

Table 1: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 1 & 2

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
1	1,000	z1	18	20	0.036761	7.25E-09	16	17	0.022648	4.54E-09	20	22	0.039549	3.91E-09
		z2	18	20	0.021058	4.69E-09	17	19	0.015737	3.34E-09	19	21	0.019494	6.52E-09
		z3	23	25	0.01702	5.41E-09	19	21	0.014624	3.95E-09	19	21	0.013438	7.03E-09
		z4	20	22	0.015559	5.71E-09	16	18	0.013967	7.39E-09	19	21	0.014209	6.84E-09
		z5	16	18	0.014585	9.67E-09	16	18	0.013136	3.47E-09	18	20	0.01365	7.62E-09
		z6	17	19	0.016183	9.95E-09	18	20	0.016541	6.91E-09	18	20	0.013137	8.69E-09
		z7	17	19	0.015243	6.3E-09	17	19	0.015253	3.3E-09	18	20	0.014263	6.51E-09
		z8	16	18	0.0154	4.67E-09	15	17	0.013126	6.33E-09	18	20	0.012781	4.07E-09
		z9	17	19	0.016007	7.87E-09	17	19	0.014944	4.5E-09	19	21	0.013411	4.07E-09
		z10	14	16	0.012724	8.36E-09	14	16	0.012692	4.65E-09	16	18	0.012463	7.12E-09
	10,000	z1	19	21	0.068664	4.21E-09	37	39	0.150035	6.87E-09	20	22	0.065675	4.37E-09
		z2	18	20	0.075766	8.03E-09	17	19	0.075087	7.52E-09	20	22	0.069855	8.94E-09
		z3	19	21	0.077525	6.09E-09	18	20	0.073019	3.83E-09	19	21	0.061777	4.81E-09
		z4	19	21	0.075728	4.45E-09	18	20	0.084838	3.14E-09	18	20	0.063899	5.85E-09
		z5	17	19	0.069937	6.03E-09	17	19	0.058019	2.91E-09	19	21	0.06157	8.46E-09
		z6	18	20	0.068096	5.73E-09	20	22	0.085963	6.25E-09	19	21	0.07183	4.51E-09
		z7	18	20	0.069688	3.7E-09	17	19	0.074412	7.76E-09	19	21	0.061944	5.54E-09
		z8	16	18	0.063916	9.54E-09	16	18	0.068354	5.71E-09	19	21	0.060545	4.53E-09
		z9	18	20	0.070141	4.56E-09	17	19	0.073624	5.35E-09	19	21	0.062038	4.89E-09
		z10	15	17	0.059065	5.18E-09	15	17	0.060293	4.22E-09	17	19	0.068867	7.93E-09
	100,000	z1	19	21	0.493843	6.86E-09	—	—	—	—	—	—	—	—
		z2	19	21	0.526418	4.77E-09	—	—	—	—	21	23	0.470643	5.41E-09
		z3	20	22	0.491625	7.7E-09	—	—	—	—	21	23	0.459705	3.85E-09
		z4	19	21	0.474267	4.91E-09	—	—	—	—	20	22	0.441913	5.67E-09
		z5	18	20	0.445452	4.05E-09	18	20	0.440851	3.75E-09	20	22	0.401647	7.81E-09
		z6	19	21	0.469903	3.34E-09	—	—	—	—	—	—	—	—
		z7	18	20	0.4556	7.29E-09	25	27	0.690836	7.58E-09	20	22	0.415227	6.46E-09
		z8	17	19	0.428587	5.65E-09	16	18	0.40087	9.98E-09	20	22	0.40307	5E-09
		z9	18	20	0.45313	9.26E-09	—	—	—	—	21	23	0.458229	4.74E-09
		z10	15	17	0.388235	9E-09	16	18	0.406762	3.83E-09	18	20	0.376073	8.82E-09
2	1,000	z1	6	8	0.006587	6.52E-10	6	8	0.006603	3.26E-09	21	23	0.018055	3.64E-09
		z2	5	7	0.007581	6.03E-09	6	8	0.007437	2.11E-10	20	22	0.017478	4.16E-09
		z3	5	7	0.005959	3E-09	6	8	0.006788	2.98E-10	19	21	0.017544	8.36E-09
		z4	5	7	0.006457	1.36E-09	6	8	0.008042	2.31E-10	19	21	0.018713	5.47E-09
		z5	4	6	0.005274	2.69E-09	4	6	0.00505	2.52E-09	16	18	0.013618	8.71E-09
		z6	6	8	0.007447	2.26E-10	6	8	0.0068	1.1E-09	20	22	0.01882	7.05E-09
		z7	5	7	0.006165	2.98E-10	8	10	0.008471	7.99E-10	19	21	0.018228	5.48E-09
		z8	4	6	0.006526	6.14E-10	4	6	0.005305	6.11E-10	15	17	0.015545	6.47E-09
		z9	6	8	0.006733	2.36E-09	8	10	0.008612	8.68E-10	19	21	0.015714	6.23E-09
