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RELIABILITY MODELLING AND ANALYSIS OF A NON REPAIRABLE SERIES-PARALLEL SYSTEM

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Abstract: Performance of a non repairable system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. The present study deals with the reliability modeling of a non repairable series-parallel system under three types of failures. Type I failure is unit failure from the same subsystem, type II is a failure of one unit from each subsystem while type III is a failure of all units from the same subsystem due to common cause. To each failure, the failed units are replaced with identical new ones. Markov models are developed and differential equations are derived to obtain the steady state availability, busy period of repairmen, profit function and mean time to system failure. Graphs are drawn to highlight the behavior of the results.

Keywords: reliability; series-parallel; non repairable, replacement.

2010 AMS Subject Classification: 68M15.

1. INTRODUCTION

Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Systems operating under normal conditions may experience random failures and cease functioning abruptly. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order. Availability and profit of an industrial system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. Improving the reliability and availability of system/subsystem, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieve be maintaining reliability and availability at highest order. To achieve high production and profit, the system should remain operative for maximum possible duration. It is important to consider profit as well as the quality requirement. There is an extensive literature on reliability analysis of series-parallel systems under various situations such as Coit and Smith [1], Hu et al [2],kolowrocki [3], Juang et al [4], Levitin [5], Liang and Chen [6], Li et al [8], Moghaddam [9] and Sun et al [11]. Many of these models are based on the idea of reliability optimization or redundancy allocation. Zakarian and Kusiak [12] studied the analytical method to address system availability as a measure of system performance. Levitin and Lisnianski [7] investigated the optimization of the number of machine in parallel and maintenance scheduling of a serial-parallel system. Ramirez-Marquez et. al [10]

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investigated redundancy allocation for series-parallel systems using a Max-Min approach. Bulama et. Al [13] analyzed some reliability characteristics of a repairable warm standby system. In this paper, we considered a non repairable series-parallel system and derived its corresponding mathematical models using linear first order differential equations method. The focus of our analysis is primarily to capture the effect of both failure and replace rates on reliability measures of system effectiveness such as MTSF, availability, busy period and profit based on assumed numerical values given to the system parameters.The organization of the paper is as follows. In Section 2, is description of the system. Some reliability characteristics of the system are derived in Sections 3. The results of our numerical simulations are presented and discussed in Section 4. Finally, we make a concluding remark in Section 5.

1.1 Nomenclature

β_i / λ_i =Failure rate of unit A_i / B_i, $i = 1, 2$ of subsystem A and B respectively.

η = Common cause failure rate of subsystem A and B

$\delta_1 / \delta_2 / \delta_3 / \delta_4$:=Replacement rate of unit A_1 and B_1 / Replacement rate of unit A_1 and A_2 / Replacement rate of unit B_1 and B_2 / Replacement rate of the system

$R_1 / R_2 / R_3$:= Repairman assigned to replace units in subsystem A/ Repairman assigned to replace units in subsystem B/ Repairman assigned to replace units in subsystem A and B

$C_0 / C_1 / C_2 / C_3 / C_4 / C_5$:Revenue per-unit time by the system when it is operative / cost per-unit time when R_1 repairman is replacing A_1 and A_2 / cost per-unit time when R_3 repairman is replacing A_1 and A_2 due to common cause failure / cost per-unit time when R_2 repairman is replacing B_1 and B_2 / cost per-unit time when R_3 repairman is replacing B_1 and B_2 due to common cause failure / cost per-unit time when R_3 repairman is replacing A_1 and B_1.

2. SYSTEM DESCRIPTION

The system consists of two subsystems A and B in series. Subsystems A and B have three units each A_1, A_2 and A_3 and B_1, B_2 and B_3 in cold standby. Assume that all switchover times are instantaneous and switching is perfect, e.g. never fails and never does any damage. Each of the unit fails independently of the state of the others. At initial state, two units from each subsystem are working. Whenever one of these units from either subsystem A or B fails with parameter β_i or λ_i, it is immediately replaced by a cold standby unit. Such failure is considered as type I failure (i.e failure of a unit from any of the subsystems). Once another unit breaks down from the other subsystem different from the subsystem of earlier failure, such failure is considered as type II failure and two failed units are replace immediately with parameter δ_1, or from the subsystem similar to the earlier failure, the failure is type III and two failed units are replace immediately with parameter δ_2 if from subsystem A and δ_3 if from subsystem B. It assumed the system can failed at common cause failure. System failure occur when all the units or units from the same subsystem have failed. The entire system is replace at common cause failure with parameter δ_4.

3. RELIABILITY MODELLING OF THE SYSTEM

Following Markov assumptions, the Kolmogorov's differential equations are obtained for the calculation of state probabilities through the state transitions diagram of the system shown in Figure 1. State S_0 is the initial state where the units work perfectly.

