Department of Electronic & Telecommunication Engineering University of Moratuwa

BM4151 – Biosignal Processing

MATLAB Assignment 2
Wiener & Adaptive Filtering

Palihakkara A.T. 170418E

This report is submitted in partial fulfillment of the requirements for the module BM4151 Biosignal processing.

01 November 2021

List of Figures

1. Weiner Filtering

Weiner filters are a set of optimum filters that gives the best possible filter of a given order considering the statistical characteristics of the signals comparing to the conventional FIR and IIR filters.

Optimization done by minimizing the mean square error between the filtered signal and a desired signal to be obtained. For Weiner filters weights are calculated using the Weiner-Hopf equation.

$$w_0 = \Phi_X^{-1}\theta_{Xy}$$

Where, w_0 is the optimum weight matrix, Φ_X is the autocorrelation of the sampled signal and θ_{Xy} is the cross-correlation between the desired signal and sampled signal (noisy signal). In Weiner filter, it assumes that noise component and the signal component is independent to each other. With that assumption we can write the Weiner-Hopf equation as,

$$w_0 = (\Phi_Y + \Phi_N)^{-1}\theta_{Xy}$$

With this simplified equation we can estimate the filter coefficients using the desired signal and the noisy signal.

1.1 Discrete-time domain implementation of the Wiener filter.

The given ideal ECG signal is added with 10 dB Gaussian white noise and a low frequency sinusoidal noise to obtain the noisy ECG signal.

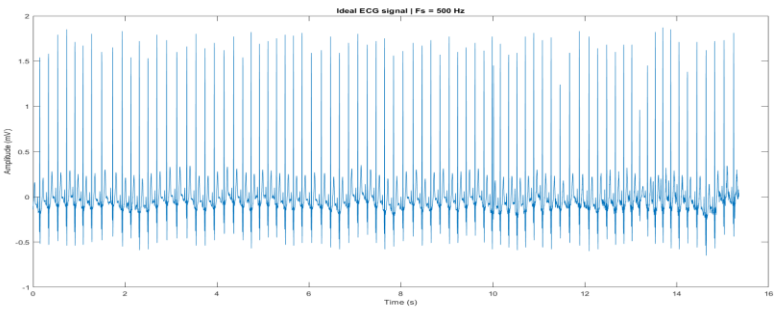

Figure 1: Ideal ECG signal

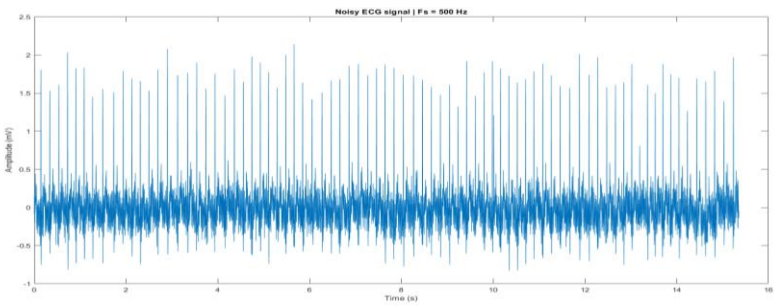

Figure 2: Noisy ECG signal

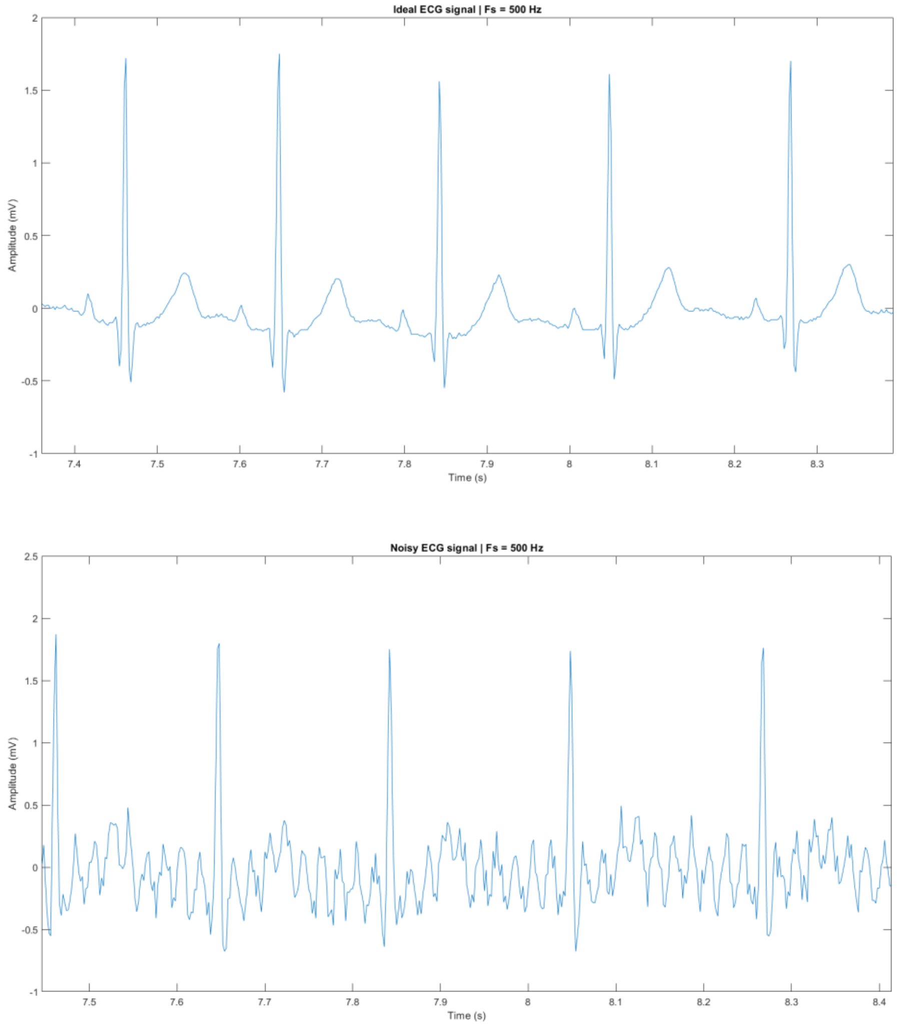

Figure 3: Ideal & Noisy ECG pulses (zoomed)

According to Figure 3, we can see that the ideal signal is corrupted with the added noise.

Part 1 – Extract single pulse from the signal.

A single ECG pulse was selected randomly from the ideal ECG signal as the desired signal for the Weiner filter. In the meantime, noisy signal part is obtained from the end of the T wave to the beginning of the P wave of the following ECG pulse, which is called as the isoelectric segment.

To have the signals in same length, the isoelectric segment is replicated into several times.

Sample length of the ECG pulse = 84

Sample length of the isoelectric segment = 21

So, the isoelectric segment is replicated by 4 times.