		z10	4	6	0.006213	1.35E-09	5	7	0.006023	6.88E-10	16	18	0.015788	7.14E-09
	10,000	z1	6	8	0.028325	1.41E-09	—	—	—	—	—	—	—	—
		z2	6	8	0.027008	1.37E-10	—	—	—	—	20	22	0.103719	6.03E-09
		z3	5	7	0.02477	6.68E-09	—	—	—	—	19	21	0.094654	4.66E-09
		z4	5	7	0.023423	3E-09	6	8	0.031557	4.76E-10	20	22	0.100849	5.66E-09
		z5	4	6	0.02045	6.03E-09	5	7	0.022135	1.9E-09	17	19	0.086231	9.2E-09
		z6	6	8	0.026568	4.78E-10	—	—	—	—	—	—	—	—
		z7	6	8	0.030225	4.88E-09	10	12	0.040806	2.5E-10	20	22	0.094794	5.32E-09
		z8	4	6	0.019762	1.34E-09	5	7	0.023957	5.22E-10	16	18	0.080683	6.78E-09
		z9	6	8	0.032765	5.05E-09	10	12	0.042314	3.74E-10	20	22	0.103286	4.03E-09
		z10	4	6	0.021618	2.99E-09	7	9	0.031875	2.72E-10	17	19	0.089897	7.74E-09
	100,000	z1	6	8	0.174608	4.27E-09	—	—	—	—	—	—	—	—
		z2	6	8	0.171625	4.16E-10	—	—	—	—	—	—	—	—
		z3	6	8	0.176654	2.04E-10	—	—	—	—	—	—	—	—
		z4	5	7	0.153172	9.14E-09	—	—	—	—	—	—	—	—
		z5	5	7	0.154385	1.84E-10	5	7	0.168487	4.49E-09	17	19	0.557022	4.71E-09
		z6	6	8	0.181897	1.45E-09	—	—	—	—	—	—	—	—
		z7	7	9	0.215196	1.48E-10	8	10	0.291893	4.13E-10	20	22	0.647656	6.3E-09
		z8	4	6	0.126974	4.07E-09	5	7	0.140061	2.57E-10	17	19	0.542378	7.06E-09
		z9	7	9	0.222925	1.53E-10	8	10	0.287844	1.85E-10	21	23	0.734054	7.15E-09
		z10	4	6	0.139459	9.1E-09	8	10	0.223636	3.39E-10	18	20	0.565397	8.59E-09

Table 2: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 3 & 4

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
3	1,000	z1	6	8	0.005942	2.4E-10	7	9	0.00622	4.13E-09	20	22	0.017711	5.97E-09
		z2	5	7	0.005086	2.51E-10	7	9	0.006095	3.08E-09	20	22	0.015857	5.67E-09
		z3	5	7	0.005473	2.91E-10	7	9	0.007453	3.35E-09	20	22	0.016378	4.95E-09
		z4	5	7	0.006134	2.98E-10	7	9	0.006467	3.04E-09	20	22	0.014669	4.09E-09
		z5	5	7	0.005361	1.08E-10	5	7	0.005651	5.1E-09	18	20	0.015148	8.68E-09
		z6	5	7	0.005871	1.13E-10	7	9	0.006562	4.87E-09	20	22	0.020166	6.51E-09
		z7	12	14	0.01285	6.55E-09	21	23	0.01828	3.66E-09	18	20	0.018488	3.29E-09
		z8	4	6	0.005565	5.5E-09	4	6	0.005418	6.27E-09	18	20	0.024221	4.35E-09
		z9	12	14	0.01189	8.84E-09	21	23	0.017708	4.77E-09	18	20	0.016622	4.4E-09
		z10	10	12	0.010439	7.58E-09	17	19	0.016126	8.9E-09	15	17	0.017367	4.83E-09
	10,000	z1	6	8	0.024162	7.58E-10	7	9	0.031274	4.18E-09	21	23	0.082867	5.67E-09
		z2	5	7	0.021552	7.93E-10	7	9	0.028923	6.38E-09	20	22	0.079572	6.89E-09
		z3	5	7	0.02279	9.2E-10	7	9	0.02858	8.99E-10	21	23	0.097106	6.31E-09
		z4	5	7	0.020435	9.42E-10	7	9	0.028707	8.73E-09	20	22	0.080728	7.88E-09
		z5	5	7	0.021897	3.42E-10	6	8	0.02198	6.43E-09	19	21	0.09699	9.65E-09
		z6	5	7	0.023938	3.56E-10	7	9	0.030868	5.8E-09	21	23	0.099906	4.5E-09
		z7	13	15	0.066552	3.82E-09	22	24	0.083569	4.03E-09	18	20	0.098144	9.47E-09
		z8	5	7	0.021195	1.74E-10	5	7	0.022752	8.08E-09	19	21	0.071794	4.84E-09
		z9	13	15	0.069197	5.16E-09	22	24	0.085593	3.87E-09	19	21	0.083308	3.01E-09
		z10	11	13	0.056696	4.42E-09	19	21	0.080104	3.62E-09	16	18	0.094939	4.58E-09
	100,000	z1	6	8	0.180678	2.4E-09	—	—	—	—	—	—	—	—
		z2	5	7	0.139855	2.51E-09	—	—	—	—	21	23	0.784733	7.39E-09
