$$\mathcal{R} = (r^2+a^2-a\xi)^2 - \Delta J$$

$$\Delta = r^2-2mr+a^2$$
$$J = \eta+(a-\xi)^2$$

$$\mathcal{R} = (r^2+a^2-a\xi)^2 - (r^2-2mr+a^2)[\eta+(a-\xi)^2] = 0$$

$$m' = \frac{dm}{dr}$$

$$\partial_r R = 2(r^2+a^2-a\xi)2r - (2r-2m-2rm')[\eta+(a-\xi)^2] = 0$$

$$\partial_r R = 4r(r^2+a^2-a\xi) - 2[r-m-rm')[\eta+(a-\xi)^2] = 0$$

$$\partial_r R = 4r(r^2+a^2-a\xi) - 2[r-m(1+\tfrac{r}{m}m')[\eta+(a-\xi)^2] = 0$$

$$\partial_r R = 4r(r^2+a^2-a\xi) - 2(r-mf)[\eta+(a-\xi)^2] = 0$$

$$f = 1 + \frac{r}{m}m'$$

$$\begin{cases} R = (r^2+a^2-a\xi)^2-(r^2-2mr+a^2)[\eta+(a-\xi)^2] = 0 & (\mathrm{I}) \\ \partial_r R = 4r(r^2+a^2-a\xi)-2(r-mf)[\eta+(a-\xi)^2] = 0 & (\mathrm{II}) \end{cases}$$

$$m' = \frac{dm}{dr}$$

$$f = 1 + \frac{r}{m} m'$$

From (II) we have

$$[\eta+(a-\xi)^2] = \frac{2r(r^2+a^2-a\xi)}{(r-mf)} \qquad (\mathrm{III})$$

Replacing in (I) we obtain

$$(r^2+a^2-a\xi)^2-(r^2-2mr+a^2)\frac{2r(r^2+a^2-a\xi)}{(r-mf)} = 0$$

Here we assume that

$$r^2+a^2-a\xi \neq 0$$

$$(r^2+a^2-a\xi) = \frac{2r(r^2-2mr+a^2)}{(r-mf)}$$

$$a\xi = r^2 + a^2 - \frac{2r(r^2-2mr+a^2)}{(r-mf)}$$

$$\xi = \frac{r^2+a^2}{a} - \frac{2r(r^2-2mr+a^2)}{a(r-mf)}$$

$$\xi = \frac{(r^2+a^2)(r-mf) - 2r(r^2-2mr+a^2)}{a(r-mf)}$$

Replacing in (III) we obtain η;

$$\left[\eta + (a-\xi)^2\right] = \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -(a-\xi)^2 + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\left[a - \frac{(r^2+a^2)(r-mf) - 2r(r^2-2mr+a^2)}{a(r-mf)}\right]^2 + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\frac{\left[a^2(r-mf) - (r^2+a^2)(r-mf) + 2r\,(r^2-2mr+a^2)\right]^2}{a^2(r-mf)^2} + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\frac{\left[-r^2\,(r-mf) + 2r\,(r^2-2mr+a^2)\right]^2}{a^2(r-mf)^2} + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\frac{r^2\left[-r(r-mf) + 2\,(r^2-2mr+a^2)\right]^2}{a^2(r-mf)^2} + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\frac{r^2\left[\;r^2 + m(f-4)r + 2a^2\right]^2}{a^2(r-mf)^2} + \frac{2r(r^2+a^2-a\xi)}{(r-mf)}$$

$$\eta = -\frac{r^2\left[\;r^2 + m(f-4)r + 2a^2\right]^2}{a^2(r-mf)^2} + \frac{2r}{(r-mf)}\left[\frac{2r(r^2-2mr+a^2)}{(r-mf)}\right]$$

$$\eta = -\frac{r^2\left[\;r^2 + m(f-4)r + 2a^2\right]^2}{a^2(r-mf)^2} + \frac{4r^2(r^2-2mr+a^2)}{(r-mf)^2}$$

$$\eta = \frac{r^2\left\{4a^2(r^2-2mr+a^2) - \left[\;r^2 + m(f-4)r + 2a^2\right]^2\right\}}{a^2(r-mf)^2}$$

$$\eta = \frac{r^2\{4a^2(r^2-2mr+a^2)-[r^4+m^2(f-4)^2r^2+4a^4+2m(f-4)r^3+4a^2r^2+4ma^2(f-4)r]\}}{a^2(r-mf)^2}$$

$$\eta = \frac{r^3\{-r^3-2m(f-4)r^2+(4a^2-m^2(f-4)^2-4a^2)r-8ma^2-4ma^2(f-4)\}}{a^2(r-mf)^2}$$

$$\eta = -\frac{r^3\{r^3+2m(f-4)r^2+m^2(f-4)^2r+4ma^2(f-2)\}}{a^2(r-mf)^2}$$

$$\eta = -\frac{r^3\{r[r+m(f-4)]^2+4ma^2(f-2)\}}{a^2(r-mf)^2}$$

$$\eta = \frac{r^3\{4ma^2(2-f)-r[r-m(4-f)]^2\}}{a^2(r-mf)^2}$$

Parameters of the shadow

$$\xi = \frac{(r^2+a^2)(r-mf) - 2r(r^2-2mr+a^2)}{a(r-mf)}$$

$$\eta = \frac{r^3\{4ma^2(2-f) - r[r-(4-f)m]^2\}}{a^2(r-mf)^2}$$

$$m' = \frac{dm}{dr}$$

$$f = 1 + \frac{m'}{m} r$$