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## \* Basics of R software :-

R is a software for data analysis of statistical computing.

This software is used for effective data handling & output storage is possible.

It is capable of graphical display.

It is a free software.

$$\textcircled{1} \quad 2^2 + \sqrt{25} + 35$$

$$\rightarrow 2^2 + \sqrt{25} + 35 \\ = 44$$

$$\textcircled{2} \quad 2 \times 5 \times 3 + 62 \div 5 + \sqrt{49}$$

$$\rightarrow 2 \times 5 \times 3 + 62 \div 5 + \sqrt{49} \\ = 49.4$$

$$\textcircled{3} \quad \sqrt{76} + 4 \times 2 + 9 \div 5$$

$$\rightarrow \sqrt{76} + 4 \times 2 + 9 \div 5 \\ = 9.262829$$

$$\textcircled{4} \quad 42 + |-10| + 7^2 + 3 \times 9$$

$$\rightarrow 42 + \text{abs}(-10) + 7^2 + 3 \times 9 \\ = 128$$

$$x = 20 ; y = 30$$

$$\text{Find, } x+y ; x^2+y^2 ; \sqrt{y^3-x^3} ; \text{abs}(x-y)$$

$$\begin{aligned} x+y &= 50 \\ x^2+y^2 &= 1300 \end{aligned}$$

$$\begin{aligned} \sqrt{y^3-x^3} &= 137.8405 \\ \text{abs}(x-y) &= 10 \end{aligned}$$

calculate the following:

$$c(2,3,4,5)^2$$

$$4 \quad 9 \quad 16 \quad 25$$

$$c(4,5,6,8)*3$$

$$12 \quad 15 \quad 18 \quad 24$$

$$c(2,3,5,7)*c(-2,-3,-5,-4)$$

$$-4 \quad -9 \quad -25 \quad -28$$

$$c(2,3,5,7)*c(8,9)$$

$$16 \quad 27 \quad 40 \quad 63$$

$$c(1,2,3,4,5,6) \wedge c(2,3)$$

$$1 \quad 8 \quad 9 \quad 64 \quad 25 \quad 216$$

Find the sum, prod, min, max of the given values

5, 8, 6, 7, 9, 10, 15, 5

$x = c(5, 8, 6, 7, 9, 10, 15, 5)$

length(x)  
= 8

max(x)  
= 15

min(x)  
= 5

sum(x)

= 65

prod(x)

= 11340000

$$\begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 8 & 0 & 0 \end{bmatrix}$$

Matrix calculation.

$\text{E} \times \text{S} + \text{X} \times \text{C} = \text{M}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$x \leftarrow \text{matrix}(\text{nrow}=4, \text{ncol}=2, \text{data}=c(1, 2, 3, 4, 5, 6, 7, 8))$

$$x = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \quad y = \begin{bmatrix} 2 & 4 & 10 \\ -2 & 8 & -11 \\ 10 & 6 & 12 \end{bmatrix}$$

Find  $xy$ ,  $x*y$ ,  $2x+3y$

(\*)  $x*y = 0$

(\*\*)  $2x+3y$

$x \leftarrow \text{matrix}(\text{nrow}=3, \text{ncol}=3, \text{data}=c(1, 2, 3, 4, 5, 6, 7, 8, 9))$

$y \leftarrow \text{matrix}(\text{nrow}=3, \text{ncol}=3, \text{data}=c(2, -2, 10, 4, 8, 6, 10, -11, 12))$

$x + y$

$$\begin{bmatrix} 3 & 8 & 17 \\ 0 & -3 & \\ 13 & 12 & 21 \end{bmatrix}$$

$x * y$

$$\begin{bmatrix} 2 & 186 & 70 \\ -4 & 40 & -88 \\ 30 & 36 & 108 \end{bmatrix}$$

$2 * x + 3 * y$

$$\begin{bmatrix} 8 & 20 & 44 \end{bmatrix}$$

7

9

10

12

14

15

16

17

18

19

2

2000

## Problems on PDF &amp; CDF

Q1) Can the following be pdf?

