BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

08. Accretion

Edward Larrañaga

Outline for Part 1

Accretion Basics

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Process of matter falling into the potential well of a gravitating object.

Accretion Regimes

1. Spherical Accretion
2. Cylindrical Accretion
3. Accretion Disk
4. Two-Stream Accretion

Spherical Accretion

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  c_s: speed of sound in matter
  v_{rel}: relative velocity between accretor and matter
- $v_{rel} \ll c_s$
- If the accretor is a BH,

$$v_{rel} = v$$

  v: velocity of accreting matter (i.e. the BH doesn't move!)

Cylindrical Accretion

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- $v_{rel} \geq c_s$

Accretion Disk

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- Matter spirals down into the accretor

Two-Stream Accretion

- Quasi-spherically symmetric inflow coexist with an accretion disk

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Gas moves very slowly. The description is made in terms of fluid mechanics equations.

First and simplest description: To avoid the disintegration of the accretion structure, the outward force due to radiation pressure must be counterbalanced by the gravitational force.

Radiation Pressure

Outward energy flux at distance r from the center

$$F = \frac{L}{4\pi r^2}$$

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Then, the outwards momentum flux (or pressure) is

$$P_{rad} = \frac{F}{c} = \frac{L}{4\pi r^2 c}$$

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$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

Interaction with protons is negligible because it is lower by a factor of $\left(\frac{m_p}{m_e}\right)^2 \sim 3 \times 10^6$

Gravitational Force

The gravitational force between the central object M and one electron-proton pair is

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$$\vec{f}_g = -\frac{GM(m_e + m_p)}{r^2}\hat{r} \sim -\frac{GMm_p}{r^2}\hat{r}$$

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Eddington Lumionisty: Maximum luminosity of a source *M* powered by spherical accretion.

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L_{44}: Central Source Luminosity in units of $10^{44}\ \mathrm{erg \cdot s^{-1}}$ (typical value for AGN's)
$M_E = 8 \times 10^5 L_{44} M_\odot$: *Eddington's Mass*. Minimum mass for a given luminosity

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AGN's have typically $L \sim 10^{43} - 10^{47}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

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Black holes with $M \sim 10^5 - 10^9 M_{\odot}$

Estimation of the Central Mass from its Luminosity

Name	z	L_{bol}		M_{BH}	ref.	Type
3C 120	0.033	45.34	I	7.42	1	SY1
3C 390.3	0.056	44.88	I	8.55	1	SY1
Akn 120	0.032	44.91	I	8.27	1	SY1
F 9	0.047	45.23	F	7.91	1	SY1
IC 4329A	0.016	44.78	I	6.77	1	SY1
Mrk 79	0.022	44.57	I	7.86	1	SY1
Mrk 110	0.035	44.71	F	6.82	1	SY1
Mrk 335	0.026	44.69	I	6.69	1	SY1
Mrk 509	0.034	45.03	I	7.86	1	SY1
Mrk 590	0.026	44.63	I	7.20	1	SY1
Mrk 817	0.032	44.99	I	7.60	1	SY1
NGC 3227	0.004	43.86	I	7.64	1	SY1
NGC 3516	0.009	44.29	I	7.36	3	SY1
NGC 3783	0.010	44.41	I	6.94	2	SY1
NGC 4051	0.002	43.56	I	6.13	1	SY1
NGC 4151	0.003	43.73	I	7.13	1	SY1
NGC 4593	0.009	44.09	I	6.91	3	SY1
NGC 5548	0.017	44.83	I	8.03	1	SY1
NGC 7469	0.016	45.28	I	6.84	1	SY1
PG 0026+129	0.142	45.39	I	7.58	1	RQQ
PG 0052+251	0.155	45.93	F	8.41	1	RQQ
PG 0804+761	0.100	45.93	F	8.24	1	RQQ
PG 0844+349	0.064	45.36	F	7.38	1	RQQ
PG 0953+414	0.239	46.16	F	8.24	1	RQQ
PG 1211+143	0.085	45.81	F	7.49	1	RQQ
PG 1229+204	0.064	45.01	I	8.56	1	RQQ
PG 1307+085	0.155	45.83	F	7.90	1	RQQ
PG 1351+640	0.087	45.50	I	8.48	1	RQQ
PG 1411+442	0.089	45.58	F	7.57	1	RQQ
PG 1426+015	0.086	45.19	I	7.92	1	RQQ
PG 1613+658	0.129	45.66	I	8.62	1	RQQ
PG 1617+175	0.114	45.52	F	7.88	1	RQQ
PG 1700+518	0.292	46.56	F	8.31	1	RQQ
PG 2130+099	0.061	45.47	I	7.74	1	RQQ
PG 1226+023	0.158	47.35	I	7.22	1	RLQ
PG 1704+608	0.371	46.33	I	8.23	1	RLQ