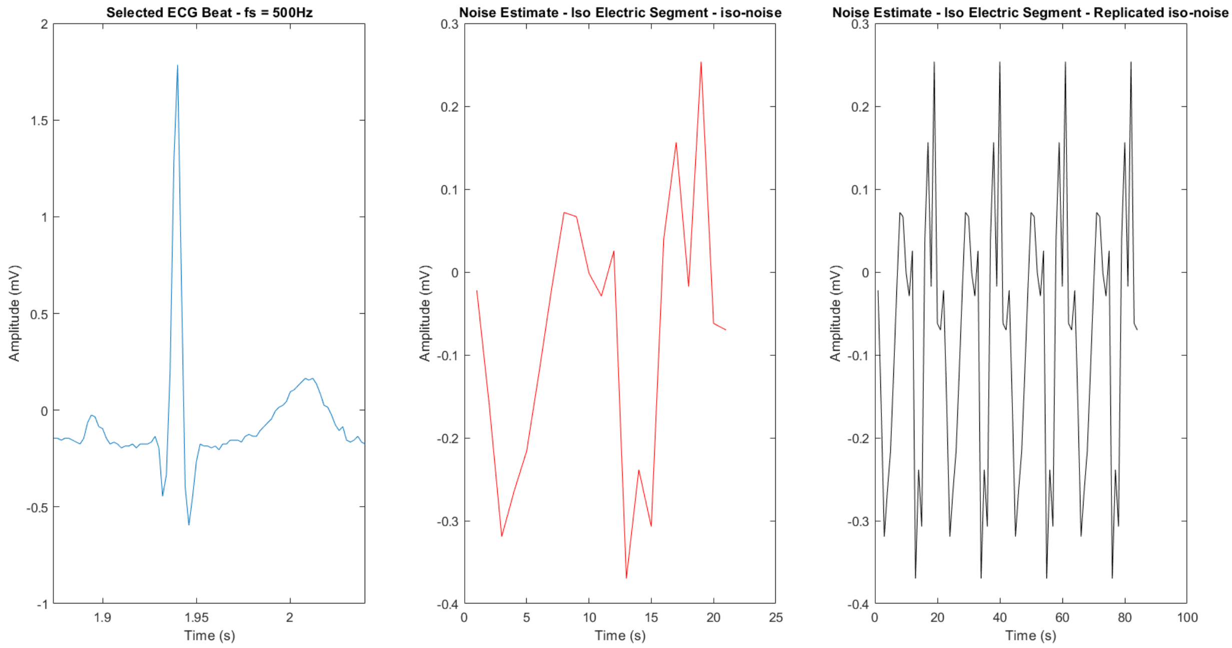

Figure 4: Selected ECG pulse & the elongated isoelectric segment

Then, a weight matrix obtained from the Weiner-Hopf Equation for an arbitrary selected filter order of 15.

Table 1: Weight Matrix | Order = 15

0.6096	0.3056	-0.1153	-0.1519	-0.0675	0.0949	0.1057	0.0582	0.0009	0.0098	0.0181	0.0022	0.0257	0.0145	-0.0104

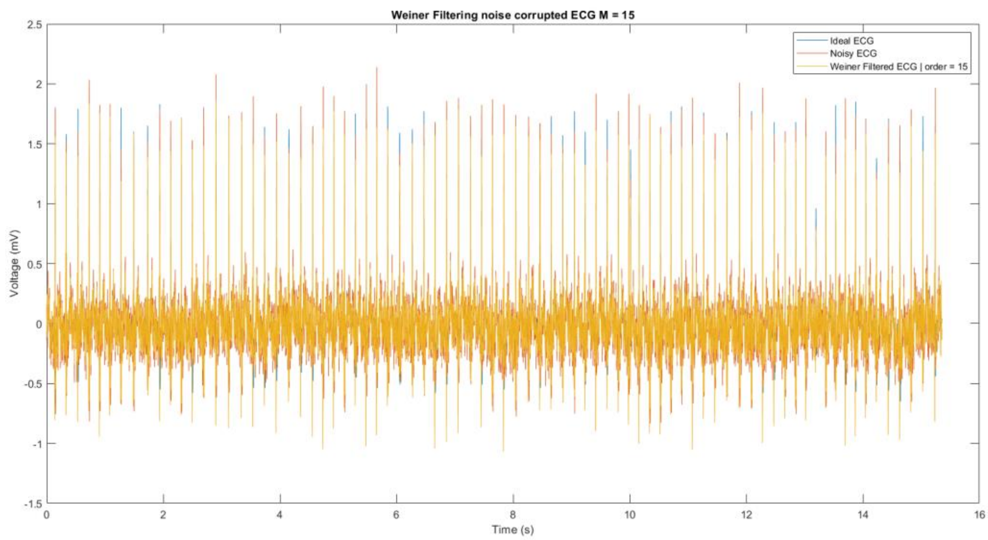

Figure 5: Weiner Filtered signal | Order = 15

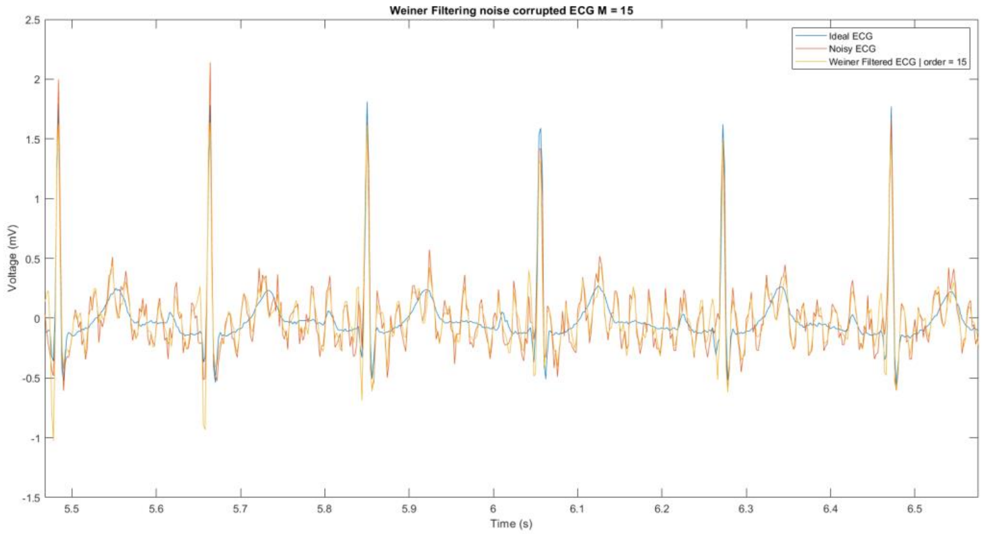

Figure 6: Weiner Filtered signal (zoomed) | Order = 15

Then the optimum filter order is obtained using the Mean Square Error values of the filtered signal and the desired signal for different Order values ranging from 2 to 55.

