02. Accretion Basics

ACCRETION

- Process of matter falling into the potential well of a gravitating object.

→ Accretion of matter without angular momentum:
  * spherical accretion
  * determined by the relation between
    c_s: speed of sound in the matter
    V_{rel}: relative velocity between accretor and matter.

If the accretor is a B.H. $\Rightarrow$ $V_{rel} \approx V$

Velocity of accreting matter. (BH does not move!).

→ Accretion of matter with angular momentum:
  * Accretion Disk

ACCRETION REGIMES

* Spherical:
  • $V_{rel} \ll c_s$
  • No significant angular momentum

* Cylindrical:
  • Small angular momentum
  • $V_{rel} \gtrsim c_s$

* Disk:
  • Angular momentum is enough to form an accretion disk.

* Two-Stream Accretion:
  • Quasi-spherically symmetric inflow coexists with an accretion disk.

INTRODUCTION TO SPHERICAL ACCRETION

In this introduction, we will describe a completely ionized hydrogen gas and we will assume isotropy and stability in the source of radiation, e.g. in the accretion structure.
To avoid the disintegration of the accretion structure, the outward force due to radiation pressure must be counterbalanced by the gravitational force.

The outward energy flux at distance r from the center is:

$$F = \frac{L}{4\pi r^2}$$

where L is the bolometric luminosity (in $erg \cdot s^{-1}$).
Since photons have

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) \quad \text{and} \quad p^2 = 0 = \frac{E^2}{c^2} - |\vec{p}|^2 ,$$

the outward momentum flux or pressure is

$$P_{rad} = \frac{F}{c} = \frac{L}{4\pi r^2 c} .$$

The radiation force on a single electron is obtained by multiplying by the cross section σ_e

$$\vec{f}_{rad} = \sigma_e \frac{L}{4\pi r^2 c} \hat{r} .$$

Here σ_e is the Thomson cross-section

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$$

representing the interaction of the electron with a photon. The interaction with protons is negligible because it is lower by a factor of $\left(\frac{m_p}{m_e}\right)^2 \approx 3 \times 10^6$.

The gravitational force between the central object M and an electron-proton pair is

$$\vec{f}_g = \frac{GM(m_e+m_p)}{r^2}\,\hat{r} \approx \frac{GMm_p}{r^2}\,\hat{r}.$$

In order to avoid the disintegration of the accretion structure, we need

$$|f_{rad}| \leq |f_g|$$

$$\sigma_e \frac{L}{4\pi r^2 c} \leq \frac{GMm_p}{r^2}$$

Note that this force depends on $\frac{1}{r^2}$ just as gravity !

From here we have the condition on luminosity

$$L \leq \frac{4\pi G M m_p c}{\sigma_e} \sim 6.31 \times 10^4\, M \text{ erg s}^{-1} \sim 1.26 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{erg}\cdot\text{s}^{-1}$$

II. Eddington Luminosity

We define Eddington's luminosity as the critical value

$$L_E = \frac{4\pi G m_p c}{\sigma_e} M = 1.3 \times 10^{38} \frac{M}{M_\odot} \text{ erg s}^{-1}$$

which corresponds to the maximum luminosity of a source of mass M powered by spherical accretion.

III. Mass of the Central Object

In order to estimate the mass of the central object, e.g. a black hole, from observational data we write

$$M \geqslant \frac{L \sigma_e}{4\pi G m_p c}$$

$$M \geqslant \frac{L}{1.26 \times 10^{38}} M_\odot \sim 8 \times 10^5 L_{44} M_\odot$$

where L_{44} is the central source luminosity in units of 10^{44} erg. s^{-1} (which is characteristic of AGN's)
The critical value is known as Eddington's mass

$$M_E = 8 \times 10^5 L_{44} M_\odot$$

and corresponds to the minimum mass for a given luminosity.

In AGNs, is observed $L \sim 10^{43} - 10^{47}$ erg s^{-1}

Therefore we infer $M \sim 10^5 - 10^9 M_\odot$

IV. EDDINGTON ACCRETION RATE

The luminosity is a fraction of the relativistic energy of the accreting mass, $E=mc^2$. (the other fraction of the mass goes into the BH, growing its mass)

$$L \propto \frac{dE}{dt} = \frac{dm}{dt}c^2 \qquad \text{but} \qquad \frac{dm}{dt} = \frac{dM}{dt} = \dot{M}$$

$$L \propto \dot{M}c^2$$

$$L = \eta \dot{M} c^2$$

η: efficiency, it is expected $\eta \sim 0.1 = 10\%$ *

Accretion produces radiation by conversion of gravitational potential. A mass m located at a distance r from the central source M has the potential energy

$$U = \frac{GMm}{r}$$

The rate of convertion of potential energy to radiation is

$$L \simeq \frac{dU}{dt} = \frac{GM}{r}\frac{dm}{dt}$$

$$L = \frac{GM}{r}\dot{M}$$

and thus the efficiency is $\eta = \frac{GM}{rc^2}$.

