–	21	23	0.183199	5.79E-09
		z5	7	9	0.037446	1.1E-09	–	–	–	–	–	–	–	–
		z6	7	9	0.040569	1.28E-09	–	–	–	–	–	–	–	–
8	1,000	z1	14	16	0.013145	2.53E-09	19	21	0.015142	4.51E-09	16	18	0.015983	7.33E-09
		z2	13	15	0.011644	3.02E-09	18	20	0.015376	5.13E-09	15	17	0.014381	6.78E-09
		z3	11	13	0.010263	6.73E-09	16	18	0.013695	4.76E-09	13	15	0.01355	9.43E-09
		z4	13	15	0.011276	2.74E-09	18	20	0.015435	4.45E-09	15	17	0.012986	5.92E-09
		z5	14	16	0.012295	3.22E-09	19	21	0.014737	6.93E-09	16	18	0.015932	8.16E-09
		z6	14	16	0.013548	8.16E-09	16	18	0.012236	8.85E-09	14	16	0.014473	8.64E-09
	10,000	z1	14	16	0.058877	8.01E-09	17	19	0.071658	4.69E-09	17	19	0.085409	3.39E-09
		z2	13	15	0.053878	9.55E-09	19	21	0.068069	5.03E-09	16	18	0.086833	5.8E-09
		z3	12	14	0.055133	4.62E-09	17	19	0.064957	5.01E-09	14	16	0.075968	8.11E-09
		z4	13	15	0.05626	8.66E-09	19	21	0.073393	4.57E-09	16	18	0.087378	5.08E-09
		z5	15	17	0.068114	2.21E-09	20	22	0.073496	6.38E-09	17	19	0.09017	3.34E-09
		z6	15	17	0.066863	5.6E-09	19	21	0.073209	7.47E-09	15	17	0.078666	4.64E-09
9	1,000	z1	14	16	0.020469	3.38E-09	22	24	0.03124	5.1E-09	17	19	0.031742	5.11E-09
		z2	13	15	0.023402	3.18E-09	21	23	0.031896	4.34E-09	18	20	0.030163	8.28E-09
		z3	12	14	0.021335	7.92E-09	21	23	0.029192	3.78E-09	18	20	0.033684	5.24E-09
		z4	12	14	0.018372	1.94E-09	20	22	0.028301	7.45E-09	17	19	0.02955	9.93E-09
		z5	12	14	0.019083	3.64E-09	20	22	0.025039	4.94E-09	17	19	0.027181	5.31E-09
		z6	11	13	0.017712	5.97E-09	20	22	0.028376	6.04E-09	17	19	0.024195	6.41E-09
	10,000	z1	15	17	0.116284	1.97E-09	21	23	0.182182	9.75E-09	20	22	0.192527	4.8E-09
		z2	14	16	0.10961	1.86E-09	22	24	0.155455	7.99E-09	19	21	0.192352	6.74E-09
		z3	13	15	0.100289	4.63E-09	22	24	0.191745	5.27E-09	18	20	0.167451	5.77E-09
		z4	12	14	0.111041	6.15E-09	22	24	0.153829	3.69E-09	18	20	0.175169	9.66E-09
		z5	13	15	0.165186	2.12E-09	21	23	0.149497	5.47E-09	18	20	0.172095	5.03E-09
		z6	12	14	0.096057	3.48E-09	20	22	0.146935	3.96E-09	18	20	0.172834	4.93E-09
10	1,000	z2	14	16	0.014524	2.28E-09	21	23	0.023109	4.45E-09	18	20	0.021997	6.12E-09
		z3	13	15	0.015234	2.04E-09	22	24	0.022636	7.02E-09	18	20	0.021251	4.32E-09
		z4	14	16	0.014873	1.91E-09	20	22	0.019614	9.01E-09	19	21	0.020221	4.12E-09
		z5	13	15	0.013688	6.31E-09	21	23	0.02173	8.92E-09	19	21	0.024237	3.68E-09
		z6	13	15	0.014566	2.77E-09	19	21	0.02147	8.13E-09	17	19	0.020174	3.26E-09
		z2	14	16	0.113189	7.2E-09	–	–	–	–	17	19	0.1052	9.63E-09
	10,000	z3	13	15	0.072997	6.45E-09	–	–	–	–	18	20	0.107023	9.26E-09
		z4	14	16	0.085927	6.04E-09	–	–	–	–	19	21	0.118409	4.78E-09
		z5	14	16	0.060437	3.68E-09	22	24	0.151209	6.34E-09	19	21	0.118739	4.35E-09
		z6	13	15	0.063288	8.77E-09	20	22	0.093252	4.79E-09	17	19	0.106091	7.05E-09