		z3	5	7	0.132149	2.91E-09	—	—	—	—	19	21	0.766242	7.39E-09
		z4	5	7	0.137804	2.98E-09	9	11	0.288251	8.23E-09	22	24	0.705285	5.63E-09
		z5	5	7	0.129873	1.08E-09	6	8	0.143563	7.31E-09	20	22	0.534168	9.26E-09
		z6	5	7	0.134165	1.13E-09	—	—	—	—	—	—	—	—
		z7	14	16	0.485152	2.23E-09	22	24	0.630109	8.07E-09	19	21	0.642433	7.17E-09
		z8	5	7	0.136125	5.5E-10	6	8	0.142574	9.09E-09	20	22	0.525555	5.34E-09
		z9	14	16	0.476908	3.01E-09	—	—	—	—	—	—	—	—
		z10	12	14	0.409385	2.58E-09	20	22	0.560778	4.1E-09	17	19	0.567985	4.35E-09
4	1,000	z1	6	8	0.005619	2.87E-09	7	9	0.006823	1.19E-10	20	22	0.021271	4.63E-09
		z2	5	7	0.00595	9.89E-09	6	8	0.005469	5.21E-10	20	22	0.018736	5.87E-09
		z3	5	7	0.005407	3.92E-09	6	8	0.005786	3.94E-10	20	22	0.021209	3.86E-09
		z4	5	7	0.005703	1.46E-09	6	8	0.005812	2.31E-10	19	21	0.017197	6.7E-09
		z5	4	6	0.005901	1.98E-09	4	6	0.00612	1.85E-09	16	18	0.016049	8.84E-09
		z6	6	8	0.007933	5.59E-10	7	9	0.006046	1.54E-10	18	20	0.017072	9.81E-09
		z7	6	8	0.00682	1.52E-09	10	12	0.008768	4.15E-09	19	21	0.019414	5.55E-09
		z8	4	6	0.006465	4.2E-10	4	6	0.004756	4.18E-10	15	17	0.013598	6.25E-09
		z9	6	8	0.006017	1.58E-09	10	12	0.007923	4.82E-09	19	21	0.018768	6.64E-09
		z10	4	6	0.005274	9.07E-10	7	9	0.006779	3.38E-09	16	18	0.014634	6.93E-09
	10,000	z1	6	8	0.025622	9.09E-09	8	10	0.033583	2.22E-10	21	23	0.089595	8.04E-09
		z2	6	8	0.022932	3.13E-10	6	8	0.024428	2.93E-09	21	23	0.084526	6.5E-09
		z3	6	8	0.023912	1.24E-10	6	8	0.024945	2.8E-09	20	22	0.085129	6.81E-09
		z4	5	7	0.020152	4.63E-09	6	8	0.024826	5E-09	17	19	0.071388	4.43E-09
		z5	4	6	0.017415	6.25E-09	5	7	0.02025	1.95E-09	17	19	0.07388	9.76E-09
		z6	6	8	0.02415	1.77E-09	7	9	0.031434	1.05E-10	20	22	0.08945	5.43E-09
		z7	6	8	0.025838	4.79E-09	11	13	0.037641	1.13E-10	20	22	0.080899	5.62E-09
		z8	4	6	0.017359	1.33E-09	5	7	0.020348	5.16E-10	16	18	0.060966	6.95E-09
		z9	6	8	0.026715	5.01E-09	10	12	0.034902	1.05E-10	20	22	0.074722	4.89E-09
		z10	4	6	0.01742	2.87E-09	8	10	0.029054	1.07E-10	17	19	0.066232	7.72E-09
	100,000	z1	7	9	0.174274	2.87E-10	—	—	—	—	—	—	—	—
		z2	6	8	0.15497	9.89E-10	—	—	—	—	21	23	0.593699	3.56E-09
		z3	6	8	0.156447	3.92E-10	—	—	—	—	21	23	0.576189	4.95E-09
		z4	6	8	0.160227	1.46E-10	—	—	—	—	21	23	0.554794	7.76E-09
		z5	5	7	0.129201	1.98E-10	5	7	0.1435	4.93E-09	17	19	0.436677	5.36E-09
		z6	6	8	0.182174	5.59E-09	—	—	—	—	—	—	—	—
		z7	7	9	0.185465	1.52E-10	10	12	0.269967	2.61E-10	20	22	0.502666	9.89E-09
		z8	4	6	0.117002	4.2E-09	5	7	0.117484	2.69E-10	17	19	0.458408	7.29E-09
		z9	7	9	0.186866	1.58E-10	6	8	0.20446	8.05E-09	19	21	0.533535	4.36E-09
		z10	4	6	0.117006	9.07E-09	8	10	0.181251	3.38E-10	18	20	0.471752	8.59E-09

Table 3: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 5 & 6

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
5	1,000	z1	7	9	0.00676	1.54E-09	11	13	0.007697	6.09E-09	20	22	0.015367	3.91E-09
		z2	6	8	0.006472	1.5E-09	10	12	0.008317	5.67E-09	19	21	0.015108	6.52E-09
		z3	6	8	0.006201	1.57E-09	10	12	0.008339	3.42E-09	19	21	0.014058	7.03E-09