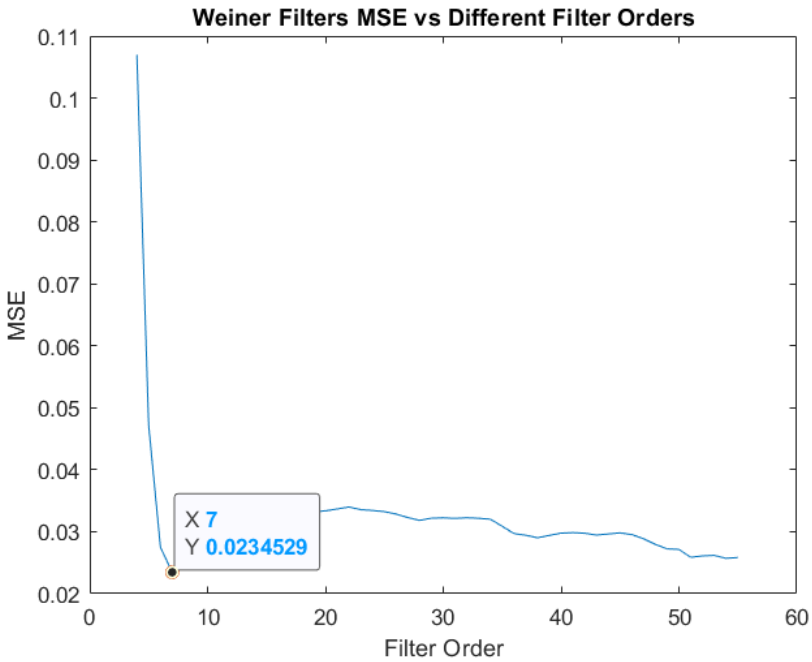

Figure 7: Weiner Filter MSE vs Filter Order

The order which has the lowest Mean Square Error value is considered as the Optimum filter order.

Obtained optimum filter order = 7

The magnitude and the phase response of the obtained optimum order filter is as follows,

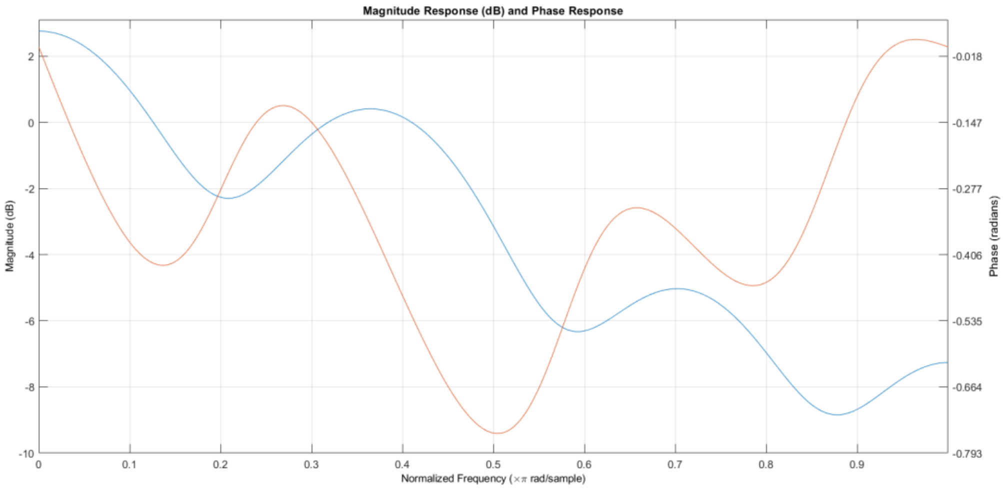

Figure 8: Magnitude (Blue) and Phase (Orange) Response of the Optimum Order Filter

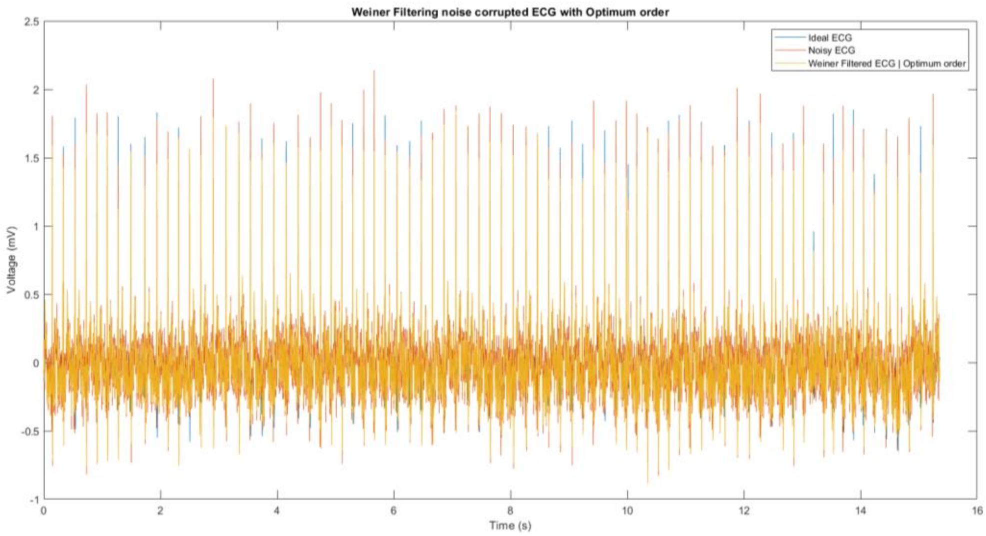

Figure 9: Weiner Filtered signal | Optimum Order = 7

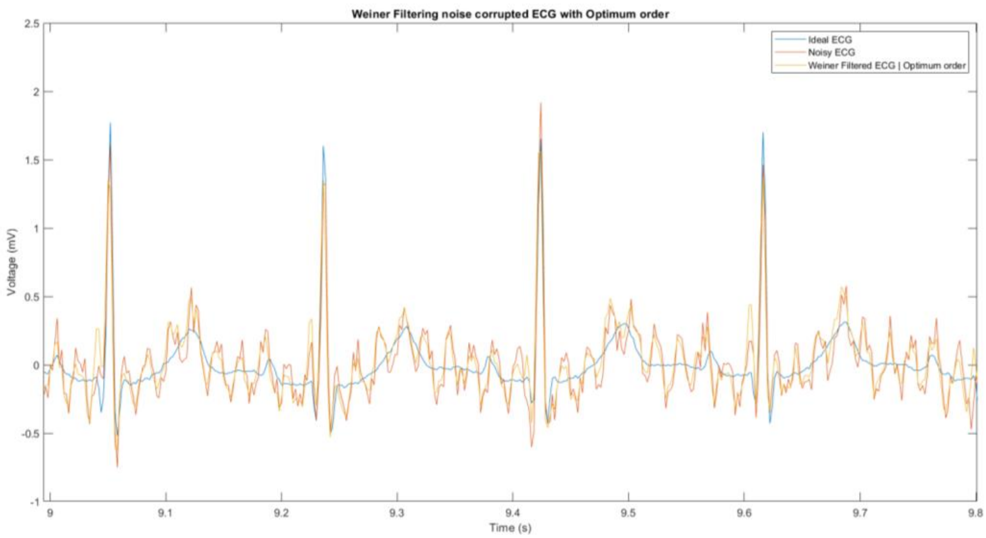

Figure 10: Weiner Filtered signal (zoomed) | Optimum Order = 7

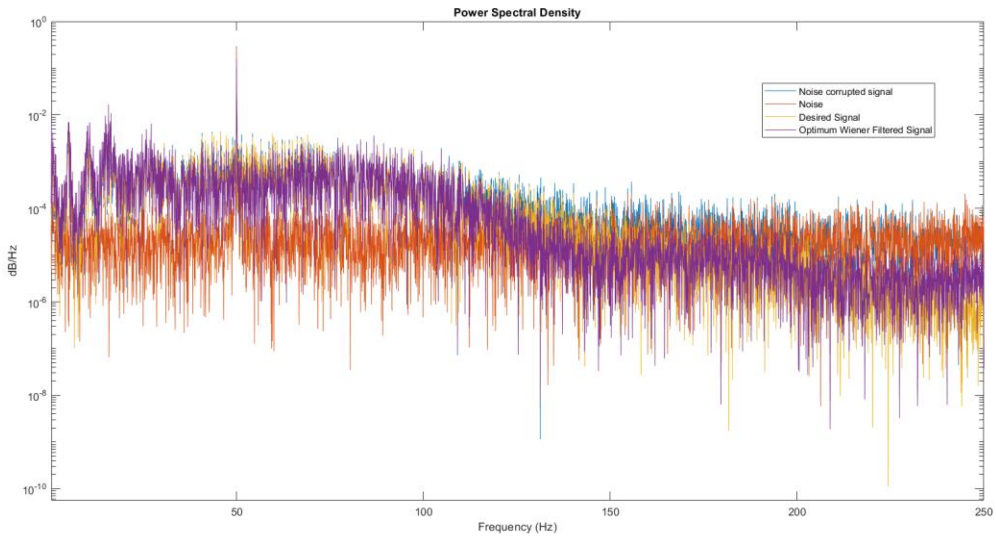

Figure 11: Power Spectral Density

Looking into the Figure 11, we can see that the optimum order Weiner Filter is able to remove the high frequency but couldn’t remove the low frequency noise. 50 Hz high spike shows that the filter was unable to remove the added sinusoidal noise.

Part 2 – Using a linear model of a pulse.

In practical scenarios, it’s hard to find ideal ECG pulse signals. Because of that we have a linear modeled ECG pulse to train and obtain the optimum filter order.

Created linear model of the ECG pulse can be seen in the Figure 12.

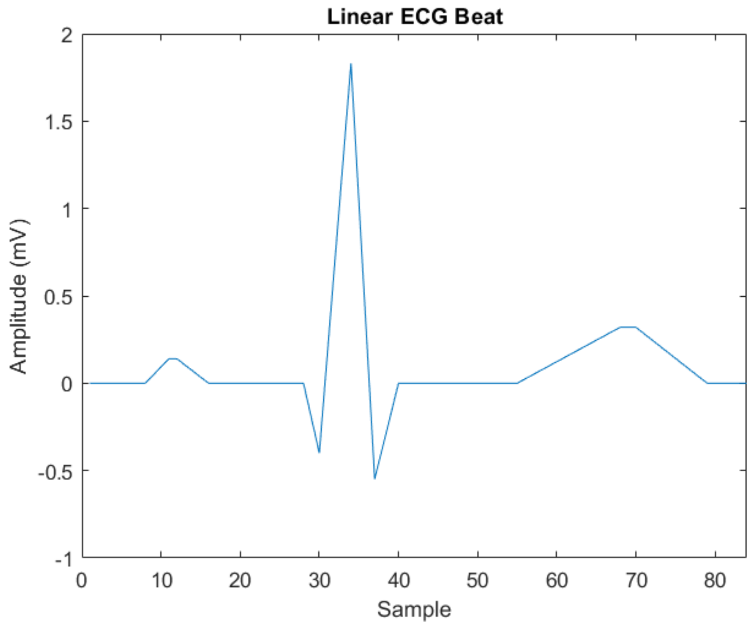

Figure 12: Linear model of the ECG pulse

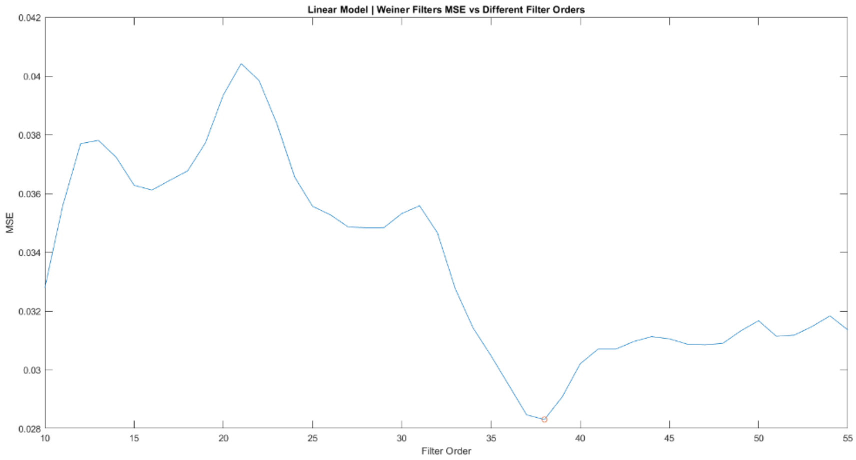

Figure 13: Weiner Filter MSE vs Filter Order | Linear Model

Obtained optimum filter order for the linear model = 38

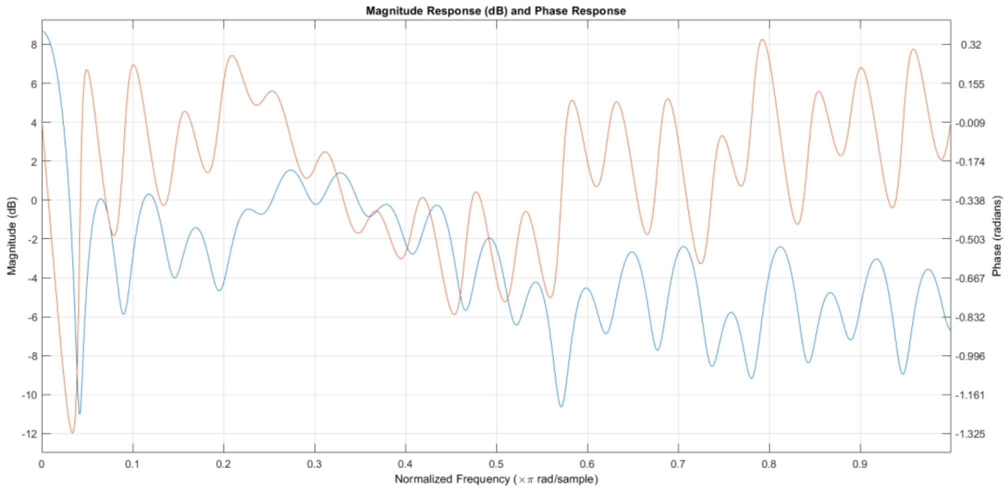

Figure 14: Magnitude (Blue) and Phase (Orange) Response of the Optimum Order Filter | Linear Model

