Let (M, τ_{dis}) 0-dimesinal connected Top-manifold.
Let $p, q \in M$ with $p \neq q$.
Sinc M is connected that there exist continous function

$$\begin{aligned} c : [0,1] &\longrightarrow M \\ 0 &\longmapsto p \\ 1 &\longmapsto q \end{aligned}$$

Let $U = c^{-1}(\{p\})$
Since $\{P\}$ is open in M (discreate top) and c is continous, U is open in $([0,1], \tau_{sub})$ 1

Note that $M \setminus \{P\}$ is an subset of M, that is open under discreate topology.

$V = c^{-1}(M \setminus \{P\})$ is open in $[0,1]$ 2

claim: $V \cap U \neq \emptyset$ 3

$$\begin{aligned} c^{-1}(M \setminus \{P\}) \cap c^{-1}(\{P\}) &= c^{-1}((M \setminus \{P\}) \cap \{P\}) \quad \text{(lemma1)} \\ &= c^{-1}(\emptyset) \quad \text{(lemma 2)} \\ &= \emptyset \end{aligned}$$

claim: $V \cup U = [0,1]$ — 4

$V \cup U = c^{-1}(M \setminus \{p\}) \cup c^{-1}(\{p\})$ (lemma 3)

$= c^{-1}((M \setminus \{p\}) \cup \{p\})$

$= c^{-1}(M)$

$= [0,1]$?

Now we found that two non-empty opensets such that,

$V \cap U = \phi$ and $V \cup U = [0,1]$

But, we know that $[0,1]$ is connected. So, this is a contradiction.

Thefore, $p = q$

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Lemma1 : $c^{-1}(U_1 \cap U_2) = c^{-1}(U_1) \cap c^{-1}(U_2)$

Let $x \in c^{-1}(U_1) \cap c^{-1}(U_2)$

Then $x \in c^{-1}(U_1)$ and $x \in c^{-1}(U_2)$

$c(x) \in U_1$ and $c(x) = U_2$

Thus, $c(x) = U_1 \cap U_2$

$x \in c^{-1}(U_1 \cap U_2)$

Thus, $c^{-1}(U_1 \cap U_2) \supseteq c^{-1}(U_1) \cap c^{-1}(U_2)$

Let $y \in c^{-1}(U_1 \cap U_2)$. Then $c(y) \in U_1 \cap U_2$

Thus $c(y) \in U_1$ and $c(y) \in U_2$

$y \in c^{-1}(U_1)$ and $y \in c^{-1}(U_2)$

Thus, $y \in c^{-1}(U_1) \cap c^{-1}(U_2)$

Hence, $c^{-1}(U_1) \cap c^{-1}(U_2) \supseteq c^{-1}(U_1 \cap U_2)$

Therfore $c^{-1}(U_1 \cap U_2) = c^{-1}(U_1) \cap c^{-1}(U_2)$

Lemma 2: $c^{-1}(\phi) = \phi$

Assume the contray $c^{-1}(\phi) \neq \phi$. Then we can choose that $x \in c^{-1}(\phi)$. Thus, $c(x) \in \phi$. This is contradiction. Thefore $c^{-1}(\phi) = \phi$.

Lemma 3: $c^{-1}(U \cup V) = c^{-1}(U) \cup c^{-1}(V)$

Let $x \in c^{-1}(U \cup V)$. Then, $c(x) \in U \cup V$

$c(x) \in U$ or $c(x) \in V$

$x \in c^{-1}(U)$ or $x \in c^{-1}(V)$

$x \in c^{-1}(U) \cup c^{-1}(V)$

Thus, $c^{-1}(U \cup V) \subseteq c^{-1}(U) \cup c^{-1}(V)$

Let $y \in c^{-1}(U) \cup c^{-1}(V)$. So,

$y \in c^{-1}(U)$ or $y \in c^{-1}(V)$

$c(y) \in U$ or $c(y) \in V$

Thus, $c(y) \in U \cap V$. Thus, $y \in c^{-1}(U \cup V)$

Hence, $c^{-1}(U) \cup c^{-1}(V) \subseteq c^{-1}(U \cup V)$

Thefore, $c^{-1}(U) \cup c^{-1}(V) = c^{-1}(U \cap V)$

Lemma 4: $c: X \longrightarrow Y$ is continuous function

$c^{-1}(M) = [0,1]$

Let $x \in c^{-1}(M)$

$c(x) \in M$

$x \in$

Let $y \in [0,1]$

$c(y) \in M$

$y \in c^{-1}(M)$

Lemm 5: $c^{-1}(M) = [0,1]$

Recall the definition of preimage

$$c^{-1}(M) := \{x \in M \mid c(x) \in M\}$$

Let $a \in [0,1]$. Then $c(a) \in M$. Thus, $a \in c^{-1}(M)$

Hence, $[0,1] \subseteq c^{-1}(M)$

Let $b \in c^{-1}(M)$. So, $c(b) \in M$, then $b \in [0,1]$

$$c: [0,1] \longrightarrow M$$