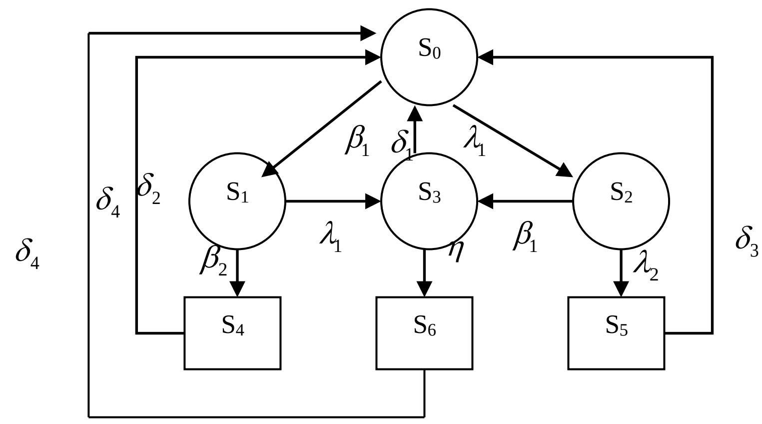

Figure 1. Transition diagram of the system

State S_0 : Units A_1, A_2, B_1, B_2 , are working, unit A_3 and B_3 are in standby, the system is working, the repairmen are not available.

State S_1 : Unit A_1 failed, units A_2 , A_3, B_1, B_2 , are working, unit B_3 is in standby, the system is working, the repairmen are not available.

State S_2 : Unit B_1 failed, units A_1 , A_2 , B_2 , B_3 are working, unit A_3 is in standby, the system is working, the repairmen are not available.

State S_3 : Unit A_1 and B_1 failed, units A_2 , A_3 , B_2 , B_3 are working, the system is working, repairman R_3 is busy replacing both A_1 and B_1.

State S_4 : Unit A_1 and A_2 failed, units A_3, B_1, B_2 are idle, unit B_3 is in standby, the system failed, repairman R_1 is busy replacing A_1 and A_2 .

State S_5 : Unit B_1 and B_2 failed, units A_1, A_2 , B_3 are idle, unit A_3 is in standby, the system failed, repairman R_2 is busy replacing A_1 and A_2 .

State S_6 : Unit A_1 and B_1 failed, units A_2 , A_3 , B_1 , B_2 failed due to common cause failure, the system failed, R_3 is busy replacing both the failed units.

3.1 Availability and Busy period

Let $Q(t)$ be the probability that the system in state i at time t . Relating the state of the system at time t and $t+dt$ from Fig. 1 above, the steady-state for the system can be expressed in the form:

$$\frac{d}{dt}(Q(t)) = TQ(t) \tag{1}$$

For the analysis of availability case of system, we use the following procedure to obtain the steady-state availability, busy period and profit function. In steady-state, the derivatives of the state probabilities become zero and we obtain:

$$TQ(t) = 0 \tag{2}$$

Let T be the time to failure of the system for system. The explicit expression for the steady-state availability is as follows:

$$A_T(\infty) = Q_0(\infty) + Q_1(\infty) + Q_2(\infty) + Q_3(\infty) \tag{3}$$

From state 1 to 6 the repairmen are busy in those states replacing the failed units due type I, type II and type III. Let $B_{R1}(\infty)$ and $B_{R2}(\infty)$ be the respective probabilities that the R_1 and R_2 repairmen are busy in the replacing of totally failed units due to type I failure, $B^{\bullet}{}_{R3}(\infty)$ and $B^{**}{}_{R3}(\infty)$ be the respective probabilities that the R_3 repairman is busy in the replacing of totally failed units due to type II and type III failure respectively. Using (2) and (3) above, the explicit expressions for the steady-state busy period of repairmen are as follows:

The steady-state busy period of R_1 due to type I failure is given by :

$$B_{R1}(\infty) = Q_4(\infty) \tag{4}$$

The steady-state busy period of R_2 due to type I failure is given by :

$$B_{R2}(\infty) = Q_5(\infty) \tag{5}$$

The steady-state busy period of R_3 due to type II failure is given by :

$$B^{*}{}_{R3}(\infty) = Q_3(\infty) \tag{6}$$

The steady-state busy period of R_3 due to type III failure of A_1 and A_2 is given by :

$$B^{**}{}_{R3}(\infty) = Q_6(\infty) \tag{7}$$

The steady-state busy period of R_3 due to type III failure of B_1 and B_2 is given by :

$$B^{**}{}_{R3}(\infty) = Q_6(\infty) \tag{8}$$

3.2 Profit Analysis

The failed units are subjected to replacement as can be observed in states S_1 to S_6. The repairmen are busy in those states replacing the failed units, the total profit per unit time incurred to the system in the steady-state is

