BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

09. Accretion in Binary Systems

Edward Larrañaga

Outline for Part 1

Accretion by Roche Lobe Overflow

Binary Systems

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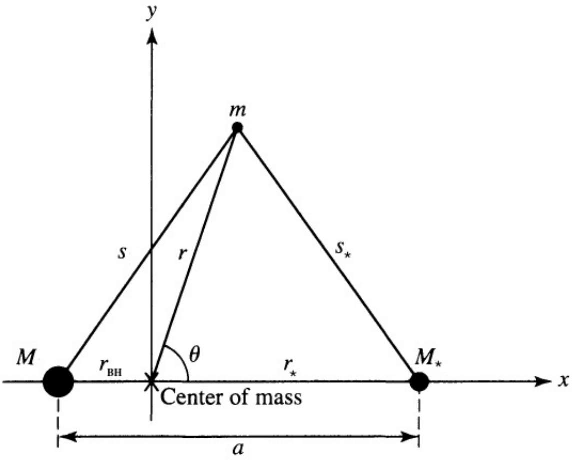
- This process may dissipate orbital and rotational energy until the system reaches the state of minimum energy for constant angular momentum: *synchronous rotation in circular orbits*

Binary System Geometry



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Center of mass:

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Angular velocity of the system:

$$\omega = \frac{v_*}{r_*} = \frac{v_{BH}}{r_{BH}}$$

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Kepler's third law

$$\omega^2 = \left(\frac{2\pi}{\tau}\right)^2 = \frac{G(M+M_*)}{a^3} = \frac{G(M+M_*)}{(r_{BH}+r_*)^3}$$

Lagrange Points

Lagrange Points:

$$\begin{cases} \frac{\partial \Phi}{\partial x} & = 0 \\ \frac{\partial \Phi}{\partial y} & = 0 \end{cases}$$

Lagrange Points

The location of the point L_1 from M_{BH} and from M_* are given by the approximations

$$b \approx a\left[0.500 - 0.227\log_{10}(q)\right]$$

$$b_* \approx a\left[0.500 + 0.227\log_{10}(q)\right]$$

B. Peterson. *An Introduction to Active galactic Nuclei*. Cambrdige University Press. (1997)
B. W. Carroll and D. A. Ostlie. *An Introduction to Modern Astrophysics*. Addison-Wesley (1996)

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- Since pressure is due to the weight of the overlaying layer of material, density has a constant value along the equipotential.

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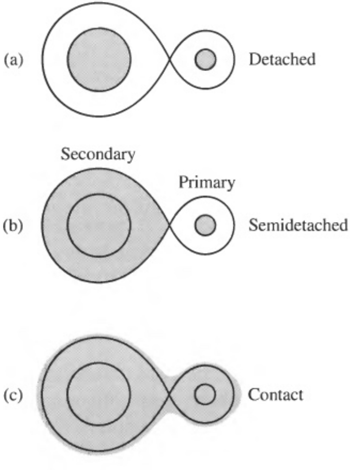

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$q = \frac{M_*}{M}$

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For $0.1 \leq q \leq 0.8$ there is the Paczynski approximation

$$\frac{R_{*R}}{a} \approx \frac{2}{3^{4/3}\left(\frac{q}{1+q}\right)^{1/3}}$$

$$\frac{R_{*R}}{a} \approx 0.462\left(\frac{M_*}{M + M_*}\right)^{1/3}$$

Self-sustainability of the Roche Overflow

$$\begin{aligned}\text{Roche overflow} &\longrightarrow \text{change } q\\ &\longrightarrow \text{change } a, \tau\\ &\longrightarrow \text{Roche lobe ?}\end{aligned}$$

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If Roche lobe shrinks $\longrightarrow$ overflow continues!

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$$a = \frac{J^2}{G} \frac{M + M_*}{M^2 M_*^2}$$

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For conservative systems: BH grows putting more mas near the CM and the star moves in a wider orbit, increasing a, in order to conserve J.

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  The Roche overflow is rapid and violent but stops when q is smaller than $\frac{5}{6}$.

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The transfer of mass continues if

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  - Tidal forces on the star
  - Wind (magnetically linked to the star)

Outline for Part 2

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$v_{||}$: parallel component of the velocity with respect to the line of centers.
$v_{\perp}$: perpendicular component of the velocity with respect to the line of centers.

