04. Binary Systems.

Mass Transfer

MATTER TRANSFER IN BINARY SYSTEMS

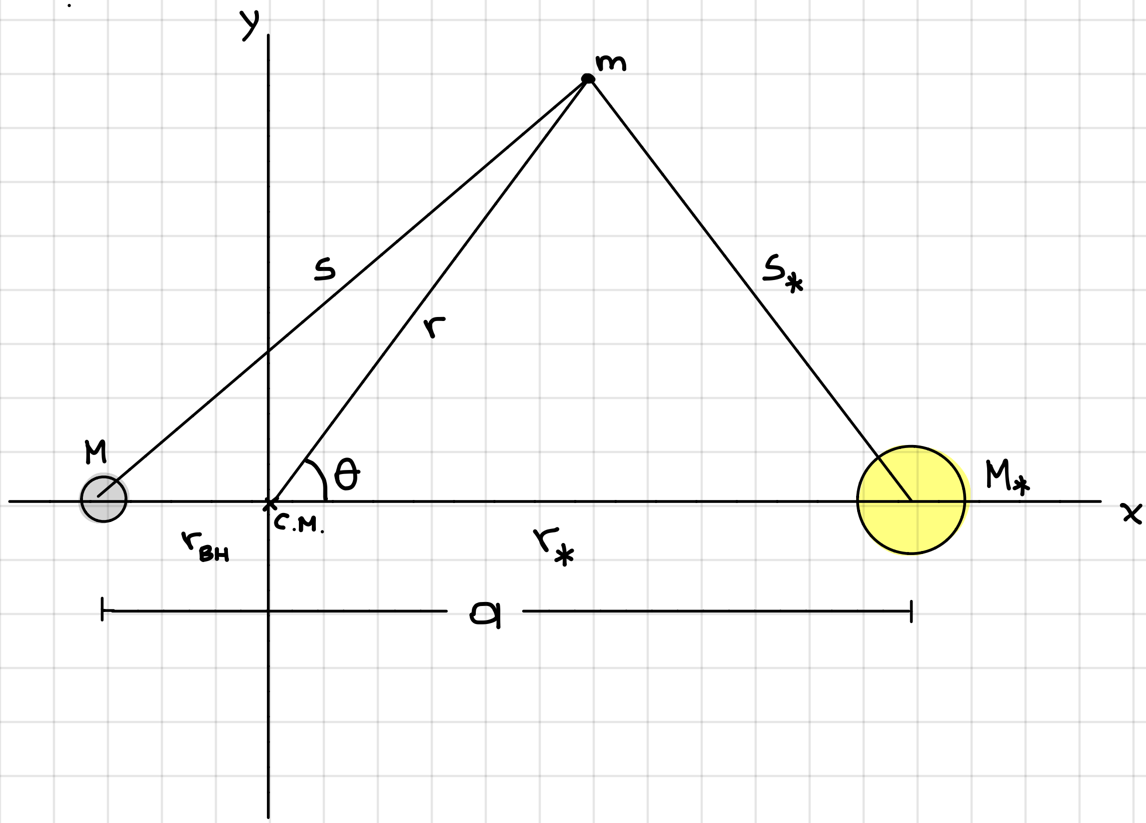
Matter transfer in binary systems can be produced by two mechanisms:

i) If the star increase its radius (stellar evolution) or if the separation between the members shrinks, some of the outer layers can be removed (Roche lobe overflow)

ii) Some mass of the star can be ejected by the stellar wind and it can be captured by the companion. (Stellar wind accretion)

Roche Lobe Overflow

For the configuration



we have that Kepler's third law is

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 = \frac{G(M+M_*)}{a^3}$$

giving

$$a^3 = GM\left(1+\frac{M_*}{M}\right)\frac{P^2}{4\pi^2}$$

$$a = \sqrt[3]{\frac{GM_\odot}{4\pi^2}}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3}P^{2/3}$$

where $q = \dfrac{M_*}{M}$.

