Maple

Ashan J

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Contents

Chapter 1

Introduction to Maple

This is temporary file

1.1 What is Maple?

- Maple is a Symbolic Computation System or Computer Algebra System which can be used to obtain exact analytical solutions to many mathematical problems, including integrals, systems of equations, differential equations, and problems in linear algebra.
- It also has the capability of plotting functions in 2D and 3D and displaying animations.
- Maple can perform calculations in binding speed, but one has to be responsible for making these calculations meaningfully and mathematically correct.

1.2 Using a Maple Worksheet

The following figure shows the Maple window with a blank Maple worksheet and this window contains:

- a menu bar across the top with menus:
- a tool bar immediately below the menu bar, with button-based short cuts to common operations;
- a context bar directly below the tool bar, with controls specific to the task being performed;

- a window, containing a Maple prompt [>, called a worksheet;

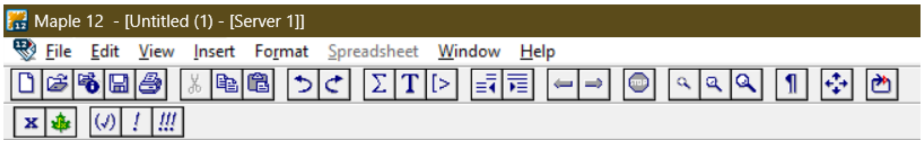
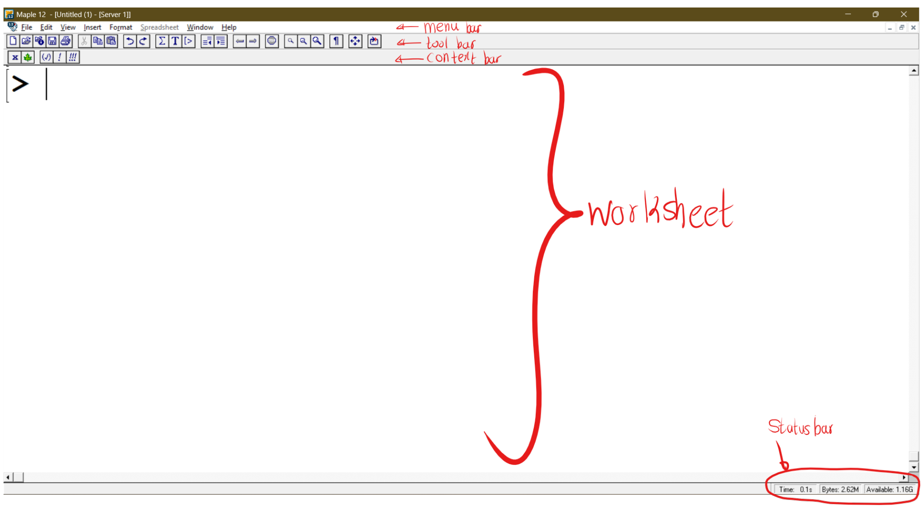


Figure 1.1: The menu bar, tool bar, and context bar.

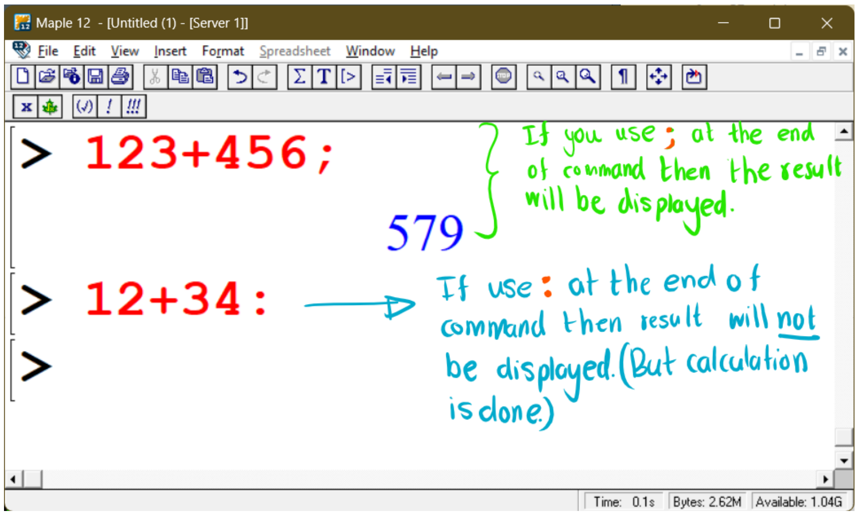
- a status bar at the bottom, with boxes marked Ready, Time and Memory



From the File menu, select the options **Save** or **Save As** to save the active Maple classic worksheet. Maple classic worksheets are saved with the extension ".mws", but in the standard interface, Maple worksheets are saved with the extension ".mw"

1.3 Entering Maple Commands

- The " > " is the command prompt in Maple. That is where you type your commands or statements.
- Every command in Maple should end with a semicolon(;) or a colon(:). (If you use a semicolon then the result of the command will be displayed. If you use a colon then the result will not be displayed.)



- If you want to make any comments you can use the text format by clicking on the box `T` in the tool bar or use the symbol `#`.

1.4 Arithmetic operations

Arithmetic operators follow the same precedence rules as in Mathematics, and these are brackets, of, division, multiplication, addition and subtraction (BODMAS). Usual arithmetic operations can perform easily with Maple.

```
[> 312+121;
```


```
> 125-45;
```

The `*` key is used for multiplication, `/` for division and `^` for the power.

```
[> 13*267;
```

```
> 13*267;
```

3471

```
[> 565/5;
```

```
> 565/5;
```

113

```
[> 561/5;
```

```
> 561/5;
```

$$\frac{561}{5}$$

```
[> 125-45; 13*267; 12345/5; #Three arithmetic operations
```

```
> 125-45; 13*267; 12345/5; #Three arithmetic operations
```

80
3471
2469

```
[> 2^5;
```

```
> 2^5;
```

32

```
[> 2^(-5);
```

```
> 2^(-5);
```

$$\frac{1}{32}$$

```
[> 3^40;
```

```
> 3^40;
```

12157665459056928801

Remark. ***Don't use commas when you type large numbers in Maple. - For example: Compute the product 102,136,543 & 20,077,410 .***

```
[> 102136543*20077410;
```

```
> 102136543*20077410;
```

2050637249793630

1.5 Operations

Maple adheres to the same order of operations that we use in Mathematics. By inserting parentheses, we can change this order.

```
[> 2+3*4-5*6;
[> 2+(3*4-5)*6;
[> (2+3)*4-5*6;
```

```
> 2+3*4-5*6;
                                                -16
> 2+(3*4-5)*6;
                                                 44
> (2+3)*4-5*6;
                                                -10
```

```
[> 29/(100-11*3^2);
```

```
> 29/(100-11*3^2);
                                                 29
```

```
[> (3^4-2^6)/(3^2-2^3);
```

```
> (3^4-2^6)/(3^2-2^3);
                                                 17
```

1.5.1 Exercises

Exercise 1.1.

1. Calculate the followings

i. $1428 + 456 - 41$
ii. $421 \times 240 \div 55$
iii. $(128 - 691 + 458) \times 8$
iv. $2214875(201 \times 11 - 55)$
v. $201 \div (2012 - 1)$

vi. $21^{4^{2^3}}$

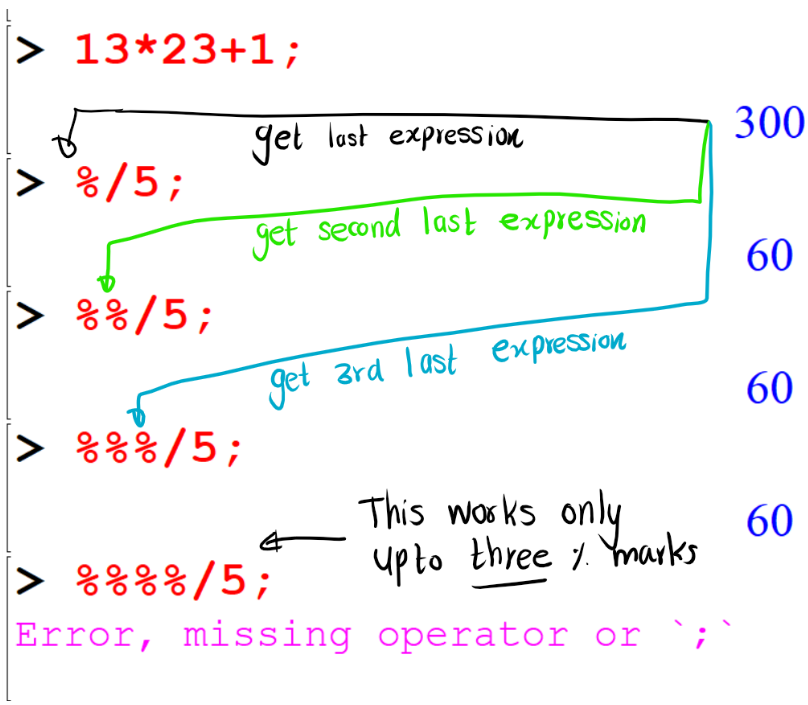
Exercise 1.2.

i. Compute 3^{400}.
ii. Find the command to find the length (number of digits) of a number.
iii. How many digits are there in the number 3^{400} ?
iv. Does the above command give correct answer to the fractional numbers?

1.6 Shortcut to retyping

One shortcut that we use often in Maple to retype is the % key. This refers to most recently executed result.

```
[> 13*23+1;
[> %/5;
[> %%/5;
[> %%%/5;
```




```
[> 12540*4;
[> %/4;
[> %/4;
```

```
> 12540*4;
```

gets the last expression

$$50160$$

```
> %/4;
```

gets the last expression

$$12540$$

```
> %/4;
```

$$3135$$

```
>
```

1.7 Fractions and Decimals

By simply entering a fraction Maple automatically reduce it.

```
[> 45/4;
```

```
> 45/4;
```

$$\frac{45}{4}$$

```
[> 148/24;
```

```
> 148/24;
```

$$\frac{37}{6}$$

```
[> 25/15;
```

```
> 25/15;
```

$$\frac{5}{3}$$

```
[> 2/3+3/7;
```

```
> 2/3+3/7;
```

$$\frac{23}{21}$$

```
[> 3/2+4/5-1/3;
```

```
> 3/2+4/5-1/3;
```

$$\frac{59}{30}$$

You can do calculations with decimal numbers also.

```
[> 25.361+124.6;
```

```
> 25.361+124.6;
```

$$149.961$$

```
[> 2.138*0.013;
```

```
> 2.138*0.013;
                                        0.027794
```

```
[> 56.101/0.102;
```

```
> 56.101/0.102;
                                        550.0098039
```

1.8 Roots

```
[> sqrt(16);
```

```
> sqrt(16);
                                        4
```

```
[> sqrt(30);
```

```
> sqrt(30);
```

$$\sqrt{30}$$

```
[> evalf(sqrt(30));
```

```
> evalf(sqrt(30));
                                        5.477225575
```

The `evalf` command numerically evaluates expressions (or subexpressions) involving constants (for example, `Pi`, `exp(1)`) and mathematical functions (for example, `exp`, `ln`, `sin`).

```
[> 30^(1/2);
[> evalf(%);
```

```
> 30^(1/2);
```

$$\sqrt{30}$$

```
> evalf(%);
```

$$5.477225575$$

1.8.1 Exercise

Exercise 1.3. Compute the following:

i. $11 + \sqrt{31}$
ii. $\sqrt[3]{64}$
iii. $\sqrt{2}^{\sqrt{3}}$

Exercise 1.4. Calculate 120^8

i. Divide the answer by 10^8
ii. Divide the answer in part i. by 2048×8

1.9 Pi Vs pi

π is a constant in Mathematics and is recognized by maple and typed as `Pi` **(Note the capitalization of "p" but not "i')**.

```
[> Pi;
[> evalf(%);
```

```
> Pi;
```

$$\pi$$

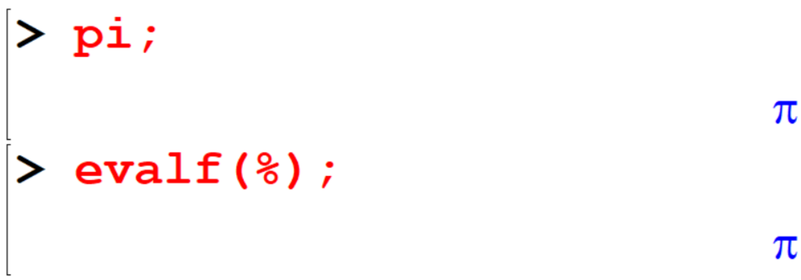
```
> evalf(%);
```

$$3.141592654$$

```
[> evalf(2*Pi);
```

If you use `pi` then `evalf` command does not return π numerically.

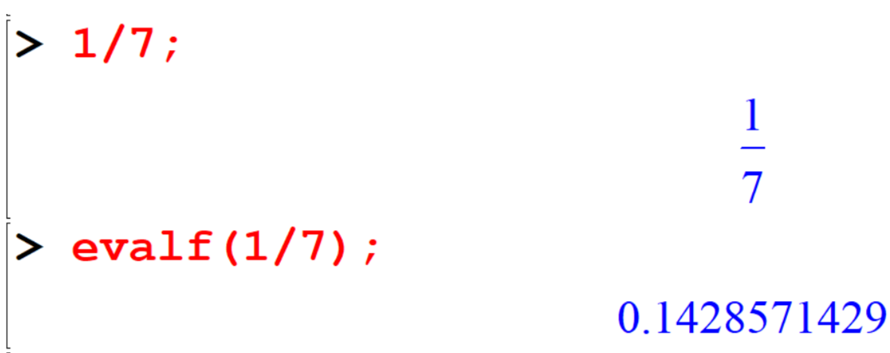
```
[> pi;
[> evalf(%);
```



1.10 Rational Numbers

Maple usually leaves fractions in fraction form. However, we can force it to express fractions in decimal form using the `evalf` command.

```
[> 1/7;
[> evalf(1/7);
```




```
[> 25/35;
[> evalf(%);
```

```
> 25/35;
```

$$\frac{5}{7}$$

```
> evalf(%);
```

$$0.7142857143$$

Maple displays 10 decimal places as a default. If this is not enough and for better precision you can specify the exact number of decimal places as a second parameter to the `evalf` command.

Note that the second parameter normally represents the number of non-zero digits in the answer.

```
[> evalf(1/7,100);
```

```
> evalf(1/7,100);
0.1428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571429
```

```
[> evalf(29/3,5);
```

```
> evalf(29/3,5);
```

$$9.6667$$

Here, if you want to calculate the answer for 4 decimal places the command should be `evalf(29/3,5)` and if you want the answer to be 5 decimal places the command should be `evalf(29/3,6)`.

```
[> evalf(1/15,5);
```

```
> evalf(1/15,5);
```

$$0.066667$$

```
[> evalf(11/19,5);
```

```
> evalf(11/19,5);
                                                   0.57895
```

Fractions are also rational numbers because their decimal expansions always have repeating blocks of digits. By looking at the decimal representation of a rational number you can see the repeating cycle.

```
[> evalf(1/35);
```

```
> evalf(1/35);
                                              0.02857142857
```

Here we can not see the repeat cycle. But if we calculate $\frac{1}{35}$ for more decimal places we can see the repeating cycle.

```
[> evalf(1/35, 100);
```

```
> evalf(1/35, 100);
        0.02857142857142857142857142857142857142857142857142857142857142857142857142857142857142857142857142857
```

1.10.1 Exercises

Exercise 1.5. Evaluate the value of correct up to 5 decimal places.