The expected profit per-unit time in steady state is

$$P_F(\infty) = C_o A_V(\infty) - C_1 B_{R1}(\infty) - C_2 B^{**}{}_{R3}(\infty) - C_3 B_{R2}(\infty) - C_4 B^{**}{}_{R3}(\infty) - C_5 B^{*}{}_{R3}(\infty) \tag{9}$$

3.3 Mean time to system failure

To obtain an expression for MTSF, we delete the rows and columns of absorbing state of matrix M and take the transpose to produce a new matrix, say M .The expected time to reach an absorbing state is obtained from

$$MTSF = E\left[T_{P(0)\to P(absorbing)}\right] = P(0)(-M^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \tag{10}$$

$$M = \begin{bmatrix} -(\beta_1+\lambda_1) & \beta_1 & \lambda_1 & 0 \\ 0 & -(\beta_2+\lambda_1) & 0 & \lambda_1 \\ 0 & 0 & -(\beta_1+\lambda_2) & \beta_1 \\ \delta_1 & 0 & 0 & -(\delta_1+\eta) \end{bmatrix} \tag{11}$$

4. NUMERICAL EXAMPLE

For the graphical study of system behavior, the following set of parameters values are fixed throughout the simulations:

$\beta_1 = 0.2$, $\beta_2 = 0.3$, $\beta_3 = 0.2$, $\lambda_1 = 0.2$, $\lambda_2 = 0.4$, $\lambda_3 = 0.1$, $\eta = 0.3$, $\delta_1 = 0.6$, $\delta_2 = 0.5$, $\delta_3 = 0.5$, $\delta_4 = 0.7$, $C_0 = 500,000$, $C_1 = 10,000$, $C_2 = 20,000$, $C_3 = 10,000$, $C_4 = 20,000$, $C_5 = 40,000$

