

## **Financial Mathematics**

i.

Five company stocks were chosen for the portfolio, in which daily closed values were obtained going back one year from December 12<sup>th</sup>, 2023, to December 12<sup>th</sup>, 2024.

Amazon.com Inc. is an American multinational technology company involved in e-commerce, cloud computing, online advertising, digital streaming, and artificial intelligence. Their revenue was recorded at \$514 billion in 2022.

Oxford BioMedica plc collaborates with the world's most innovative pharmaceutical and biotechnology companies, providing viral vector development in gene and cell therapy and whose mission is to make cell and gene therapy a universally accessible clinical option.

Taiwan Semiconductor Mfg. Co. Ltd. is a Taiwanese multinational semiconductor contract manufacturing and design company. It is currently the world's most valuable semiconductor company and provides its products to many businesses around the world.

UiPath Inc. is a global software company that makes robotic automation software. Its headquarters are in New York, and it is in the technological business.

Intel Corporation is an American multinational corporation and technology company which designs manufactures and sells computer components and related products for business and consumer markets across the globe.

Portfolio Optimisation is the process of finding a portfolio that maximises or minimises certain target quantities. This can be done with multiple assets across the general market using the historic daily return of stocks. The overall objective is to maximise profit whilst minimising the total risk of losing money. The Capital Asset Pricing Model (CAPM) will be used with the view of optimising a portfolio.

A portfolio is a collection of investments aimed to maximise the return for a given level of risk or minimise the risk for a target return, ensuring a profit is made after a period of time. For each company stock, the daily return and volatility were calculated. The closed stock prices, including one year's worth of daily prices, were downloaded and used to calculate the daily return.

Annualised Mean & SD		
Stock	Mean	SD
AMZN	0.48586969119764	0.27689195
OXB	0.89403485800400	0.42094233
PATH	-0.36238354702097	0.52768848
TSM	0.73255879055721	0.40044267
INTC	-0.64705969962443	0.51132379

Figure 1: Annualised Mean and Standard Deviation for each Stock Return.

The variance between the asset returns was calculated using the Variance-Covariance Matrix function in the data analysis pack. The table represents the risk level of each asset and the interrelationship between them.

The results were then transposed across the table and showed a positive covariance across the results between all stocks apart from OXB-TSM, which means that all assets generally move in the same direction. The negative results between OXB and TSM suggest the assets generally move in opposite directions. This suggests that when the return of one of the assets increases, the other would tend to decrease. Assets which are in the same industry sometimes move in the same direction as there is a direct relationship between them due to them having shared factors within market trends and economic conditions.

Annualised Variance-Covariance Matrix					
Stock	AMZN	OXB	PATH	TSM	INTC
AMZN	0.07636370	0.00248097435	0.049842923415	0.0422162354977	0.0574067513461153
OXB	0.00248097	0.17648650535	0.0195664279896	-0.008122185988	0.0184797960803001
PATH	0.04984292	0.01956642798	0.277345755735	0.0433283314408	0.0650591479830015
TSM	0.04221623	-0.00812218598	0.0433283314408	0.1597154744197	0.074420289270772
INTC	0.05740675	0.01847979608	0.065059147983	0.0744202892707	0.260410378612411

Figure 2: Variance-Covariance Matrix for Looking at the Correlation Between Each of the Five Stocks.

ii.

A portfolio was made with equal weight between the assets before optimising the portfolio. The expected return was calculated by

Matrix multiplication(transpose(asset weights) mean asset returns)

The standard deviation of the portfolio was calculated by

Square root (matrix multiplication (matrix multiplication(transpose(Asset weights  
))Variance-Covariance Matrix)Asset weights))

The solver function add-on on Excel was used to determine portfolio risk and the percentage of each asset in the portfolio. The data analysis solver package was used to find the portfolio with the highest target return using certain constraints. These constraints included the sum of the asset weights being equal to one and each asset being equal to or more than zero to avoid the values being negative.

