

Baby Rudin

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Chapter 1

The Real and Complex Number Systems

1.1 Exercise

Exercise 1.1. If r is rational $r \neq 0$ and x is irrational, prove that $r + x$ and rx are irrational.

Suppose that $0 \neq r \in \mathbb{Q}$ and $x \in \mathbb{R} \setminus \mathbb{Q}$

- **Claim 1:** $r + x$ is irrational.

Assume the contrary that $r + x \in \mathbb{Q}$. Since, \mathbb{Q} is field,

$$x = (r + x) - r \in \mathbb{Q}$$

. This is a contradiction. Thus, $r + x$ is contradiction.

- **Claim 2:** rx is irrational.

Assume the contrary that $rx \in \mathbb{Q}$. Since, \mathbb{Q} is field,

$$\left(\frac{1}{x}\right)rx = \left(\frac{1}{x}\right)xr = \left(\frac{1}{x}x\right)r = r \in \mathbb{Q}$$

. This is a contradiction. Thus, rx is contradiction.

Exercise 1.2 (1:R2). Prove that there is no rational number whose square is 12.

Proof. First observe that

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

Claim 1 : $\forall n \in \mathbb{N} 3|n^2 \implies 3|n$.

Let $n \in \mathbb{N}$. Suppose that $3|n^2$. We know that 3 is prime. Then by number theory results we can get $3|n$ or $3|n$. We are done.

Claim 2: $\sqrt{3}$ is irrational.

We use indirect proof. Assume contrary, $\sqrt{3}$ is rational. In other words,

$$\sqrt{3} = \frac{p}{q} \text{ for some } q \in \mathbb{Z}, q \neq 0 \text{ and } p, q \text{ have no common factors.}$$

Thus,

$$3q^2 = p^2$$

Then, by above claim we can get that $3|p$. Thus, there exists $k \in \mathbb{Z}$ such that $3k = p$. Thus,

$$3q^2 = p^2$$

$$3q^2 = (3k)^2$$

$$3q^2 = 9k^2$$

$$q^2 = 3k^2$$

So, both p and q have common factor 3. This contradicts our assumption. Therefore, $\sqrt{3}$ is irrational. So we are done proof of claim 2.

Since $\sqrt{3}$ is not a

□

Exercise 1.3. Let E be a nonempty subset of an ordered set. Suppose α is a lower bound of E and β is an upper bound of E . Prove that $\alpha \leq \beta$.

Let $\emptyset \neq E$ is a subset of an ordered set.

Chapter 2

Basic Topology

Chapter 3

Numerical Sequences and Series

Chapter 4

Continuity