		z4	6	8	0.005654	1.58E-09	10	12	0.007987	4.6E-09	19	21	0.015544	6.84E-09
		z5	5	7	0.006229	1.57E-09	9	11	0.007522	4.95E-09	18	20	0.014218	7.62E-09
		z6	4	6	0.004118	8.92E-10	9	11	0.00724	2.82E-09	18	20	0.014127	8.69E-09
		z7	5	7	0.005704	2.27E-10	5	7	0.005422	5.77E-09	18	20	0.012952	6.51E-09
		z8	4	6	0.005326	8.95E-10	9	11	0.007176	2.65E-09	18	20	0.012761	4.07E-09
		z9	5	7	0.006035	5.87E-10	6	8	0.006561	1.19E-10	19	21	0.014354	4.07E-09
		z10	3	5	0.004241	1.58E-09	4	6	0.004547	1.53E-09	16	18	0.012745	7.12E-09
	10,000	z1	7	9	0.027813	4.86E-09	12	14	0.039537	1.7E-10	20	22	0.069167	4.37E-09
		z2	6	8	0.022781	4.74E-09	11	13	0.037809	1.57E-10	20	22	0.064303	8.94E-09
		z3	6	8	0.021258	4.97E-09	11	13	0.033413	1.3E-10	19	21	0.062573	4.81E-09
		z4	6	8	0.02195	4.99E-09	11	13	0.034558	1.27E-10	18	20	0.059762	5.85E-09
		z5	5	7	0.018217	4.98E-09	10	12	0.030144	1.53E-10	19	21	0.06161	8.46E-09
		z6	4	6	0.014701	2.82E-09	6	8	0.025164	2.84E-09	19	21	0.068317	4.51E-09
		z7	5	7	0.017739	7.19E-10	6	8	0.020381	1.12E-10	19	21	0.061574	5.54E-09
		z8	4	6	0.015018	2.83E-09	9	11	0.028777	8.37E-09	19	21	0.065709	4.53E-09
		z9	5	7	0.01815	1.86E-09	6	8	0.020543	1.25E-09	19	21	0.076442	4.89E-09
		z10	3	5	0.01215	5E-09	4	6	0.015282	4.84E-09	17	19	0.061914	7.93E-09
	100,000	z1	8	10	0.172356	1.54E-10	—	—	—	—	—	—	—	—
		z2	7	9	0.147858	1.5E-10	—	—	—	—	21	23	0.476847	5.41E-09
		z3	7	9	0.151455	1.57E-10	—	—	—	—	21	23	0.500895	3.85E-09
		z4	7	9	0.165042	1.58E-10	—	—	—	—	20	22	0.447635	5.67E-09
		z5	6	8	0.133655	1.57E-10	8	10	0.15607	3.36E-10	20	22	0.399647	7.81E-09
		z6	4	6	0.093881	8.92E-09	—	—	—	—	—	—	—	—
		z7	5	7	0.109231	2.27E-09	6	8	0.145751	5.3E-09	20	22	0.473855	6.46E-09
		z8	4	6	0.100766	8.95E-09	9	11	0.161947	5.6E-10	20	22	0.406266	5E-09
		z9	5	7	0.107399	5.87E-09	10	12	0.238174	5.39E-10	21	23	0.450178	4.74E-09
		z10	4	6	0.08967	1.58E-10	5	7	0.092156	1.51E-10	18	20	0.390821	8.82E-09
6	1,000	z1	8	10	0.012043	9.74E-09	17	19	0.019679	8.73E-09	16	18	0.021584	4.96E-09
		z2	12	14	0.016164	4.6E-09	18	20	0.020792	9.61E-09	16	18	0.019559	5.33E-09
		z3	12	14	0.015438	2.09E-09	18	20	0.019869	8.83E-09	14	16	0.017468	3.33E-09
		z4	11	13	0.01453	3.88E-09	18	20	0.021325	3.8E-09	14	16	0.019084	3.83E-09
		z5	11	13	0.019421	2.14E-09	17	19	0.019787	9.85E-09	14	16	0.018743	4.62E-09
		z6	13	15	0.016606	2.29E-09	17	19	0.022128	3.74E-09	16	18	0.021872	3.86E-09
		z7	11	13	0.016568	8.52E-09	19	21	0.022795	2.91E-09	15	17	0.019811	2.41E-09
		z8	9	11	0.011429	3.3E-09	17	19	0.020545	5.81E-09	12	14	0.017294	4.21E-09
		z9	10	12	0.013478	4.07E-09	16	18	0.018605	7.85E-09	15	17	0.014079	5.09E-09
		z10	7	9	0.009764	9.74E-09	14	16	0.017425	2.99E-09	12	14	0.013235	4.81E-09
	10,000	z1	10	12	0.068632	5.02E-09	15	17	0.096181	5.21E-09	17	19	0.117131	6.27E-09
		z2	11	13	0.066346	2.46E-09	15	17	0.088532	3.35E-09	17	19	0.11579	9.41E-09
		z3	10	12	0.062165	6.81E-09	13	15	0.074248	3.21E-09	15	17	0.105101	9.16E-09
		z4	10	12	0.068928	2.04E-09	14	16	0.079345	2.98E-09	15	17	0.098544	7.04E-09