Exercise 1.6. Find the area of a circle with radius 10cms

Exercise 1.7. Check whether the followings are rational numbers or not

i. $\frac{1}{49}$
ii. π
iii. $\sqrt{2}$

Exercise 1.8. How many digits are repeating in $\frac{1}{212}$

1.11 Complex Numbers

In Maple, complex arithmetic is normally done automatically with `I` standing for $\sqrt{-1}$ (for example, if you square `I` you will get `-1`, not `I^2`)

```
[> (-4+7*I)+(5-10*I);
```

```
> (-4+7*I)+(5-10*I);
```

$$1 - 3\,I$$

```
[> 5*I-(-9+I);
```

```
> 5*I-(-9+I);
```

$$9 + 4\,I$$

```
[> (1-5*I)*(-9+2*I);
```

```
> (1-5*I)*(-9+2*I);
```

$$1 + 47\,I$$

```
[> (3-I)/(2+7*I);
```

```
> (3-I)/(2+7*I);
```

$$\frac{-1}{53} - \frac{23}{53}\,I$$

But Maple does not always automatically evaluate an expression involving complex numbers. For example, it may leave an expression as the product of some complex numbers or as an expression involving a root of a complex number.

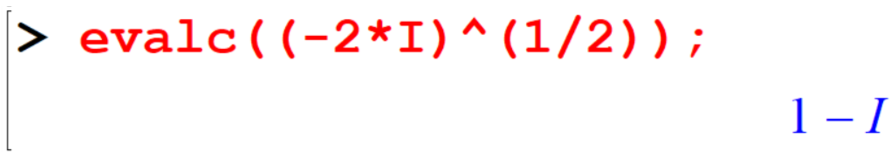
```
[> (-2*I)^(1/2);
```

```
> (-2*I)^(1/2);
```

$$\sqrt{-2\,I}$$

The function `evalc` to force Maple to evaluate as a complex number.

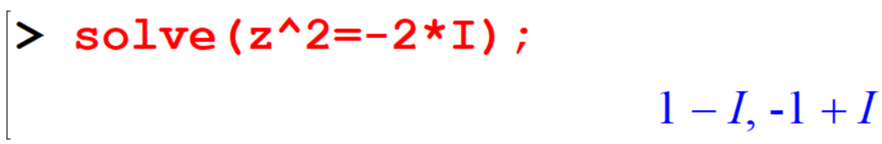
```
[> evalc((-2*I)^(1/2));
```



Note that `evalc` does not give you both the square roots of `-2*I` it only gives the *principal value* of the root.

If you want to find the roots, use solve command as follows.

```
[> solve(z^2=-2*I);
```



1.11.1 Exercise

Exercise 1.9. Simplify the following:

1. $(-3+3i)+(7\text{–}2i)$
2. $(5+3i)-(3-i)$
3. $(1+2i)(1-2i)4.(56-8i) \div (14+10i)$

Exercise 1.10.

2. Simplify $(2i)^{\frac{1}{2}}$ by using `evalc` and solve commands.

Exercise 1.11.

3. Multiply the following and obtain the answer in standard form:

$$(2-\sqrt{-100})(1+\sqrt{-36})$$

1.12 MAPLE Help

Maple contains a complete online help system you can use to find information about specific topic easily and to explore the wide range of commands available. To get the information about commands, which you learn in MAPLE, you can use either one of the following.

- Topic search function
- F1 key
- `?` In front of the command

Chapter 2

Basic Algebra

2.1 Variables and Expressions

Study how we write the following expressions in Maple.

```
[> 3*x;
```

```
> 3*x;
```

$$3\,x$$

```
[> 2+3*x;
```

```
> 2+3*x;
```

$$2 + 3\,x$$

```
[> x^2;
```

```
> x^2;
```

$$x^2$$

```
[> 1/(x-2);
```

2.1.1 Exercise

Exercise 2.1. Generate the following expressions using maple commands.

i) $(2x+3)(4x+5)$
ii) $9x^2-4$
iii) $\frac{2x}{5}+\frac{y}{5}$
iv) $\frac{\sqrt{2x+1}}{3+y}$

2.2 Assigning values to variables

Maple will store things (numbers, expressions, functions,...) in 'containers' or 'variables'. This process is called assignment, and you can assign values to the variables by using `:=` or using the command `assign`. After assigning a value to a variable it becomes a constant and Maple will remember that value from that point.

- **Method I**

```
[> x:=5;
```

```
[> x^2;
```

```
[> 2*x+23;
```

```
> x:=5;
                                        x := 5
> x^2;
                                          25
> 2*x+23;
                                          33
```

- **Method II**

```
[> assign(y,5);

[> y^2;
```

```
> assign(y,5);
> y^2;
                                          25
```

You can also define variables in a descriptive manner. However they must begin with a character and any blank spaces must not be included. Note that maple is case sensitive.

```
[> Age:=21;

[> age;

[> Age;

[> Exam_No:=2012300;

[> New Age:=Age+4; # There should not be any blank spaces when giving a name for the variable

[> New_Age:=Age+4;
```

If you want to reset the predefined value for variables you have to use following methods.

- **Method I**: `restart` command

```
[> restart;
[> Age;
```

- **Method II** : `unassign`
  The `unassign` command unassigns all the unevaluated names given as input.

```
[> x:=5;
[> x+20;
[> unassign('x');
[> x+20;
```

2.3 Substituting Values

To substitute numbers (or other expressions) in the place of variables in an algebraic expression without permanently changing the values of the variable, we use the command, `subs(variable, expression)`.

Try to use help

```
[> ?subs
```

```
> x;
> subs(x=2,x+5);
> subs(x=2,x^2-5*x+4);
```

It is convenient to make substitution by giving name for an expression.

```
> expr:=(2*x+1)/(5-3*x);
> subs(x=3,expr);
```

You can even substitute other variables or expressions to a variable in an expression.

```
> subs(x=a,expr);
> subs(x=a+1,expr);
```

You can also substitute more than one variable for a expression.

```
[> expr1:=(7*x-3*y)/(x^2-y^2);
```

```
[> subs({x=3,y=2},expr1);
```

```
[> subs({x=a-2,y=a+2},expr1);
```

```
[> expr2:=x^2+2*y^2+z^2;
```

```
[> subs({x=1,y=2,z=3},expr2);
```

2.4 Factoring expressions

Maple can factorize an integer into primes using the command `ifactor`.

```
[> ifactor(18);
```

```
[> ifactor(525);
```

```
[> ifactor(2^8-1);
```

2.5 Expanding expressions

One of the most important things Maple can do is to calculate with expressions as well as numbers and we use the command `expand` to get the expansion of an expression.

2.6 Finding the degree and leading coefficient of polynomials

The highest power of the variable that occurs in the polynomial is called the **degree** of a polynomial. The **leading term** is the term with the highest power, and its coefficient is called the **leading coefficient**.

If x is a single indeterminate, the `degree` and `ldegree` commands compute the degree and low degree respectively, of the polynomial p in x.

```
[> p:=(x+5)^5;
[> expand(%);
[> degree(p,x);
[> ldegree(p,x);
```

```
> p:=(x+5)^5;
```
$$p := (x+5)^5$$
```
> expand(%);
```
$$x^5 + 25\,x^4 + 250\,x^3 + 1250\,x^2 + 3125\,x + 3125$$
```
> degree(p,x);
```
$$5$$
```
> ldegree(p,x);
```
$$0$$

The `coeff(p, x^n)` command finds the coefficient of x^n in the polynomial p where p is the polynomial in x and n is the integer corresponds to the power. When finding the leading coefficient, n corresponds to the highest power in the polynomial.

```
[> coeff(p,x^5);
```

```
[> coeff(p,x,5); #Another way of finding leading coefficient
```

```
> coeff(p,x^5);
```
$$1$$
```
> coeff(p,x,5); #Another way of finding leading coefficient
```
$$1$$

2.6.1 Exercise

Exercise 2.2. Generate the expression $x^3 + \sqrt{x}$ using maple commands and find the value of the expression when $x = 4$.

Exercise 2.3. PQR is an isosceles triangle where $Q = PR = 2r + 3$ and $QR = r + 3$. Find an expression for the perimeter of the PQR triangle in terms of r, giving your answer in its simplest form and find the perimeter when $r = 8$.

Exercise 2.4. Factorize the following expressions:

i. $3x^2 + 8x + 5$
ii. $x^4 - 3x^2 + 2$
iii. $36^{13}b^{10} - 40a^{11}b^{11}$
iv. $24389x^{12} - 2197$

Exercise 2.5. Expand the following expressions, find the degree, lower degree and the leading coefficients.

i. $(5x - 11)^{13}$
ii. $(x^2 + x + 1)$^{10})

2.7 Simplifying Expressions

2.7 Simplifying Expressions

```
[> 3*(x-1)+7*(x+2)-5*(x+11);
```

Maple will automatically simplify some simple expressions. However, more complicated expressions will not be simplified. So, we need to give a command to simplify such expressions.

```
[> 3*(x-1)^2+7*(x+2)^3-5*(x+11)^4;
[> %=simplify(%);
```

```
[> simplify((x^2-y^2)/(x-y));
```

Chapter 3

Matrices

There are several ways to define a matrix in Maple. Use the help command to find the general definition of a matrix. To work with Matrices first load the package `linalg` Which include most linear algebra related commands.

3.1 Defining matrices

There are several ways to define Matrix. Let's look at them through examples.

You can use maple help.

```
> ? matrix
```

```
> restart;
> with(linalg):
> M1 :=matrix (3, 3, [1,-2,-3,2,-2,2,3,-3,3]);
```

```
> restart;
> with(linalg):
> M1 :=matrix (3, 3, [1,-2,-3,2,-2,2,3,-3,3]);
```

$$M1 := \begin{bmatrix} 1 & -2 & -3 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix}$$

```
>
```

```
> M2:= matrix(3,4,[1,1/2,-1/3,1/4,2,1/2,-3/4,5/4,3,3/5,3/7,3/8]);
```

```
> M2:= matrix(3,4,[1,1/2,-1/3,1/4,2,1/2,-3/4,5/4,3,3/5,3/7,3/8]);
```

$$M2 := \begin{bmatrix} 1 & \frac{1}{2} & \frac{-1}{3} & \frac{1}{4} \\ 2 & \frac{1}{2} & \frac{-3}{4} & \frac{5}{4} \\ 3 & \frac{3}{5} & \frac{3}{7} & \frac{3}{8} \end{bmatrix}$$

```
>
```

```
[> A:= matrix(3,3,[[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

```
> A:= matrix(3,3,[[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$A := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
[> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

```
> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$B := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
>
```

```
[> Matrix (2,2,fill=a);
```

```
> Matrix (2,2,fill=a);
```

$$\begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

Uppercase

```
> |
```

```
[> Matrix (2,2,symbol=a);
```

```
> Matrix (2,2,symbol=a);
```

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

```
>
```

```
[> f:=i->x*i-1;
[> M:=Matrix(2,f);
```

```
> f:=i->x*i-1;
```

$$f := i \to x\, i - 1$$

```
> M:=Matrix(2,f);
```

$$M := \begin{bmatrix} x-1 & x-1 \\ 2\,x-1 & 2\,x-1 \end{bmatrix}$$

```
>
```

```
[> g:=(i,j)->x*(i+j-1);
[> M1:=Matrix(3,g);
```

```
> g:=(i,j)->x*(i+j-1);
```

$$g := (i,j) \to x\,(i+j-1)$$

```
> M1:=Matrix(3,g);
```

$$M1 := \begin{bmatrix} x & 2\,x & 3\,x \\ 2\,x & 3\,x & 4\,x \\ 3\,x & 4\,x & 5\,x \end{bmatrix}$$

```
>
```

To define a diagonal matrix.

```
[> C:=diag(1,2,3);
```

```
> C:=diag(1,2,3);
```

$$C := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

```
[> diag(1,1,1);
```

```
> diag(1,1,1);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To define lower triangular matrix.

```
[> M_1:=Matrix(3,[[x],[y,y],[z,z,z]],shape=triangular[lower]);
```

```
> M_1:=Matrix(3,[[x],[y,y],[z,z,z]],shape=triangular[lower]);
```

$$M\_1 := \begin{bmatrix} x & 0 & 0 \\ y & y & 0 \\ z & z & z \end{bmatrix}$$

```
[> M_2:=Matrix(3,[[1,2,3],[4,5,6],[7,8,9]],shape=triangular[upper]);
```

```
> M_2:=Matrix(3,[[1,2,3],[4,5,6],[7,8,9]],shape=triangular[upper]);
```

$$M\_2 := \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{bmatrix}$$

To define zero matrix

```
[> M_3:=Matrix(4,4,shape=zero);
```

```
> M_3:=Matrix(4,4,shape=zero);
```

$$M\_3 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To define identity matrix

```
[> M_3:=Matrix(3,3,shape=identity);
```

```
> M_3:=Matrix(3,3,shape=identity);
```

$$M\_3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find an entry of a matrix, follow the name of the matrix by indices inside a square bracket.

```
[> A:= matrix(3,3,[[1,-3,-2],[2,-3,5],[-4,2,6]]);
[> A[3,2];
```

```
> A:= matrix(3,3,[[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$A := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
> A[3,2];
```

$$2$$

```
[> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
[> B[1,2];
```

```
> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$B := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
> B[1,2];
```

$$-3$$

Matrix Operation

3.1.1 Addition and Scalar Multiplication

Algebric expressions with matrices are evaluated by the command `evalm`. scalar multiplication of matrices done by the usual symbol `*`.

```
[> evalm(A+B);
```

```
> evalm(A+B);
```

$$\begin{bmatrix} 2 & -6 & -4 \\ 4 & -6 & 10 \\ -8 & 4 & 12 \end{bmatrix}$$

```
[> evalm(2*A+3*B);
```

```
> evalm(2*A+3*B);
```

$$\begin{bmatrix} 5 & -15 & -10 \\ 10 & -15 & 25 \\ -20 & 10 & 30 \end{bmatrix}$$

3.1.2 Matrix Multiplication

For matrix multiplication the symbol `&*` is used.

```
[> evalm(A&*B);
```

```
> evalm(A&*B);
```

$$\begin{bmatrix} 3 & 2 & -29 \\ -24 & 13 & 11 \\ -24 & 18 & 54 \end{bmatrix}$$

3.1.3 Transpose

To get the transpose of a matrix the command `transpose` is used.

