Artin's Algebra

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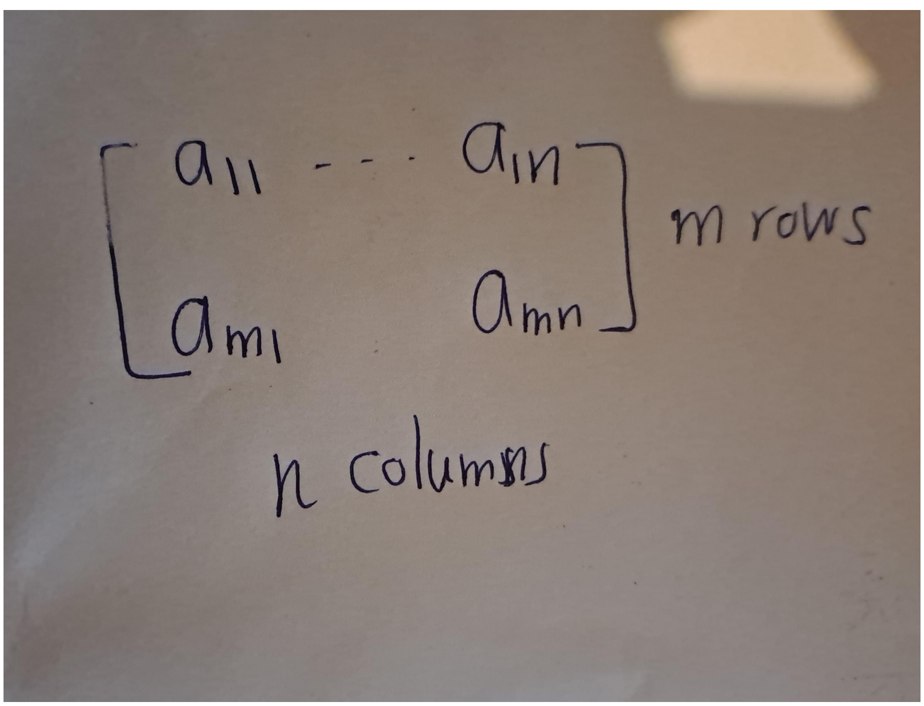
Chapter 1

Matrices

1.1 Basic Operations

Let m and n be positive integers. An $m \times n$ matrix is a collection of mn numbers arranged in a rectangular array.



::: {.example #unnamed-chunk-1}

$$A := \begin{bmatrix} 8 & 0 & 3 \\ 78 & -5 & 2 \end{bmatrix}$$

A is 2×3 matrix.(two rows and three columns)

::: The numbers in a matrix are the matrix entries. They may be denoted by a_{ij}, where i and j are indices (integers) with $1 < i < m$ and $1 < j < n$. The index i represents the row index, and j represents the column index. So a_{ij} is the entry that appears in the ith row and jth column of the matrix.

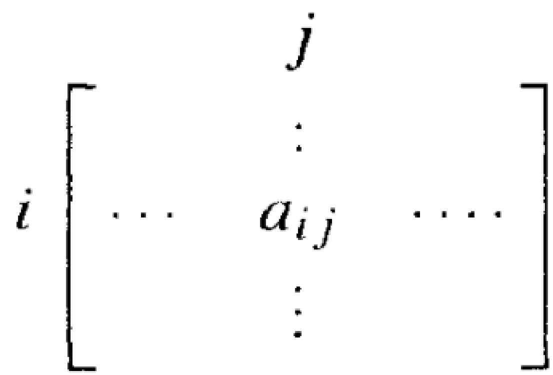


Figure 1.1:

Chapter 2

Group Theory

2.1 Laws of Compositions

2.1.1 Exercises

Exercise 2.1. Let S be a set. Prove that the law of composition defined by $ab = a$ for all a and b in S is associative? For which sets does this law have an identity?

Solution: Let $a, b, c \in S$. Now consider following

$$(ab)c = (ac) = a = (ab) = a(bc)$$

Thus, the given law of composition is associative.

If the given law of composition has an identity element whenever every element $a \in S$ has a multiplicative inverse./ In other words, for every element $a \in S$, there exists an element $e \in S$ such that

$$ae = ea = a.$$

Thus, $ae = a = ea = e$. So, the identity element is the same as every element in S, and the law has an identity for all sets S. Thus, only singletons sets have this given law of compositions have identity.

2.2 Groups and Subgroups

Definition 2.1 (Group)**.** A group is a set G together with a law of composition that has the following properties: