Manifolds

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Chapter 1

Basic Theroms and Definitions

Definition 1.1 (Topology)**.** A topology on a set X is a collection $\mathcal{T}$ of subsets of X such that

(T1) ϕ and X are in $\mathcal{T}$;

(T2) Any union of subsets in $\mathcal{T}$ is in $\mathcal{T}$;

(T3) The finite intersection of subsets in $\mathcal{T}$ is in $\mathcal{T}$.

A set X with a topology $\mathcal{T}$ is called a topological space. Denoted by $(X, \mathcal{T})$. An element of $\mathcal{T}$ is called an open set.

Definition 1.2. A subset $U \subset M$ is referred to as open in M if $U \in \mathcal{T}$. A subset $A \subset M$ is termed closed if $M \setminus A \in \mathcal{T}$.

Definition 1.3 (Continuity)**.** If both $(M, \mathcal{T}_M)$ and $(N, \mathcal{T}_N)$ are topological spaces, a map $f : M \to N$ is termed continuous if

$$f^{-1}(V) \in \mathcal{T}_M \text{ for all } V \in \mathcal{T}_N$$

. In other words, the preimages of open sets must be open.

Definition 1.4 (Homemorphism)**.** A map $f : M \to N$ between two topological spaces is called homemorphism if it has following propoties. - f is a bijection, - f is continuous, - the inverse function f^{-1} is continuous.

Two topological spaces M and N are called homeomorphic if there exists a homeomorphism between them.

Definition 1.5 (Hausdorff Space)**.** A topological space $(X, \mathcal{T})$ is called a Hausdorff space if

(H1) $\forall x, y \in X$ such that $x \neq y$, $\exists U_x, U_y \in \mathcal{T}$ such that $x \in U_x$, $y \in U_y$, and $U_x \cap U_y = \emptyset$.

i.e., for every pair of distinct points x, y in X, there are disjoint neighborhoods U_x and U_y of x and y respectively.

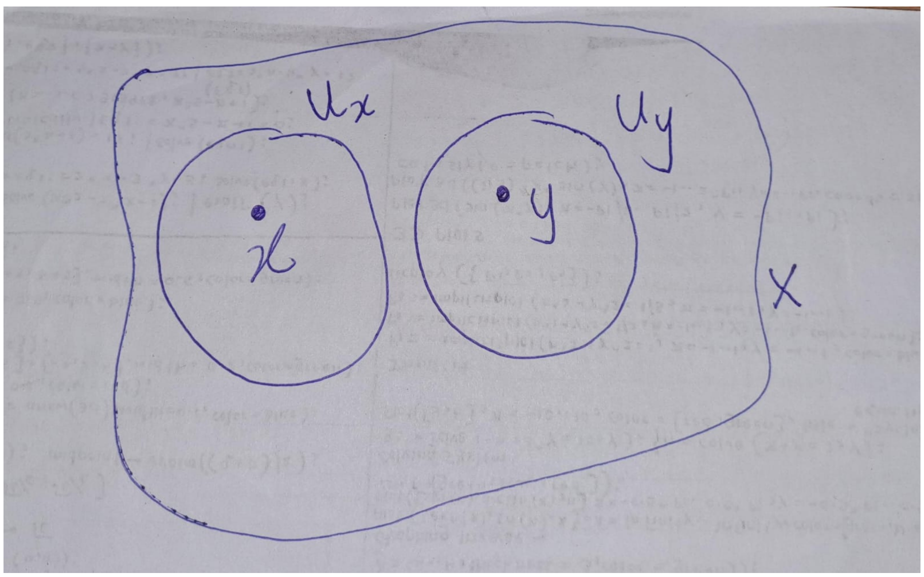

Figure 1.1:

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A space $X$ is said to have a **countable basis at the point $x$** if there is a counta
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Chapter 2

Manifolds

2.1 Topological Manifolds