```
[> transpose(A);
```

```
> transpose(A);
```

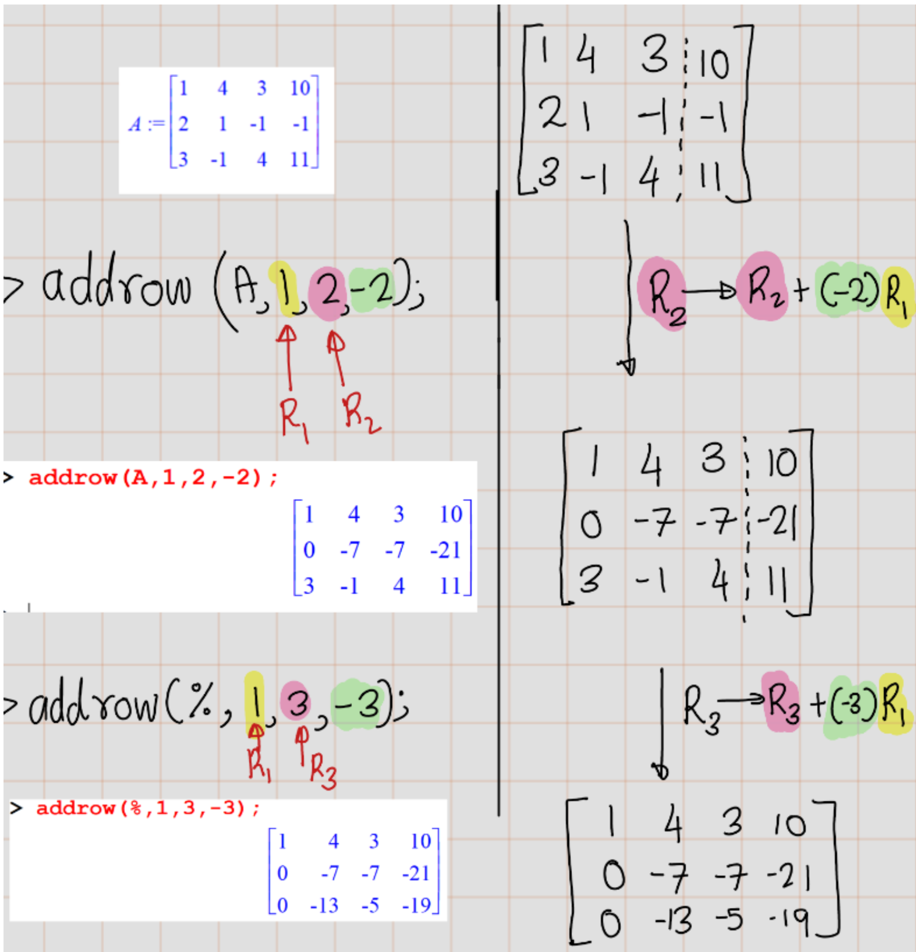
$$\begin{bmatrix} 1 & 2 & -4 \\ -3 & -3 & 2 \\ -2 & 5 & 6 \end{bmatrix}$$

3.2 Row Opreations

In your linear algebra class you will learn elementary row operations on matrices here we will use Maple to do the same thing. Let's define a new matrix A.

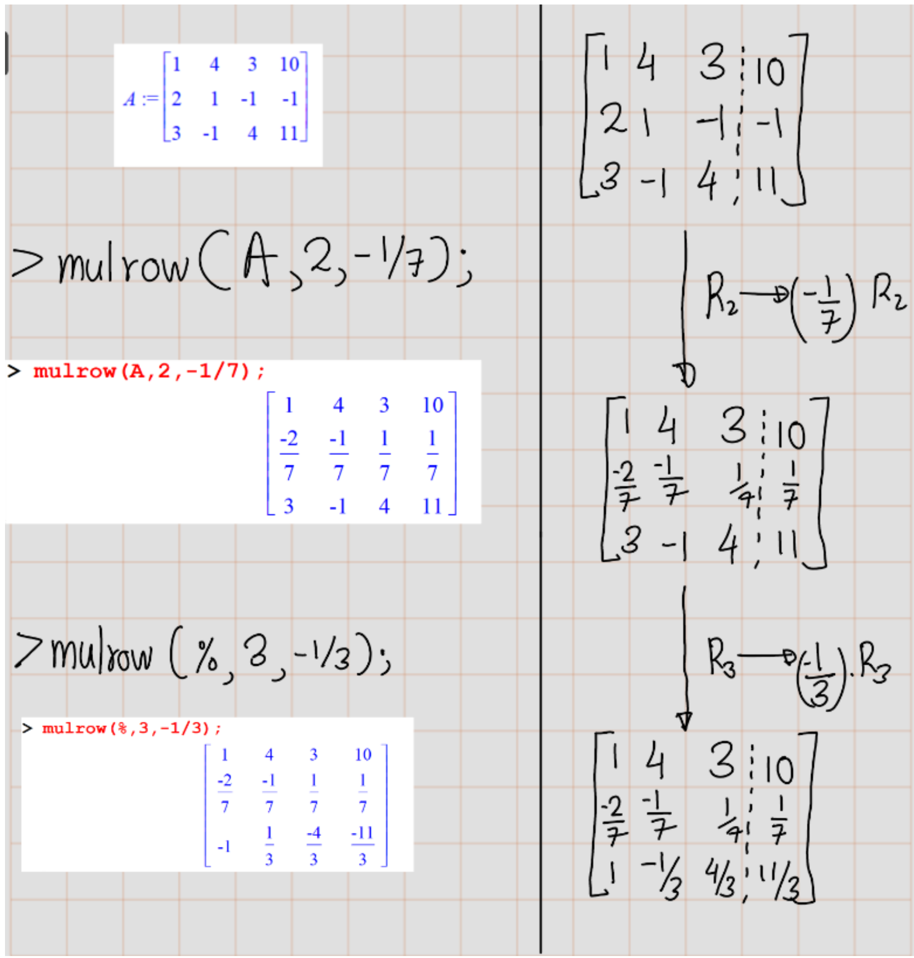
- **addrow(A,r1,r2,m)**
  Returns a copy of a matrix A in which row r2 replaced with by row(A,r2)+m*row(A,r1)

```
[> A:=matrix([[1,4,3,10],[2,1,-1,-1],[3,-1,4,11
[> addrow(A,1,2,-2);
[> addrow(%,1,3,-3);
```



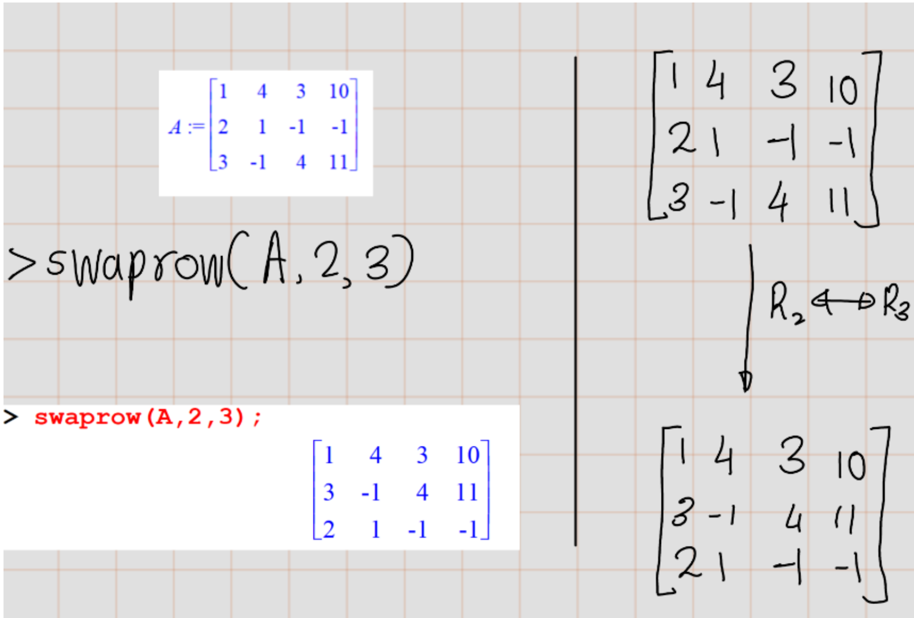
- **mulrow(A,row,expr)**
  Returns a matrix A in which has the same entries as A with the r^{th} row mutiplied by *expr*

```
[> mulrow(A,2,-1/7);
[> mulrow(%,3,-1/3);
```



- **swaprow(A,r1,r2)**
  This command interchange row r1 and r2 of A

```
[> swaprow(A,2,3);
```



Similarly, you can learn `addcol`, `mulcol`,`swapcol` commands by your self.

3.3 Determinanat of Matrix

To find the determinate of a matrix, maple has a special command `det`.

```
[> M1 :=matrix (3, 3, [1,-2,-3,2,-2,2,3,-3,3]);
[> det(M1);
```

```
> M1 :=matrix (3, 3, [1,-2,-3,2,-2,2,3,-3,3]);
```

$$M1 := \begin{bmatrix} 1 & -2 & -3 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix}$$

```
> det(M1);
```

$$0$$

```
[> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
[> det(B);
```

```
> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$B := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
> det(B);
```

$$84$$

3.3.1 Inverse of a matrix

The `inverse` command is used to find the inverse of a square matrix, if exists.

```
[> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
[> inverse(B);
```

```
> B:=matrix([[1,-3,-2],[2,-3,5],[-4,2,6]]);
```

$$B := \begin{bmatrix} 1 & -3 & -2 \\ 2 & -3 & 5 \\ -4 & 2 & 6 \end{bmatrix}$$

```
> inverse(B);
```

$$\begin{bmatrix} \frac{-1}{3} & \frac{1}{6} & \frac{-1}{4} \\ \frac{-8}{21} & \frac{-1}{42} & \frac{-3}{28} \\ \frac{-2}{21} & \frac{5}{42} & \frac{1}{28} \end{bmatrix}$$

```
>
```

```
[> inverse([[2,-1],[3,2]]);
```

```
> inverse([[2,-1],[3,2]]);
```

$$\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{bmatrix}$$

```
>
```

3.4 Exercise

Exercise 3.1. Performs the indicated computations.

A. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 4 \\ -2 & 5 & 8 \end{bmatrix}$

B. $5\begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \\ -6 & 1 & 5 \end{bmatrix} - 3\begin{bmatrix} -2 & 1 & 4 \\ 5 & 0 & 7 \\ 2 & -1 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 2 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 3 & 1 & 5 \\ 0 & 6 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 7 & 1 \\ 2 & 0 & 3 \\ 1 & 0 & 0 \\ 0 & 5 & 6 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 0 & 7 & 4 \\ 2 & 3 & 0 \end{bmatrix}$

F. $3\begin{bmatrix} -2 & 1 \\ 0 & 4 \\ 2 & 3 \end{bmatrix}$

G. $\begin{bmatrix} 1 & 0 & 3 & -1 & 5 \\ 2 & 1 & 6 & 2 & 5 \end{bmatrix}\begin{bmatrix} 7 & 1 \\ 2 & 3 \\ -1 & 0 \\ 5 & 6 \\ 2 & 3 \end{bmatrix}$

H. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 5 & 6 \\ 2 & 4 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

Exercise 3.2. Determine the given matrices are invertible. If they are, compute the inverse.

A. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$

C. $\begin{bmatrix} a & a \\ b & b \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 5 & 7 & 0 \\ 2 & 1 & 3 \\ 1 & 0 & 0 \\ 0 & 4 & 3 \end{bmatrix}$

F. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{bmatrix}$

G. $\begin{bmatrix} 1 & -3 & 0 & 2 \\ 3 & -12 & -2 & -6 \\ -2 & 10 & 2 & 5 \\ -1 & 6 & 1 & 3 \end{bmatrix}$

Exercise 3.3. Compute the 3×3 matrix whåse entries are given by the function y^{ij}, Where $i = j = 1, 2, 3$.

Exercise 3.4. Compute the matrix, $\begin{bmatrix} x & x^2 \\ x^2 & x^3 \end{bmatrix}$ by defining a suitable function as in the exercise 3.3

Exercise 3.5. Which of the following matrices are skew-symmetric? (A square matrix a symmetric if $A^T = -A$, where A^T is the transpose of A.)

A. $\begin{bmatrix} 1 & -6 \\ 6 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -6 \\ 6 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 2 & -2 & 2 \\ 2 & 2 & -2 \\ 2 & 2 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 1 & -2 & 0 \end{bmatrix}$

Exercise 3.6. Convert the following matrix in to an upper triangular matrix.

$$\begin{bmatrix} 2 & 3 & -1 \\ -3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Exercise 3.7. Convert the following matrices into a identity matrix.

A. $\begin{bmatrix} 2 & 7 & 3 \\ 1 & 3 & 2 \\ 3 & 7 & 9 \end{bmatrix}$

B. $\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$

Chapter 4

System of Linear Equations and Matrices

A linear system is a collection of first degree equations. A solution to a system consists of one or more sets of specific values that our common solutions to each of the individual equations. Here is a simple example which we can solve quite easily using the `solve` command.

4.1 Augment of a matrix

```
[> with(linalg):
[> A:=matrix([[1,2],[2,3]]);
```

Output:

```
> with(linalg):
> matrix([[1,2],[2,3]]);
```

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

```
[> B:=matrix([[3,4,5],[6,7,8]]);
```

Output:

```
> B:=matrix([[3,4,5],[6,7,8]]);
```

$$B := \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

```
augment(A,B);
```

Output:

```
> augment(A,B);
```

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 6 & 7 & 8 \end{bmatrix}$$

The function augment joins two or more matrices together horizontally. The matrices and vectors must have the same number of rows.

4.2 Solving Linear Systems

There are three types of systems of linear equations, and three types of solutions.

- An **independent system** has exactly one solution.
- An **inconsistent system** has no solution.
- A **dependent system** has infinitely many solutions.

Let's try to solve linear system.

Independent Systems:

$$\begin{array}{rcrcrcr} 2x & + & 5y & - & 4z & = & 9 \\ 3x & + & 5y & + & 2z & = & 12 \\ 4x & - & y & + & 5z & = & -3 \end{array}$$

$$\underbrace{\begin{bmatrix} 2 & 5 & -4 \\ 3 & 5 & 2 \\ 4 & -1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 9 \\ 12 \\ -3 \end{bmatrix}}_{B}$$

4.2.1 Method I

A linear system is a collection of first degree equations. A solution to a system consists of one or more set of specific values that our common solution to each of the individual equations. Here is a simple example which we can solve quite easily using the solve command.

```
[> sys:={2*x+5*y-4*z=9,3*x+5*y+2*z=12,4*x-y+5*z=-3};
[> solve(sys,{x,y,z});
```

Output:

```
> sys:={2*x+5*y-4*z=9,3*x+5*y+2*z=12,4*x-y+5*z=-3};
```

$$sys := \{2\,x+5\,y-4\,z=9,\ 3\,x+5\,y+2\,z=12,\ 4\,x-y+5\,z=-3\}$$

```
> solve(sys,{x,y,z});
```

$$\{x=\frac{-33}{37},\ y=\frac{99}{37},\ z=\frac{24}{37}\}$$

Maple will automatically use fractions, however you can force decimal answers using the ***evalf*** command.

```
[> evalf(%);
```

Output:

```
> evalf(%);
```

$$\{x=-0.8918918919,\ y=2.675675676,\ z=0.6486486486\}$$

4.2.2 Method II

We can also convert the above system of equations to a matrix system.

```
[> A:=genmatrix(sys,[x,y,z],b);
[> evalm(b);
```

Output:

```
> A:=genmatrix(sys,[x,y,z],b);
```

$$A := \begin{bmatrix} 2 & 5 & -4 \\ 3 & 5 & 2 \\ 4 & -1 & 5 \end{bmatrix}$$

```
> evalm(b);
```

$$[9, 12, -3]$$

In this case ,

- A is the coefficient matrix, and
- b is vector representing the constant values.

The command `evalm(b)` evaluated b as a matrix (a vector is a $n \times 1$ matrix). In other words, this command simply writes out what the vector b looks like.

```
[> linsolve(A,b);
```

Output:

```
> linsolve(A,b);
```

$$\left[\frac{-33}{37}, \frac{99}{37}, \frac{24}{37}\right]$$

4.2.3 Method III

Another way to solve a matrix equation $Ax = b$ is to left multiply both sides by the inverse matrix A^{-1}, if it exists, to get the solution $x = A^{-1}b$.

```
[>  inverse(A);
[> evalm(inverse(A)&*b);
```

Output:

```
> inverse(A);
```

$$\begin{bmatrix} \frac{9}{37} & \frac{-7}{37} & \frac{10}{37} \\ \frac{-7}{111} & \frac{26}{111} & \frac{-16}{111} \\ \frac{-23}{111} & \frac{22}{111} & \frac{-5}{111} \end{bmatrix}$$

```
> evalm(inverse(A)&*b);
```

$$\left[\frac{-33}{37}, \frac{99}{37}, \frac{24}{37}\right]$$

The `evalm` command forces a matrix computation and express the result.

Dependent Systems:

Some systems are dependent which means there are an infinite number of solutions. The form of these solutions will entail the use of parameters. Let's look an example using two different methods; `solve` and `linsolve`.

Example 4.1.

$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & 3 \\ x & + & y & - & 2z & = & -4 \\ 2x & - & y & - & z & = & -1 \end{array}$$

```
[> restart;
[> with(linalg):
[> sys:={x-2*y+z=3,x+y-2*z=-4,2*x-y-z=-1};
[> solve(sys,{x,y,z});
```

```
> restart;
> with(linalg):
> sys:={x-2*y+z=3,x+y-2*z=-4,2*x-y-z=-1};
```

$$sys := \{x - 2\,y + z = 3,\, x + y - 2\,z = \text{-}4,\, 2\,x - y - z = \text{-}1\}$$

```
> solve(sys,{x,y,z});
```

output:

$$\{x = -\frac{5}{3} + z,\, y = -\frac{7}{3} + z,\, z = z\}$$

```
[> A:=genmatrix(sys,[x,y,z],b);
[> evalm(b);
[> linsolve(A,b);
```

output:

```
> A:=genmatrix(sys,[x,y,z],b);
```

$$A := \begin{bmatrix} 1 & \text{-}2 & 1 \\ 1 & 1 & \text{-}2 \\ 2 & \text{-}1 & \text{-}1 \end{bmatrix}$$

```
> evalm(b);
```

$$[3, \text{-}4, \text{-}1]$$

```
> linsolve(A,b);
```

$$\left[-\frac{5}{3} + \_t_1,\, -\frac{7}{3} + \_t_1,\, \_t_1\right]$$

In both cases, the solution contains a parameter. The `solve` command express it in terms actual variable used,and the `linsolve` command use the funny t character to distinguish it from a variable you might have defined yourself.

Inconsistent System

An Inconsistent system has no solution. Maple generally, refers to answer questions which have no answer.

Example 4.2.

$$\begin{array}{ccccccl} x & + & y & - & 3z & = 10 \\ x & + & y & - & z & = 1 \\ x & + & y & + & z & = 8 \end{array}$$

Let's try to solve this system.

```
[> restart;
[> with(linalg):
[> sys:={x+y-3*z=10,x+y-z=1,x+y+z=8};
[> solve(sys,{x,y,z});
```

Output:

```
> restart;
> with(linalg):
> sys:={x+y-3*z=10,x+y-z=1,x+y+z=8};
```

$$sys := \{x + y - 3\,z = 10, x + y - z = 1, x + y + z = 8\}$$

```
> solve(sys,{x,y,z});
> |
```

This method does not work. It does not return anything.

Let's try `linsolve` method.

```
[> A:=genmatrix(sys,[x,y,z],b);
[> linsolve(A,b);
```

Output:

```
> A:=genmatrix(sys,[x,y,z],b);
```

$$A := \begin{bmatrix} 1 & 1 & -3 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

```
> linsolve(A,b);
>
>
```

This method also does not works.

```
[> evalm(inverse(A)&*b);
```

Output:

```
> evalm(inverse(A)&*b);
Error, (in linalg:-inverse) singular matrix
>
```

4.2.3.1 Automatic Reduction

```
[> sys:={3*x+5*y+2*z=12,2*x+5*y-4*z=9,4*x-y+5*z=-3};
[> evalm(b);
[> C:=augment(A,b);
```

Output:

```
> sys:={3*x+5*y+2*z=12,2*x+5*y-4*z=9,4*x-y+5*z=-3};
```

$$sys := \{2x + 5y - 4z = 9, 3x + 5y + 2z = 12, 4x - y + 5z = -3\}$$

```
> evalm(b);
```

$$[10, 1, 8]$$

```
> C:=augment(A,b);
```

$$C := \begin{bmatrix} 1 & 1 & -3 & 10 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 8 \end{bmatrix}$$

4.2.4 Method IV

An even faster method is simplifying let maple to do all the work for us. The `gausselim` command will perform all of the steps of Gaussian eliminations and reduce an augmented matrix to **row echelon form**.

```
[> sys:={3*x+5*y-2*z=12,2*x+5*y-4*z=9,4*x-y+5*z=-3};
[> A:=genmatrix(sys,[x,y,z],b);
[> evalm(b);
[> C:=augment(A,b);
[> gausselim(C);
```

```
> sys:={3*x+5*y-2*z=12,2*x+5*y-4*z=9,4*x-y+5*z=-3};
```

$$sys := \{2\,x+5\,y-4\,z=9,\ 3\,x+5\,y-2\,z=12,\ 4\,x-y+5\,z=-3\}$$

```
> A:=genmatrix(sys,[x,y,z],b);
```

$$A := \begin{bmatrix} 2 & 5 & -4 \\ 3 & 5 & -2 \\ 4 & -1 & 5 \end{bmatrix}$$

```
> evalm(b);
```

$$[9, 12, -3]$$

```
> C:=augment(A,b);
```

$$C := \begin{bmatrix} 2 & 5 & -4 & 9 \\ 3 & 5 & -2 & 12 \\ 4 & -1 & 5 & -3 \end{bmatrix}$$

```
> gausselim(C);
```

$$\begin{bmatrix} 2 & 5 & -4 & 9 \\ 0 & -11 & 13 & -21 \\ 0 & 0 & \frac{23}{22} & \frac{36}{11} \end{bmatrix}$$

```
> |
```

4.2.5 Method V

The command `gaussjord` does the same thing. It performs all the steps of Gauss Jordan elimination and reduces and reduces an augmented matrix into reduced row echelon form.

```
[> gaussjord(C);
```

```
> gaussjord(C);
```

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-75}{23} \\ 0 & 1 & 0 & \frac{129}{23} \\ 0 & 0 & 1 & \frac{72}{23} \end{bmatrix}$$

```
>
```

4.3 Exercise

Exercise 4.1. Solve the following system of equations.

$$\begin{array}{rcrcrl} x_1 & + & 2x_2 & + & 3x_3 & = 9 \\ 2x_1 & - & x_2 & + & x_3 & = 8 \\ 3x_1 & & & - & x_3 & = 3 \end{array}$$

Exercise 4.2. A small manufacturing plant makes three types of inflatable boats: one-person, two-person, and four-person models. Each boat requires the services of three departments, as listed in the table. The cutting, assembly, and packaging departments have available a maximum of 380, 330, and 120 labor hours per week, respectively.

Department	One Person Boat	Two-Person Boat	Four-Person Boat
Cutting	0.5 hr	1.0 hr	1.5 hr
Assembling	0.6 hr	0.9 hr	1.2 hr
Packaging	0.2 hr	0.3 hr	0.5 hr

(A) How many boats of each type must be produced each week for the plant to operate at full capacity?
(B) How is the production schedule in part A affected if the packaging department is no longer used?
(C) How is the production schedule in part A affected if the four-person boat is no longer produced?

Chapter 5

Intergration

The `int` command is used to compute both definite and indefinite integrals of Maple expressions as shown by the following examples.

5.1 Indefinite Integration

```
[> int(x^2,x);
```

```
> int (x^2,x);
```

$$\frac{x^3}{3}$$

```
[> int(sin(2*t),t);
```

```
> int(sin(2*t),t);
```

$$-\frac{1}{2}\cos(2\,t)$$

```
[> f:=x->(3*x-6)/(x^2-4);
[> int(f(x),x);
```

```
> f:=x->(3*x-6)/(x^2-4)  ;
```

$$f := x \to \frac{3\,x-6}{x^2-4}$$

```
> int(f(x),x);
```

$$3\ln(x+2)$$

```
[> Int(sin(x),x);
```

```
> Int(sin(x),x);
```

$$\int \sin(x)\,dx$$

```
[> Int(3*x^2+2)^(5/3),x);
```

```
> Int(x*(3*x^2+2)^(5/3),x);
```

$$\int x\,(3\,x^2+2)^{(5\,/\,3)}\,dx$$

```
[> Int(x*(3*x^2+2)^(5/3),x)=int(x*(3*x^2+2)^(5/3),x);
```

```
> Int(x*(3*x^2+2)^(5/3),x)=int(x*(3*x^2+2)^(5/3),x);
```

$$\int x\,(3\,x^2+2)^{(5\,/\,3)}\,dx = \frac{(3\,x^2+2)^{(8\,/\,3)}}{16}$$

Notice that Maple doesn't include a constant of integration for indefinite integrals. Where there are constants, Parameters other variable around, Maple assume that you mean to take the integral as the variable you specify changes, and that all other letters in the expression represent constants.

5.2 Definite Integration

```
[> Int(g(x),x=a..b);
```

```
> Int(g(x),x=a..b);
```

$$\int_a^b \mathrm{g}(x)\,dx$$

```
[> Int(x^2*exp(x),x=0..2)=int(x^2*exp(x),x=0..2);
```

```
> Int(x^2*exp(x),x=0..2)=int(x^2*exp(x),x=0..2);
```

$$\int_0^2 x^2\,\mathbf{e}^x\,dx = -2+2\,\mathbf{e}^2$$

```
[> f := x -> x*sin(x);
[> int(f(x),x=0..Pi);
```

```
> f := x -> x*sin(x);
```

$$f := x \to x\sin(x)$$

```
> int(f(x),x=0..Pi);
```

$$\pi$$

Some functions can not be integrated analytically, but the definite integrals of such functions still have meaning and MAPLE can determine them.

```
[> int(exp(-x^2)*ln(x),x);
```

```
> int(exp(-x^2)*ln(x),x);
```

$$\int \mathbf{e}^{(-x^2)} \ln(x)\, dx$$

```
[> int(exp(-x^2)*ln(x),x=0..infinity);
[> evalf(%);
```

```
> int(exp(-x^2)*ln(x),x=0..infinity);
```

$$-\frac{\sqrt{\pi}\,\gamma}{4} - \frac{1}{2}\sqrt{\pi}\ln(2)$$

```
> evalf(%);
```

$$-0.8700577270$$

5.3 Integration by Parts

The `intparts` command is used to integrate by parts, and the command exist in `student` package.

The `intparts` command has two arguments: the first is the expression to be anti-differentiated, and the second is the choice for, the piece which is to be differentiated.

$$\int uD(v)dx = uv - \int vD(u)dx$$

Example 5.1. Method of integration by parts to compute $\int x\cos(x)$

```
[> restart;
[> with(student):
[> p1:= Int(x*cos(x),x);
[> p2:= intparts(p1,x);
[> p3:= value(p2);
[> p1 = p3 + C;
```

```
> restart;
> with(student):
> p1:= Int(x*cos(x),x);
```

$$p1 := \int x\cos(x)\,dx$$

```
> p2:= intparts(p1,x);
```

$$p2 := x\sin(x) - \int \sin(x)\,dx$$

```
> p3:= value(p2);
```

$$p3 := x\sin(x) + \cos(x)$$

```
> p1 = p3 + C;
```

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x) + C$$

```
[> restart;
[> with(student):
[> Int(sqrt(x)*ln(x),x)=intparts(Int(sqrt(x)*ln(x),x),ln(x));
```

```
> restart;
> with(student):
> Int(sqrt(x)*ln(x),x)=intparts(Int(sqrt(x)*ln(x),x),ln(x));
```

$$\int \sqrt{x}\,\ln(x)\,dx = \frac{2}{3}\ln(x)\,x^{(3/2)} - \int \frac{2\sqrt{x}}{3}\,dx$$

```
[> Int(sqrt(x)*ln(x),x)=value(intparts(Int(sqrt(x)*ln(x),x),ln(x)));
```

```
> Int(sqrt(x)*ln(x),x)=value(intparts(Int(sqrt(x)*ln(x),x),ln(x)));
```

$$\int \sqrt{x}\,\ln(x)\,dx = \frac{2}{3}\ln(x)\,x^{(3/2)} - \frac{4\,x^{(3/2)}}{9}$$

5.4 Substitutions

These are done using the `changevar` command. Look at the following example.

Example 5.2. Evaluate $\int 4\cos(\sin(x))dx$ using the substitution, $u = \cos(x)$.

```
[> Int(cos(x)^4*sin(x),x)=changevar(u=cos(x),Int(cos(x)^4*sin(x),x),u);
```

```
> Int(cos(x)^4*sin(x),x)=changevar(u=cos(x),Int(cos(x)^4*sin(x),x),u);
```

$$\int \cos(x)^4 \sin(x)\,dx = \int -u^4\,du$$

```
[> v1:=value(changevar(u=cos(x),Int(cos(x)^4*sin(x),x),u));
```

```
> v1:=value(changevar(u=cos(x),Int(cos(x)^4*sin(x),x),u));
```

$$v1 := -\frac{u^5}{5}$$

To back-substitute, we use the subs command,

```
[> subs(u=cos(x),v1);
```

```
> subs(u=cos(x), v1);
```

$$-\frac{1}{5}\cos(x)^5$$

5.5 Multiple Integration

5.5.1 Double integration (Area calculation)

```
[> Int(Int(x^2+2*x,x),x);
```

```
> Int(Int(x^2+2*x,x),x);
```

$$\iint x^2 + 2\,x\,dx\,dx$$

```
[> int(int(x^2+2*x,x),x);
```

```
> int(int(x^2+2*x,x),x);
```

$$\frac{1}{12}x^4 + \frac{1}{3}x^3$$

```
[> Int(Int(x+y^2,y=0..x),x=1..2);
```

```
> Int(Int(x+y^2,y=0..x),x=1..2);
```

$$\int_1^2 \int_0^x x + y^2\,dy\,dx$$

```
[> a:=Int(x+y^2,y=0..x);
```

```
> a:=Int(x+y^2,y=0..x);
```

$$a := \int_0^x x + y^2\,dy$$

```
[> b:=Int(a,x=1..2);
```

```
> b:=Int(a,x=1..2);
```

$$b := \int_{1}^{2} \int_{0}^{x} x + y^2 \, dy \, dx$$

```
[> Int(Int(x+y^2,y=0..x),x=1..2)=int(int(x+y^2,y=0..x),x=1..2);
```

```
> Int(Int(x+y^2,y=0..x),x=1..2)=int(int(x+y^2,y=0..x),x=1..2);
```

$$\int_{1}^{2} \int_{0}^{x} x + y^2 \, dy \, dx = \frac{43}{12}$$

```
[> A:=Doubleint(x+y^2,y=0..x,x=1..2);
[> value(A);
```

```
> A:=Doubleint(x+y^2,y=0..x,x=1..2);
```

$$A := \int_{1}^{2} \int_{0}^{x} x + y^2 \, dy \, dx$$

```
> value(A);
```

$$\frac{43}{12}$$

5.5.2 Triple integration (Volume Calculation)

```
[> Int(Int(Int(x^3*exp(y)*sin(z),x=0..2),z=-Pi..2*Pi),y=0..1)=int(int(int(x^3*exp(y)*s
```

```
> Int(Int(Int(x^3*exp(y)*sin(z),x=0..2),z=-Pi..2*Pi),y=0..1)
  =int(int(int(x^3*exp(y)*sin(z),x=0..2),z=-Pi..2*Pi),y=0..1);
```

$$\int_{0}^{1} \int_{-\pi}^{2\pi} \int_{0}^{2} x^3 \, \mathbf{e}^y \sin(z) \, dx \, dz \, dy = 8 - 8 \, \mathbf{e}$$

```
[> V:=Tripleint(x^3*exp(y)*sin(z),x=0..2,z=-Pi..2*Pi,y=0..1);
[> value(V);
```