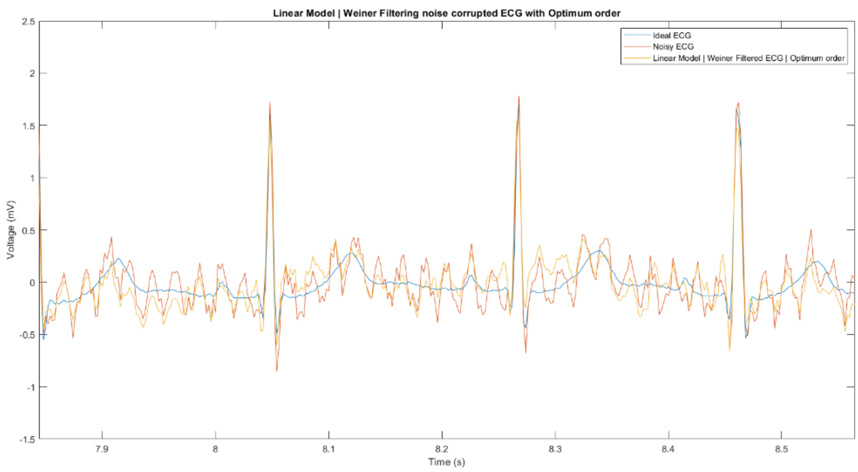

Figure 15: Linear Model | Weiner Filtered signal (zoomed) | Optimum Order = 38

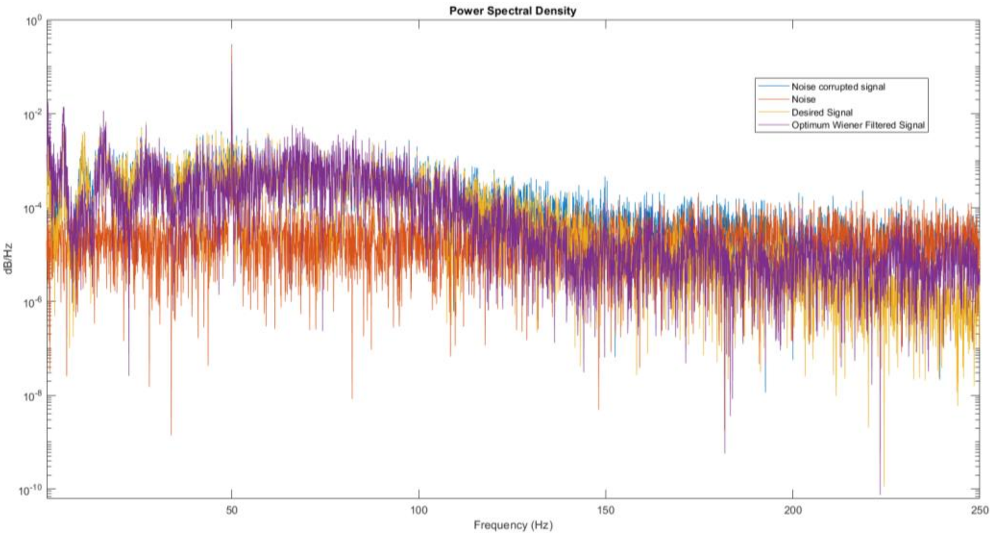

Figure 16: Linear Model | Power Spectral Density

When comparing the Figure 11 and Figure 16, we can see that the high frequency removal is low with the Linear Model of the ECG pulse than the extracted ideal ECG pulse.

1.2 Frequency domain implementation of the Wiener filter

Following equation is used to find the filter coefficients in the frequency domain implementation of the Weiner Filter.

$$W(f) = \frac{S_{YY}(f)}{S_{YY}(f) + S_{NN}(f)}$$

In here we need to feed the ideal signal and the noise to obtain the weight matrix, $S_{YY}(f)$ is the power spectral density of the signal y(n) which is equivalent to the squared value of the absolute value of the Fourier Transform of signal y(n).

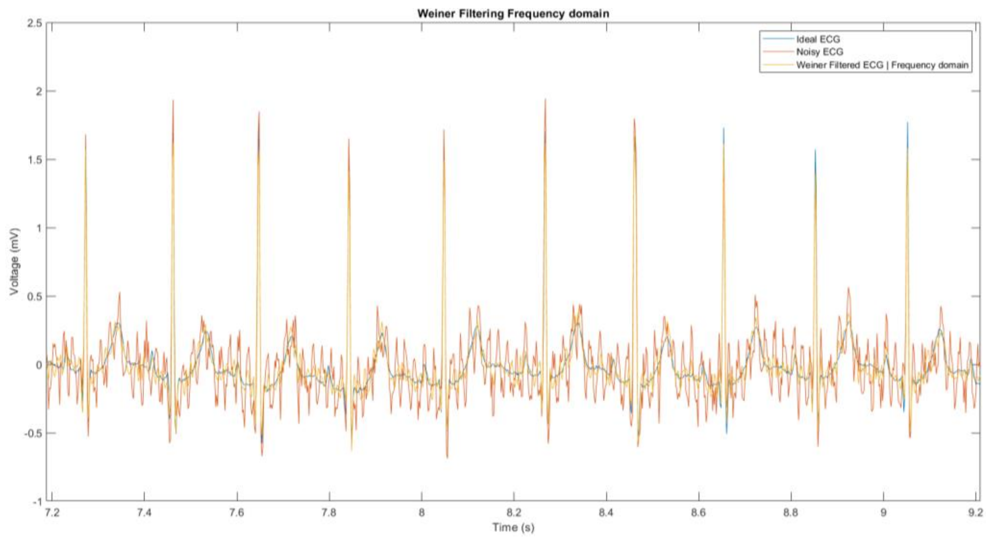

Figure 17: Frequency domain Weiner Filtered Signal (zoomed)

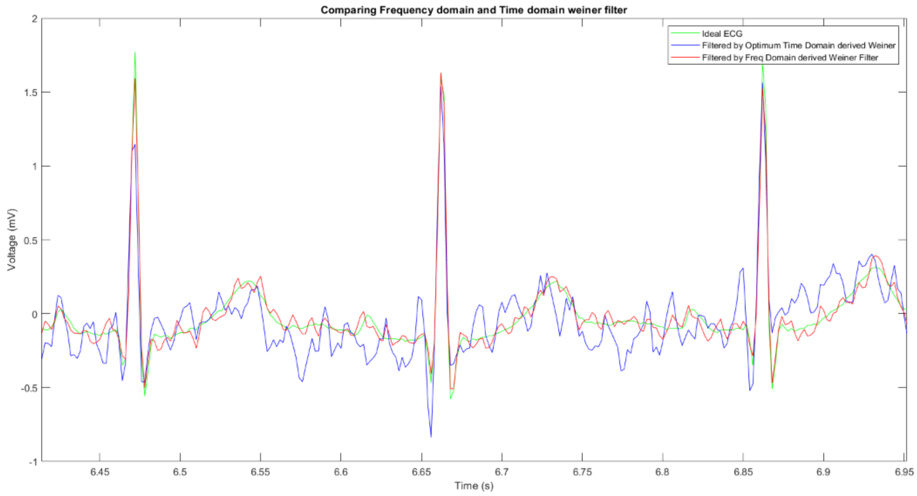

Figure 18: Comparison between the time domain and frequency domain implementation of Weiner Filter