		z5	9	11	0.062373	6.68E-09	15	17	0.085929	5.87E-09	14	16	0.092003	9.78E-09
		z6	11	13	0.077969	7.51E-09	14	16	0.090732	7.75E-09	15	17	0.100506	9.25E-09
		z7	10	12	0.061319	4.53E-09	15	17	0.084891	6.17E-09	16	18	0.109527	8.97E-09
		z8	7	9	0.059258	6.88E-09	15	17	0.09133	3.93E-09	15	17	0.102201	9.48E-09
		z9	9	11	0.057741	2E-09	14	16	0.083351	3.78E-09	15	17	0.099291	9.57E-09
		z10	9	11	0.061101	5.02E-09	10	12	0.066903	6.41E-09	13	15	0.102688	9.08E-09
	100,000	z1	9	11	0.380941	2.06E-09	—	—	—	—	—	—	—	—
		z2	9	11	0.387209	7.41E-09	—	—	—	—	17	19	0.980548	2.54E-09
		z3	9	11	0.393752	3.28E-09	—	—	—	—	17	19	0.888177	2.96E-09
		z4	8	10	0.37006	5.54E-09	15	17	0.692001	7.26E-09	15	17	0.878389	2.29E-09
		z5	8	10	0.36469	1.99E-09	16	18	0.620307	3.15E-09	18	20	0.883358	3.6E-09
		z6	10	12	0.488956	3.77E-09	—	—	—	—	—	—	—	—
		z7	9	11	0.453113	2.13E-09	14	16	0.594301	7.05E-09	16	18	0.81286	6.73E-09
		z8	10	12	0.490746	9.51E-09	12	14	0.47552	4.93E-09	17	19	0.842603	3.75E-09
		z9	7	9	0.317164	2.53E-09	15	17	0.676554	5.94E-09	18	20	1.074665	3.68E-09
		z10	8	10	0.362125	2.06E-09	15	17	0.607184	6.93E-09	14	16	0.786929	7.73E-09

Table 4: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 7 & 8

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ v_k\ $
7	1,000	z1	8	10	0.010557	2.91E-09	9	11	0.01233	5.05E-10	21	23	0.025551	6.53E-09
		z2	8	10	0.010211	3.69E-09	10	12	0.014542	4.39E-09	19	21	0.027144	4.95E-09
		z3	8	10	0.009844	3.89E-09	11	13	0.01505	1.84E-10	21	23	0.023852	7.66E-09
		z4	8	10	0.011307	4.09E-09	11	13	0.013188	3.71E-10	21	23	0.029021	5.6E-09
		z5	8	10	0.009348	4.72E-09	12	14	0.015335	4.02E-10	21	23	0.025973	8.92E-09
		z6	8	10	0.009322	3.29E-09	11	13	0.013053	2.56E-10	21	23	0.025945	4.2E-09
		z7	8	10	0.008962	5.48E-09	12	14	0.016211	1.53E-10	20	22	0.027174	6.02E-09
		z8	8	10	0.010708	4.83E-09	12	14	0.014927	1.88E-10	21	23	0.026292	7.59E-09
		z9	8	10	0.009526	5.67E-09	12	14	0.013969	2.21E-10	21	23	0.023062	8.7E-09
		z10	8	10	0.009539	4.96E-09	12	14	0.015007	2.64E-10	21	23	0.024413	5.78E-09
	10,000	z1	7	9	0.040889	6.7E-10	—	—	—	—	21	23	0.127687	4.23E-09
		z2	7	9	0.040382	8.54E-10	—	—	—	—	20	22	0.128844	6.96E-09
		z3	7	9	0.038999	9.02E-10	—	—	—	—	22	24	0.150906	3.76E-09
		z4	7	9	0.038577	9.5E-10	—	—	—	—	21	23	0.183199	5.79E-09
		z5	7	9	0.037446	1.1E-09	—	—	—	—	—	—	—	—
		z6	7	9	0.038935	7.61E-10	—	—	—	—	22	24	0.130399	8.58E-09
		z7	7	9	0.040569	1.28E-09	—	—	—	—	—	—	—	—
		z8	7	9	0.054202	1.13E-09	—	—	—	—	—	—	—	—
		z9	7	9	0.036501	1.33E-09	—	—	—	—	—	—	—	—
		z10	7	9	0.037553	1.16E-09	—	—	—	—	—	—	—	—
	100,000	z1	7	9	0.320677	6.73E-10	—	—	—	—	—	—	—	—
		z2	7	9	0.275229	8.58E-10	—	—	—	—	—	—	—	—
		z3	7	9	0.323518	9.06E-10	—	—	—	—	—	—	—	—
		z4	7	9	0.271228	9.55E-10	—	—	—	—	—	—	—	—
		z5	7	9	0.26768	1.1E-09	—	—	—	—	—	—	—	—
		z6	7	9	0.267562	7.64E-10	—	—	—	—	—	—	—	—
		z7	7	9	0.25155	1.29E-09	—	—	—	—	—	—	—	—
		z8	7	9	0.258587	1.13E-09	—	—	—	—	—	—	—	—
		z9	7	9	0.261322	1.33E-09	—	—	—	—	—	—	—	—
		z10	7	9	0.250103	1.16E-09	—	—	—	—	—	—	—	—
8	1,000	z1	14	16	0.013145	2.53E-09	19	21	0.015142	4.51E-09	16	18	0.015983	7.33E-09
