Lab 10: Maximum Likelihood and Optimization

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Loading the data

```
library(MASS)
```

```
## Warning: package 'MASS' was built under R version 3.1.3
```

```
data(cats)
summary(cats)
```

```
##  Sex         Bwt             Hwt       
##  F:47   Min.   :2.000   Min.   : 6.30  
##  M:97   1st Qu.:2.300   1st Qu.: 8.95  
##         Median :2.700   Median :10.10  
##         Mean   :2.724   Mean   :10.63  
##         3rd Qu.:3.025   3rd Qu.:12.12  
##         Max.   :3.900   Max.   :20.50  
```

Question 1

Fitting a gamma distribution to the cats' heart weights by maximum likelihood. In this I wrote a functon for the negative log-likelihood, its gradient, and found the minimum using the optim() function.

```
# get the log likelihood
ll.func <- function(theta){
  return(-sum(dgamma(cats$Hwt,shape=theta[1],scale=theta[2],log=T)))
}
library(numDeriv)
```

```
## Warning: package 'numDeriv' was built under R version 3.1.3
```

```
# function that returns another function
grad.ll <- function(theta) {
  grad(func=ll.func,x=theta)
}
theta0=c(20,0.5)
# finding the mimum
fit <- optim(theta0,ll.func,grad.ll,method="BFGS",hessian=TRUE)
```

Question 2

Here I verify that the gradient of the negative log-likelihood at the maximum likelihood estimate is close to 0.

```
output<-fit$par
grad.ll(output)
```

```
## [1] 0.0006194483 0.0285247108
```

Question 3

The negative inverse Hessian matrix of the log likelihood is an estimate of the covariance matrix of the estimated parameters. In order to find this for cast' heart weights, we just invert the Hessian matrix outputted by the optim() function. The reason we dont need to take the negative is becasue we optimize over the the negative log-likelihood.

```
hes <- fit$hessian
fisher.info<-solve(hes)
fisher.info
```

```
##           [,1]         [,2]
## [1,]  5.630558 -0.145265979
## [2,] -0.145266  0.003841628
```

Question 4

In order to find the sample confidence intervals for the paramters in the gamma distrubtion, we use the covariance matrix that we found in question 3. From the covariance matrix we get the variances for each of the parameters.

```
sig<-sqrt(diag(fisher.info))
upper1<-output[1]+1.96*sig[1]
lower1<-output[1]-1.96*sig[1]
cat('Confidence Interval for the shape [',lower1,',',upper1,']' )
```

```
## Confidence Interval for the shape [ 15.64807 , 24.94976 ]
```

```
upper2<-output[2]+1.96*sig[2]
lower2<-output[2]-1.96*sig[2]
cat('Confidence Interval for the scale [',lower2,',',upper2,']' )
```

```
## Confidence Interval for the scale [ 0.4022208 , 0.6451858 ]
```