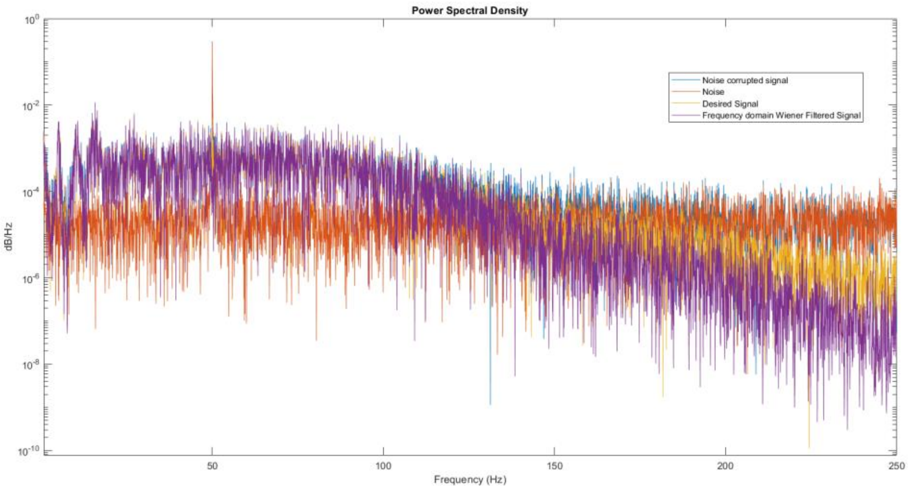

Figure 19: PSD of the frequency domain implementation of Weiner Filter

When comparing the PSDs we can see that the Frequency domain implementation Weiner Filter is able to remove both high and low frequency noise from the noisy signal. In Figure 19 and Figure 18 we can see that the PSD and the signals are very similar to the desired ECG signal. Also, the frequency domain implementation of Weiner Filter has less computations compared to the other one.

1.3 Effect on non-stationary noise on the Wiener filtering

In this section nonstationary noise is added to signal. 50 Hz sinusoidal noise to the first half of the signal and 100 Hz sinusoidal noise to the other part of the signal.

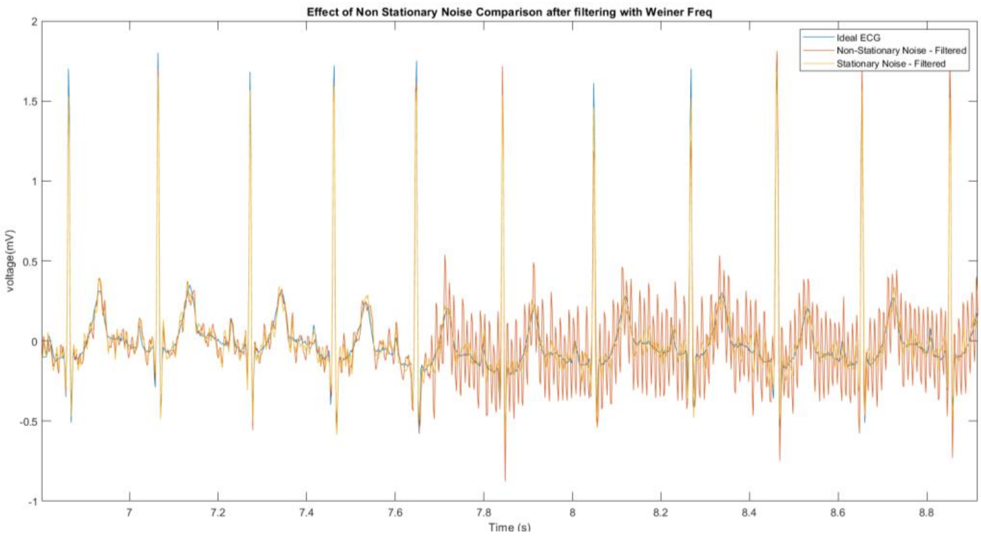

Figure 20: Frequency domain Weiner Filtered nonstationary noise added signal

Though the filter could suppress the low frequency noise, it was unable to remove the added high frequency noise.

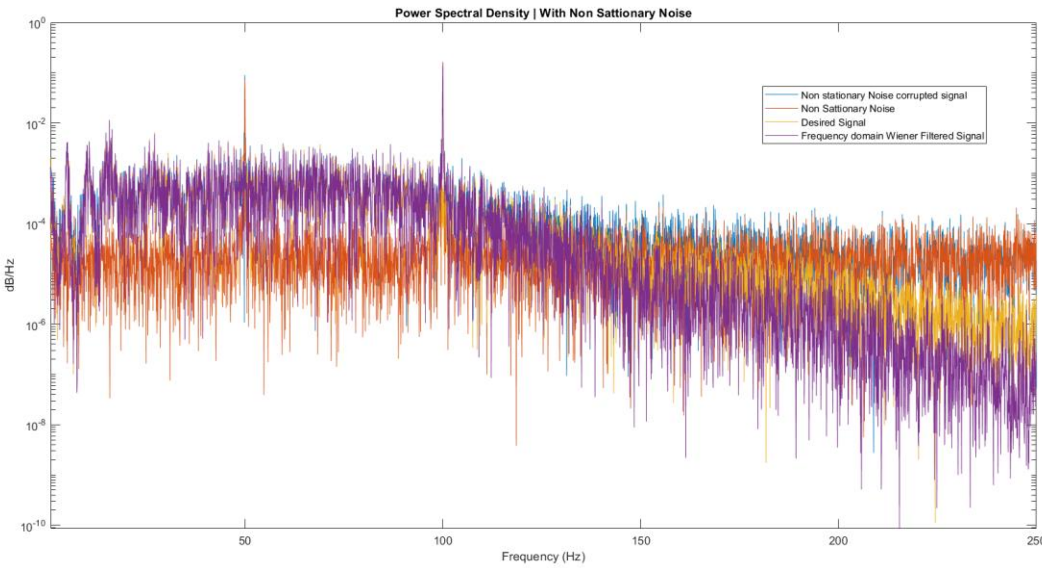

Figure 21: PSD of the Filtered nonstationary noisy signal

According to the Figure 21, we can prove the above statement that the filtered signal couldn’t remove the 100 Hz noise as we can it as a clear spike though it removed the 50 Hz noise.

2. Adaptive Filtering

Adaptive filters can adapt to changes in the noise with time and is able to give the optimum filter. Since Weiner-Hopf equation takes considerable time to obtain the inverse of the matrix, Adaptive filters use two different types of algorithms to find the optimum filter. Those algorithms are Least Mean Square (LMS) algorithm and the Recursive Least Square (RLS) algorithm.

For the experiment the algorithms are tested with a created sawtooth wave with 0.5 width. The sawtooth signal is added with gaussian white noise and sinusoidal noise to obtain the noisy signal.

