MATLAB assignment 2

Wiener and adaptive filtering

- **This is a MATLAB exercise to enhance the understanding of concepts learnt during lectures.**
- **Use data files and function files (.mat & .m) that are attached with this document as instructed.**

Submission document	**Submission method**	**Notes**
Report	Upload the softcopy to Moodle	Should include observations and discussions with relevant plots to support your answers.
MATLAB scripts	Upload a single ZIP file including all the .m files to Moodle	Name each script according to question number. Also, include necessary comments on the scripts for better read-ability.

Introduction

This exercise allows;

- The implementation of the Wiener filter.
- The implementation of adaptive noise canceller based on LMS and RLS algorithms and analyse their behaviour with non-stationary noise.

1. Weiner filtering (on stationary signals)

Data construction

- Ideal signal $y_i(n)$: `idealECG.mat`. Sampling frequency = 500 Hz
- Input signal $x(n) = y_i(n) + \eta(n)$
- Where $\eta(n) = \eta_{wg}(n) + \eta_{50}(n)$
- $\eta_{wg}(n)$: white Gaussian noise such that SNR is 10 dB with respect to $y_i(n)$
- $\eta_{50}(n) = 0.2\sin(2\pi f_1 n) \quad where\ f_1 = 50$

1.1. Discrete-time domain implementation of the Wiener filter

Calculate the optimum weight vector $\mathbf{w}_0 = (\Phi_{\mathbf{Y}} + \Phi_{\mathbf{N}})^{-1}\Theta_{\mathbf{Y}y}$ given the signals in Part 1 and Part 2 and parameters:

a) For an arbitrary filter order
b) Find the optimum filter order and its coefficients. Plot magnitude response
c) Obtain the filtered signal $\hat{y}(n)$ using filter obtained in above.
d) Plot the spectra of $y_i(n),\ \eta(n), x(n), \hat{y}(n)$ on the same plot.
e) Interpret spectra and the magnitude response of the filter.

Part 1

- Desired signal: single beat of $y_i(n)$
- Noise signal: Extract the signal segment from the T wave of any chosen ECG beat to the P wave of the next ECG beat (isoelectric segment) of $x(n)$.

Part 2

- Desired signal: Construct a linear model having the same length and a comparable morphology to that of a single ECG beat of the ideal signal $y_i(n)$. For example, see Figure 1
- Noise signal: Extract the signal segment from the T wave of any chosen ECG beat to the P wave of the next ECG beat (isoelectric segment) of $x(n)$.

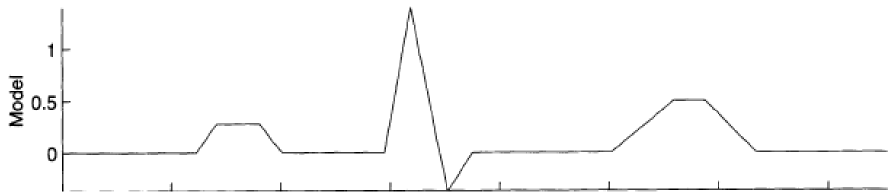

Figure 1

1.2. Frequency domain implementation of the Wiener filter

$$\boldsymbol{W(f)} = \frac{\boldsymbol{S_{YY}(f)}}{\boldsymbol{S_{YY}(f)} + \boldsymbol{S_{NN}(f)}}$$

Where $S_{ZZ}(f)$ is the PSD of a signal $z(n)$. Alternatively, $S_{ZZ}(f)$ is the power of the Fourier transform of the template $z(n)$. Therefore, $S_{ZZ}(f) = (absoulte(Fourier\ transform[z(n)]))^2$.

a) Implement the frequency domain version of the Wiener filter. $S_{\hat{Y}}(f) = W(f)S_X(f)$.
b) Compare the filtered signal with Part 1 or Part 2 with the aid of a plot and mean squared error with respect to the ideal signal $y(n)$.

1.3. Effect on non-stationary noise on the Wiener filtering

This section explores the behaviour of the Wiener filter when the noise characteristics are non-stationary. This is demonstrated by changing the 50 Hz noise to 100 Hz halfway through the signal duration.

<u>*Data construction*</u>

$\eta(n) = \eta_{wg}(n) + \eta_{50}(n) + \eta_{100}(n)$

$\eta_{wg}(n)$: white Gaussian noise such that SNR is 10 dB with respect to $y_i(n)$
50 Hz noise such that: $\eta_{50}(n) = 0.2\sin(2\pi f_1 n)\ \ where\ f_1 = 50$ and $0 \le n < T/2$
100 Hz noise such that: $\eta_{100}(n) = 0.3\sin(2\pi f_2 n)\ \ where\ f_2 = 125$ and $T/2 \le n \le T$
Where T is the total time.

a) Apply any Wiener filter derived out of sections 1.1, 1.2 on the new $x(n)$
b) Plot the filtered time-domain signal and interpret the result.

2. Adaptive filtering

This section is based on the ANC and LMS sections of the 6_*Optimum and adaptive filters.pdf*

Data construction

Sampling frequency (f_s)= 500 Hz
Number of samples (N) = 5000

$y_i(n)$: Sawtooth waveform with a width of 0.5.
$\eta(n)$: Non-stationary noise: use same as in section 1.3.
$r(n) = a(\eta_{wg}(n) + \eta_{50}(nf_{50} + \phi_1) + \eta_{100}(nf_{100} + \phi_2))$, where a and ϕ_1, ϕ_2 are arbitrary constants.

Use the following code to generate signals

```
N = 5000; % Number of points
t = linspace(0,5,N)'; % Time vector with fs = 500 Hz
s = sawtooth(2*pi*2*t(1:N,1),0.5);  % Sawtooth signal
n1 = 0.2*sin(2*pi*50*t(1:N/2,1)-phi);   % Sinusoid at 50 Hz
n2 = 0.3*sin(2* pi*100*t(N/2+1:N,1)-phi);   % Sinusoid at 100 Hz
nwg = s - awgn(s,snr,'measured'); % Gaussian white noise
```

2.1. LMS method

Implement the LMS method as per the equation

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu e(n)\mathbf{R}^T(n)$$

a) Explore the rate of adaptation by varying the rate of convergence μ and the order of the adaptive filter $(M-1)$. To quantify you may calculate the mean squared error with respective to desired signal.

2.2. RLS method

Implement the LMS method as per the equations:

$$\mathbf{k}(n) = \frac{\lambda^{-1}\mathbf{p}(n-1)\mathbf{r}(n)}{1 + \lambda^{-1}\mathbf{r}^T(n)\mathbf{p}(n-1)\mathbf{r}(n)}$$
$$\mathbf{p}(n) = \lambda^{-1}\mathbf{p}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{r}^T(n)\mathbf{p}(n-1)$$
$$\alpha(n) = x(n) - \mathbf{w}^T(n-1)\mathbf{r}(n)$$
$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{k}(n)\alpha(n)$$
$$\alpha(n) = \hat{s}(n) = x(n) - \mathbf{w}^T(n)\mathbf{r}(n)$$

Initialisations:

$P(0) = \delta^{-1}I$, where δ is a small constant and I is the identity matrix
$\mathbf{w}(0) = 0$ or any tap weight vector if necessary

a) Explore the rate of adaptation by varying the forgetting factor λ and the order of the adaptive filter $(M-1)$. To quantify you may calculate the mean squared error with respective to desired signal.

b) Compare the performance of LMS and RLS algorithms.

c) Test LMS and RLS algorithms using the $y_i(n)$ as the `idealECG.mat`

Reference plot for 2.1