[a] Column (1) Name, (2) redshift, (3) log of the bolometric luminosity (ergs s^{-1}), (4) method for bolometric luminosity estimation (I: flux integration; F: SED fitting), (5) black hole mass estimate from reverberation mapping (for Kaspi et al. (2000) sample, where black hole mass is log mean of rms FWHM and mean FWHM mass, in solar masses), (6) reference for black hole mass estimation, and (7) AGN type.

References. — (1) Kaspi et al. (2000), (2) Onken & Peterson (2002), (3) Ho (1999).

J.-H. Woo, C. M. Urry. *AGN Black Hole Masses and Bolometric Luminosities*. *Astrophys. J.* 579 (2002) 530-544

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η: Efficiency of the process

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Accretion produces radiation by conversion of gravitational potential.

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$$\eta \sim 0.1 - 0.2$$

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Using this point, one obtains an efficiency of $\eta \sim 0.1$

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$\dot{M}_E$: Maximum possible accretion rate for a mass $M_8 = \frac{M}{10^8 M_\odot}$.

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2. $\dot{M}_E$ can be exceeded with non-spherical models.

Growth Time

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For $L \sim L_E$, the BH grows exponentially on time scales of the order $\sim 10^8\ \mathrm{yr}$.

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$\bar{\nu}$: frequency of a typical (average) photon

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$$F = \sigma T_{eff}^4$$

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$$T_{eff} = 1.01 \times 10^6 M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left(\frac{r}{r_S} \right)^{-3/4}$$

compactness

One way to estimate the compactness of a source is using the luminosity and the effective surface temperature.

$$r_{BB} = \sqrt{\frac{L}{4\pi\sigma T_{eff}^4}}$$

Example

Consider a system in our galaxy with $L = 10^{37}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

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Eddington limit, the mass of the central object must be

$$M \geq \frac{L}{1.26 \times 10^{38}} M_\odot \sim \frac{10^{37}}{10^{38}} M_\odot \sim 0.1 M_\odot$$

Example

If the radiation is in the optical-UV,

https://rechneronline.de/spectrum

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$$T_{eff} \sim T_c = \frac{10^{15}}{5.88 \times 10^{10}} \sim 10^5\mathrm{K}$$

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Typical size of a Star!
https://rechneronline.de/spectrum

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If the radiation is in soft X-rays at 1 keV,

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$$T_{eff} \sim T_c = \frac{10^{17}}{5.88 \times 10^{10}} \sim 10^7\mathrm{K}$$

https://rechneronline.de/spectrum

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Typical size of a neutron star or a BH! https://rechneronline.de/spectrum

Virial Temperature

T_{th}: Temperature reached by the accreted material if all the gravitational energy is transformed into thermal energy.

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If the accretion flow is optically thick, the radiation reaches thermal equilibrium with the accreted material before escaping out to the observer,

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In general,

$$T_{eff} \lesssim T_{rad} \lesssim T_{th}$$

Outline for Part 2

Hydrodynamics Description of Spherical Accretion

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Assumptions:

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- Local velocity of the gas and local speed of sound
- Spectrum of the emitted radiation by the accretion structure

Hydrodynamics Equations

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$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Continuity Equation

Hydrodynamics Equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Continuity Equation

ρ: Mass density

$\vec{v}$: Velocity of the gas

Hydrodynamics Equations

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P + \rho \vec{g} + \vec{\nabla} \cdot \vec{\sigma}$$