Portfolio 1 Highest Exp Return		Portfolio 2 0.85 Exp Return		Portfolio 7 0.6 Exp Return		Portfolio 8 0.55 Exp Return		Portfolio 13 0.3 Exp Return		Portfolio 14 0.25 Exp Return	
Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight
AMZN	0.000000	AMZN	0.000000	AMZN	0.50811	AMZN	0.49858	AMZN	0.43877	AMZN	0.42681
OXB	1.000000	OXB	0.72730	OXB	0.27211	OXB	0.26006	OXB	0.20162	OXB	0.18993
PATH	0.000000	PATH	0.000000	PATH	0.04672	PATH	0.05755	PATH	0.13130	PATH	0.14605
TSM	0.000000	TSM	0.27270	TSM	0.17307	TSM	0.15588	TSM	0.07384	TSM	0.05743
INTC	0.000000	INTC	0.000000	INTC	0.000000	INTC	0.02794	INTC	0.15447	INTC	0.17978
Sum	1.000000	Sum	1.000000	Sum	1.000000	Sum	1.000000	Sum	1.000000	Sum	1.000000
Exp Return	0.8940349	Exp Return	0.85000	Exp Return	0.60000	Exp Return	0.55000	Exp Return	0.30000	Exp Return	0.25000
Std Dev	0.4201030	Std Dev	0.31939	Std Dev	0.22154	Std Dev	0.22230	Std Dev	0.23992	Std Dev	0.24593
Sharpe Ratio	2.1245621	Sharpe Ratio	2.65662	Sharpe Ratio	2.70149	Sharpe Ratio	2.46742	Sharpe Ratio	1.24415	Sharpe Ratio	1.01044
Portfolio 3 0.8 Exp Return		Portfolio 4 0.75 Exp Return		Portfolio 9 0.5 Exp Return		Portfolio 10 0.45 Exp Return		Portfolio 15 0.2 Exp Return		Portfolio 16 0.15 Expected Return	
Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight
AMZN	0.66995	AMZN	0.22108	AMZN	0.48662	AMZN	0.47468	AMZN	0.41485	AMZN	0.40289
OXB	0.52452	OXB	0.44576	OXB	0.24837	OXB	0.23668	OXB	0.17825	OXB	0.16656
PATH	0.00000	PATH	0.00000	PATH	0.07230	PATH	0.08705	PATH	0.16080	PATH	0.17555
TSM	0.40554	TSM	0.33316	TSM	0.13947	TSM	0.12306	TSM	0.04102	TSM	0.02461
INTC	0.00000	INTC	0.00000	INTC	0.05325	INTC	0.07855	INTC	0.20509	INTC	0.23039
Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000
Exp Return	0.80000	Exp Return	0.75000	Exp Return	0.50000	Exp Return	0.45000	Exp Return	0.20000	Exp Return	0.15000
Std Dev	0.27261	Std Dev	0.24662	Std Dev	0.22402	Std Dev	0.22668	Std Dev	0.25265	Std Dev	0.26002
Sharpe Ratio	2.92909	Sharpe Ratio	3.03498	Sharpe Ratio	2.22525	Sharpe Ratio	1.97855	Sharpe Ratio	0.78567	Sharpe Ratio	0.57110
Portfolio 5 0.7 Exp Return		Portfolio 6 0.65 Exp Return		Portfolio 11 0.4 Exp Return		Portfolio 12 0.35 Exp Return		Portfolio 17 0.1 Expected Return		Portfolio 18 0.05 Exp Return	
Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight	Stock	Weight
AMZN	0.37221	AMZN	0.50960	AMZN	0.46269	AMZN	0.45073	AMZN	0.39092	AMZN	0.37113
OXB	0.36700	OXB	0.29341	OXB	0.22500	OXB	0.21331	OXB	0.15487	OXB	0.14145
PATH	0.00000	PATH	0.00386	PATH	0.10180	PATH	0.11655	PATH	0.19030	PATH	0.20587
TSM	0.26079	TSM	0.19313	TSM	0.10665	TSM	0.09025	TSM	0.00821	TSM	0.00000
INTC	0.00000	INTC	0.00000	INTC	0.10386	INTC	0.12917	INTC	0.25570	INTC	0.28155
Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000	Sum	1.00000
Exp Return	0.70000	Exp Return	0.65000	Exp Return	0.40000	Exp Return	0.35000	Exp Return	0.10000	Exp Return	0.05000
Std Dev	0.22920	Std Dev	0.22231	Std Dev	0.23025	Std Dev	0.23468	Std Dev	0.26860	Std Dev	0.27654
Sharpe Ratio	3.04759	Sharpe Ratio	2.91708	Sharpe Ratio	1.73075	Sharpe Ratio	1.48502	Sharpe Ratio	0.36754	Sharpe Ratio	0.17538

Figure 3: Portfolios for Different Expected Returns to Create the Efficient Frontier Curve.

The process was repeated to create several portfolios. The Efficient Frontier Curve (EFC) was created from the result of multiple portfolios with different asset weights, expected returns, Standard Deviations and Sharpe Ratios.

An efficient portfolio has the highest reward for a given level of risk, or the lowest risk for a given reward, which can be seen in the EFC. The best portfolio on the EFC is considered a subjective choice and depends on an investor's risk preferences. Each dot in the figure represents a different portfolio of the same assets but with different weights. This affects the volatility and expected return of portfolios.