```
> V:=Tripleint(x^3*exp(y)*sin(z),x=0..2,z=-Pi..2*Pi,y=0..1);
```

$$V := \int_0^1 \int_{-\pi}^{2\pi} \int_0^2 x^3 \, e^y \sin(z) \, dx \, dz \, dy$$

```
> value(V);
```

$$8 - 8\,e$$

Note: Since there is often more than one way to find an indefinite integral, it may happen that the answer you obtain by doing the techniques seen in class is different than the one obtained with Maple.

5.6 Exercise

Exercise 5.1. Integrate the following expressions,

i. $2x^2 + \frac{x}{2}$
ii. $x^3(3x^2+2)^{\frac{5}{3}}$
iii. e^{-x^2}

Exercise 5.2. Find the definite integral of the following over the given intervals,

i. $\int_2^{10} x^3(3x^2+2)^{\frac{5}{3}}\,dx$
ii. $\int_0^{\frac{\pi}{2}} \cos(x)\sin(x^2)\,dx$
iii. $\int_0^{\pi} e^x \cos(x)\,dx$
iv. $\int_0^{\pi} e^{2\sin(x)} \cos(x)\,dx$

Exercise 5.3. Use of the method of integration by parts to compute,

i. $\int xe^{-2x}\,dx$
ii. $\int x\ln(x)\,dx$

Exercise 5.4. Evaluate the following expressions

i. $\int_{-3}^{3}\int_{-3}^{3} 3x^2 + 5y^2 + 4\,dxdy$
ii. $\int_{-3}^{3}\int_{-3}^{3} 9\ln(x) - 5y^2\,dxdy$

Note: Since there is often more than one way to find an indefinite integral, it may happen that the answer you obtain by doing the techniques seen in class is different than the one obtained with Maple. Compare the answers obtained by Maple and the answer you obtained manually for the integral, $\int \cos(3x)\,dx$

There are two main ways to use the definite integral. - The easiest one to understand is as a means for computing areas (and volumes). - The second way the definite integral is used is as a sum. That is, we use the definite integral to “add things up”.

5.7 Computing the area from the integral

Example 5.3. Find the area under the curve $x^2 \sin(x)$

```
[> int(x^2*sin(x),x=-Pi..Pi);
```

```
[> plot(x^2*sin(x),x=-Pi..Pi);
```

```
[> a1:=int(x^2*sin(x),x=-Pi..0);
```

```
‘[> a2:=int(x^2*sin(x),x=0..Pi);
```

```
[> abs(a1)+a2;
```

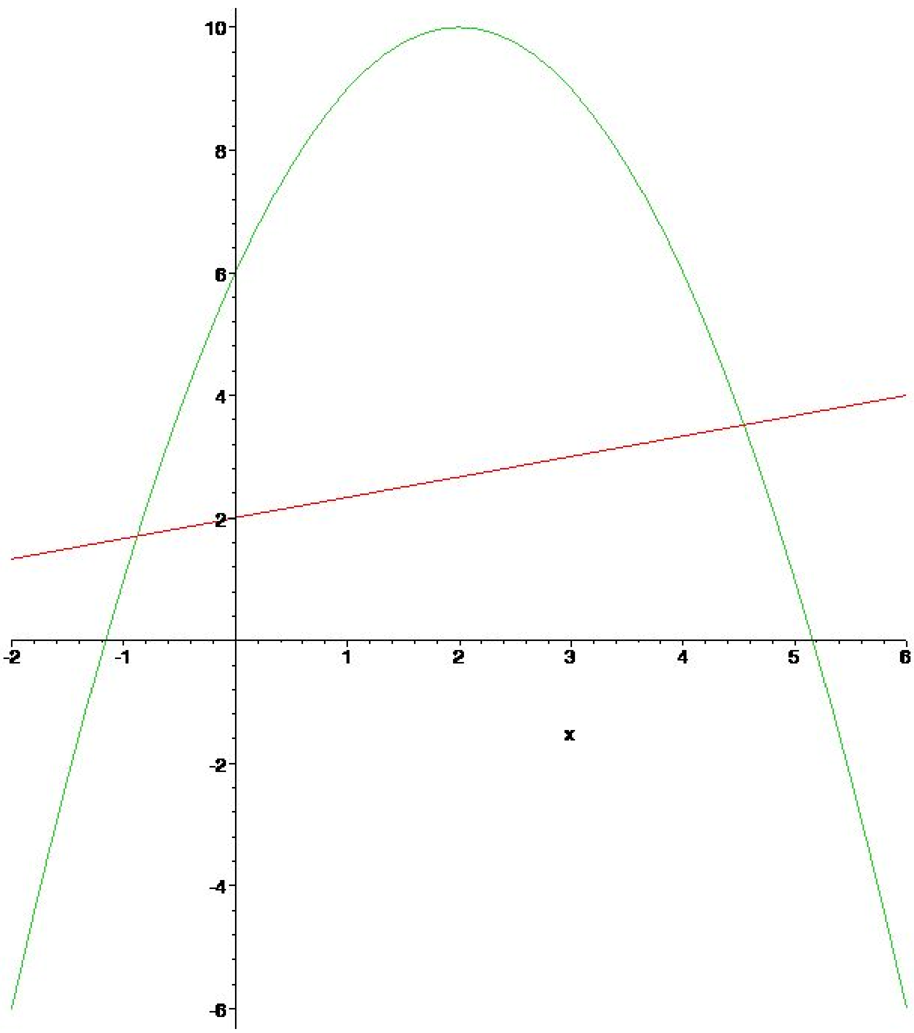
Example 5.4. Find the area bounded by the curves $-x^2 + 4x + 6$ and $\frac{x}{3} + 2$

```
[> f := x-> -x^2+4*x+6;
[> g := x-> x/3+2;
[> plot({f(x),g(x)},x=-2..6);
```

```
> f := x-> -x^2+4*x+6;
  g := x-> x/3+2;
  plot({f(x),g(x)},x=-2..6);
```

$$f := x \to -x^2 + 4\,x + 6$$

$$g := x \to \frac{1}{3}x + 2$$




```
[> a := fsolve(f(x)=g(x),x=-2..0);
[> b := fsolve(f(x)=g(x),x=4..6);
[> Area:=int(f(x)-g(x),x=a..b);
```

```
> a := fsolve(f(x)=g(x),x=-2..0);
```

$$a := -0.8798034327$$

```
> b := fsolve(f(x)=g(x),x=4..6);
```

$$b := 4.546470099$$

```
>
> Area:=int(f(x)-g(x),x=a..b);
```

$$Area := 26.62893493$$

```
[> evalf(solve(-x^2+4*x+6-x/3-2,x));
```

```
[> Area:=int(f(x)-g(x),x=-.8798034327..0)+int(f(x)-g(x),x=0..4.546470099);
```

Example 5.5. Find the area of the region enclosed between the two curves $f(x) = 2x + 5$ and $g(x) = x^2 + 2$ from $x = 0$ to $x = 6$.

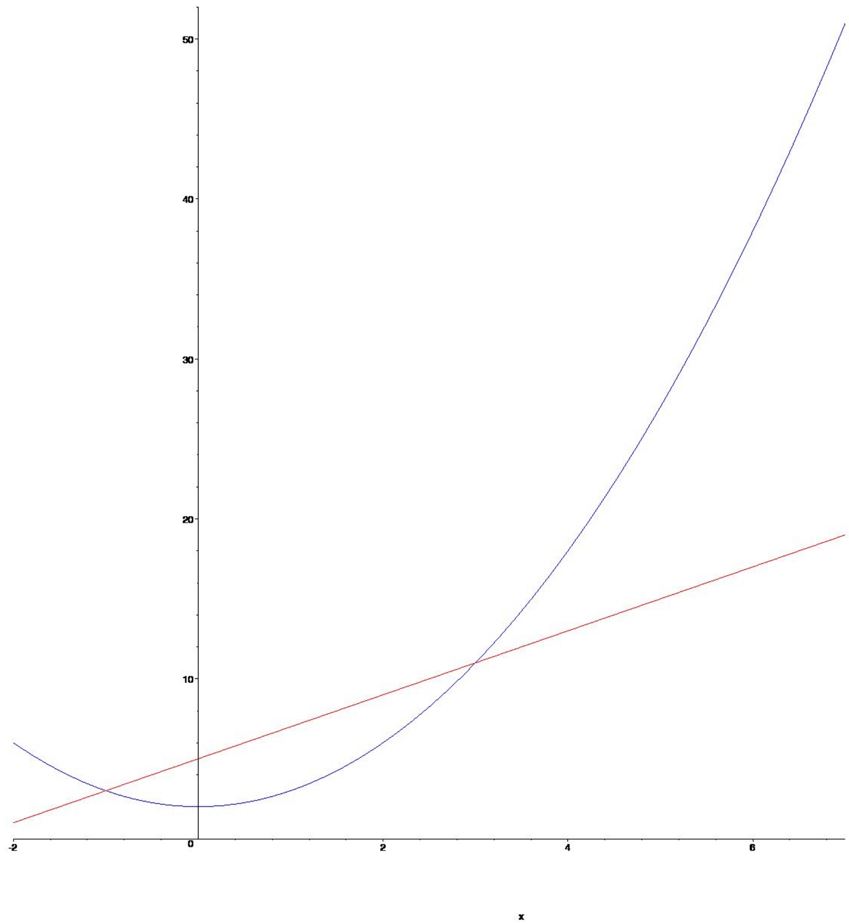
```
[> g:=x->x^2+2;f:=x->2*x+5;
[> plot({f(x),g(x)},x=-2..7,color=[red,blue]);
```

```
>g:=x->x^2+2;f:=x->2*x+5;
```

$$g := x \to x^2 + 2$$

$$f := x \to 2\,x + 5$$

```
>plot({f(x),g(x)},x=-2..7,color=[red,blue]);
```




```
[> solve(x^2+2-2*x-5,x);
```

```
[> Area:=int(f(x)-g(x),x=0..3)+int(g(x)-f(x),x=3..6);
```

5.8 Exercise

Exercise 5.5. Area under the curve $10 - x^2$ in the interval [0, 4].

Exercise 5.6. Area bounded by the curves $f(x) = \cos(x)$ and $y = \frac{1}{2}$ in the interval 0 to 8.

Exercise 5.7. Area bounded by the curves $f(x) = \frac{1}{4}x^2 - 4$ and $g(x) = \frac{1}{4}x + 1$.

Chapter 6

Differential Equations

6.1 Introduction to differential equations

In this Maple session, we see some of the basic tools for working with differential equations in Maple. First, we need to load the `DEtools` library: We can find the derivative of a given function by using `diff` command. - The `diff` command computes the partial derivative of a given expression with respect to the variables given. - The `Diff` command returns the unevaluated function. Now consider the following examples.

Example 6.1. Find the derivative of $y = xe^x$ with respect to x.

```
[> with(DEtools):
[> y:=x->x*exp(-x);
[> Diff(y(x),x);
[> diff(y(x),x);
[> Diff(y(x),x)=diff(y(x),x);
```

```
[> with(DEtools):
[> y:=x->x*exp(-x);
```

$$y := x \to x\,\mathbf{e}^{(-x)}$$

```
[> Diff(y(x),x);
```

$$\frac{d}{dx}(x\,\mathbf{e}^{(-x)})$$

```
[> diff(y(x),x);
```

$$\mathbf{e}^{(-x)} - x\,\mathbf{e}^{(-x)}$$

```
[> Diff(y(x),x)=diff(y(x),x);
```

$$\frac{d}{dx}(x\,\mathbf{e}^{(-x)}) = \mathbf{e}^{(-x)} - x\,\mathbf{e}^{(-x)}$$

Example 6.2. Consider the follwing function h of two varibels.

$$h(x, y) = 5x^2 + 2x^2y + 3xy^2 + 12yx + \frac{3y^3}{x}$$

Now we are going to differentiate h partially. The next few commands have been done in order to understand the process.

First, we define the function h.

```
[> h :=(x,y)-> 5*x^2+2*x^2*y+3*x*y^2+12*y*x+3*y^3/x;
```

Then,

```
[> h :=(x,y)-> 5*x^2+2*x^2*y+3*x*y^2+12*y*x+3*y^3/x;
```

$$h := (x, y) \to 5\,x^2 + 2\,x^2\,y + 3\,x\,y^2 + 12\,y\,x + \frac{3\,y^3}{x}$$

```
[> Diff( h(x,y),x);
[> diff( h(x,y),x);
[> Diff( h(x,y),x)=diff( h(x,y),x);
```

```
> Diff( h(x,y),x);
```

$$\frac{\partial}{\partial x}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)$$

```
> diff( h(x,y),x);
```

$$10\,x+4\,y\,x+3\,y^2+12\,y-\frac{3\,y^3}{x^2}$$

```
> Diff( h(x,y),x)=diff( h(x,y),x);
```

$$\frac{\partial}{\partial x}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)=10\,x+4\,y\,x+3\,y^2+12\,y-\frac{3\,y^3}{x^2}$$

```
[> Diff(h(x,y),y);
[> diff(h(x,y),y);
[> Diff(h(x,y),y)=diff(h(x,y),y);
```

```
> Diff(h(x,y),y);
```

$$\frac{\partial}{\partial y}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)$$

```
> diff(h(x,y),y);
```

$$2\,x^2+6\,y\,x+12\,x+\frac{9\,y^2}{x}$$

```
> Diff(h(x,y),y)=diff(h(x,y),y);
```

$$\frac{\partial}{\partial y}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)=2\,x^2+6\,y\,x+12\,x+\frac{9\,y^2}{x}$$

Here we are going to differentiate the function h with respect to x and y, using them both in one command. Here the function h should be differentiate first with respect to x, and then again with respect to y.

```
[> diff( h(x,y),x,y);
```

```
> Diff(h(x,y),x,y)=diff( h(x,y),x,y);
```

$$\frac{\partial^2}{\partial y\,\partial x}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)=4\,x+6\,y+12-\frac{9\,y^2}{x^2}$$

Now, the function h should be differentiate first with respect to y, and then again with respect to x.

```
[> diff( h(x,y),y,x);
```

```
> Diff( h(x,y),y,x)=diff( h(x,y),y,x);
```

$$\frac{\partial^2}{\partial x\,\partial y}\left(5\,x^2+2\,x^2\,y+3\,x\,y^2+12\,y\,x+\frac{3\,y^3}{x}\right)=4\,x+6\,y+12-\frac{9\,y^2}{x^2}$$

First define the function in Maple. Then you can find the derivative as follows.

Example 6.3. Define the differential equation $y' = y(4-y$

```
[> diff(y(t),t) = y(t)*(4-y(t));
```

We cannot drop the '(t)' from the dependent variable y. Maple treats 'y' and '$y(t)$' differently, and our equation is for \(y(t)\)

6.1.1 Exercise

Exercise 6.1. Find the derivatives of the following functions with respect to x.

1. $y = \frac{x^2+\tan(x^2)}{5x^3+9}$
2. $y = x^3 \sin(\cos^2(x))$
3. $y = (x-4)^2 \ln(x) + \sin(xe^x)$
4. $y = 3x^3 \sin^{-1}(x)$
5. $y = \sec^3(x)\cos(2x) + cosec^2(x)$
6. $y = x\ln(x) + \cos(2x)e^{5x}$
7. $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

6.2 Higher derivatives

Following examples show, how to find higher derivatives of functions. ### Method 1

You can find the second derivative by differentiating twice.

