Baby Rudin

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Contents

Chapter 1

The Real and Complex Number Systems

1.1 Exercise

Exercise 1.1. If r is rational $r \neq 0$ and x is irrational, prove that $r + x$ and rx are irrational.

Suppose that $0 \neq r \in \mathbb{Q}$ and $x \in \mathbb{R} \setminus \mathbb{Q}$

- **Claim 1**: $r + x$ is irrational.
  Assume the contray that $r + x \in \mathbb{Q}$. Since, $\mathbb{Q}$ is field,
  $$x = (r + x) - r \in \mathbb{Q}$$
  . This is a contradiction. Thus, $r + x$ is contradiction.
- **Claim 2**: rx is irrational.
  Assume the contray that $rx \in \mathbb{Q}$. Since, $\mathbb{Q}$ is field,
  $$\left(\frac{1}{x}\right) rx = \left(\frac{1}{x}\right) xr = \left(\frac{1}{x}x\right) r = x \in \mathbb{Q}$$
  . This is a contradiction. Thus, $r + x$ is contradiction.

Exercise 1.2 (1:R2)**.** Prove that there is no rational number whose square is 12.

Proof. First observe that

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$