Equity Option Greeks and their dynamics

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1 Introduction

The greeks are sensitivities of the option value to different underlying variables, such as the underlying asset value, sensitivity to volatility etc. Here, we will go through each of the Greeks and analyse their

dynamics - how they change with respect to other variables and why they take certain values.

1.1 Delta

The simplest sensitivity is the Delta (Δ) - the change in the option value with respect to the underlying. In the case of equity options this underlying would be the stock, in caplets it would be the forward rate. In the case of swaptions, this underlying would be the swap rate. An easy way to calculate this

Delta would be to take the first derivative of the Black-Scholes-Merton formula w.r.t the underlying value:

$$\frac{\delta V_C}{\delta S} = e^{-qT}N(d_1) \tag{1}$$

$$\frac{\delta V_P}{\delta S} = -e^{-qT}N(-d_1) \tag{2}$$

Simply put, the Delta shows us how the option value will vary for different values of the underlying, and takes on a different sign for calls and puts. For calls: $0 < \Delta_C < 1$, and for puts: $-1 < \Delta_P < 0$

Traders can use the Delta to decide how many shares to use to hedge the option against small movements in the underlying. A Delta hedged portfolio looks like the following:

$$\Pi = V - \Delta S \tag{3}$$

Given that a Call has a positive delta, we need to hedge the option by shorting the shares, whereas since a put has a negative delta, the above equation switches signs, meaning we need to buy shares in order to remain delta hedged.

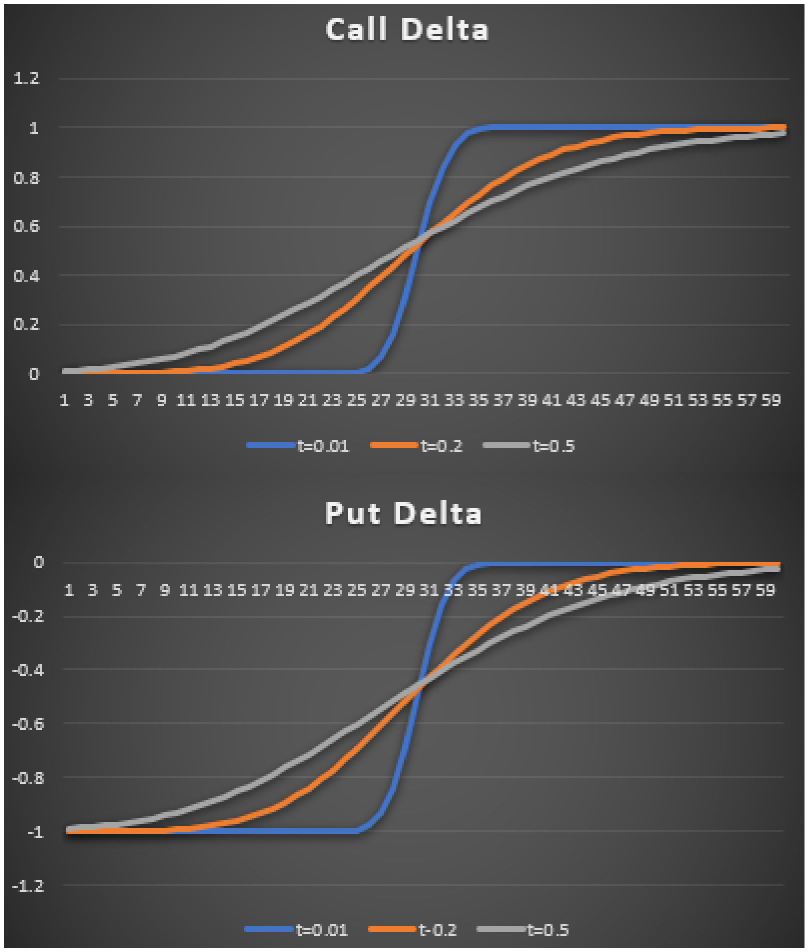


Figure 1: Call vs Put Deltas for Option with Strike K = 30

We can see that as the option expiry gets closer, the option value becomes very sensitive when ITM, as any changes in the stock price will change the intrinsic value of the option, whereas with longer dated options, the option price is less sensitive in comparison until you get deeper ITM.

1.2 Gamma

What Figure 1 shows is that the Delta does not remain constant over the range of stock prices, but changes. The rate of change of delta is known as the Gamma (convexity in the case of interest rate derivatives). This can be shown as the second derivative of the option price w.r.t the underlying i.e. the derivative of the delta w.r.t the underlying:

$$\frac{\delta \Delta}{\delta S} = \frac{\delta^2 V}{\delta S^2} = \phi(d_1)e^{-qT}\frac{\delta d_1}{\delta S} = \frac{\phi(d_1)e^{-qT}}{S\sigma\sqrt{T}} \tag{4}$$

The Gamma is positive for both Calls and Puts. From Figure 1, we can see that the highest rate of change of the Delta is when the option is ATM, with a decreasing Gamma as we move further ITM/OTM. We can show the Gamma for the above option:

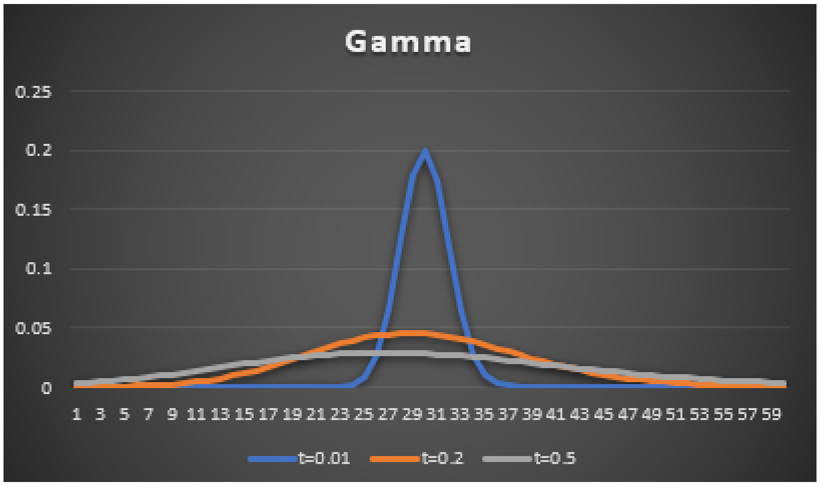


Figure 2: Gamma for Option with Strike K = 30

As we saw in Figure 1, the Delta changes far more for smaller changes in the stock price when the time to maturity is low, and hence we observe a much higher Gamma with lower maturity. This is because smaller price changes will heavily affect the probability of the option expiring ITM, as for close to expiry options, the delta is close to equal to this probability. Gamma becomes quite important

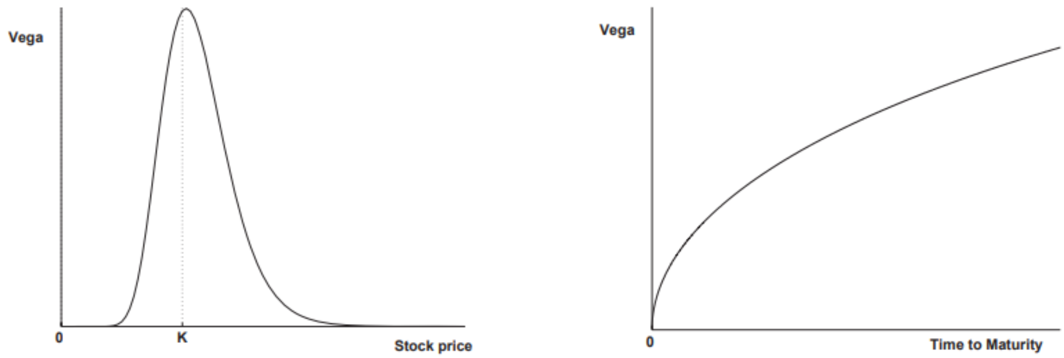
when looking to delta hedge positions because a higher gamma means smaller changes will throw off the delta hedge. Traders can utilise the gamma to understand how often they will need to re-balance the portfolio to remain hedged, and can aim to gamma hedge their portfolios as well.

1.3 Vega

Vega is the sensitivity of the option price to the volatility of the underlying. Mathematically, as the volatility increases, the price of the option increases. Volatility does not determine the probability of being ITM/OTM, because you could be equally likely to end up ITM as you could OTM with higher volatility. Rather, it increases the kurtosis of the bell curve associated with the probability, i.e. if you were to end up ITM, you could take on more extreme values as compared to before, thus increasing the potential payoff of the option, and so increasing its price:

$$\frac{\delta V}{\delta \sigma} = S_0 e^{-qT} \phi(d_1)\sqrt{T} = K e^{-rT} \phi(d_2)\sqrt{T} \tag{5}$$

With that, the Vega is at its highest when the option is close to ATM, as changes in volatility would result in different potential payoffs, and usually increases with time to maturity:



(a) Vega with stock price (b) Vega with Time

Figure 3: Vega of both Call and Put

1.4 Theta

Theta Θ is the sensitivity of the option price w.r.t the time left till expiry/maturity. Even if all the other variables remain unchanged, the option price will still see a change in value, and this is due to Theta. Usually, this change is negative, as time passes, the option loses its time value.

$$\frac{\delta V_C}{\delta T} = rKe^{-rT}N(d_2) \tag{6}$$

$$\frac{\delta V_P}{\delta T} = \frac{S_0\phi(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2) \tag{7}$$

The time decay, or Theta is close to 0 for deep ITM and OTM, and is very negative for ATM or close to ATM. The reason for ATM having the highest time decay is because their value comes from extrinsic, or the time value of the option. As options get closer to expiry, this time decay becomes more significant, so options that are close to ATM and are getting closer to expiry will see a higher theta than something like an ITM option. OTM options also dont tend to have as much extrinsic value as ATM options due to the fact that they are OTM, so they dont carry as much value close to expiry as ATM options do.