Conservation of Momentum (ignoring radiation pressure)

Hydrodynamics Equations

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$\vec{g}$: Acceleration due to gravity

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Conservation of Momentum (ignoring radiation pressure)

P: Pressure

$\vec{g}$: Acceleration due to gravity

If self-gravity of the accretion structure is negligible,

$$\vec{g} = -\vec{\nabla} \Phi = -\frac{GM}{r^2} \hat{r}$$

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$$\sigma_{ij}=2\eta\tau_{ij}$$

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$$\tau_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x^j}+\frac{\partial v_j}{\partial x^i}-\frac{2}{3}\frac{\partial v_k}{\partial x^k}\delta_{ij}\right)$$

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$$\eta=\rho\nu$$

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(we will assume $\nu = 0$ for spherical accretion!)

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(we will assume $\nu = 0$ for spherical accretion!)

* No-viscosity: Euler equation
* Viscosity: Navier-Stokes equation

Hydrodynamics Equations

$$P = P(\rho)$$

Equation of state

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Usually a polytropic: $P \propto \rho^\gamma$

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Equation of state

Usually a polytropic: $P \propto \rho^{\gamma}$

$1 \leq \gamma \leq \frac{5}{3}$
$\gamma = 1$: Iosthermic flow
$\gamma = \frac{5}{3}$: Adiabatic flow

Hydrodynamics Equations

$$\rho\frac{d\varepsilon}{dt} = -P\vec{\nabla}\cdot\vec{v} + 2\eta\left[S_{ij}S_{ij} - \frac{1}{3}(\vec{\nabla}\cdot\vec{v})^2\right] + Q$$

Energy Balance

Hydrodynamics Equations

$$\rho\frac{d\varepsilon}{dt} = -P\vec{\nabla}\cdot\vec{v} + 2\eta\left[S_{ij}S_{ij} - \frac{1}{3}(\vec{\nabla}\cdot\vec{v})^2\right] + Q$$

Energy Balance

ε: Internal energy per unit mass of the fluid

Hydrodynamics Equations

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Energy Balance

ε: Internal energy per unit mass of the fluid

Q: Net heat exchanged by an element of the fluid per unit time per unit volume

$$S_{ij} = \tau_{ij} - \frac{1}{3}(\vec{\nabla} \cdot \vec{v})\delta_{ij}$$

Hydrodynamics Equations

$$T = \frac{\mu m_H}{k_B} \frac{P}{\rho}$$

Perfect gas temperature

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$m_H \sim m_p$: Hydrogen mass

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μ: Mean molecular weight

$\mu = 1$ for neutral Hydrogen

$\mu = \frac{1}{2}$ for fully ionized Hydrogen

Spherical Accretion Hydrodynamics

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Spherical symmetry and steady state:

Spherical Accretion Hydrodynamics

Spherical symmetry and steady state:

$$\rho = \rho(r)$$

$$\vec{v} = v(r)\hat{r}$$

Continuity Equation

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$$\dot{M} = 4\pi r^2 \rho v$$

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Conservation of Momentum

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P + \rho \vec{g}$$

$$v \frac{dv}{dr} + \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{GM}{r^2} = 0$$

Equations Governing Spherical Accretion

$$\dot{M} = 4\pi r^2 \rho v$$

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{GM}{r^2} = 0$$

$$P \propto \rho^\gamma$$

Local Speed of Sound

$$c_s^2 = \gamma \frac{P}{\rho}$$

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$$c_s^2 = \gamma \frac{P}{\rho}$$

Sonic radius: The gas moves with the speed of sound,

$$r_s = \frac{GM}{2c_s^2}$$

Behavior of the gas in the accretion structure

For $r \gg r_s$

Behavior of the gas in the accretion structure

For $r \gg r_s$

$$c_s \approx c_\infty \left[1 - \frac{\gamma - 1}{4}\frac{r_{acc}}{r}\right] \approx c_\infty$$

$$v \approx \frac{c_\infty}{16}\left(\frac{2}{5 - 3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}}\left(\frac{r_{acc}}{r}\right)^2\left[1 - \frac{1}{2}\frac{r_{acc}}{r}\right] \approx 0$$

$$\rho \approx \rho_\infty \left[1 - \frac{1}{2}\frac{r_{acc}}{r}\right] \approx \rho_\infty$$