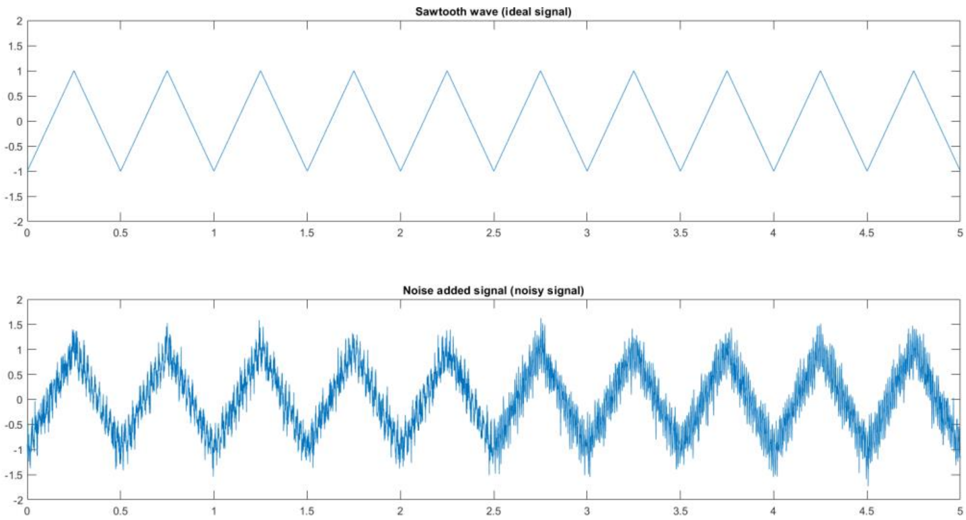

Figure 22: Raw & Noisy sawtooth signal

2.1 Least Mean Square Algorithm

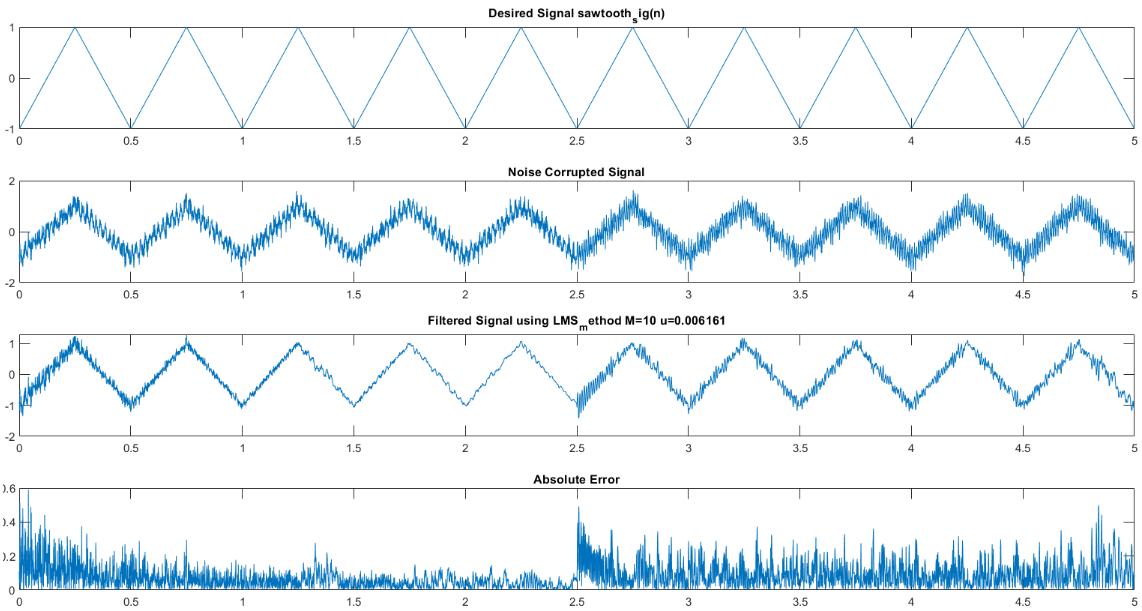

Figure 23: Adaptive Filter using LMS

Used arbitrary constants are,

$$a = 1.611, \ \Phi_1 = \frac{1}{6} \quad , \ \Phi_2 = \frac{1}{2}$$

For a given $r(n)$, different optimum filters can be obtained by changing the convergence factor μ, the filter order M and the MSE values. MSE value is calculated between the filtered signal and the desired signal.

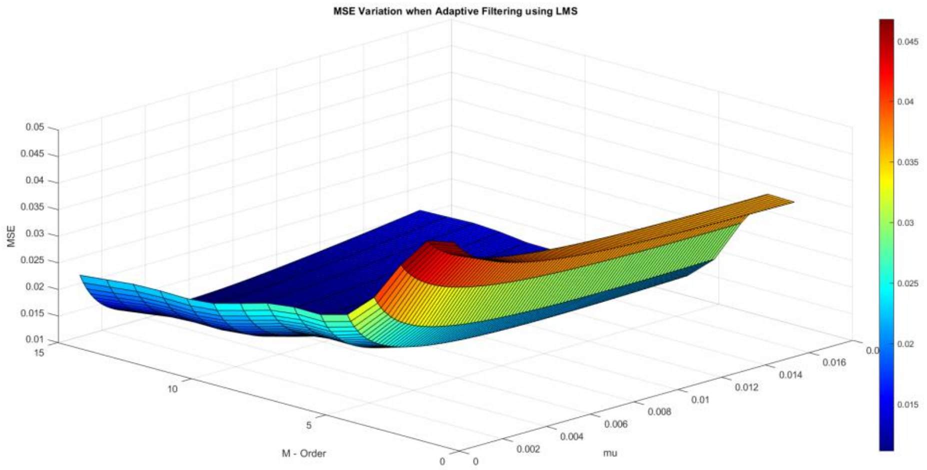

Figure 24: MSE variation w.r.t. Filter order and mu | LMS

Obtained, **Minimum MSE = 0.010678** is with Order = 15 $\mu = 0.0064686$

For very small values of μ the filter takes considerable time to converge with large errors. For very large values of μ, the filter cannot make fine adjustments and may not be able to converge, therefore the error increases. The error decreases when increasing order up to the optimum value and then increases again.

2.2 Recursive Least Square Algorithm

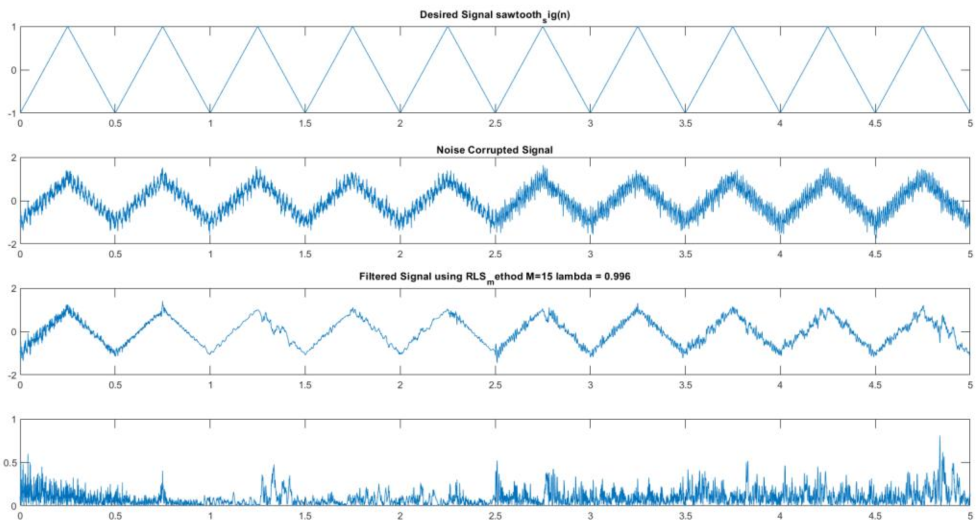

Figure 25: Adaptive Filter using RLS

In this repetitive process, initial values for matrices should be provided.