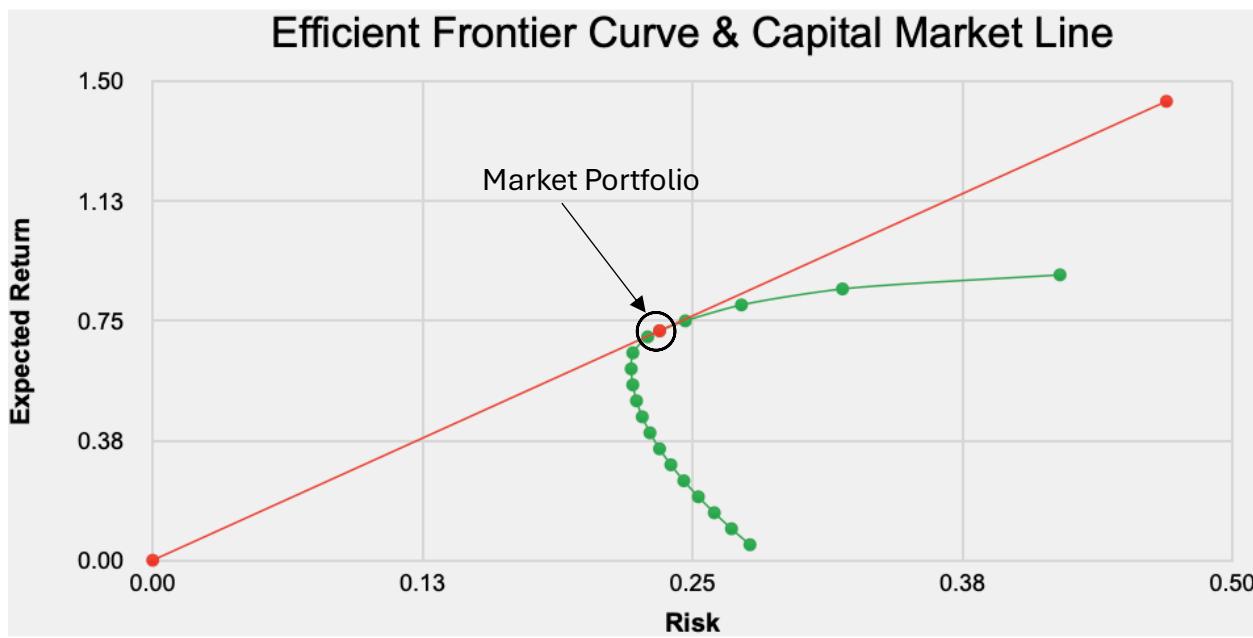


Figure 4: The Efficient Frontier Curve and Capital Market Line for the Portfolio with the Highest Sharpe Ratio. The Market Portfolio is Highlighted.

iii.

During the solver function process, a range of Sharpe Ratios was calculated from the results of different expected portfolio returns and volatilities calculated in part ii. The highest Sharpe Ratio had a value of 3.06, which was a result of an expected return of 0.719 and a volatility of 0.234.

The Capital Market Line (CML) includes a risk-free value of 1.5%, which allows it to become a straight line and includes different weights, as seen in the figure. The market portfolio is the point at which the CML touches the EFC. The economic significance of the CML is that investors can maximise returns by following the straight line of the CML as they are more efficient than any other portfolios for the same level of risk. A portfolio above the CML is rare and considered superior. A portfolio below the CML indicates underperformance, which should not be considered by investors. The CML is the relationship between risk and return for portfolios combining a risk-free asset. It can be calculated by

$$\text{Volatility} = \text{Weight} \times \text{optimal volatility}$$

$$\text{Exp. Return} = w \times \text{optimal return} + (1-w) \times \text{risk-free rate}$$

The optimal volatility and optimal return are both found in the portfolio with the highest Sharpe Ratio, and the weight can be chosen as any value. In this case, the weights were chosen as zero, one and two.

The Sharpe Ratio is calculated by

$$(\text{Expected Return} - \text{Risk-Free}) / \text{Standard Deviation}$$

This value helps investors evaluate the performance of a portfolio by considering both the return and the level of risk taken to achieve that return. A higher Sharpe Ratio of above one indicates better risk-adjusted returns and is considered favourable. A Sharpe Ratio of zero suggests the investment equals the risk-free rate. A negative result shows that the investment underperforms the risk-free rate.

Sharpe Ratio	Risk	Expected Return
2.12	0.42	0.89
2.66	0.32	0.85
2.93	0.27	0.80
3.03	0.25	0.75
3.06	0.23	0.72
3.05	0.23	0.70
2.92	0.22	0.65
2.70	0.22	0.60
2.47	0.22	0.55
2.23	0.22	0.50
1.98	0.23	0.45
1.73	0.23	0.40
1.49	0.23	0.35
1.24	0.24	0.30
1.01	0.25	0.25
0.79	0.25	0.20
0.57	0.26	0.15
0.37	0.27	0.10
0.18	0.28	0.05

Figure 5: The Resulting Sharpe Ratio, Volatility and Expected Return for Each Created Portfolio.

iv.

Linear Regression Analysis was used to calculate the *beta* for each asset in the portfolio. Other methods, such as Slope and Formula Methods, will produce the same results. The *beta* coefficient is a measure of the volatility, or systematic risk, of an asset in a portfolio in comparison to the market as a whole. Other values can be seen from Linear Regression Analysis, such as the significance and R squared value. Each asset is compared with the daily return of the market through the S&P500 closed values from the same historical dates as the assets in the portfolio.

AMZN								
<b>Regression Statistics</b>								
Multiple R	0.6759014276789							
R Square	0.4568427399384							
Adjusted R Square	0.4546613854803							
Standard Error	0.0128808107723							
Observations	251							
<b>ANOVA</b>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.0347477658399	0.0347477658399	209.4307682304	0.000000000000000			
Residual	249	0.0413129062518	0.0001659152861					
Total	250	0.0760606720917						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.0002402130151	0.00082135236152	0.2924603695655	0.770178203926	-0.001377470742	0.001857896772	-0.0013774707421	0.0018578967723
S&P 500	1.5249736068414	0.10537609107527	14.471723056723	0.000000000000000	1.3174315105009	1.732515703182	1.3174315105009	1.7325157031820
OXB								
<b>Regression Statistics</b>								
Multiple R	0.1190930852813							
R Square	0.0141831629618							
Adjusted R Square	0.0102240591986							
Standard Error	0.0263809714718							
Observations	251							
<b>ANOVA</b>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.0024932037831	0.0024932037831	3.582417589969	0.0595538857070			
Residual	249	0.1732929582938	0.0006959556557					
Total	250	0.1757861620770						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.0030956443064	0.0016821979299	1.8402378526601	0.0669233873721	-0.0002175063535	0.006408795148	-0.0002175063535	0.0064087951482
S&P 500	0.4084865607719	0.2158189963049	1.892725530221	0.059553885707	-0.0165769091871	0.833550030731	-0.0165769091871	0.8335500307316
PATH								
<b>Regression Statistics</b>								
Multiple R	0.4593176673454							
R Square	0.2109727195356							
Adjusted R Square	0.2078039352767							
Standard Error	0.0295865065451							
Observations	251							
<b>ANOVA</b>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.0582801963218	0.0582801963218	66.57844217182	0.000000000000000			
Residual	249	0.21796498101737	0.0008753613695					
Total	250	0.2762451773392						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	-0.003623921810	0.0018866007310	-1.920873744503	0.055889521277	-0.007339651505	0.000091807883	-0.0073396515051	0.0000918078836
S&P 500	1.9749649257778	0.24204302535115	8.1595613957994	0.000000000000000	1.4982522637934	2.451677587762	1.4982522637934	2.4516775877621
TSM								
<b>Regression Statistics</b>								
Multiple R	0.5828482004684							
R Square	0.3397120247892							
Adjusted R Square	0.3370602658526							
Standard Error	0.0205388966148							
Observations	251							
<b>ANOVA</b>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.0540419605894	0.0540419605894	128.1081851377	0.000000000000000			
Residual	249	0.1050397222651	0.0004218462741					
Total	250	0.1590816828545						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.0008020684209	0.0013096746420	0.6124180732767	0.540820049050	-0.001777384071	0.003381520913	-0.0017773840712	0.0033815209130
S&P 500	1.9017982016553	0.16802580819917	11.318488641941	0.000000000000000	1.5708651776920	2.232731225618	1.5708651776920	2.232731225618