$$\textcircled{1} \quad f(x) = \begin{cases} 2-x & ; 1 \leq x \leq 2 \\ 0 & \text{o.w.} \end{cases}$$

$$\rightarrow \int f(x) dx = 1$$

$$= \int_1^2 (2-x) dx$$

$$= \int_1^2 2 dx - \int_1^2 x dx$$

$$= 2x \Big|_1^2 - \frac{x^2}{2} \Big|_1^2$$

$$= (4-2) - (2-0.5)$$

$$\neq 1$$

$$\text{Using, } \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\textcircled{2} \quad f(x) = \begin{cases} 3x^2 & ; 0 \leq x \leq 1 \\ 0 & \text{o.w.} \end{cases}$$

$$\rightarrow \int f(x) dx = 1$$

$$= \int_0^1 3x^2 dx$$

$$= \left[ \frac{3x^3}{3} \right]_0^1$$

Find;  $P(x \leq 2)$ ;  $P(2 \leq x < 4)$ ;  $P(\text{at least } 4)$ ;  $P(3 < x \leq 6)$

$$\begin{aligned} P(x \leq 2) &= P(0) + P(1) + P(2) \\ &= 0.1 + 0.1 + 0.2 \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(2 \leq x < 4) &= P(2) + P(3) \\ &= 0.2 + 0.2 \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(\text{at least } 4) &= P(4) + P(5) + P(6) \\ &= 0.1 + 0.2 + 0.1 \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(3 < x \leq 6) &= P(4) + P(5) \\ &= 0.1 + 0.2 \\ &= 0.3 \end{aligned}$$

Q.4]	1]	x	0	1	2	3	4	5	6
		$P(x)$	0.1	0.1	0.2	0.2	0.1	0.2	0.1

$$\begin{aligned} F(x) &= 0 && \text{if } x < 0 \\ &= 0.1 && \text{if } 0 \leq x < 1 \\ &= 0.2 && \text{if } 1 \leq x < 2 \end{aligned}$$

3

(2)	$x$	10	12	14	16	18
	$P(x)$	0.2	0.35	0.15	0.25	0.1
	$P(x) = 0.2$	$10 \leq x < 12$	$P(x) = 0.35$	$12 \leq x < 14$	$P(x) = 0.15$	$14 \leq x < 16$
	$= 0.55$		$10 \leq x < 12$	$12 \leq x < 14$	$14 \leq x < 16$	
	$= 0.70$			$12 \leq x < 14$		
	$= 0.90$			$14 \leq x < 16$		
	$= 1.0$			$x \geq 18$		

## Probability Distribution &amp; Binomial Distribution

Find the PDF of the following CDF and draw the graph.

$x$	10	20	30	40	50
$p(x)$	0.15	0.25	0.3	0.2	0.1

$$\rightarrow p(x) = \begin{cases} 0 & \text{if } x < 10 \\ 0.15 & 10 \leq x < 20 \\ 0.40 & 20 \leq x < 30 \\ 0.70 & 30 \leq x < 40 \\ 0.90 & 40 \leq x < 50 \\ 1.0 & x \geq 50 \end{cases}$$

\* Commands \*

$x = c(10, 20, 30, 40, 50)$

$prob = c(0.15, 0.25, 0.3, 0.2, 0.1)$

$cumsum(prob)$

[1] 0.15 0.40 0.70 0.90 1.00

$plot(x, cumsum(prob), xlab = "values", ylab = "probability",$   
 $main = "graph of CDF", "s")$

From the graph it is clear that

### BINOMIAL DISTRIBUTION

- 12] Suppose there are 12 MCQ's in a test, each question has 5 options & only one of them is correct. Find a probability of having
- give correct answers.
  - At most 4 correct ans.

$\rightarrow$  It is given that  $n=12$ ,  $p=\frac{1}{5}$ ,  $q=1-p=\frac{4}{5}$   
 $x = \text{total no. of correct answers}$   
 $x \sim B(n, p)$

- 5 correct answers.

$\rightarrow$   $n=12$ ,  $p=\frac{1}{5}$ ,  $q=\frac{4}{5}$ ;  $x=5$   
 $\text{dbinom}(5, 12, 1/5) = 0.0931$   
 $[1] 0.0931$

- At most 4 correct answers.

$\rightarrow$   $n=12$ ,  $p=\frac{1}{5}$ ,  $q=\frac{4}{5}$ ;  $x=4$   
 $\text{dbinom}(4, 12, 1/5) = 0.9274$   
 $[1] 0.9274$

- 13] There are 10 members in a committee, the probability of any member attending a meeting is 0.9, find the probability

- 7 members attended
- At least 5 members attended
- At most 6 members attended

$\rightarrow$  It is given that  $n=10$ ,  $p=0.9$ ,  $q=0.1$   
 $x = \text{total no. of members attended}$   
 $x \sim B(n, p)$

- = members attended

$\rightarrow$   $n=10$ ,  $p=0.9$ ,  $q=0.1$ ;  $x=7$   
 $\text{dbinom}(7, 10, 0.9) = 0.0573$   
 $[1] 0.0573$

- At least 5 members attended

$\rightarrow$   $n=10$ ,  $p=0.9$ ,  $q=0.1$ ;  $x=5$   
 $1 - \text{dbinom}(5, 10, 0.9) = 0.9983$   
 $[1] 0.9983$