```
[> restart;
[> y:=x->x*exp(-x);
[> Diff(y(x),x,x);
[> d1:=diff(y(x),x);
[> d2:=diff(d1,x);
```

```
> restart;
> y:=x->x*exp(-x);
```

$$y := x \to x\,\mathbf{e}^{(-x)}$$

```
> Diff(y(x),x,x);
```

$$\frac{d^2}{dx^2}\left(x\,\mathbf{e}^{(-x)}\right)$$

```
> d1:=diff(y(x),x);
```

$$d1 := \mathbf{e}^{(-x)} - x\,\mathbf{e}^{(-x)}$$

```
> d2:=diff(d1,x);
```

$$d2 := -2\,\mathbf{e}^{(-x)} + x\,\mathbf{e}^{(-x)}$$

.

6.2.1 Method 2

You can get the same result by using the following command.

```
[> d1_2:=diff(y(x),x$2);
```

```
> d1_2:=diff(y(x),x$2);
```

$$d1\_2 := -2\,\mathbf{e}^{(-x)} + x\,\mathbf{e}^{(-x)}$$

Also you can find higher derivatives by changing n. `x$n` As an example the fifth derivative of the above function as follows.

```
[> d1_5:=diff(y(x),x$5);
```

```
> d1_5:=diff(y(x),x$5);
```

$$d1\_5 := 5\,\mathbf{e}^{(-x)} - x\,\mathbf{e}^{(-x)}$$

Exercise

Certainly! Let's express the given expressions in LaTeX and compute their derivatives:

Example 6.4. Let $f(x) = x^2 \sin(kx^3)$.

(i) Compute the 3rd derivative, f_{xxx} of $f(x)$.
(ii) Evaluate f_{xxx} at $x = 1$ and $k = 3$.

Example 6.5. Check whether the following functions satisfy the given equations:

(i) If $y = (1 + \sin(x))^2$, then $\frac{dy}{dx} - \cos(x) = 2\cos(x)\sin(x) + \cos(x)$.
(ii) If $y = x^2 \sin(x)$, then $\frac{8y}{x} - 4\frac{dy}{dx} + x\frac{d^2y}{dx^2} = -(x^2 - 2)\sin(x)$.

6.3 Homogenous Equations

We can check whether a given first order differential equation is homogeneous or not by using the command `odeadvisor` after loading `DEtools` package.

Example 6.6.

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}$$

```
[> restart;
[> with(DEtools):
[> Eq:=diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2;
[> odeadvisor(Eq);
```

```
> restart;
> with(DEtools):
> Eq:=diff(y(x),x)=(y(x)^2+2*x*y(x))/x^2;
```

$$Eq := \frac{d}{dx}\,\mathrm{y}(x) = \frac{\mathrm{y}(x)^2 + 2\,x\,\mathrm{y}(x)}{x^2}$$

```
> odeadvisor(Eq);
```

$$[[\_homogeneous, class\ A], \_rational, \_Bernoulli]$$

::: {.example #unnamed-chunk-47}

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

:::

```
[> Eq2:=diff(y(x),x)=(x*y(x))/(x^2+y(x)^2);
[> odeadvisor(Eq2);
```

```
> Eq2:=diff(y(x),x)=(x*y(x))/(x^2+y(x)^2);
```

$$Eq2 := \frac{d}{dx}\,\mathrm{y}(x) = \frac{x\,\mathrm{y}(x)}{x^2 + \mathrm{y}(x)^2}$$

```
> odeadvisor(Eq2);
```

$$[[\_homogeneous, class\ A], \_rational, \_dAlembert]$$

6.4 Solving Differential Equations

The command `dsolve` is used to solve an ordinary differential equation.

Example 6.7. Solve the differential equation, $\frac{dy}{dx} = -2xy$

```
[> restart;
[> with(DEtools):
[> ODE1:=diff(y(x),x)=-2*x*y(x);
[> dsolve(ODE1,y(x));
```

```
> restart;
> with(DEtools):
> ODE1:=diff(y(x),x)=-2*x*y(x);
```

$$ODE1 := \frac{d}{dx}\,\mathrm{y}(x) = -2\,x\,\mathrm{y}(x)$$

```
> dsolve(ODE1,y(x));
```

$$\mathrm{y}(x) = \_C1\,\mathrm{e}^{(-x^2)}$$

Maple returned the general solution. `_C1` denotes, of course, an arbitrary constant. Maple can handle initial value problems also. suppose we have the initial condition $y(0) = 2$.

```
[> dsolve({ODE1,y(0)=2},y(x));
```

```
> dsolve({ODE1,y(0)=2},y(x));
```

$$\mathrm{y}(x) = 2\,\mathrm{e}^{(-x^2)}$$

Example 6.8. Example 2: Solve the initial value problem, $\frac{dy}{dx} = 8x^3y^2, \quad y(0) = \frac{1}{2}$

```
[> dsolve( {diff(y(x), x) = 8*x^3*y(x)^2,y(0)=1/2},y(x));
```

```
> dsolve( {diff(y(x), x) = 8*x^3*y(x)^2,y(0)=1/2},y(x));
```

$$\mathrm{y}(x) = -\frac{1}{2\,(-1+x^4)}$$

Example 6.9. Solve the initial value problem, $\frac{dy}{dx} + \frac{y}{x} = 1, \quad y(1) = -1$

```
[> restart;
[> with(DEtools):
[> ODE2:=diff(y(x), x)+y(x) / x =1 ;
[> dsolve({ODE2,y(1)=-1},y(x));
```

```
> restart;
> with(DEtools):
> ODE2:=diff(y(x),x)+y(x)/x=1;
```

$$ODE2 := \left(\frac{d}{dx}\,\mathrm{y}(x)\right) + \frac{\mathrm{y}(x)}{x} = 1$$

```
> dsolve({ODE2,y(1)=-1},y(x));
```

$$\mathrm{y}(x) = \frac{x}{2} - \frac{3}{2\,x}$$

Example 6.10. Solve the initial value problem, $y' = y(4-y), \quad y(0) = 1$

```
[> ODE3:=diff(y(t),t)=y(t)*(4-y(t));
[> sol:=dsolve({ODE3,y(0)=1},y(t));
```

```
> ODE3:=diff(y(t),t)=y(t)*(4-y(t));
```

$$ODE3 := \frac{d}{dt}\,\mathrm{y}(t) = \mathrm{y}(t)\,(4 - \mathrm{y}(t))$$

```
> sol:=dsolve({ODE3,y(0)=1},y(t));
```

$$sol := \mathrm{y}(t) = \frac{4}{1 + 3\,\mathbf{e}^{(-4\,t)}}$$

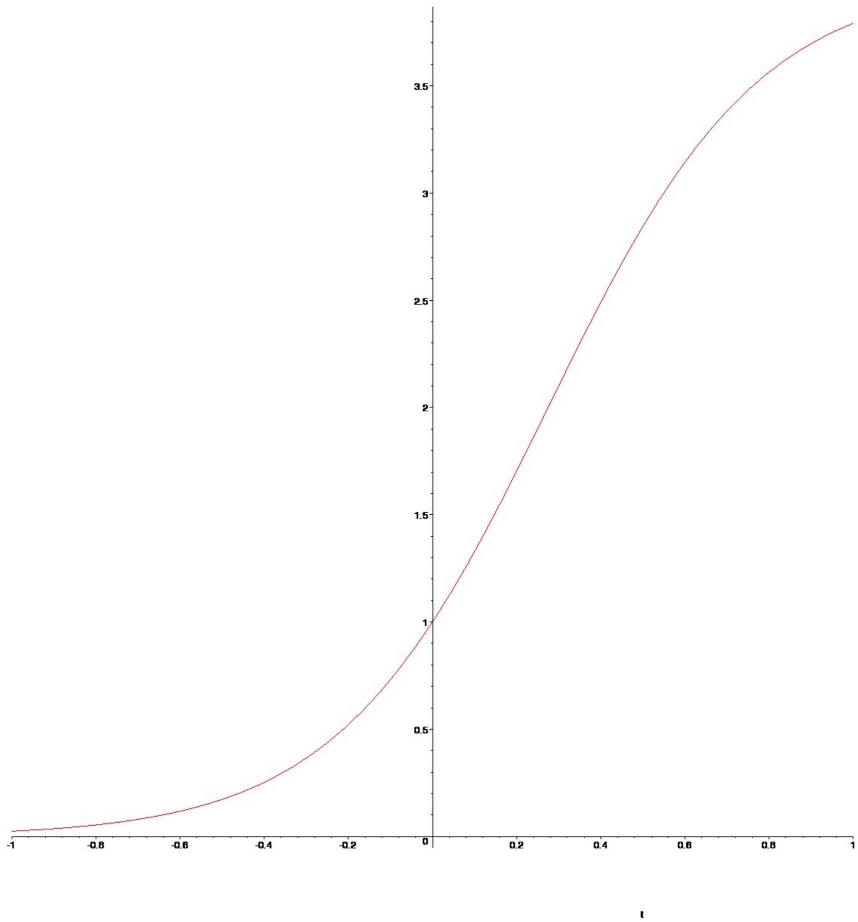
How do we plot this solution?

```
[> rhsSol:=rhs(sol);
[> plot(rhsSol,t=-1..1);
```

```
> rhsSol:=rhs(sol);
```

$$rhsSol := \frac{4}{1 + 3\,\mathbf{e}^{(-4\,t)}}$$

```
> plot(rhsSol,t=-1..1);
```



6.5 Direction Fields

A set of short line segment representing the tangent lines can be constructed for a large number of points. This collection of line segment is known as the direction fields of the differential equations.

In Maple you can use the `DEplot` command, but make sure to load the `DEtools` package.

`DEplot(deqns, vars, xrange, options)`

- `deqns` - list or set of first order ordinary differential equations, or a single differential equation of any order
- `vars` - dependent variable, or list or set of dependent variables
- `xrange` - range of the independent variable

Example 6.11. Let see how to graph the direction fields associated with the equation,

$$y' = y(4 - y), y(0) = 1$$

```
[> restart;
[> with(DEtools):
[> ODE3:=diff(y(t),t)=y(t)*(4-y(t));
[> dsolve({ODE3,y(0)=1},y(t));
[> DEplot(ODE3,y(t),t=-1..1,y=-1..5);
```



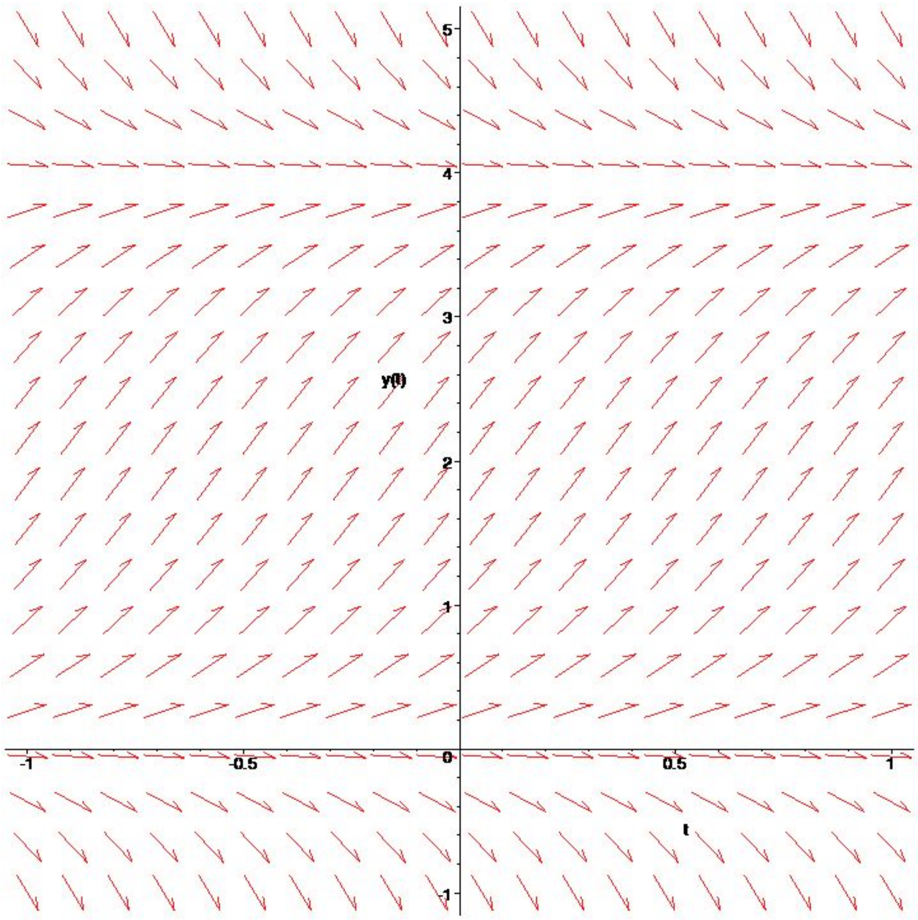
```
> restart;
> with(DEtools):
> ODE3:=diff(y(t),t)=y(t)*(4-y(t));
```


$$ODE3 := \frac{d}{dt}\mathrm{y}(t) = \mathrm{y}(t)\,(4 - \mathrm{y}(t))$$


```
> dsolve({ODE3,y(0)=1},y(t));
```


$$\mathrm{y}(t) = \frac{4}{1 + 3\,\mathbf{e}^{(-4\,t)}}$$

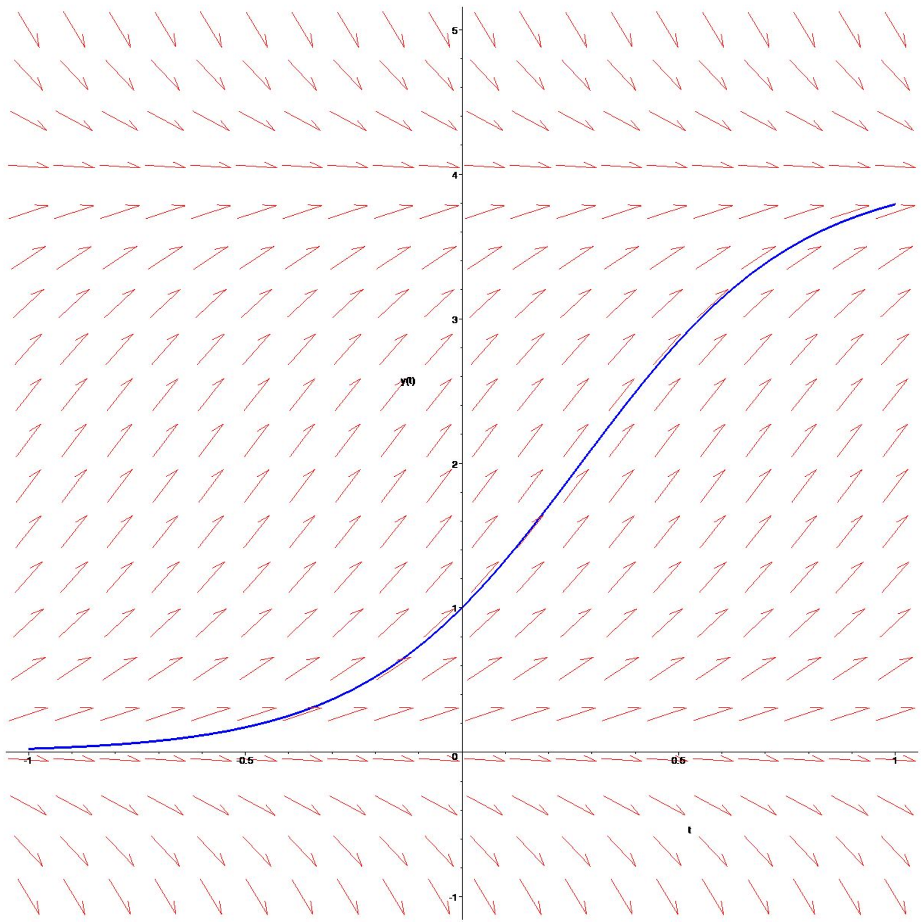