		z2	13	15	0.011644	3.02E-09	18	20	0.015376	5.13E-09	15	17	0.014381	6.78E-09
		z3	11	13	0.010263	6.73E-09	16	18	0.013695	4.76E-09	13	15	0.01355	9.43E-09
		z4	13	15	0.011276	2.74E-09	18	20	0.015435	4.45E-09	15	17	0.012986	5.92E-09
		z5	14	16	0.012295	3.22E-09	19	21	0.014737	6.93E-09	16	18	0.015932	8.16E-09
		z6	13	15	0.012982	7.65E-09	19	21	0.014835	4.28E-09	16	18	0.015475	4.81E-09
		z7	14	16	0.013548	8.16E-09	16	18	0.012236	8.85E-09	14	16	0.014473	8.64E-09
		z8	14	16	0.012695	3.78E-09	19	21	0.016126	7.68E-09	16	18	0.015562	9.45E-09
		z9	14	16	0.011986	9.77E-09	18	20	0.016402	5.46E-09	15	17	0.014401	3.53E-09
		z10	14	16	0.013313	4.51E-09	19	21	0.020132	8.35E-09	17	19	0.01597	3.01E-09
	10,000	z1	14	16	0.058877	8.01E-09	17	19	0.071658	4.69E-09	17	19	0.085409	3.39E-09
		z2	13	15	0.053878	9.55E-09	19	21	0.068069	5.03E-09	16	18	0.086833	5.8E-09
		z3	12	14	0.055133	4.62E-09	17	19	0.064957	5.01E-09	14	16	0.075968	8.11E-09
		z4	13	15	0.05626	8.66E-09	19	21	0.073393	4.57E-09	16	18	0.087378	5.08E-09
		z5	15	17	0.068114	2.21E-09	20	22	0.073496	6.38E-09	17	19	0.09017	3.34E-09
		z6	14	16	0.055157	5.25E-09	19	21	0.075769	8.97E-09	17	19	0.09626	2.73E-09
		z7	15	17	0.066863	5.6E-09	19	21	0.073209	7.47E-09	15	17	0.078666	4.64E-09
		z8	15	17	0.056879	2.59E-09	19	21	0.075977	4E-09	16	18	0.082456	5.99E-09
		z9	15	17	0.061671	6.7E-09	20	22	0.08161	7.27E-09	16	18	0.095701	6.35E-09
		z10	15	17	0.06033	3.09E-09	20	22	0.104739	4.2E-09	18	20	0.097309	3.17E-09
	100,000	z1	15	17	0.398271	5.5E-09	—	—	—	—	18	20	0.688944	5.49E-09
		z2	14	16	0.380177	6.55E-09	20	22	0.666958	6.09E-09	16	18	0.577961	9.14E-09
		z3	13	15	0.345049	3.17E-09	18	20	0.657546	5.26E-09	15	17	0.534358	6.98E-09
		z4	14	16	0.378672	5.94E-09	19	21	0.610211	4.12E-09	17	19	0.62249	3.57E-09
		z5	15	17	0.400157	6.99E-09	—	—	—	—	18	20	0.664593	6.8E-09
		z6	15	17	0.388646	3.6E-09	20	22	0.571283	7.15E-09	17	19	0.76903	7.4E-09
		z7	16	18	0.420976	3.84E-09	—	—	—	—	19	21	0.768806	6.77E-09
		z8	15	17	0.385743	8.2E-09	—	—	—	—	18	20	0.700174	7.41E-09
		z9	16	18	0.416979	4.6E-09	—	—	—	—	—	—	—	—
		z10	15	17	0.54679	9.77E-09	—	—	—	—	18	20	0.725455	3.82E-09

Table 5: Numerical results of TTHZP, IHZPM & EHZPM methods for problems 9 & 10

Problems	Dimension	IP	TTHZP				IHZPM				EHZPM			
			Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $	Iter	Funeval	Time(s)	$\ \Phi_k\ $
9	1,000	z1	14	16	0.020469	3.38E-09	22	24	0.03124	5.1E-09	17	19	0.031742	5.11E-09
		z2	13	15	0.023402	3.18E-09	21	23	0.031896	4.34E-09	18	20	0.030163	8.28E-09
		z3	12	14	0.021335	7.92E-09	21	23	0.029192	3.78E-09	18	20	0.033684	5.24E-09
		z4	12	14	0.018372	1.94E-09	20	22	0.028301	7.45E-09	17	19	0.02955	9.93E-09
		z5	12	14	0.019083	3.64E-09	20	22	0.025039	4.94E-09	17	19	0.027181	5.31E-09
		z6	13	15	0.019934	9.05E-09	20	22	0.027777	5.22E-09	19	21	0.028153	5.11E-09
		z7	11	13	0.017712	5.97E-09	20	22	0.028376	6.04E-09	17	19	0.024195	6.41E-09
		z8	11	13	0.018268	8.36E-09	19	21	0.024042	6.28E-09	16	18	0.025918	8.01E-09