- At most 6 members attended

$\rightarrow$   $n=10$ ,  $p=0.9$ ,  $q=0.1$ ;  $x=6$   
 $\text{dbinom}(6, 10, 0.9) = 0.01279$   
 $[1] 0.01279$

\* CDF \*

14] Find the CDF by Draw the graph.

x	0	1	2	3	4	5	6
$P(x)$	0.1	0.1	0.2	0.2	0.1	0.2	0.1
$\rightarrow P(x) =$	0	if $x < 0$					
	= 0.1	$0 \leq x \leq 1$					
	= 0.2	$1 \leq x \leq 2$					
	= 0.4	$2 \leq x \leq 3$					
	= 0.6	$3 \leq x \leq 4$					
	= 0.7	$4 \leq x \leq 5$					
	= 0.9	$5 \leq x \leq 6$					

10 - MARKET

COMMANDS \*

\* ~~NON-HABITUAL COMMAND~~

$x = c(0, 1, 2, 3, 4, 5, 6)$

$\text{prob} = c(0.1, 0.1, 0.2, 0.2, 0.1, 0.2, 0.1)$

$\text{cumsum}(\text{prob})$

[1] 0.1 0.2 0.4 0.6 0.7 0.9 1.0

$\text{plot}(x, \text{cumsum}(\text{prob}), \text{xlab} = \text{"values"}, \text{ylab} = \text{"probability"}, \text{main} = \text{"graph of CDF", "s"})$

(1, 2) showing  $\text{graph of CDF}$

~~graph of CDF~~  
~~(1, 2)~~

10-9: 10-11

10-10: 10-11

10-11: 10-12

10-12: 10-13

10-13: 10-14

10-14: 10-15

10-15: 10-16

10-16: 10-17

10-17: 10-18

10-18: 10-19

10-19: 10-20

10-20: 10-21

10-21: 10-22

## PRACTICAL - 04

## 11. BINOMIAL DISTRIBUTION

- ① Find the complete binomial distributions when  
 $n=5$ ,  $p=0.1$

Solution :-

Note :-

①  $P(X=\infty) = \text{dbinom}(x, n, P)$

$$n=5, p=0.1$$

dbinom(0:5, 5, 0.1)

$$= 0.59049 \quad 0.32805 \quad 0.7230 \quad 0.00810 \quad 0.00045 \quad 0.00001$$

- $$\begin{aligned} \textcircled{B} & \text{ Find the probability of exactly 10 successes in 100 trials with } p=0.1 \\ \rightarrow & n = 100, \quad p = 0.1, \quad x = 10 \\ & \text{ binom } (10, 100, 0.1) \\ = & 0.1318653 \end{aligned}$$

- ⑧ x follows binomial distribution with  $n=12$ ,  $p=0.2$

Given

  - ①  $P(x=s)$
  - ②  $P(x \leq s)$
  - ③  $P(x > 7)$
  - ④  $P(5 < x < 7)$

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By Ashay

solutions :-

$$\begin{aligned}
 n &= 12, P = 0.25 \\
 n &= 12, P = 0.25, \lambda = 5 \\
 \rightarrow & \text{dbinom}(12, 5, 0.25) \\
 &= 0.103244
 \end{aligned}$$

$$\text{② } u=12, p=0.25, x \leq 5$$

Binomial

$$P(X \leq 5) = \sum_{k=0}^{12} \binom{12}{k} (0.25)^k (0.75)^{12-k}$$

$$\text{Pois}(12, 0.25) = \frac{e^{-12} \cdot 12^{12}}{12!} = 0.084$$

$$\begin{aligned}
 & P(X \geq 5) = 1 - P(X \leq 4) \\
 & = 1 - \sum_{x=0}^{4} \binom{12}{x} (0.25)^x (0.75)^{12-x} \\
 & = 1 - \left( \binom{12}{0} (0.25)^0 (0.75)^{12} + \binom{12}{1} (0.25)^1 (0.75)^{11} + \dots + \binom{12}{4} (0.25)^4 (0.75)^8 \right) \\
 & = 1 - (0.0001 + 0.0012 + \dots + 0.0414) \\
 & = 1 - 0.0414 = 0.9586
 \end{aligned}$$

NOTE :-

$$\begin{aligned} \textcircled{1} \quad P(X > 7) &= 1 - P(X \leq 7) \\ &= 1 - \text{binom}(6, 6, p) \end{aligned}$$

$$\textcircled{2} \quad P(X \geq 7) = 1 - P(X \leq 5) \quad \text{since } P(X \leq 5) = \text{probability of } X \text{ being less than or equal to 5}$$

The probability of salesman makes a sale to customer

is 0.15; find the probability:  
1) no sale for 10 customers. (b) (iii)