Behavior of the gas in the accretion structure

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$$\rho \approx \rho_\infty \left[1 - \frac{1}{2} \frac{r_{acc}}{r}\right] \approx \rho_\infty$$

$r_{acc} = \frac{2GM}{c_\infty^2}$: Accretion radius

Behavior of the gas in the accretion structure

For $r \ll r_s$

Behavior of the gas in the accretion structure

For $r \ll r_s$

$$v \approx \sqrt{\frac{2GM}{r}} = v_{ff}$$

$$\rho \approx \rho(r_s) \left(\frac{r_s}{r} \right)^{\frac{3}{2}}$$

Temperature of the gas in the accretion structure

$$T = \frac{\mu m_H}{k_B} \frac{P}{\rho}$$

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$$T = T(r_s) \left(\frac{r_s}{r} \right)^{\frac{3}{2}(\gamma - 1)}$$

Temperature of the gas in the accretion structure

First law of thermodynamics:

$$d\varepsilon = dQ - PdV$$

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Bremsstrahlung (free-free) radiation

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$\frac{dQ}{dt} = -\alpha_{ff} T^{1/2} \rho$

Temperature of the gas in the accretion structure

First law of thermodynamics:

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Bremsstrahlung (free-free) radiation

$\frac{dQ}{dt} = -\alpha_{ff} T^{1/2} \rho$

$\alpha_{ff} \approx 5 \times 10^{20}\ \mathrm{erg\ cm^3\ g^{-2}\ s^{-1}\ K^{-1/2}}$ for Hydrogen.

Temperature of the gas in the accretion structure

$$\frac{dT}{dr} = -\frac{T}{r} - \alpha_{ff}\rho(r_s)\sqrt{\frac{r_s}{2GM}}\frac{T^{1/2}}{r}\left(\frac{2\mu m_H}{3k_B}r_s\right) + \frac{2\mu m_H}{3k_B}\frac{dQ}{dr}$$

Temperature of the gas in the accretion structure

If there is only Bremsstrahlung radiation,

$$\frac{dT}{dr} = -\frac{T}{r} - \alpha_{ff}\rho(r_s)\sqrt{\frac{r_s}{2GM}}\frac{T^{1/2}}{r}\left(\frac{2\mu m_H}{3k_B}r_s\right)$$

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Therefore,

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Therefore,

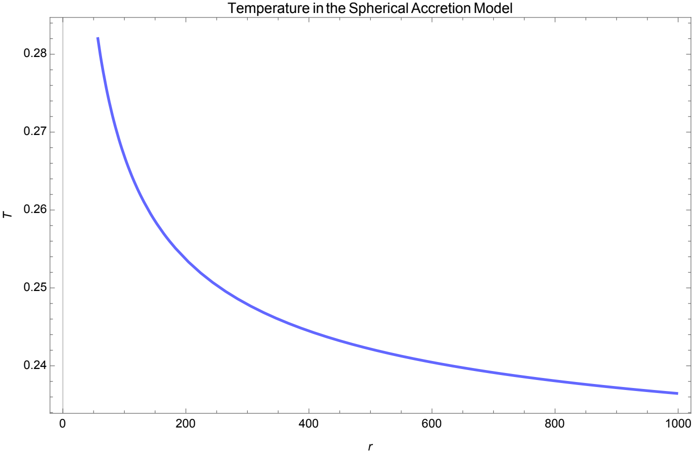
$$\frac{dT}{dr} < 0$$

The temperature of the flow decreases as the gas approaches the BH (*cooling flow*).

Temperature of the gas in the accretion structure

$$T = \left[-\frac{4}{K} + \sqrt{\frac{16}{K^2} + \frac{4}{K\sqrt{r}} + C} \right]^2$$

Temperature of the gas in the accretion structure



Next Lecture

09. Accretion Disks