In order to estimate the order of η, remember that the innermost stable orbit around Schwarzschid is $3r_s = \frac{6GM}{c^2}$ and hence, a particle falling from this point has

$$U = \frac{GMm}{3r_s} = \frac{GM}{6GM} mc^2 = 0{,}16\ mc^2$$

or $\eta \sim 0{,}1$.

For a neutron star $R \sim 10 Km$ $M \sim M_\odot$

$\underbrace{\Delta E_{acc}}_{\to \text{ radiation!}} \sim 10^{20}$ erg per accreted gram

For comparison: "burning" of $H \to He$ gives

$\Delta E_{nuc} = 0{,}007 mc^2 = 0.7\%\, mc^2$

i.e. $\Delta E_{nuc} \sim 6 \times 10^{18}$ erg per gram burned !!!

It is usual to consider particles falling from $5r_s$ because this point is where most of the optical/UV continuum radiation. From this point we obtain $\eta = 0.1$ too.

It is important to note that the efficiency of fusion of Hydrogen to Helium has an efficiency of $\eta = 0.007$.

Now, using $c = 3\times10^{10}$ cm/s and luminosity in units of 10^{44} erg. s^{-1} we may write

$$\dot{M} = \frac{L}{\eta c^2} = 1.11\times10^{23}\,\frac{L_{44}}{\eta}\left[\frac{gr}{s}\right]$$

Using now $M_\odot = 1.98\times10^{30}\,Kg = 1.98\times10^{33}\,gr$ ✓
$1\,yr = 3.157\times10^{7}\,s$

$$\dot{M} = 1.77\times10^{-3}\,\frac{L_{44}}{\eta}\left[\frac{M_\odot}{yr}\right]$$

$$\dot{M} = 1.77\times10^{-2}\,L_{44}\left(\frac{\eta}{0.1}\right)^{-1}\left[\frac{M_\odot}{yr}\right]$$

* In AGNs $L_{acc} = \eta\dot{M}c^2 \sim 10^{47}$ erg s^{-1}

If $\eta = 0.007$ (as in H→He "burning")

we get $\dot{M} \sim 250\,M_\odot\,yr^{-1}$ ← Too Much!!

If $\eta = 0.1 = 10\%$ we get $\dot{M} \sim 20 M_\odot yr^{-1}$ ← Acceptable?

For the Eddington accretion we have

$$L_E = \eta \dot{M}_E c^2$$

$$\dot{M}_E = \frac{L_E}{c^2 \eta}$$

$$\dot{M}_E = \frac{1.51\times 10^{38} \frac{M}{M_\odot} \text{ erg } s^{-1}}{(3\times 10^{10} \text{ cm } s^{-1})^2 \cdot 0.1} \left(\frac{\eta}{0.1}\right)^{-1}$$

$$\dot{M}_E = 1.67\times 10^{16} \frac{M}{M_\odot} \left(\frac{\eta}{0.1}\right)^{-1} \left[\frac{gr}{s}\right]$$

$$\dot{M}_E = 2,67\times 10^{-8} \frac{M}{M_\odot} \left(\frac{\eta}{0.1}\right)^{-1} \left[\frac{M_\odot}{yr}\right]$$

$$\dot{M}_E \simeq 3 \, M_8 \left(\frac{\eta}{0.1}\right)^{-1} [M_\odot \, yr^{-1}]$$ where $M_8 = \frac{M}{10^8 M_\odot}$

This represents the maximum possible accretion rate for the mass M.

What happens if $\dot{M} > \dot{M}_E$?

* Supposing perfect spherical symmetry, we must take into account a careful determination of η.

  If $\eta < 0.1 \rightarrow$ the outward flux is diminished.

* If we assume other symmetry $\rightarrow$ the model changes!

* $\dot{M}_E$ can be exceeded with non-spherical models.

GROWTH TIME

$$\dot{M} = \frac{dM}{dt} = \frac{L}{c^2\eta}$$ Mass growth of the B.H.

$$\frac{dM}{dt} = \frac{L}{L_E}\frac{4\pi G c m_p}{\eta c^2 \sigma_e}$$

$$\int \frac{dM}{M} = \frac{L}{L_E}\frac{4\pi G m_p}{\eta c \sigma_e}\int dt$$

$$M(t) = M_0 \exp\left[\frac{L}{L_E}\frac{4\pi G m_p}{\eta c \sigma_e} t\right] = M_0 \exp\left[\frac{t}{t_{growth}}\right]$$

$$t_{growth} = \frac{\eta c \sigma_e}{4\pi G m_p}\frac{L_E}{L}$$

$$t_{growth} \simeq 3.7 \times 10^8 \eta \frac{L_E}{L} \text{ yr}$$

For $L \simeq L_E$, the BH grows exponentially on times scales of the order $\sim 10^8$ yr !!