INTC						
Regression Statistics						
Multiple R	0.5275426469827					
R Square	0.2783012443855					
Adjusted R Square	0.2754028558087					
Standard Error	0.0274185219467					
Observations	251					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	1	0.0721849430043	0.0721849430043	96.01930073025	0.0000000000000	
Residual	249	0.1871920610897	0.0007517753457			
Total	250	0.2593770040941				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	-0.005000413599	0.0017483579371	-2.860062858758	0.004594870820	-0.008443869014	-0.001556958184
S&P 500	2.197972189776	0.2243070499892	9.7989438578987	0.0000000000001	1.7561911851504	2.639753194401
					1.7561911851504	2.639753194401

Figure 6: Linear Regression Analysis for Each Stock, Calculating the Systematic Risk (Beta).

The *beta* values show a value greater than zero for all the stocks, showing that they all have a relationship with the market. The stocks AMZN, PATH, TSM and INTC all have a *beta* value of >1, meaning they are more volatile than the market as a whole. OXB has a *beta* value of <1, meaning it is less volatile than the market as a whole. This means OXB tends to fluctuate less than the overall market movement, but the rest of the assets tend to fluctuate more, making them riskier. Therefore, OXB would tend to be a more stable stock, performing more consistently even during more volatile market conditions. The negative is that the potential upside in strong markets may be limited compared to the other four stocks. The stocks with the highest *beta* values have the potential for greater returns however, they are more at risk of significant losses if the market downturns. Tech companies are said to generally have a high *beta* value in today's current market, especially emerging businesses.

Regression Systematic Risk	
Stock	Beta
AMZN	1.5249736068414
OXB	0.4084865607719
PATH	1.9749649257778
TSM	1.9017982016553
INTC	2.197972189776

Figure 7: Beta Values for Each of the Five Stocks.

Value at Risk (VaR) is an estimate, with a given degree of confidence, of how much one can lose from a portfolio over a given time. A VaR of 5% calculates the most a portfolio will lose at a 95% confidence level over a set period of time. VaR is calculated assuming the normal market circumstances, which means that extreme market conditions such as market crashes are not considered. The calculation requires at least the current price of all assets, their volatilities and the correlations between them. VaR is calculated as

$$\text{VaR} = \Delta S \left( \mu \delta t - \sigma \delta t^{1/2} \alpha(1 - c) \right).$$

Each asset contributes to the estimated VaR depending on its weight within the portfolio and its *beta* value. A higher weight means the asset contributes more to the portfolio's overall risk; a higher *beta* value will amplify the portfolio's systematic risk and therefore contribute more to the portfolio's VaR.

When looking at the optimised portfolio, the OXB asset has the highest weight at 40%, which will significantly affect the VaR value. The beta value is 0.41, which means it will be quite stable along with the overall market.

The AMZN asset weighs 31% within the portfolio, and TSM at 29%. Both assets have a *beta* value of >1, meaning they are more volatile than the market and may affect the VaR more than the OXB stock due to them having a lower weight and *beta* value.

Calculating VAR	
Exp Return	0.7190782547420
Exp Volatility	0.2346952965873
Days	5
Confidence Level	0.95
Z(1-Alpha)	-1.64
Asset Value (£)	2000
<b>VAR(5%) £</b>	
<b>28.48</b>	

Figure 8: VaR score at 95% Confidence

The returning VaR value shows a 95% confidence level that after five days, an investment of £2000 will lose a maximum of £28.48. The Z(1-Alpha) score represents the confidence level, which can be adjusted to percentages up to 99%.

## V.

The volatility of a single asset in the portfolio was estimated using ARCH/GARCH and its extensions using R. The chosen asset was OXB, and the best model was chosen based on various outcomes.

The problem with certain time series models is that they fail to capture one of the typical characteristics of financial data. This is known as heteroscedasticity, which means that the variance of a time series process is not constant over time. The Autoregressive Conditional Heteroscedastic (ARCH) model attempts to solve this problem. The ARCH and Generalised Autoregressive Conditional Heteroscedastic (GARCH) have become popular in finance as their flexibility allows them to test important ideas. First, the OXB stock was plotted as a time series figure to look at the daily returns for one year.