87 More than 3 sale in 20 customer

27 More than 3 live in 20 houses

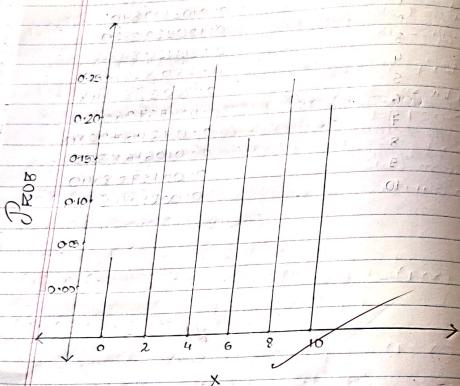
x-values

## Probability

0	0.0060466176
1	0.0403107840
2	0.1209323520
3	0.2149908480
4	0.2508226560
5	0.2006581248
6	0.1114767360
7	0.0424673280
8	0.0106168320
9	0.0015728640
10	0.0001048576

A1

58  
plot(x, prob, "u")  
plot(x, cumprob, "s")



### PRACTICAL 5 Normal Distribution

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- i)  $P[X=x] = \text{dnorm}(x, \mu, \sigma)$
- ii)  $P[X \leq x] = \text{pnorm}(x, \mu, \sigma)$
- iii)  $P[X \geq x] = 1 - \text{pnorm}(x, \mu, \sigma)$
- iv)  $P[X_1, X_2, \dots, X_n] = \text{pnorm}(x_1, \mu_1, \sigma_1) \dots \text{pnorm}(x_n, \mu_n, \sigma_n)$
- v) go find the value of  $K$  so that  $P[X \leq K] = 0.99$   
 $(P(0.99, \mu, \sigma))$
- vi) go generate  $n$  random numbers:  
 $(\mu, \sigma)$  (mean, standard deviation)  
 $[n = 50, \sigma^2 = 100]$
- vii) i)  $P[X \leq 40]$   
ii)  $P[42 \leq X \leq 60]$   
iii)  $P[X > 55]$   
iv)  $P[X \leq 12] = 0.7$ ;  $K = 8$
- xNN  $(\mu = 100, \sigma^2 = 36) \Rightarrow \sigma = 6$   $[z = 1.96] \approx 0.97$
- viii) i)  $P[X \leq 110] = 0.92$   
ii)  $P[X \leq 95]$  (Continuous from minus infinity to 95)  
iii)  $P[X > 115] = 0.05$  ( $z = 1.96$ )  
iv)  $P[95 \leq X \leq 105] = 0.4$  ( $z = 1.77$ )
- v)  $P[X \leq K] = 0.4$ ;  $K = 9$

$P(X \leq 110)$

- >  $a = \text{pnorm}(110, 100, 6)$
- >  $\text{cat}("P(X \leq 110) \text{ is : } ", a)$
- >  $P(X \leq 110) \text{ is : } 0.9522096$

$P(X \leq 95)$

- >  $b = \text{pnorm}(95, 100, 6)$
- >  $\text{cat}("P(X \leq 95) \text{ is : } ", b)$
- >  $P(X \leq 95) \text{ is : } 0.2023284$

$P(X > 115)$

- >  $c = 1 - \text{pnorm}(115, 100, 6)$
- >  $\text{cat}("P(X > 115) \text{ is : } ", c)$
- >  $P(X > 115) \text{ is : } 0.006209665$

$P(95 \leq X \leq 105)$

- >  $d = \text{pnorm}(95, 100, 6) - \text{pnorm}(105, 100, 6)$
- >  $\text{cat}("P(95 \leq X \leq 105) \text{ is : } ", d)$
- >  $P(95 \leq X \leq 105) \text{ is : } 0.5953432$

$P(X \leq 192) = 0.49$ ,  $K \text{ is } 9$

- >  $e = \text{qnorm}(0.4, 100, 6)$
- >  $\text{cat}("P(X \leq K) = 0.4, K \text{ is : } ", e)$
- >  $P(X \leq K) = 0.4, K \text{ is : } 98.47992$

3y

- >  $x = \text{mean } (10, 60, 5)$
- >  $x$
- (C) 52.4310 59.9626 63.42084 61.2825 61.9102  
51.8524 66.9836 62.5194 61.3294 62.3123
- >  $\text{ave} = \text{mean}(x)$
- >  $\text{ave}$
- (C) 60.44768
- >  $\text{median} = \text{mediana}(x)$
- >  $\text{median}$
- (C) 61.6196
- >  $n = 10$
- >  $\text{variancia} = (n-1) \times \text{var}(x) / n = 1 \Rightarrow 20.39836$
- >  $\text{variancia}$
- (C) 20.39836
- >  $sd = \sqrt{\text{variancia}}$
- >  $sd$
- (C) 4.5142

Draw the graph of standard normal distribution.