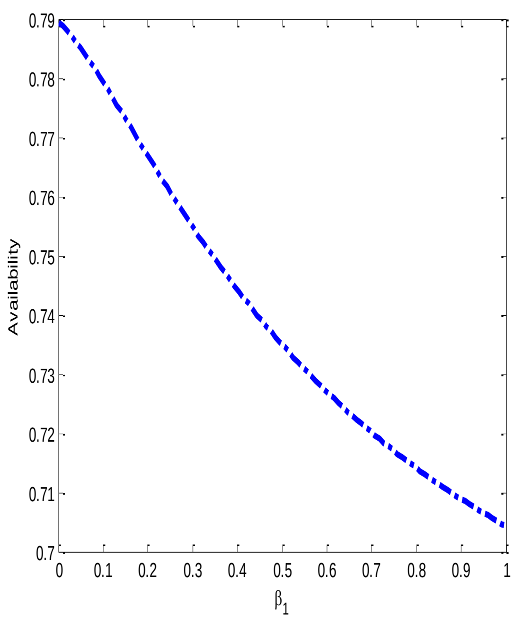

Fig. 2 Availability against β_1

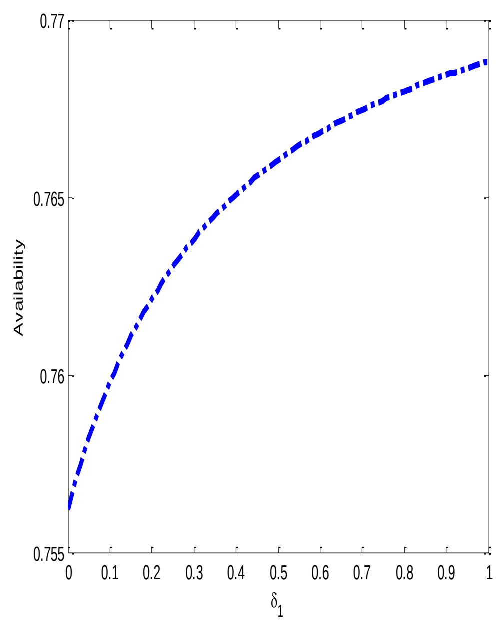

Fig. 3 Availability against δ_1

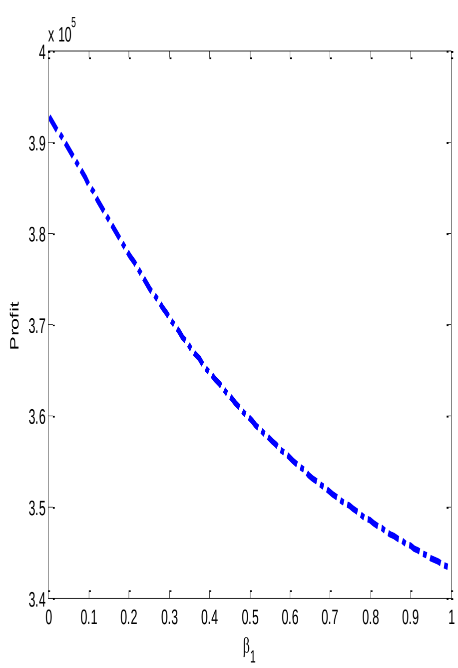

Fig. 4 Profit against β_1

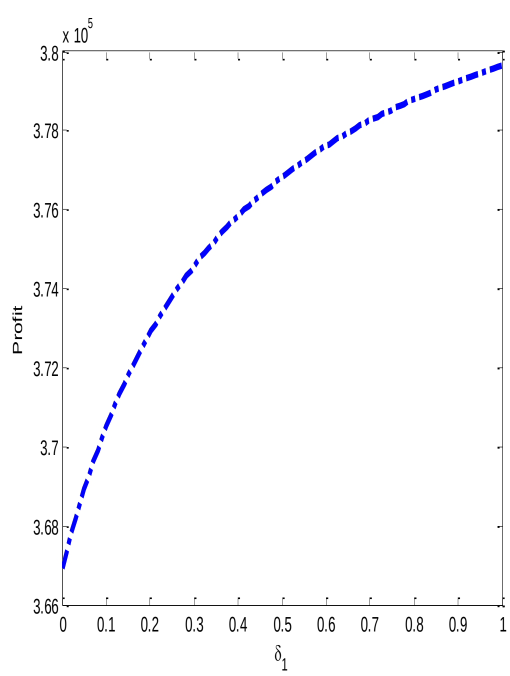

Fig. 5 Profit against δ_1

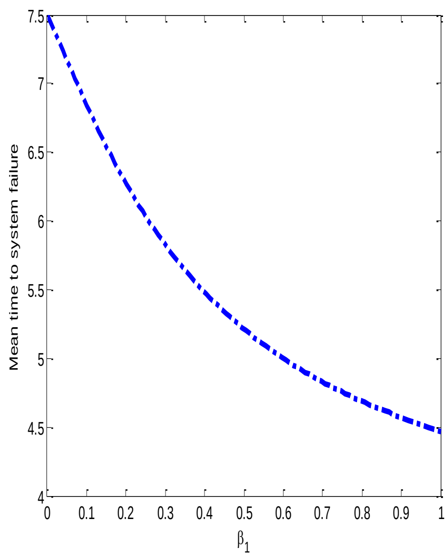

Fig. 6 MTSF against β_1

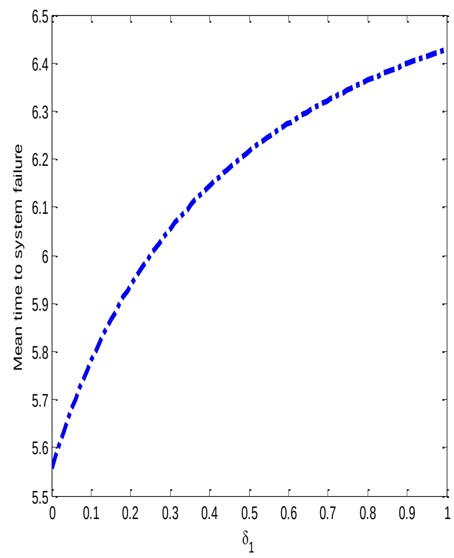

Fig. 7 MTSF against δ_1

Effect of β_1 on steady-state availability, profit MTSF and busy period can be observed in Figures 2,4 and 6. From these Figures, it is evident that the steady-state availability, profit and MTSF decrease with increase in β_1. Results of steady-state availability, profit, MTSF and busy period of repairmen with respect to δ_1 are given in Figures 3, 5 and 7. It is evident from these Figures that as δ_1 increases, the steady-state availability, profit and MTSF also increase while busy period of repairmen decrease with increase in δ_1.

5. CONCLUDING REMARKS

We studied the availability and profit of a series-parallel system attended by three repairmen. Explicit expressions for the steady-state availability, busy period and profit function have been derived. Parametric investigation of various system parameters on availability and profit function and their effect o availability, busy period and profit function have been captured. Results in Figures 2 – 7 have shown that availability and profit studied in the paper increases with increase in replacement rate and decreases with increase in failure rate. Maintaining high or required level of reliability and availability is often an essential requisite for improving system reliability and generated profit. Maintenance managers, reliability engineers and system designers are faced with the challenges of competition and market globalization on maintenance system to improve efficiency and reduce operational costs. Models developed in this paper are found to be highly beneficial to engineers, maintenance managers, system designers and plant management for proper maintenance analysis, decision, and evaluation of performance and for safety of the system as a whole.

Conflict of Interests

The authors declare that there is no conflict of interests.

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