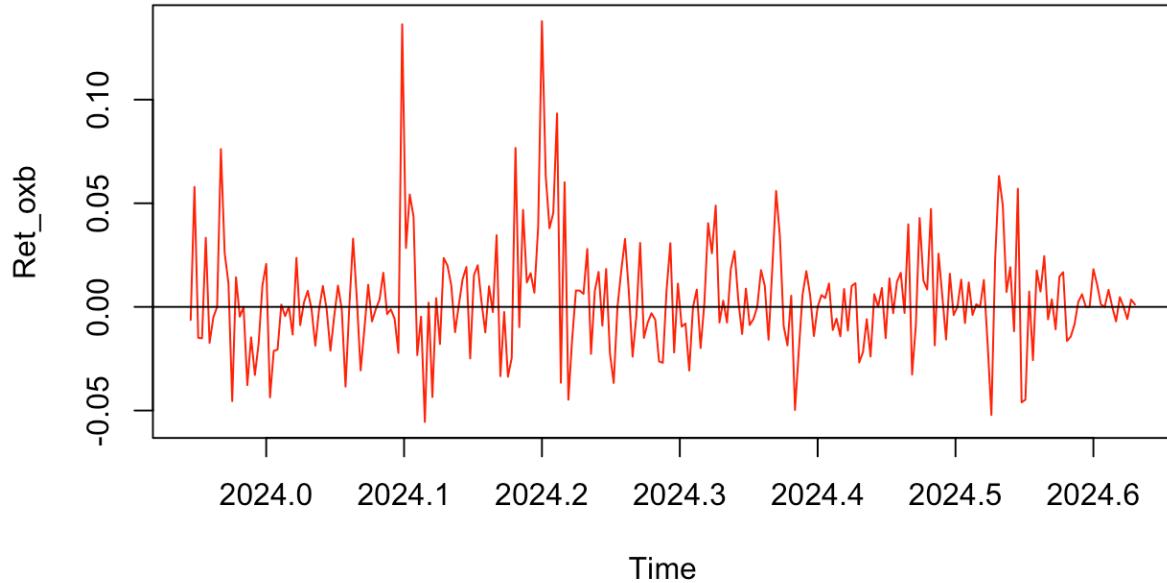


Figure 9: Time Series Plot of Daily Returns for the OXB Stock.

The time series extension was used in RStudio to test the different GARCH models against the dataset. The models were compared by looking for the Akaike information criterion (AIC), with the lowest AIC value suggesting the best-fitted model for the data.

	df	AIC
g10	2	-1097.119
g01	2	-1117.703
g11	3	<b>-1124.238</b>
g20	3	-1094.425
g21	4	-1124.649
g22	5	-1120.269
g02	3	-1122.318
g12	4	-1116.321

Figure 10: AIC Values for each of the Tested Models Against the OXB Dataset.

Based on the results from the AIC, model G11 was the best model to fit against the dataset. This model includes one lag each for the autoregressive term and the moving average term. It is used to model time series where variance changes over time and can capture high and low volatility within the OXB stock data. The model does not incorporate additional features or external regressors and assumes a simple structure for volatility dynamics. Additional diagnostic tests, such as the Ljung-Box test can be examined to evaluate the model's fit

through the summary function in RStudio. The p-value in the Box-Ljung Test ( $p = 0.8843$ ) suggests the model has successfully captured volatility clustering.

```
Call:  
garch(x = ts(Ret_oxb), order = c(1, 1), trace = FALSE)  
  
Model:  
GARCH(1,1)  
  
Residuals:  
    Min     1Q Median     3Q    Max  
-2.43413 -0.42534  0.04458  0.59099  6.09596  
  
Coefficient(s):  
            Estimate Std. Error t value Pr(>|t|)  
a0 1.632e-04 3.137e-05 5.203 1.96e-07 ***  
a1 3.474e-01 6.903e-02 5.033 4.84e-07 ***  
b1 4.846e-01 7.242e-02 6.691 2.21e-11 ***  
---  
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1  
  
Diagnostic Tests:  
    Jarque Bera Test  
  
data: Residuals  
X-squared = 491.14, df = 2, p-value < 2.2e-16  
  
Box-Ljung test  
  
data: Squared.Residuals  
X-squared = 0.02117, df = 1, p-value = 0.8843
```

Figure 11: Summary of the Model with the Lowest AIC (G11)