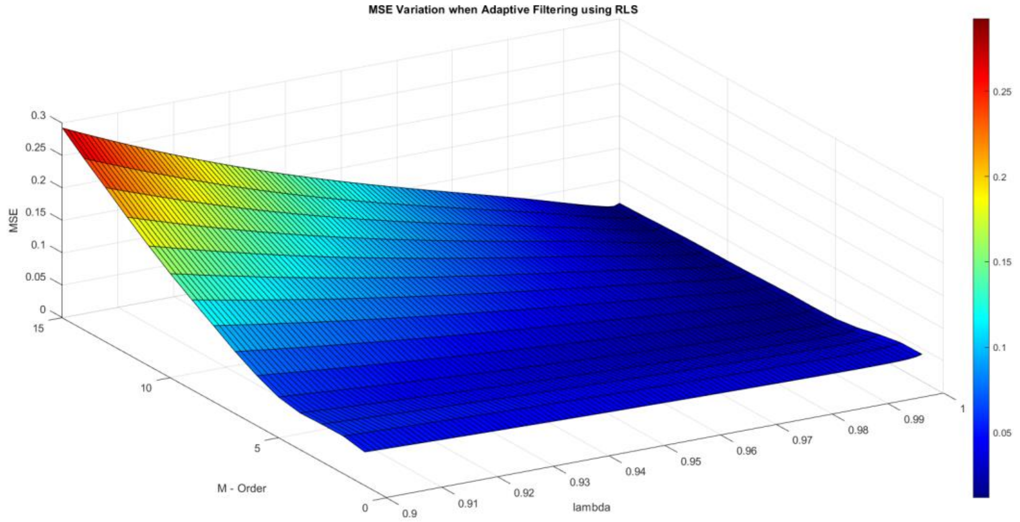

Figure 26: MSE variation w.r.t. Filter order and mu | RLS

Obtained, **Minimum MSE = 0.011856** is with Order = 10 $\lambda = 0.9098$

The value of λ should be near to 1 so that contribution from previous repetitions is considerable. As in Figure 26, for large filter orders, the values of λ should be closer to 1 to reduce the error from previous samples.

Comparing the performance of LMS and RLS Algorithms

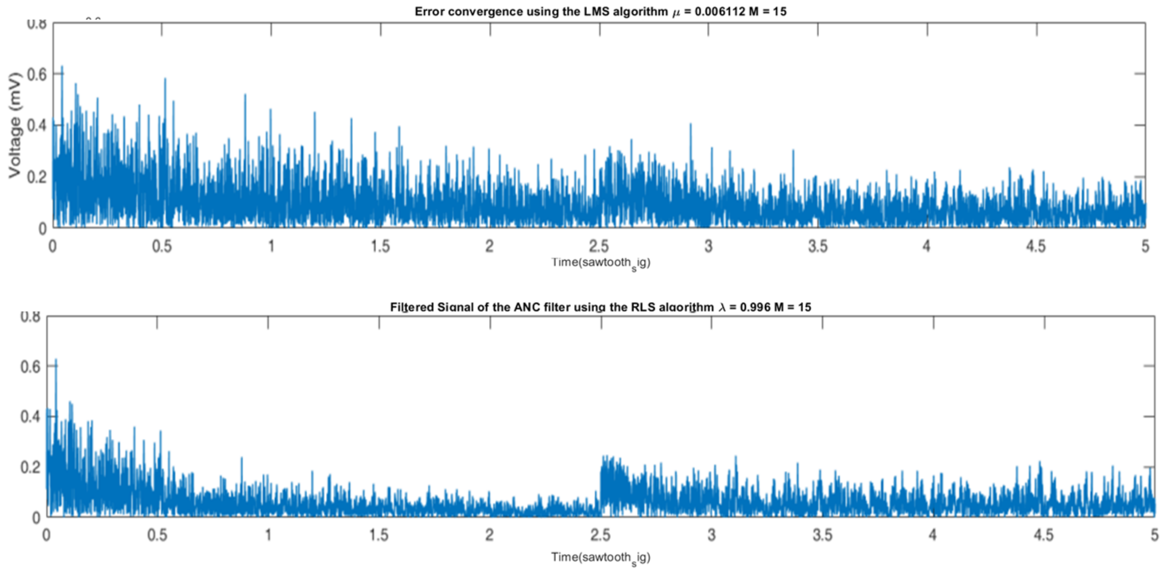

Figure 27: Performance comparison of LMS and RLS

By looking into the Figure 27 absolute error plots, we can see that the RLS algorithm converges faster than the LMS algorithm.

Filtering ECG wave with non-stationary noise using adaptive filters.

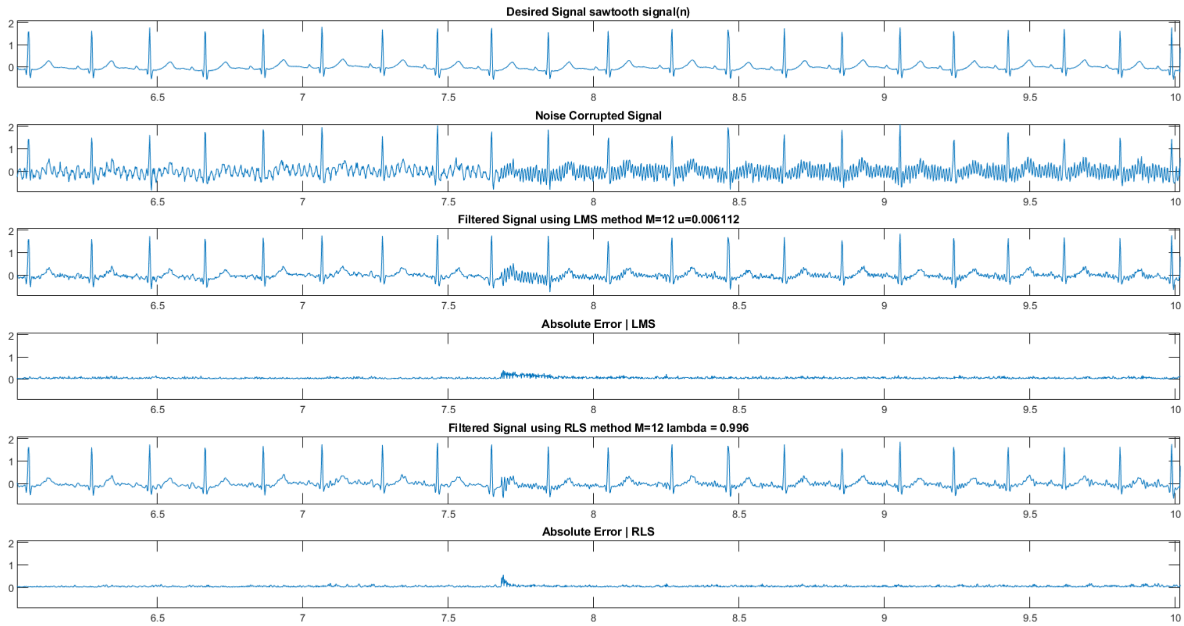

Figure 28: ECG signal filtering using LMS and RLS algorithm in Adaptive filters

By looking into the Figure 28, we can see that both algorithms were able to remove the low frequency components but, the LMS algorithm couldn’t suppress high frequency components compared to the RLS algorithm.