		z9	13	15	0.018318	1.9E-09	20	22	0.026954	6.09E-09	17	19	0.02582	6.95E-09
		z10	10	12	0.016309	7.3E-09	17	19	0.024152	8.74E-09	15	17	0.025734	4.73E-09
	10,000	z1	15	17	0.116284	1.97E-09	21	23	0.182182	9.75E-09	20	22	0.192527	4.8E-09
		z2	14	16	0.10961	1.86E-09	22	24	0.155455	7.99E-09	19	21	0.192352	6.74E-09
		z3	13	15	0.100289	4.63E-09	22	24	0.191745	5.27E-09	18	20	0.167451	5.77E-09
		z4	12	14	0.111041	6.15E-09	22	24	0.153829	3.69E-09	18	20	0.175169	9.66E-09
		z5	13	15	0.165186	2.12E-09	21	23	0.149497	5.47E-09	18	20	0.172095	5.03E-09
		z6	14	16	0.100772	5.29E-09	22	24	0.164789	4.73E-09	20	22	0.190027	3.68E-09
		z7	12	14	0.096057	3.48E-09	20	22	0.146935	3.96E-09	18	20	0.172834	4.93E-09
		z8	12	14	0.108705	4.88E-09	20	22	0.169326	7.11E-09	17	19	0.167154	7.6E-09
		z9	13	15	0.118699	6.01E-09	20	22	0.145191	6.89E-09	17	19	0.1808	5.15E-09
		z10	11	13	0.083155	4.25E-09	18	20	0.13145	9.9E-09	16	18	0.158308	4.49E-09
	100,000	z1	15	17	0.91504	6.24E-09	—	—	—	—	—	—	—	—
		z2	14	16	0.867538	5.88E-09	—	—	—	—	—	—	—	—
		z3	14	16	0.765665	2.7E-09	—	—	—	—	20	22	1.654807	4.88E-09
		z4	13	15	0.751449	3.59E-09	—	—	—	—	18	20	1.346615	4.4E-09
		z5	13	15	0.729771	6.72E-09	21	23	1.106345	3.9E-09	19	21	1.453386	3.97E-09
		z6	15	17	0.823708	3.08E-09	—	—	—	—	—	—	—	—
		z7	13	15	0.697483	2.03E-09	23	25	1.602479	4.92E-09	19	21	1.407026	4.31E-09
		z8	13	15	0.701442	2.84E-09	21	23	1.197109	6.66E-09	18	20	1.352549	7.13E-09
		z9	14	16	0.755143	3.5E-09	—	—	—	—	19	21	1.375671	8.81E-09
		z10	12	14	0.665132	2.48E-09	20	22	1.033497	4.02E-09	17	19	1.360813	4.26E-09
10	1,000	z2	14	16	0.014524	2.28E-09	21	23	0.023109	4.45E-09	18	20	0.021997	6.12E-09
		z3	13	15	0.015234	2.04E-09	22	24	0.022636	7.02E-09	18	20	0.021251	4.32E-09
		z4	14	16	0.014873	1.91E-09	20	22	0.019614	9.01E-09	19	21	0.020221	4.12E-09
		z5	13	15	0.013688	6.31E-09	21	23	0.02173	8.92E-09	19	21	0.024237	3.68E-09
		z6	14	16	0.013282	1.89E-09	21	23	0.019163	7.56E-09	17	19	0.021155	5.78E-09
		z7	13	15	0.014566	2.77E-09	19	21	0.02147	8.13E-09	17	19	0.020174	3.26E-09
		z8	13	15	0.014625	4.24E-09	21	23	0.019272	7.12E-09	18	20	0.023302	9.29E-09
		z9	12	14	0.017596	7.64E-09	18	20	0.020901	6.05E-09	15	17	0.018886	9.28E-09
		z10	13	15	0.015029	2.49E-09	21	23	0.022668	5.4E-09	18	20	0.02154	6.55E-09
	10,000	z2	14	16	0.113189	7.2E-09	—	—	—	—	17	19	0.1052	9.63E-09
		z3	13	15	0.072997	6.45E-09	—	—	—	—	18	20	0.107023	9.26E-09
		z4	14	16	0.085927	6.04E-09	—	—	—	—	19	21	0.118409	4.78E-09
		z5	14	16	0.060437	3.68E-09	22	24	0.151209	6.34E-09	19	21	0.118739	4.35E-09
		z6	14	16	0.072375	5.96E-09	21	23	0.108386	4.57E-09	20	22	0.115235	5.7E-09
		z7	13	15	0.063288	8.77E-09	20	22	0.093252	4.79E-09	17	19	0.106091	7.05E-09
		z8	14	16	0.077851	2.47E-09	21	23	0.090118	3.97E-09	19	21	0.114839	4.58E-09
		z9	13	15	0.073467	4.45E-09	19	21	0.09069	6.97E-09	16	18	0.095777	8.07E-09
		z10	13	15	0.061811	7.86E-09	21	23	0.087473	8.79E-09	19	21	0.115358	4.63E-09
	100,000	z2	15	17	0.555942	4.2E-09	—	—	—	—	—	—	—	—
		z3	14	16	0.558755	3.76E-09	—	—	—	—	—	—	—	—