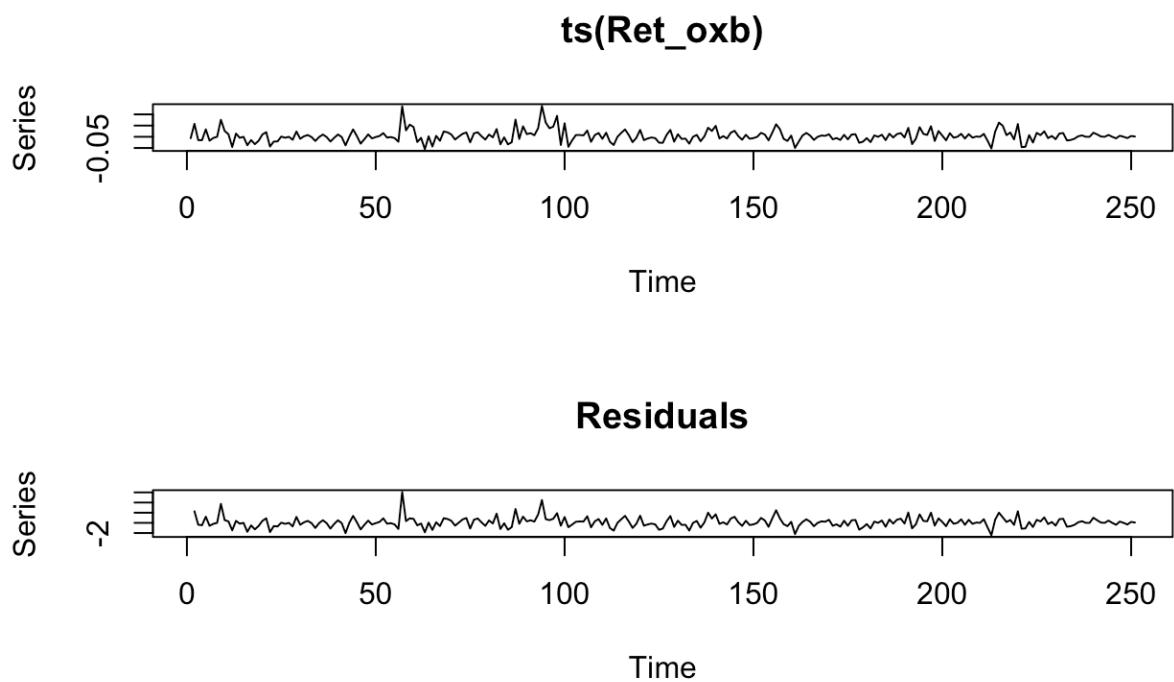


Figure 12: Plot of the returns and the residuals of the fitted GARCH model

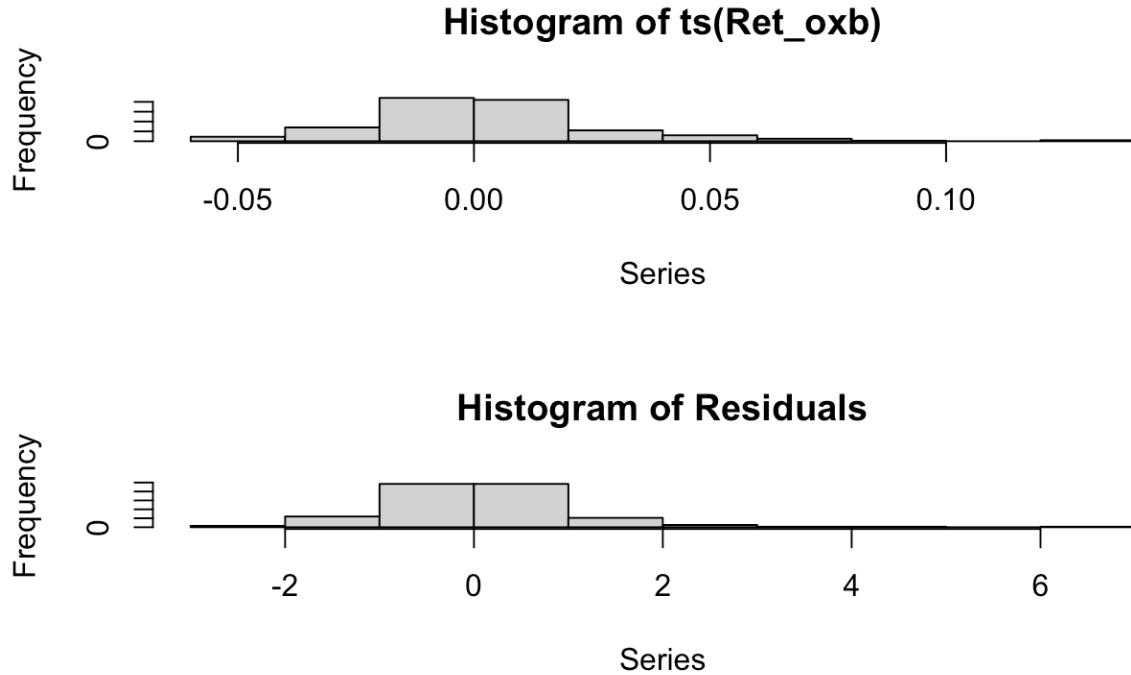


Figure 13: Histogram of the returns and the residuals of the fitted GARCH model

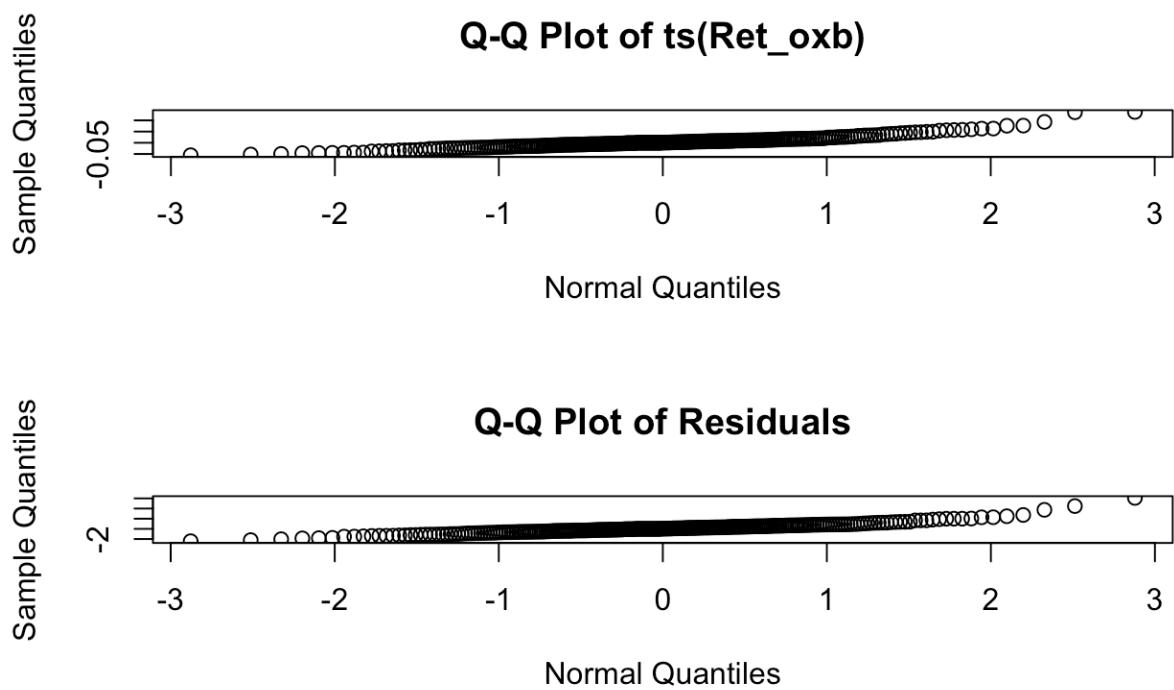


Figure 14: QQ-plot of the returns and the residuals of the fitted GARCH model