 $x = \text{sqrt}(-3:3, \text{by } \delta=0.1)$ 
 $y = \text{dnorm}(x)$ 
 $\text{plot}(x, y, \text{xlab} = "y \text{ values}", \text{ylab} = "Probability", \text{main} = \text{"standard normal graph"})$

Practical - 06

### Z-DISTRIBUTION

$H_0: \mu = 10$  against  $H_1: \mu \neq 10$ . A sample of size 400 is selected which give a mean 10.2 ad a standard deviation 2.25. Test the hypothesis at 5% level of significance.

$\rightarrow \mu_0$  (mean of population) = 10

$\mu_x$  (mean of sample) = 10.2

$s_d$  (standard deviation) = 2.25

$n$  = sample size = 400

$$> z_cal = (\mu_x - \mu_0) / (s_d / \sqrt{n})$$

$$> cat("z_cal is:", z_cal)$$

$$> zcal <- 1.77$$

$$> pvalue = 2 * (1 - pnorm(abs(zcal)))$$

$$> pvalue$$

$$> 0.079$$

Thus, hypothesis accepted.

Test the hypothesis

$H_0: \mu = 75$   $H_1: \mu \neq 75$

A sample of size 100 is selected and the sample mean 80 with the standard deviation of 3. Test the hypo. at 5% level of significance.

$\rightarrow \mu_0 = 75$

$\mu_x = 80$

$s_d = 3$

$$z_cal = (\mu_x - \mu_0) / (s_d / \sqrt{n})$$

$$z_cal = 0.8025$$

$$pvalue = 2 * (1 - pnorm(abs(zcal)))$$

$$pvalue = 0.4023$$

Thus, hypothesis accepted.

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$zcal = (\mu_x - \mu_0) / (s_d / \sqrt{n})$

$$cat("zcal is:", zcal)$$

$$zcal <- 16.6667$$

$$pvalue = p + 2 * pnorm(abs(zcal))$$

$$pvalue = 0$$

Thus, hypothesis rejected.

Test the hypothesis

$H_0: \mu = 25$  against  $H_1: \mu \neq 25$

A sample of 30 is selected test the hypothesis at 5% level of significance. A sample of following 30 is selected

20	24	27	35	30	46	20	27	10	20	30	37	35	21	27	23	24	25	26	27	28	29	30	39	27	15	19	22	20	18
----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----

$x = c(20, 24, 27, 35, \dots, 19, 20, 20, 18)$

$\mu_x = \text{mean}(x)$

$\mu_x = 26.066$

$n = \text{length}(x)$

$\text{variance} = (n-1) * \text{var}(x) / n$

$s_d = \sqrt{\text{variance}}$

$\Rightarrow s_d = 3.2798$

$\mu_0 = 25$

$$z_cal = (\mu_x - \mu_0) / (s_d / \sqrt{n})$$

$$z_cal = 0.8025$$

$$pvalue = 2 * (1 - pnorm(abs(zcal)))$$

$$pvalue = 0.4023$$

Thus, hypothesis accepted.



Shot on Oppo F11 Pro  
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Experience has shown that 20% students of college smoke. A sample of 400 students reveal that out of 400 only 80 smokers. Test the hypothesis that the experience keeps the correct proportion or not.

$$\begin{aligned} > P &= 0.2 \\ Q &= 1 - P \\ P &= 50/400 \\ n &= 400 \end{aligned}$$

$z_{\text{cal}} = (p - p_0) / (\sqrt{\sigma^2 + (p_0 \cdot g)} \ln n)$

$p_{\text{value}} = 2 * (1 - \text{pnorm}(\text{abs}(z_{\text{cal}})))$

Q) Write two random samples of size 1000 & 2000 are drawn from 2 populations with the means 67.5 & 68 respectively and with the same SD of 2.5. Test the hypothesis that the mean of two populations are equal.

$H_0$ : two populations are equal.

	A	B
population	1000	2000
Mean	67.5	68
SD	2.5	2.5

$$\begin{aligned} n_1 &= 1000 \\ n_2 &= 2000 \\ \bar{x}_1 &= 67.5 \\ \bar{x}_2 &= 68 \\ S_{\bar{x}_1} &= 2.5 \\ S_{\bar{x}_2} &= 2.5 \\ S_{\bar{y}} &= \sqrt{\frac{(n_1 - 1)S_{\bar{x}_1}^2 + (n_2 - 1)S_{\bar{x}_2}^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(1000 - 1)2.5^2 + (2000 - 1)2.5^2}{1000 + 2000 - 2}} = 0.5 \end{aligned}$$

$$y = (\bar{x}_1 - \bar{x}_2) \sqrt{\frac{(n_1 - 1)S_{\bar{x}_1}^2 + (n_2 - 1)S_{\bar{x}_2}^2}{n_1 + n_2 - 2}}$$