		z4	15	17	0.560562	3.52E-09	—	—	—	—	—	—	—	—
		z5	15	17	0.583609	2.14E-09	—	—	—	—	—	—	—	—
		z6	15	17	0.555493	3.48E-09	—	—	—	—	—	—	—	—
		z7	14	16	0.530348	5.11E-09	22	24	0.951946	8.43E-09	18	20	0.978537	5.75E-09
		z8	14	16	0.53403	7.82E-09	—	—	—	—	—	—	—	—
		z9	14	16	0.543142	2.6E-09	20	22	0.956008	6.07E-09	17	19	0.934911	8.32E-09
		z10	14	16	0.543442	4.58E-09	—	—	—	—	—	—	—	—

Table 6: TTHZP methods for problems 3, 7 & 8 with IP that are far from the solutions

TTHZP						
Problems	Dimension	IP	Iter	Funeval	Time(s)	Norm
3	1,000,000	1,000,000	10	12	5.39215	3.45E-09
		2,000,000	10	12	5.82631	3.36E-09
		3,000,000	10	12	6.03510	8.71E-09
		4,000,000	10	12	6.14975	9.48E-09
		5,000,000	10	12	6.12519	5.04E-09
		6,000,000	10	12	6.39293	9.19E-09
		7,000,000	10	12	6.39766	6.72E-09
		8,000,000	10	12	6.58383	9.47E-09
		9,000,000	11	13	6.85648	3.15E-09
		10,000,000	11	13	6.98976	2.77E-09
7	1,000,000	1,000,000	12	14	5.96538	9.24E-09
		2,000,000	13	15	6.45715	1.88E-10
		3,000,000	13	15	6.47056	6.07E-10
		4,000,000	13	15	6.40774	6.29E-10
		5,000,000	13	15	6.41550	7.80E-10
		6,000,000	13	15	6.52297	7.96E-10
		7,000,000	13	15	6.87300	7.35E-10
		8,000,000	14	16	7.08038	5.44E-10
		9,000,000	14	16	7.04773	5.25E-10
		10,000,000	14	16	7.01074	3.63E-10
8	1,000,000	1,000,000	10	12	5.84685	4.67E-10
		2,000,000	11	13	5.99018	5.28E-10
		3,000,000	11	13	6.12070	6.15E-10
		4,000,000	10	12	5.60422	6.23E-10
		5,000,000	10	12	5.53956	5.85E-10
		6,000,000	10	12	5.34310	6.26E-10
		7,000,000	10	12	5.71980	4.94E-10
		8,000,000	11	13	5.99898	5.26E-10
		9,000,000	11	13	5.91940	3.85E-10
		10,000,000	11	13	5.86425	4.90E-10

Tables 1– 5 present detailed results of the experiments conducted by displaying initial starting point "IP", number of iterations "Iter", number of function evaluations "Funeval", processing time "Time (s)", and $\|\Phi_k\|$ is the norm of the residual at the termination point. The symbol '–' represents failure of the scheme when the iterations reach 1000; however, no z_k satisfies the stopping criterion. A careful analysis of Tables 1– 5 reveals that the TTHZP method outperforms the IHZPM and EHZPM methods since the former solved all of the problems in the experiments and the latter failed to solve some of the problems during the iteration process by clear indication from the tables. Furthermore, because it has fewer iterations, the TTHZP method converges to the solution faster than the IHZPM and EHZPM methods. Indeed, the proposed method outperforms the compared methods since it takes less processing time, iterations, and function evaluations.

In Table 6, we choose Problems 3, 7, and 8 to demonstrate the behavior of the TTHZP method. We take ten different initial points with the dimension 1,000,000, which are far from the solutions and efficiently converge to the solutions of (1). However, more iterations are required when the starting point is further away from the solution. Therefore, the proposed method is competitive for solving large-scale problems.