If P is measured in seconds, we obtain a in meters as

$$a = 1.5\times10^{6} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{seg}^{2/3} \quad m.$$

or in cm;

$$a = 1.5\times10^{8} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{seg}^{2/3} \quad cm.$$

If P is measured in minutes, hours, days or years; we have

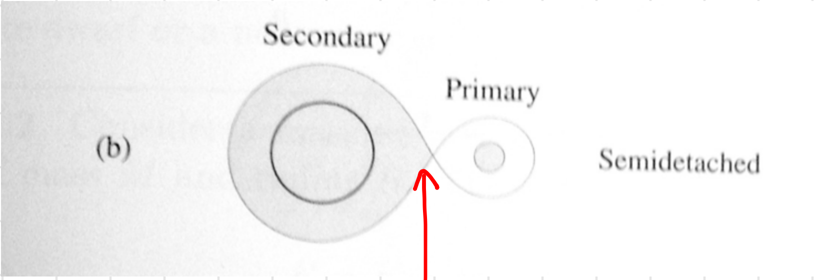
$$a = 2.3\times10^{9} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{min}^{2/3} \quad cm$$

$$a = 3.5\times10^{10} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{hr}^{2/3} \quad cm$$

$$a = 2.9\times10^{11} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{days}^{2/3} \quad cm$$

$$a = 1.5\times10^{13} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{yr}^{2/3} \quad cm$$

In a semidetached system, the star (secondary element) is very distorted from the spherical shape. This distortion can be detected as an "ellipsoidal variation" in the light curves (sometimes it also can be seen in detached systems).



Perturbations in the material in this region will push it over L_1 into the Roche lobe of the primary (B-H).
Such perturbations are always present (e.g. pressure force)

A star will fill its Roche lobe when it has a radius R_{*R} such that the corresponding sphere has the same volume of the Roche lobe. A numerical computation gives the value (Eggleton 1983),

$$\frac{R_{*R}}{a} = \frac{0.49\, q^{2/3}}{0.6\, q^{2/3} + \ln\left(1+q^{1/3}\right)}$$

For $0.1 \lesssim q \lesssim 0.8$ we can use the approximation due to Paczyński:

$$\frac{R_{*R}}{a} = \frac{2}{3^{4/3}}\left(\frac{q}{1+q}\right)^{1/3} = 0.462\left(\frac{M_*}{M+M_*}\right)^{1/3}$$

For the primary we have correspondingly,

$$\frac{r_R}{a} = \frac{0.49\, q^{-2/3}}{0.6\, q^{-2/3} + \ln\left(1+q^{-1/3}\right)}$$

or for $0.1 \lesssim q \lesssim 0.8$ the approximation

$$\frac{r_R}{a} = \frac{2}{3^{4/3}}\left(\frac{q^{-1}}{1+q^{-1}}\right)^{1/3}$$

Finally, we also remember the distance from L_1 to the black hole,

$$b \approx a\left[0.500 - 0.227 \log_{10} q\right]$$

SELF-SUSTAINABILITY OF THE ROCHE OVERFLOW

$$\text{Roche overflow} \longrightarrow \text{change } q$$
$$\downarrow$$
$$\text{change } a, P$$
$$\downarrow$$

Roche lobe grows? $\rightarrow$ overflow stops!
shrinks? $\rightarrow$ overflow continues!

Consider the total angular momentum

$$J = \left(M r_{BH}^2 + M_* r_*^2\right) \frac{2\pi}{P}$$

$$J = \left(M r_{BH}^2 + M_* r_*^2\right) \omega$$

where, from the definition of center of mass,

$$r_{BH} = \frac{M_*}{M+M_*} a$$

$$r_* = \frac{M}{M+M_*} a .$$

Then, we can write

$$J = \omega \left[\frac{M M_*^2}{(M+M_*)^2} + \frac{M_* M^2}{(M+M_*)^2} \right] a^2$$

$$J = \frac{\omega a^2}{(M+M_*)^2} M M_* (M+M_*)$$

$$J = \frac{\omega a^2}{(M+M_*)} M M_*$$

and using $\omega^2 = \frac{G(M+M_*)}{a^3}$ we get

$$\boxed{J = M M_* \sqrt{\frac{Ga}{M+M_*}}}$$

$$a = \frac{J^2}{M^2 M_*^2} \frac{M+M_*}{G} = \frac{J^2}{G} \frac{M+M_*}{M^2 M_*^2}$$