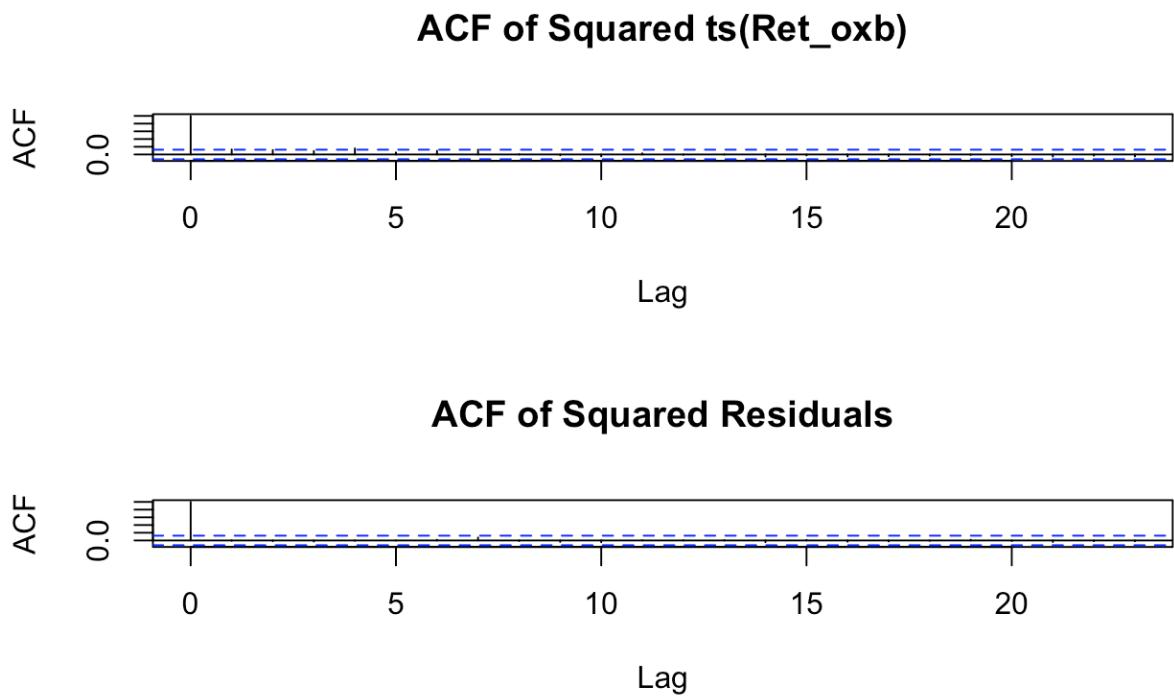


Figure 15: ACF plot of the returns and the residuals of the fitted GARCH model

**predict(g11)**

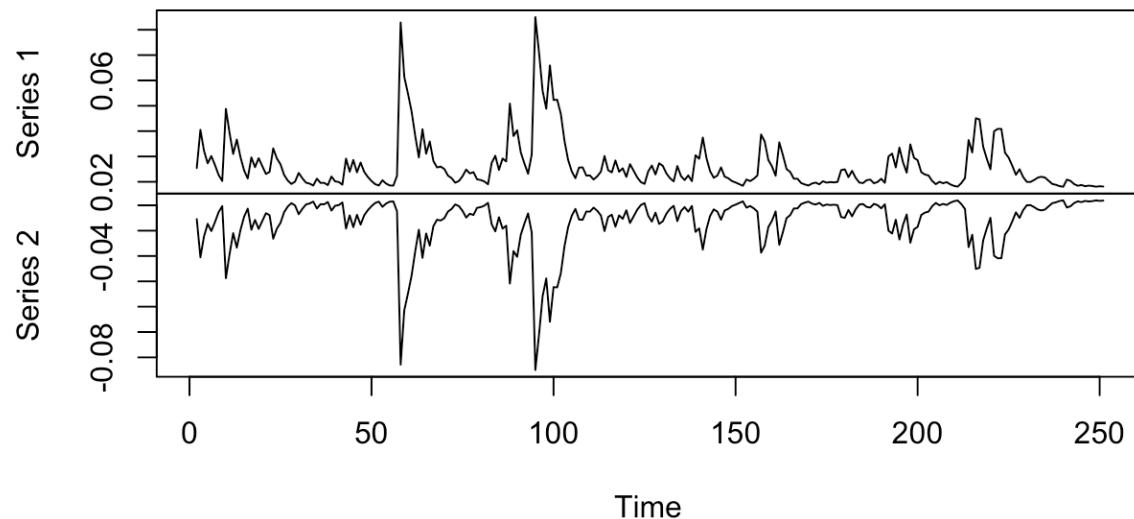


Figure 16: Predicted conditional variance and standardised residuals of the GARCH (1,1) model fitted to the return series of OXB stock.