```
> DEplot(ODE3,y(t),t=-1..1,y=-1..5);
```



Let's add a solution curve to this plot. To do this, we must specify an initial condition. Let's try $y(0) = 1$. To specify initial conditions in `DEplot`, you put them in a list, contained in square brackets,

```
[> DEplot(ODE3,y(t),t=-1..1,[[y(0)=1]],y=-1..5,linecolor=blue);
```

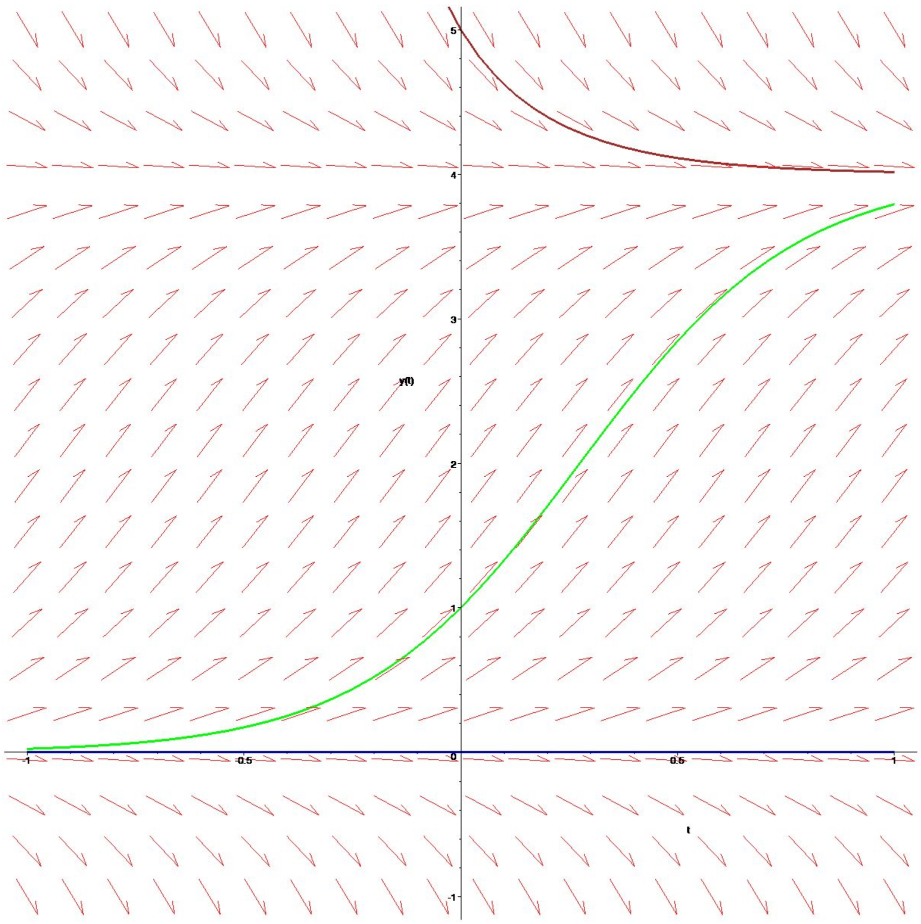
```
> DEplot(ODE3,y(t),t=-1..1,[[y(0)=1]],y=-1..5,linecolor=blue);
```



You can try out with several initial values as follows:

```
[> DEplot(ODE3,y(t),t=-1..1,[[y(0)=1],[y(0)=0],[y(0)=1],[y(0)=5]],y=-
1..5,linecolor=[blue,green,black,brown]);
```

```
> DEplot(ODE3,y(t),t=-1..1,[[y(0)=1],[y(0)=0],[y(0)=1],[y(0)=5]],y=-1..5,lineco
  lor=[blue,green,brown]);
```



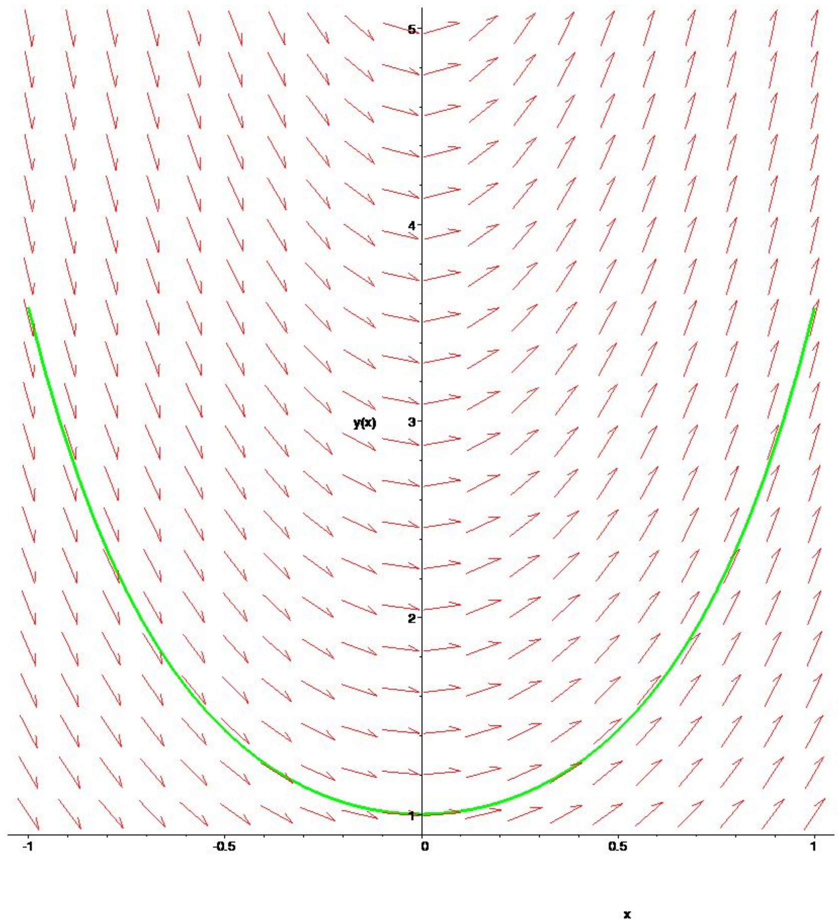
Observe how the arrows drawn in the direction field are tangent to the solution curves.

```
[> Eqn1:=diff(y(x),x)=x+2*x*y(x);
[> DEplot(Eqn1,y(x),x=1..1,[[y(0)=1/2],[y(0)=1]],y(x)=1..5,linecolor=[blue,green]);
```

```
> Eqn1:=diff(y(x),x)=x+2*x*y(x);
```

$$Eqn1 := \frac{d}{dx}\,y(x) = x + 2\,x\,y(x)$$

```
> DEplot(Eqn1,y(x),x=-1..1,[[y(0)=1/2],[y(0)=1]],y(x)=1..5,linecolor=[blue,green]);
```



6.5.1 Exercise

Exercise 6.2. Consider the ODE: $\frac{dy}{dx} = y - x^3$

(a) Find the general solution to the equation.
(b) Plot the direction fields corresponding to the equation for x and y between -2 and 2.
(c) Solve the initial value problems.

- $\frac{dy}{dx} = y - x^3$, with $y(0) = 1$
- $\frac{dy}{dx} = y - x^3$, with $y(0) = \frac{1}{2}$

(d) Plot the solution curves and the direction fields in the same graph.

Exercise 6.3.

2. Plot the direction fields for the following equations and state the stability:

(i) $\frac{dy}{dx} = y - 5$
(ii) $\frac{dy}{dx} = y(1 - y)$
(iii) $\frac{dy}{dx} = y^2(y - 3)$
(iv) $\frac{dy}{dx} = y^2 - 5y + 6$
(v) $\frac{dy}{dx} = (y - 3)^2$

Chapter 7

Equations and Functions

7.1 Solving Equations

In general we can solve different types of equations to get an exact solution. The solution may be an integer, a fraction, or it may be an expression. Using Maple we can do the same thing using the `solve` command and obtain an exact solutions for equations and inequalities.

7.2 Equations with multiple unknowns

You can use the solve command to solve equations having several variables. However you have to specify, for which variable that the equation to be solved.

7.3 Functions

Let f be a function of x. We denote it as $f(x)$, but in Maple there is a different way to define such functions.

7.3.1 Defining functions

7.3.2 Function Operations and Compositions

7.3.3 Composition of functions

7.4 Trigonometry with Maple

- The Trigonometric functions

$$\begin{array}{ccc} \sin(x) & \cos(x) & \tan(x) \\ \sec(x) & \csc(x) & \cot(x) \end{array}$$

- The Hyperbolic functions

$$\begin{array}{ccc} \sinh(x) & \cosh(x) & \tanh(x) \\ \operatorname{sech}(x) & \operatorname{csch}(x) & \coth(x) \end{array}$$

7.5 Inverse trigonometric functions

$$\begin{array}{lll} \arcsin(x) & \arccos(x) & \arctan(x) \\ \operatorname{arcsec}(x) & \operatorname{arccsc}(x) & \operatorname{arccot}(x) \\ \operatorname{arcsinh}(x) & \operatorname{arccosh}(x) & \operatorname{arctanh}(x) \\ \operatorname{arcsech}(x) & \operatorname{arccsch}(x) & \operatorname{arccoth}(x) \end{array}$$

7.6 Exercise

Exercise 7.1. Find the solutions of the following equations to 5 decimal places.

i. $2x^3 + 3x + 1 = 0$
ii. $2x^3 + 3x + \frac{1}{4} = 0$
iii. $x^2 - 13x + 10 = 0$
iv. $-3x + \frac{1}{2}x^2 = 25$

Exercise 7.2. Find the most accurate real solutions to the following equations.

i. $x^4 - 2x^3 = 7$
ii. $\frac{7}{(x-3)^2} + \frac{5}{(x+5)}$

Exercise 7.3. Express the following in the form of $y = mx + c$ using the `solve` command.

i. $3x + 4y = 2$
ii. $\frac{3y}{5} - 2x + 7 = 0$
iii. $\frac{x}{x-3} + \frac{y}{2x} = -3$

Use the Maple help to find another way to do the above.
(Hint: You have to `isolate` y in each equation.)

Exercise 7.4. Consider, $f(x) = 2x - \frac{x}{3(x+1)}$

i. Define f as a function: $f(x) = 2x - \frac{x^3}{x+1}$
ii. Evaluate $f(-\frac{1}{2})$
iii. Factor $f(x)$
iv. Simplify $f(\frac{1}{t-1})$

Exercise 7.5. Use Maple help to find out how to find `logarithms`. Then find the value of the following.

i. $\log_{10} 100$
ii. $\ln 100$
iii. $\log_3 10$
iv. $2\log_3 81 + 5\log_8 256$

Exercise 7.6. Find the value of the following trigonometric expressions for given x.

i. $\sin(\sec(x^2)) + 3x\cos^3(\frac{2x}{7})$, where $x = 71°$.
ii. $\sec^{-1}(\tanh(x+5))\cos(\sec(2x) + \sin(2x))$, where $x = 43°$.
iii. $\left(\cot^{-1}(x) + \sec^{-1}(\frac{x-3}{5})\right)^{\frac{1}{3}}$, where $x = 71^circ$.

Chapter 8

Plots

Maple Packages

In Maple we can use various packages for special applications. In order to use a package, you have to load the package first using the `with` command.

Example

```
> with(student):
> with(linalg):
```

So, when you are dealing with plots, you have to load the package `plot`.

```
> with(plots):
```

8.1 Basic Graphs

Using the plot command you can graph a function. You have to give the range of values for x otherwise Maple will use the default range.

8.2 Multiple Graphs

If you want to compare plots, you can have two or more plot windows open at the same time or you can plot more than one curve on the same set of axes. The plot command offers options which control the number of points at which the function is plotted, the number of tick marks on the axes and the placing of titles on the graph. Read the help page on plot to find out about these options.

Chapter 9

Introduction to Maple

This is temporary file

9.1 What is Maple?

- Maple is a Symbolic Computation System or Computer Algebra System which can be used to obtain exact analytical solutions to many mathematical problems, including integrals, systems of equations, differential equations, and problems in linear algebra.
- It also has the capability of plotting functions in 2D and 3D and displaying animations.
- Maple can perform calculations in binding speed, but one has to be responsible for making these calculations meaningfully and mathematically correct.

9.2 Using a Maple Worksheet

The following figure shows the Maple window with a blank Maple worksheet and this window contains:

- a menu bar across the top with menus:
- a tool bar immediately below the menu bar, with button-based short cuts to common operations;
- a context bar directly below the tool bar, with controls specific to the task being performed;

- a window, containing a Maple prompt [>, called a worksheet;

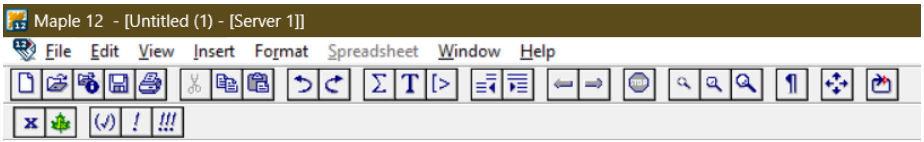
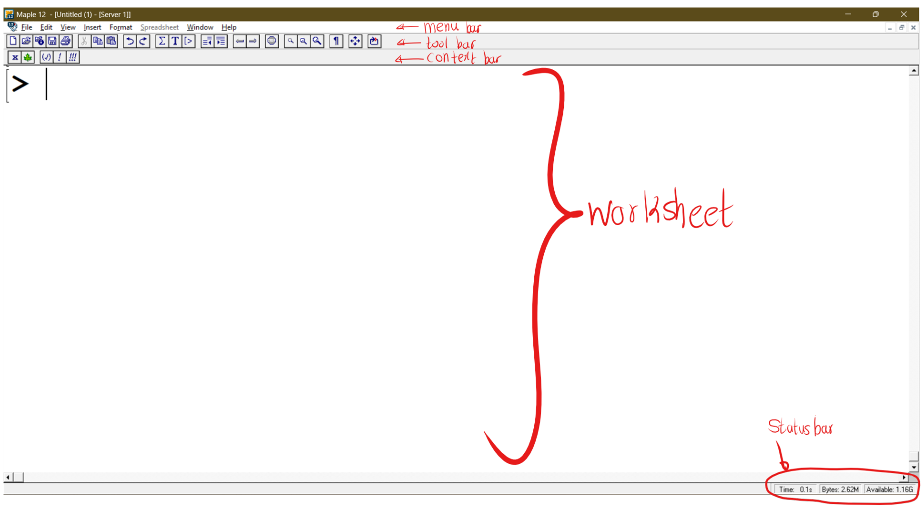


Figure 9.1: The menu bar, tool bar, and context bar.

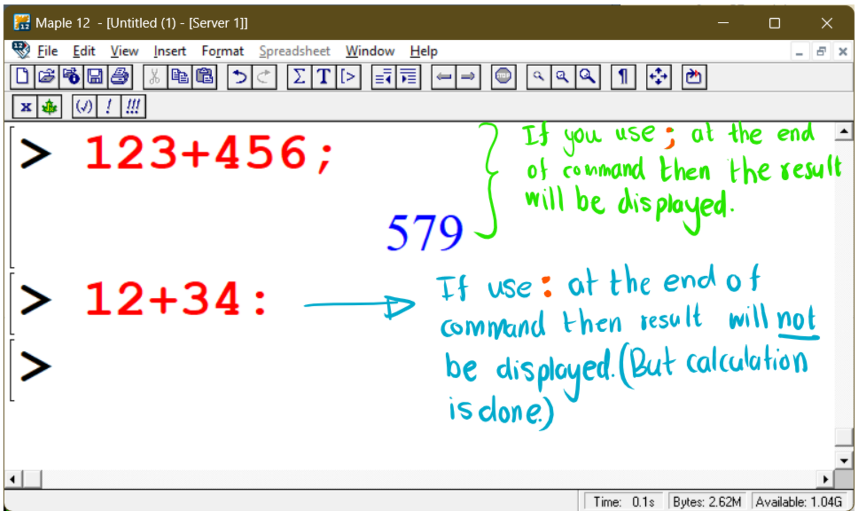
- a status bar at the bottom, with boxes marked Ready, Time and Memory



From the File menu, select the options **Save** or **Save As** to save the active Maple classic worksheet. Maple classic worksheets are saved with the extension ".mws", but in the standard interface, Maple worksheets are saved with the extension ".mw"

9.3 Entering Maple Commands

- The " > " is the command prompt in Maple. That is where you type your commands or statements.
- Every command in Maple should end with a semicolon(;) or a colon(:). (If you use a semicolon then the result of the command will be displayed. If you use a colon then the result will not be displayed.)



- If you want to make any comments you can use the text format by clicking on the box `T` in the tool bar or use the symbol `#`.

9.4 Arithmetic operations

Arithmetic operators follow the same precedence rules as in Mathematics, and these are brackets, of, division, multiplication, addition and subtraction (BODMAS). Usual arithmetic operations can perform easily with Maple.