We consider that all the mass lost by the star is accreted by the black hole. Then

$$\dot{M} + \dot{M}_* = 0 \qquad \text{with} \qquad \dot{M}_* < 0$$

Thus, differentiating,

$$\dot{a} = \frac{2J\dot{J}}{G} \frac{M+M_*}{M^2 M_*^2} + \frac{J^2}{G}\left[\frac{(\dot{M}+\dot{M}_*)}{M^2 M_*^2} - \frac{2(M+M_*)}{M^3 M_*^3}(\dot{M} M_* + M \dot{M}_*)\right]$$

$$\dot{a} = \frac{2J\dot{J}}{G} \frac{M+M_*}{M^2 M_*^2} - \frac{2J^2}{G} \frac{(M+M_*)}{M^3 M_*^3}(\dot{M} M_* + M \dot{M}_*)$$

$$\dot{a} = 2\dot{J}\frac{a}{J} - \frac{2a}{M M_*}(\dot{M} M_* + M \dot{M}_*)$$

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - \frac{2}{M_*}\left[\frac{\dot{M} M_*}{M} + \dot{M}_*\right]$$

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - \frac{2}{M_*}\left[\frac{(-\dot{M}_*)M_*}{M} + \dot{M}_*\right]$$

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} + \frac{2}{M_*}(-\dot{M}_*)\left[1 - \frac{M_*}{M}\right]$$

Conservative mass transfer implies $\dot{J}=0$; and since $\dot{M}_* < 0$ and $M_* < M$ we have

$$\frac{\dot{a}}{a} = \frac{2}{M_*}(-\dot{M}_*)\left[1 - \frac{M_*}{M}\right] > 0$$

This result implies that for conservative systems, the black hole grows, putting more mass near the center of mass, and therefore the star needs to move in a wider orbit to conserve J, so a grows.

From the equation for the Roche lobe,

$$\frac{R_{*R}}{a} = 0.462\left(\frac{M_*}{M+M_*}\right)^{1/3}$$

we have

$$\dot{R}_{*R} = 0.462\,\dot{a}\left(\frac{M_*}{M+M_*}\right)^{1/3} + 0.462\,a\,\frac{1}{3}\left(\frac{M_*}{M+M_*}\right)^{-\frac{2}{3}}\left[\frac{\dot{M}_*}{M+M_*} - \frac{M_*(\dot{M}+\dot{M}_*)}{(M+M_*)^2}\right]$$

$$\dot{R}_{*R} = 0.462\,\dot{a}\left(\frac{M_*}{M+M_*}\right)^{1/3} + 0.462\,a\,\frac{1}{3}\left(\frac{M_*}{M+M_*}\right)^{-\frac{2}{3}}\frac{\dot{M}_*}{M+M_*}$$

$$\dot{R}_{*R} = \dot{a}\frac{R_{*R}}{a} + \frac{1}{3}R_{*R}\left(\frac{M_*}{M+M_*}\right)^{-1}\frac{\dot{M}_*}{M+M_*}$$

$$\frac{\dot{R}_{*R}}{R_{*R}} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{M}_*}{M_*}$$

Using the equion of the angular momentum,

$$\frac{\dot{R}_{*R}}{R_{*R}} = 2\frac{\dot{J}}{J} + \frac{2(-\dot{M}_*)}{M_*}\left[1 - \frac{M_*}{M}\right] + \frac{1}{3}\frac{\dot{M}_*}{M_*}$$

$$\frac{\dot{R}_{*R}}{R_{*R}} = 2\frac{\dot{J}}{J} + \frac{2(-\dot{M}_*)}{M_*}\left[1 - \frac{1}{6} - \frac{M_*}{M}\right]$$

$$\frac{\dot{R}_{*R}}{R_{*R}} = 2\frac{\dot{J}}{J} + \frac{2(-\dot{M}_*)}{M_*}\left[\frac{5}{6} - \frac{M_*}{M}\right]$$

Conservative systems ($\dot{J}=0$) with $q = \frac{M_*}{M} < \frac{5}{6}$ grows the Roche lobe of the star.

However, conservative systems with $q > \frac{5}{6}$ shrinks the Roche lobe of the star.
If the angular momentum diminishes $\dot{J} < 0$; it accentuate the diminuition of the Roche lobe.
In this case, the Roche overflow is rapid and violent but it will stop once enough mass is transfered to make q smaller than 5/6.
After this limit is reached, the transfer only continues if

i) the star expands, for example if it evolves off the main sequence to become a giant. For this proccess to work, the Roche lobe must be large enough to accomodate the giant, and this implies a long-period system.
In short-period system (e.g Cataclysmic Variables CV) this mechanism does not work.

ii) the binary loses angular momentum. There are many mechanisms for losing angular momentum ($\dot{J}<0$).

- Gravitational radiation (short-period)
- Tidal force on the star
- Wind magnetically linked to the star

Referencias

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