$$y = (\bar{x}_1 - \bar{x}_2) \sqrt{\frac{(n_1 - 1)S_{\bar{x}_1}^2 + (n_2 - 1)S_{\bar{x}_2}^2}{n_1 + n_2 - 2}}$$

$$[1] -51.63978$$

$$\text{cat}("y calculated is=", y)$$

$$2 \text{ calculated is } -51.63978$$

$$\text{pvalue} = 2 * (1 - \text{pnorm}(\text{abs}(z)))$$

$$\text{pvalue} = 2 * (1 - \text{pnorm}(\text{abs}(z)))$$

$$2.417564e-07$$

$$\text{Since, pvalue} > 0.05, \text{ we accept } H_0 \text{ at } 5\% \text{ level of significance.}$$

Q) If 20% of a random sample of 400 students had defective eyesite in a school of 500 students had same defect. Is a difference of proportion is same?  $H_0$ : the proportion of the population are equal

$$n_1 = 400$$

$$n_2 = 500$$

$$p_1 = 0.2$$

$$p_2 = 0.155$$

$$p = \frac{(n_1 * p_1 + n_2 * p_2)}{(n_1 + n_2)}$$

$$[1] 0.175$$

$$q = 1 - p$$

$$q = 0.825$$

$$Z = \frac{(p_1 - p_2)}{\sqrt{p * q * ((1/n_1) + 1/n_2)}}$$

$$[1] 1.76547$$

$$\text{cat}("z calculated is=", z)$$

$$2 \text{ calculated is } 1.76547$$

$$\text{pvalue} = 2 * (1 - \text{pnorm}(\text{abs}(z)))$$

$$\text{pvalue} = 0.07748487$$

$$[1] 0.07748487$$

$$\text{Since, pvalue} > 0.05, \text{ we accept } H_0 \text{ at } 5\% \text{ level of significance.}$$

→ Four boxes of one apples in samples of 200 is collected. It is found that there are 33 bad apples in 1st sample of 20 in 5<sup>th</sup> sample. Test the hypothesis that two boxes are equivalent in terms of no. of bad apples.

→ H<sub>0</sub>: the two boxes are equivalent. H<sub>1</sub>:

$$n_1 = 200$$

$$n_2 = 200$$

$$p_1 = 33/200 = 0.165$$

$$p_2 = 20/200 = 0.100$$

$$p = (n_1 \cdot p_1 + n_2 \cdot p_2) / (n_1 + n_2)$$

$$q = 1 - p$$

$$q_1 = 0.835$$

$$q_2 = 0.895$$

$$\approx = (p_1 - p_2) / \sqrt{p_1 q_1 / n_1 + p_2 q_2 / n_2}$$

$$z = \frac{(p_1 - p_2)}{\sqrt{p_1 q_1 / n_1 + p_2 q_2 / n_2}}$$

$$z = 1.802741$$

cat(z) = 1.802741, but z = 1.802741  
 $p_{value} = 2 * (1 - pnorm(z)) = 0.0714288$

Since, pvalue > 0.05, we accept H<sub>0</sub> at 5% level of significance.

→ In a N(63.5) with SD of 2.5, in a N(60) mean weight of 60 apples in a N(63.5) with SD of 2.5. Test the hypothesis that the mean of N(63.5) is the same as the mean of N(60).

H<sub>0</sub>: means of two classes are equal.

$$u_1 = 60$$

$$u_2 = 63.5$$

$$w_x = 63.5$$

$$w_y = 60$$

$$s_{\bar{x}} = 2.5$$

$$s_{\bar{y}} = 2.5$$

$$z = (\bar{w}_x - \bar{w}_y) / \sqrt{s_{\bar{x}}^2 / n_1 + s_{\bar{y}}^2 / n_2}$$

$$p_{value} = 2 * (1 - pnorm(z)) = 0.9502$$

Given, pvalue < 0.05, we rejected the H<sub>0</sub> at 5% level of significance.

PRACTICAL-8

A Small Sample Test.

Q. D The flower stems are selected and the heights are found to be 63, 65, 68, 69, 71, 72 cm. Test the hypothesis that the mean height is 60 cm or not at 1% level of significance.

$$\rightarrow H_0: \text{Mean} = 60 \text{ cm}$$

$$> X = \{63, 65, 68, 69, 71, 72\}$$

> t-test (x)

> One Sample t-test

data:  $X = [63, 65, 68, 69, 71, 72]$   
 $t = 47.94$ , df = 6, p-value = 5.522e-09

alternative hypothesis: true mean is not equal to 60

95 percent confidence interval:

$$64.66479 - 71.62092$$

sample estimates:

mean of X

$$68.14286$$

Since, p-value is < 0.01, we reject  $H_0$  at 1% level of significance.