```
[> 312+121;
```


```
> 125-45;
```

The `*` key is used for multiplication, `/` for division and `^` for the power.

```
[> 13*267;
```

```
> 13*267;
```

$$3471$$

```
[> 565/5;
```

```
> 565/5;
```

$$113$$

```
[> 561/5;
```

```
> 561/5;
```

$$\frac{561}{5}$$

```
[> 125-45; 13*267; 12345/5; #Three arithmetic operations
```

```
> 125-45; 13*267; 12345/5; #Three arithmetic operations
```

80

3471

2469

```
[> 2^5;
```

```
> 2^5;
```

$$32$$

```
[> 2^(-5);
```

```
> 2^(-5);
```

$$\frac{1}{32}$$

```
[> 3^40;
```

```
> 3^40;
```

12157665459056928801

Remark. ***Don't use commas when you type large numbers in Maple. - For example: Compute the product 102,136,543 & 20,077,410 .***

```
[> 102136543*20077410;
```

```
> 102136543*20077410;
```

2050637249793630

9.5 Operations

Maple adheres to the same order of operations that we use in Mathematics. By inserting parentheses, we can change this order.

```
[> 2+3*4-5*6;
[> 2+(3*4-5)*6;
[> (2+3)*4-5*6;
```

```
> 2+3*4-5*6;
                                        -16
> 2+(3*4-5)*6;
                                        44
> (2+3)*4-5*6;
                                        -10
```

```
[> 29/(100-11*3^2);
```

```
> 29/(100-11*3^2);
                                        29
```

```
[> (3^4-2^6)/(3^2-2^3);
```

```
> (3^4-2^6)/(3^2-2^3);
                                        17
```

9.5.1 Exercises

Exercise 9.1.

1. Calculate the followings

i. $1428 + 456 - 41$
ii. $421 \times 240 \div 55$
iii. $(128 - 691 + 458) \times 8$
iv. $2214875(201 \times 11 - 55)$
v. $201 \div (2012 - 1)$

vi. $21^{4^{2^3}}$

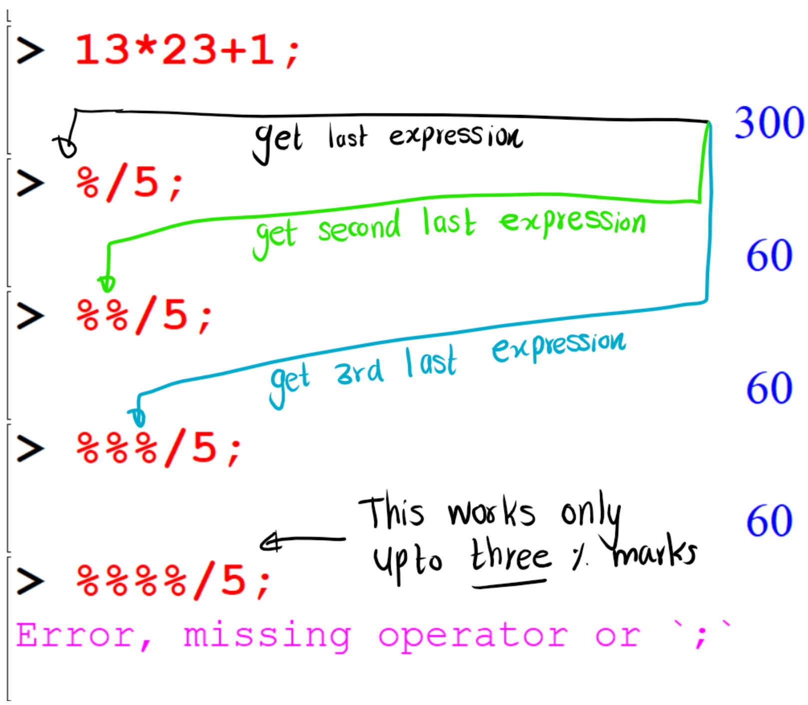
Exercise 9.2.

i. Compute 3^{400}.
ii. Find the command to find the length (number of digits) of a number.
iii. How many digits are there in the number 3^{400} ?
iv. Does the above command give correct answer to the fractional numbers?

9.6 Shortcut to retyping

One shortcut that we use often in Maple to retype is the % key. This refers to most recently executed result.

```
[> 13*23+1;
[> %/5;
[> %%/5;
[> %%%/5;
```




```
[> 12540*4;
[> %/4;
[> %/4;
```

```
> 12540*4;
```

gets the last expression

$$50160$$

```
> %/4;
```

gets the last expression

$$12540$$

```
> %/4;
```

$$3135$$

```
>
```

9.7 Fractions and Decimals

By simply entering a fraction Maple automatically reduce it.

```
[> 45/4;
```

```
> 45/4;
```

$$\frac{45}{4}$$

```
[> 148/24;
```

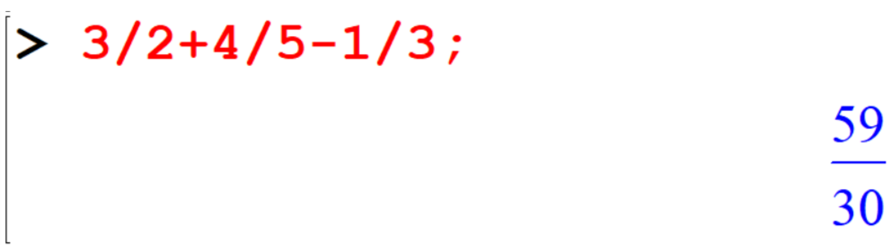
```
> 148/24;
```

$$\frac{37}{6}$$

```
[> 25/15;
```

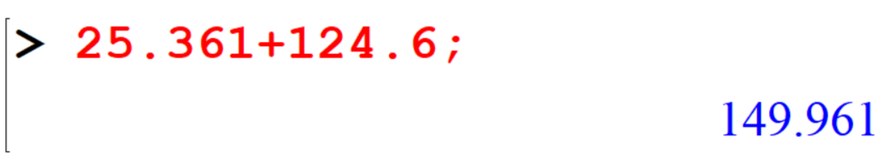

```
[> 2/3+3/7;
```


```
[> 3/2+4/5-1/3;
```



You can do calculations with decimal numbers also.

```
[> 25.361+124.6;
```




```
[> 2.138*0.013;
```

```
> 2.138*0.013;
                                        0.027794
```

```
[> 56.101/0.102;
```

```
> 56.101/0.102;
                                        550.0098039
```

9.8 Roots

```
[> sqrt(16);
```

```
> sqrt(16);
                                        4
```

```
[> sqrt(30);
```

```
> sqrt(30);
```

$$\sqrt{30}$$

```
[> evalf(sqrt(30));
```

```
> evalf(sqrt(30));
                                        5.477225575
```

The `evalf` command numerically evaluates expressions (or subexpressions) involving constants (for example, `Pi`, `exp(1)`) and mathematical functions (for example, `exp`, `ln`, `sin`).

```
[> 30^(1/2);
[> evalf(%);
```

9.8.1 Exercise

Exercise 9.3. Compute the following:

i. $11 + \sqrt{31}$
ii. $\sqrt[3]{64}$
iii. $\sqrt{2}^{\sqrt{3}}$

Exercise 9.4. Calculate 120^8

i. Divide the answer by 10^8
ii. Divide the answer in part i. by 2048×8

9.9 Pi Vs pi

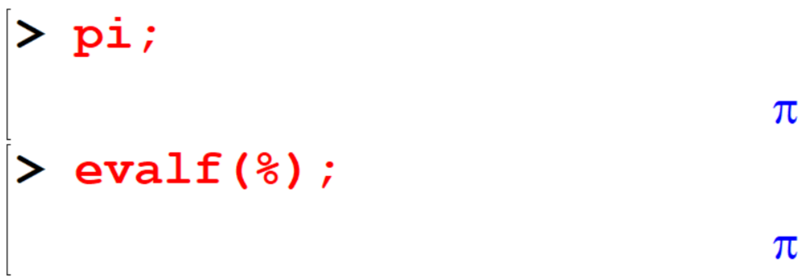
π is a constant in Mathematics and is recognized by maple and typed as `Pi` **(Note the capitalization of "p" but not "i')**.

```
[> Pi;
[> evalf(%);
```

```
[> evalf(2*Pi);
```

If you use `pi` then `evalf` command does not return π numerically.

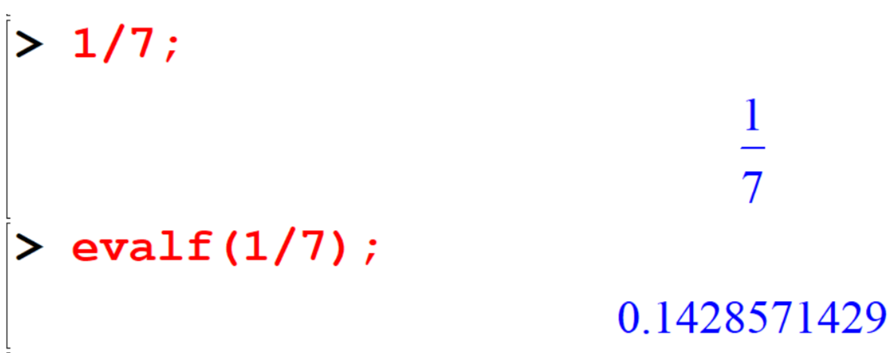
```
[> pi;
[> evalf(%);
```



9.10 Rational Numbers

Maple usually leaves fractions in fraction form. However, we can force it to express fractions in decimal form using the `evalf` command.

```
[> 1/7;
[> evalf(1/7);
```




```
[> 25/35;
[> evalf(%);
```

```
> 25/35;
```

$$\frac{5}{7}$$

```
> evalf(%);
```

$$0.7142857143$$

Maple displays 10 decimal places as a default. If this is not enough and for better precision you can specify the exact number of decimal places as a second parameter to the `evalf` command.

Note that the second parameter normally represents the number of non-zero digits in the answer.

```
[> evalf(1/7,100);
```

```
> evalf(1/7,100);
```

0.1428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571429

```
[> evalf(29/3,5);
```

```
> evalf(29/3,5);
```

$$9.6667$$

Here, if you want to calculate the answer for 4 decimal places the command should be `evalf(29/3,5)` and if you want the answer to be 5 decimal places the command should be `evalf(29/3,6)`.

```
[> evalf(1/15,5);
```

```
> evalf(1/15,5);
```

$$0.066667$$

```
[> evalf(11/19,5);
```

```
> evalf(11/19,5);
                                        0.57895
```

Fractions are also rational numbers because their decimal expansions always have repeating blocks of digits. By looking at the decimal representation of a rational number you can see the repeating cycle.

```
[> evalf(1/35);
```

```
> evalf(1/35);
                                  0.02857142857
```

Here we can not see the repeat cycle. But if we calculate $\frac{1}{35}$ for more decimal places we can see the repeating cycle.

```
[> evalf(1/35, 100);
```

```
> evalf(1/35, 100);
        0.02857142857142857142857142857142857142857142857142857142857142857142857142857142857142857142857142857
```

9.10.1 Exercises

Exercise 9.5. Evaluate the value of   correct up to 5 decimal places.

Exercise 9.6. Find the area of a circle with radius 10cms

Exercise 9.7. Check whether the followings are rational numbers or not

i. $\frac{1}{49}$
ii. π
iii. $\sqrt{2}$

Exercise 9.8. How many digits are repeating in $\frac{1}{212}$

9.11 Complex Numbers

In Maple, complex arithmetic is normally done automatically with `I` standing for $\sqrt{-1}$ (for example, if you square `I` you will get `-1`, not `I^2`)

```
[> (-4+7*I)+(5-10*I);
```

```
[> (-4+7*I)+(5-10*I);
```

$$1 - 3\,I$$

```
[> 5*I-(-9+I);
```

```
[> 5*I-(-9+I);
```

$$9 + 4\,I$$

```
[> (1-5*I)*(-9+2*I);
```

```
[> (1-5*I)*(-9+2*I);
```

$$1 + 47\,I$$

```
[> (3-I)/(2+7*I);
```

```
[> (3-I)/(2+7*I);
```

$$\frac{-1}{53} - \frac{23}{53}\,I$$

But Maple does not always automatically evaluate an expression involving complex numbers. For example, it may leave an expression as the product of some complex numbers or as an expression involving a root of a complex number.

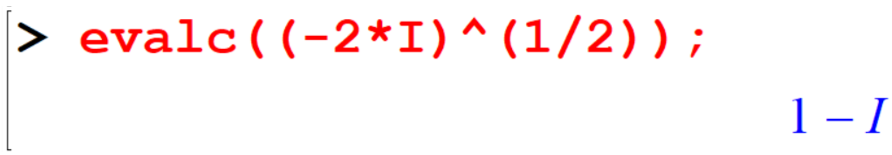
```
[> (-2*I)^(1/2);
```

```
[> (-2*I)^(1/2);
```

$$\sqrt{-2\,I}$$

The function `evalc` to force Maple to evaluate as a complex number.

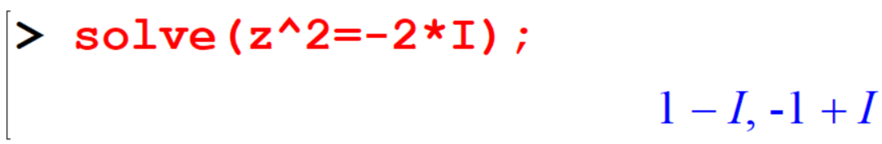
```
[> evalc((-2*I)^(1/2));
```



Note that `evalc` does not give you both the square roots of `-2*I` it only gives the *principal value* of the root.

If you want to find the roots, use solve command as follows.

```
[> solve(z^2=-2*I);
```



9.11.1 Exercise

Exercise 9.9. Simplify the following:

1. $(-3+3i)+(7–2i)$
2. $(5+3i)-(3-i)$
3. $(1+2i)(1-2i)4.(56-8i)\div(14+10i)$

Exercise 9.10.

2. Simplify $(2i)^{\frac{1}{2}}$ by using `evalc` and solve commands.

Exercise 9.11.

3. Multiply the following and obtain the answer in standard form:

$$(2-\sqrt{-100})(1+\sqrt{-36})$$

9.12 MAPLE Help

Maple contains a complete online help system you can use to find information about specific topic easily and to explore the wide range of commands available. To get the information about commands, which you learn in MAPLE, you can use either one of the following.

- Topic search function
- F1 key
- `?` In front of the command