Baby Rudin

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Contents

Chapter 1

The Real and Complex Number Systems

1.1 Exercise

Exercise 1.1. If r is rational $r \neq 0$ and x is irrational, prove that $r + x$ and rx are irrational.

Suppose that $0 \neq r \in \mathbb{Q}$ and $x \in \mathbb{R} \setminus \mathbb{Q}$

- **Claim 1**: $r + x$ is irrational.
  Assume the contray that $r + x \in \mathbb{Q}$. Since, $\mathbb{Q}$ is field,

  $$x = (r + x) - r \in \mathbb{Q}$$

  . This is a contradiction. Thus, $r + x$ is contradiction.

- **Claim 2**: rx is irrational.
  Assume the contray that $rx \in \mathbb{Q}$. Since, $\mathbb{Q}$ is field,

  $$\left(\frac{1}{x}\right) rx = \left(\frac{1}{x}\right) xr = \left(\frac{1}{x}x\right) r = x \in \mathbb{Q}$$

  . This is a contradiction. Thus, $r + x$ is contradiction.

Exercise 1.2 (1:R2)**.** Prove that there is no rational number whose square is 12.

Proof. First observe that

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

Claim 1 : $\forall n \in \mathbb{N} 3|n^2 \implies 3|n$.
Let $n \in \mathbb{N}$. Suppose that $3|n^2$ We know that 3 is prime. Then by number theory resuts we can get $3|n$ or $3|n$. We are done.

Cliam 2: $|sqrt3$ is irrartional.
We use inderect proof. Assume contary, $\sqrt{3}$ is rational. In other words,

$$\sqrt{3} = \frac{p}{q} \text{ for some } q \in \mathbb{Z}, q \neq 0 \text{ and } p, q \text{ have no comman factors.}$$

Thus,

$$3q^2 = p^2$$

Then, by above claim we can get that $3|p$. Thus, there exists $k \in \mathbb{Z}$ such that $3k = p$. Thus,

$$\begin{aligned} 3q^2 &= p^2 \\ 3q^2 &= (3k)^2 \\ 3q^2 &= 9k^2 \\ q^2 &= 3k^2 \end{aligned}$$

So, both p and q have comman factor 3.This contract our assumption. Thefore, $\sqrt(3)$ is irrational. So we are done proof of claim 2.

Since $\sqrt{3}$ is not a $\square$

Exercise 1.3. Let E be a nonempty subset of an ordered set. Suppose α is a lower bound of E and β is an upper bound of E. Prove that $\alpha \leq \beta$.

Let $\emptyset \neq E$ is sub set of ordered set.

Chapter 2

Basic Topology

Chapter 3

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Chapter 4

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