200 random sample was drawn from two different populations.  $H_0$ : there is no difference between the population means at 5% level of significance.

Sample 1:  $\{13, 10, 12, 11, 16, 15, 18, 17\}$

Sample 2:  $\{20, 15, 18, 19, 18, 10, 11, 12\}$

test the hypothesis that there is no difference between the population means at 5% level of significance.

$t = -0.36247$ , df = 13, p-value = 0.7225

alternative hypothesis: true difference in mean is not equal to 0

sample estimates:

mean of X - mean of Y

$$= 7.5 - 13.2719 = -5.7219$$

sample estimates:

mean of X - mean of Y

$$= 68.14286 - 64.66479 = 3.47807$$

Since, p-value is < 0.05 we reject  $H_0$  at 5% level of significance.

50

Q3: Following are the weights of 10 ppl before and after a diet program, test the hypothesis that the diet program is effective or not.

$$\text{Before (kg)} = \{100, 95, 96, 98, 112, 115, 123, 109, 110, 104\}$$

$$\text{After (kg)} = \{95, 80, 96, 98, 100, 105, 110, 85, 100, 101\}$$

Solution:  $H_0$ : the diet program is not effective.

$$H_a: \mu_d > 0$$

$$x = \{100, 123, 93, 96, 98, 102, 115, 104, 105, 110\}$$

$$y = \{95, 80, 95, 97, 90, 100, 110, 85, 100, 101\}$$

t-test (x, y, paired = T, alternative = "less")

Paired t-test

$$\text{data: } x \ y$$

$$t = -2.3213, df = 9, p\text{-value} = 0.04993$$

alternative hypothesis: true difference is not

mean is less than 0

$$mean = -17.89685$$

Sample estimates:

$$\text{mean of the differences} = -17.89685$$

p-value is < 0.01 we reject  $H_0$  at 5% level of significance.

margin of error = 10.00000

margin of error = 10.00000

The marks before & after training program are given below.

Before (20, 25, 32, 28, 27, 36, 35, 25)

After (20, 35, 32, 37, 37, 40, 40, 23)

t-test the hypothesis that training program is

effective or not.

$$x = \{20, 25, 32, 28, 27, 36, 37, 23\}$$

$$y = \{30, 35, 32, 37, 40, 40, 42, 33\}$$

t-test (x, y, paired = T, alternative = "greater")

Paired t-test

$$\text{data: } x \ y$$

$$t = -3.3859, df = 7, p\text{-value} = 0.000205$$

alternative hypothesis: true difference is mean

is greater than 0

90% confidence interval

$$-8.3939 < \text{diff} < 17.89685$$

Sample estimates:

$$\text{mean of the difference} = 7.50000$$

$$-5.3939$$

$$12.89685$$

$$for 90\%$$

$$-10.00000 < \text{diff} < 30.00000$$

$$10.00000 < \text{diff} < 20.00000$$

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$p_{\text{v}} = \text{chiqr-test}(x)$

$p_{\text{v}}$

Pearson's chi-squared test

data = y  
 $\chi^2 = 145.78$ , df = 9, p-value < 2.2e-16

Since,  $p_{\text{v}} < 0.05$  we reject  $H_0$  at 5% LOS.

(f) Perform ANOVA for the following data.

Varieties	Observations
A	50, 52, 54, 56, 58
B	53, 55, 53
C	60, 58, 57, 56
D	52, 54, 54, 55

$H_0$ : the mean of variety A,B,C,D are equal.

$x_1 = c(50, 52)$

$x_2 = c(53, 55, 53)$

$x_3 = c(60, 58, 57, 56)$

$x_4 = c(52, 54, 54, 55)$

d = stack(list(b1=x1, b2=x2, b3=x3, b4=x4))

names(d)

(i) "values" "ind"

one-way.test(values ~ ind, data=d, var.equal=TRUE)

One-way analysis of means

data: values and ind

F = 11.783, num df = 3, denom df = 9, p-value = 0.00

Shot on OnePlus

By Ashay

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anova = aov(values ~ ind, data=d)  
 summary(anova)

Df	Sum Sq	Mean Sq	F value	Pr(>F)
ind	3	71.06	23.688	11.73 0.001834
Residuals	9	18.14	2.019	

$p_{\text{v}} < 0.05$  we reject  $H_0$  at 5% LOS.

Perform ANOVA for the following data.