The chosen GARCH model seems to successfully predict the OXB asset return as it handles the volatility dynamics well, as seen from the alignment between the volatility predictions and the actual behaviour of the time series.

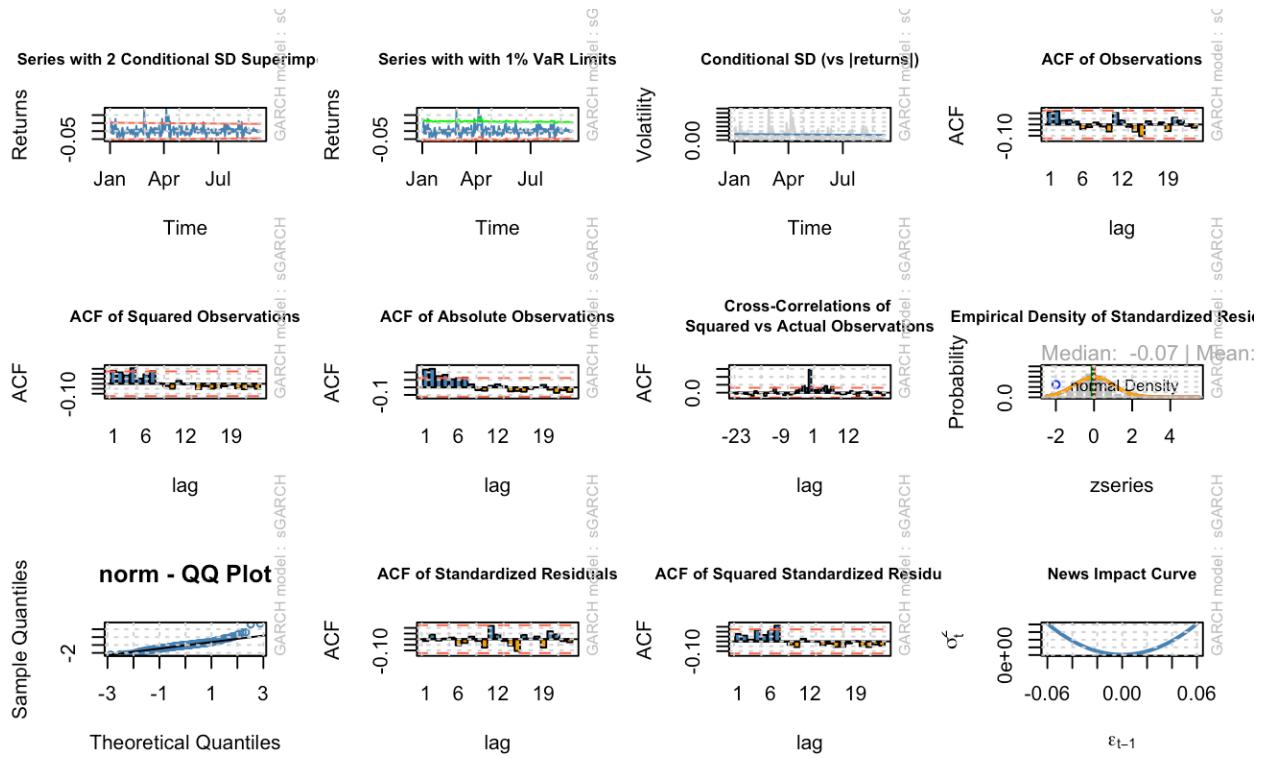


Figure 17: Output plots from a fitted ARMA(1,1), GARCH(1,1) model to the FTSE100 returns

vi.

Overall, the findings of this financial portfolio show that for the most efficient portfolio, 31% of the investment should be put in the AMZN asset, 29% into the TSM and 40% into the OXB asset. This will maximise the expected return on investment and minimise the risk. Each of these stocks comes with a level of risk, which can be compared to the overall stock market. The AMZN and TSM stocks are riskier than the average market, which suggests their value tends to drop or rise more dramatically than most stocks, and OXB is less risker than the market as a whole, meaning it will rise and fall less dramatically than the average market. Therefore, the balance in weight between these stocks is necessary to reduce the overall risk of the portfolio whilst maximising the return. There is a 95% confidence that a maximum loss of £28.48 will happen after five days from investing £2000. This can be checked with different time lengths and different amounts of investments. It is important to balance expected return and risk, depending on your intent, you can go for a riskier portfolio with a higher return potential or a portfolio with less expected return but less potential loss on investment.

There are other relevant performance measurements which can be made to ensure your investment will result in a positive outcome. Macroeconomic implications can affect the investment in assets and the overall market. Political and government policies may affect the stock more, therefore affecting your portfolio and expected return. Whilst the Capital

Asset Pricing Model and Portfolio Optimisation will provide useful and reliable results, external factors may still cause changes in the market.

## References

P. Wilmott, Paul Wilmott Introduces Quantitative Finance, 2nd ed. Hoboken, NJ, USA: Wiley, 2007.

H. Roberts, M. Stasinopoulos, and U. Pegier, Introduction to Time Series Analysis and Forecasting. September 25, 2015.

## Appendix

