A MODEL FOR SOURCE-SINK INVERSION IN GREAT TITS

Consider a population of great tits in living in two habitat types, deciduous (D) and evergreen (E) woodlands. The population includes individuals that are better adapted to deciduous and to evergreen. The habitat types have different areas, which translate to different carrying capacites: K_D for deciduous and K_E for evergreen. See Table 1 for a summary of notation.

Genetics: We assume haploid genetics underly adaptation to either deciduous or evergreen woodland.

Demography: We assume great tits are semelparous and the offspring produced in each generation are pooled and then randomly re-distributed onto the habitats. We also assume an *ideal despotic distribution* where the offspring distribute themselves preferentially into "source" habitat before filling "sink" habitat. The definitions of source and sink used in distributing offspring, however, must be relative to the genetics of the population. We choose to define the source relative to the average adaptedness of the population, i.e., when the frequency of the d allele in dispersers exceeds 0.5, ($p' > 0.5$) then deciduous habitat is the preferred habitat.

Source and *Sink*: A source is a net exporter, while a sink is a net importer. This gives the following dynamics:

- Reproduction: the number of offspring of each genotype from reproduction across all habitats

$$N'_d(t) = N_{D,d}(t)\lambda_{D,d} + N_{E,d}(t)\lambda_{E,d} \tag{1a}$$

$$N'_e(t) = N_{D,e}(t)\lambda_{D,e} + N_{E,e}(t)\lambda_{E,e} \tag{1b}$$

- Dispersal: the number of offspring distributed to each habitat type in the next generation

$$N_{D,d}(t+1) = \frac{N'_d(t)}{N'_d(t)+N'_e(t)} \begin{cases} K_D & \text{if } p' > 0.5 \\ \min\left[K_D, (N'_d(t)+N'_e(t)-K_E)\right] \end{cases} \tag{2a}$$

$$N_{D,e}(t+1) = \frac{N'_e(t)}{N'_d(t)+N'_e(t)} \begin{cases} K_D & \text{if } p' > 0.5 \\ \min\left[K_D, (N'_d(t)+N'_e(t)-K_E)\right] \end{cases} \tag{2b}$$

$$N_{E,d}(t+1) = \frac{N'_d(t)}{N'_d(t)+N'_e(t)} \begin{cases} K_E & \text{if } p' \le 0.5 \\ \min\left[K_E, (N'_d(t)+N'_e(t)-K_D)\right] \end{cases} \tag{3a}$$

$$N_{E,e}(t+1) = \frac{N'_e(t)}{N'_d(t)+N'_e(t)} \begin{cases} K_E & \text{if } p' \le 0.5 \\ \min\left[K_E, (N'_d(t)+N'_e(t)-K_D)\right] \end{cases} \tag{3b}$$

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Table 1. Notation for model.

Symbol	Meaning	Units
$N_E(t)$	Total population in Evergreen habitat at time t	[number]
$N_{E,e}(t)$	Population in E with advantaged allele$e$ at time t	[number]
$N_{E,d}(t)$	Population in E with disadvantaged d allele at time t	[number]
$N_D(t)$	Total population in Deciduous habitat at time t	[number]
$N_{D,e}(t)$	Population in D with disadvantaged allele$e$ at time t	[number]
$N_{D,d}(t)$	Population in D with advantaged d allele at time t	[number]
$\lambda_{E,e}$	Geometric growth rate of E-advantaged allele e	[]
$\lambda_{E,d}$	Geometric growth rate of E-disadvantaged allele d	[]
$\lambda_{D,d}$	Geometric growth rate of D-advantaged allele d	[]
$\lambda_{D,e}$	Geometric growth rate of D-disadvantaged allele e	[]
K_E	Carrying capacity of the Evergreen habitat	[number]
K_D	Carrying capacity of the Deciduous habitat	[number]
p	Frequency of D-advantaged allele d, i.e., $\frac{N_d}{N_d+N_e}$	[]