Types	Observations
A	6, 7, 8
B	4, 6, 5
C	8, 6, 10
D	6, 9, 9

$H_0$ : the mean of type A,B,C,D are equal

$x_1 = c(6, 7, 8)$

$x_2 = c(4, 6, 5)$

$x_3 = c(8, 6, 10)$

$x_4 = c(6, 9, 9)$

d = stack(list(b1=x1, b2=x2, b3=x3, b4=x4))

names(d)

(i) "values" "ind"

one-way.test(values ~ ind, data=d, var.equal=TRUE)

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### One-way analysis of variance

data: values and ind

$$F = 2.6667, \text{ num df} = 3, \text{ denom df} = 8, \\ p\text{ value} = 0.1189$$

anova = aov (values ~ ind, data = d)  
summary (anova)

Df	Sum Sq	Mean Sq	F value	Pr(>F)
ind	3	18	6	2.6667
Residuals	8	18	2.25	0.1189

pv > 0.05 we accept  $H_0$  at 5% loc.

Q.5

x = read.csv ("C:/Users/akshay/Desktop/MARKS.csv")

	STATS	CAL
1	40	60
2	45	48
3	42	47
4	15	20
5	37	25
6	36	27
7	49	57
8	59	58
9	20	25
10	27	27

### PRACTICAL-10

54

#### Non-Parametric test.

following are the amount of Sulphur oxide emitted by a factory:

17, 15, 20, 29, 18, 18, 22, 25, 27, 9, 24, 20, 17, 6, 24, 14, 15, 23, 24, 26

apply sign test to test the hypothesis that the population median is 21.5 against the alternative that it is less than 21.5

$H_0$  :- Population median = 21.5

$H_1$  :-  $H_0$  is less than 21.5

x = c (17, 15, 20, 29, 18, 18, 22, 25, 27, 9, 24, 20, 17, 6, 24, 14, 15, 23, 24, 26)

u = 21.5

sp = length (x > u)

sn = length (x < u)

n = sp + sn

pv = pbisnom (sp, n, 0.5)

pv

0.4119015

NOTE :- If the alternative is  $>$  median  $PV =$  pbisnom (sn, n, 0.5)

Q) For the observation 12, 19, 31, 28, 43, 40, 55, 49, 70, 63  
 18 apply sign test to test population median is 25 against the alternative > 25.

$H_0$ : The median is 25     $H_1$ : The median > 25

 $x = c(12, 19, 31, 28, 43, 40, 55, 49, 70, 63)$   
 $m = 25$   
 $sp = \text{length}(x[x > m])$   
 $sn = \text{length}(x[x < m])$   
 $n = sp + sn$   
 $p_{uv} = \text{pbinom}(sn, n, 0.5)$   
 $p_{uv}$   
 $\text{cij } 0.0546875$   
 $\text{Note:- If the alternative is < median}$   
 $p_{uv} = \text{pbinom}(sp, n, 0.5)$   
 $\text{Note:- If the alternative is > median}$   
 $p_{uv} = \text{pbinom}(sn, n, 0.5)$   
 $\text{Shot on OnePlus 6T | 6.18 GHz Octa-core (2.2 GHz Dual) | 6.43" FHD+}$   
 $\text{By Ashay}$ 

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For the following data 60, 65, 63, 83, 61, 71, 58, 51, 48, 60 test the hypothesis using wilcoxon signed rank test. For testing the hypothesis that the median is 60 against the alternative > 60.

$H_0$ : The median is 60     $H_1$ : The median > 60

$x = c(60, 65, 63, 83, 61, 71, 58, 51, 48, 60)$   
 wilcoxon test(x, alter = "greater", mu = 60)  
 wilcoxon signed rank test continuity correction  
 $v = 29$ , p value = 0.2386  
 alternative hypothesis: true location is greater than 60.

Note :- If the alternative is less than,  
 wilcoxon test(x, alter = "less", mu = 60)  
 If the alternative is not equal than  
 wilcoxon test(x, alter = "two.sided", mu = 60)

Unbiased  
Alternative is  $>$  median  $\Rightarrow$   $H_1$ : median  $< 10$

Shot on OnePlus  
By Ashay

Q2

Q2) Using Wilcoxon sign rank test, check if there is least one difference between two groups of 12 observations each. The data is given below.

$H_0$  : The median is 12.  
 $H_1$  : The median is not 12.

$$x = [12, 13, 9, 10, 15, 8, 11, 7, 6, 3, 11, 9, 10]$$

Wilcoxon test: (n = 12) →  $H_0$  is true. Wilcoxon signed rank test with confidence level 95%.

Data:  $x = [12, 13, 9, 10, 15, 8, 11, 7, 6, 3, 11, 9, 10]$   
 $H_0 \geq 25$ , p-value = 0.0581. Null hypothesis alternative hypothesis: true location is not claim 12.

Note: If the alternate in question is true, then Wilcoxon test ( $x$ ) is called "one-sided".  
Wilcoxon rank sum test is called "two-sided".