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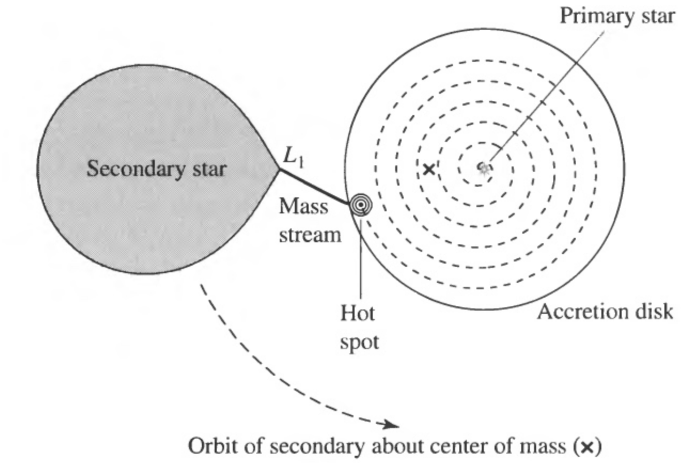

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we get

$$v_\perp \gtrsim 305.6 \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3}\, \tau_{hr}^{-1/3} \ [\mathrm{km/s}]$$

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- The particle will describe an elliptical trajectory around the BH but the presence of the star will make this ellipse to precess slowly.

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- However, angular momentum will be conserved in this collisions.
- Hence, particles will go into the trajectories with minimum energy for a given angular momentum, i.e. circular orbits!
- This process is called *circularization*.
- The radius of the resulting circular orbit is called *circularization radius*, r_{circ}.

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$$v_\phi\left(r_{circ}\right) = \sqrt{\frac{GM}{r_{circ}}}$$

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and using Kepler's third law,

$$\frac{r_{circ}}{a} = (1 + q) \left(\frac{b}{a}\right)^4$$

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Formation of the Accretion Disk

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- The redistribution of angular momentum makes particles in the outer parts move outwards (gaining angular momentum) and the particles in the inner particles spiral inwards.

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Outline for Part 3

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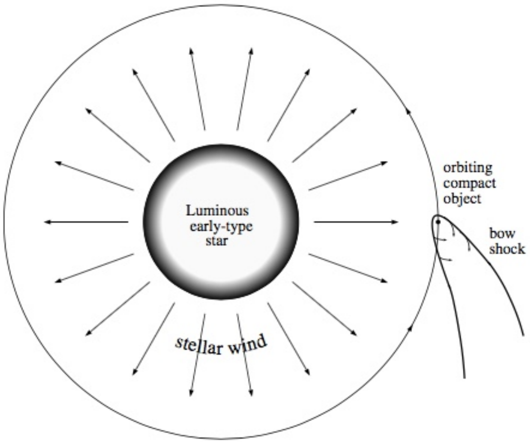
Wind Accretion



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  This implies that
  - we may neglect gas pressure
  - the flow may be considered as a collection of particles

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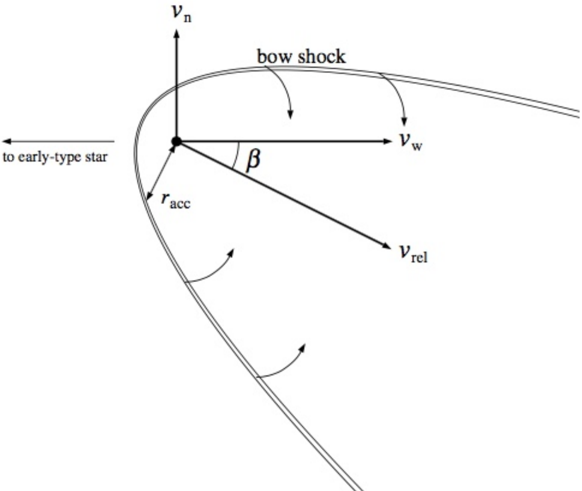

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- β: Angle in which the winds moves

$$\beta = \tan^{-1}\left(\frac{v_n}{v_w}\right)$$

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The *region of capture* is a cylinder with axis along the relative wind direction, v_{rel}, and radius

$$r_{acc} \sim \frac{2GM}{v_{rel}^2}$$

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a: Separation of the elements of the binary system.

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Remember that in the Roche lobe accretion this is $\sim b^2\omega \sim 0.500a^2\omega$. Therefore the angular momentum in the wind accretion is smaller and the chances of disk formation are smaller.

Circularization radius for the Accreting Material

The circularization radius is obtained as before

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- The value of r_{acc} used here is an approximation. The highly supersonic speed of the wind produces a strong bow shock that may change the description.

Next Lecture

10. Viscous Torques