EMITTED SPECTRUM

The continuum spectrum of the emitted radiation is characterized by a temperature T_{rad}. A typical photon has $h\bar{\nu} \sim kT_{rad}$ and thus

$$T_{rad} = \frac{h\bar{\nu}}{k}$$

For an accretion luminosity L_{acc} from a source of radius r; we define the blackbody temperature T_{eff}

$$F = \sigma T_{eff}^4$$

$$\frac{L_{acc}}{4\pi r^2} = \sigma T_{eff}^4$$

$$T_{eff} = \left(\frac{L_{acc}}{4\pi r^2 \sigma}\right)^{1/4}$$

Using

$$L_{acc} = \frac{GM}{r}\dot{M}$$

we get

$$T_{eff} = \left(\frac{GM\dot{M}}{4\pi r^3 \sigma}\right)^{1/4}$$

Introducing the Schwarzschild radius and Eddington accretion, we have

$$T_{eff} = \left[\frac{GM}{4\pi\sigma}\left(\frac{c^2}{2GM}\right)^3 \frac{L_E}{c^2\eta}\right]^{1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

$$T_{eff} = \left[\frac{c^4}{32\pi\sigma G^2 M^2 \eta}\left(1.51\times10^{38}\,\frac{M}{M_\odot}\ \mathrm{erg\ s^{-1}}\right)\right]^{1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

$$T_{eff} = \left[\frac{1.51\times10^{38}}{32\pi\sigma\eta}\left(\frac{c^4}{G^2 M_\odot^2}\right)\right]^{1/4} \left(\frac{M_\odot}{M}\right)^{1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

$$T_{eff} = \left[\frac{1.51\times10^{38}}{8\pi\sigma\eta R_{s\odot}^2}\right]^{1/4} \left(\frac{M_\odot}{M}\right)^{1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

$$T_{eff} = \left[\frac{1.51\times10^{30}}{8\pi\sigma\eta R_{s\odot}^2}\right]^{1/4} M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

$$T_{eff} = 1.01\times10^{6}\, M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E}\right)^{1/4} \left(\frac{r}{r_s}\right)^{-3/4}$$

Compactness

One way to estimate the compactness of a source is available if a large portion of the energy is emitted in X-rays rather than optical or radio.

Consider a black body spectrum with luminosity L, effective surface temperature T_{eff} and size r_{BB}

These quantities are related by

$$L = 4\pi r_{BB}^2 \sigma T_{eff}^4$$

Knowing the power output L and measuring T_B from the spectrum let us solve for r_{BB}

$$r_{BB} = \sqrt{\frac{L}{4\pi\sigma T_{eff}^4}}$$

$$\sigma = 5{,}6 \times 10^{-5} \frac{\text{erg}}{\text{cm}^2\ \text{s}\ \text{K}^4\ \text{sr}}$$

Let a binary systen in our galaxy has $L = 10^{37}\ \text{erg} \cdot \text{s}^{-1}$
From the Eddington limit, its mass must be greater than

$$M \gtrsim \frac{L}{1.26 \times 10^{38}} M_\odot \sim \frac{10^{37}}{10^{38}} M_\odot \sim 0.1\, M_\odot$$

* If the radiation is in the optical-UV; $\nu_{max} \sim 10^{15}$ Hz [2], then

$$T_{eff} \sim T_c = \frac{10^{15}\,\text{Hz}}{5.88 \times 10^{10}\ \text{Hz} \cdot \text{K}^{-1}} \sim 10^5\,\text{K}$$

The estimated size of the source is

$r_{BB} \sim 10^{12}\, cm \sim 10^{7}\, Km$ (typical site of a star)

* If the power is emitted in soft X-rays at 1 KeV; we have [2],

$$\nu_{max} \sim 10^{17}\, Hz$$

and therefore

$$T_{eff} \sim T_c = \frac{10^{17}\, Hz}{5.88 \times 10^{10}\ Hz \cdot K^{-1}} \sim 10^{7} K$$

This gives a size of

$r_{BB} \sim 10^{6}\, cm \sim 10\, Km$ (size of a neutron star or BH)

A DISK MODEL WITH DISSIPATION

A more complete derivation, including the dissipation of energy through viscosity (work done by viscous torques), yields the temperature [Bradley (1997)]

$$T_{eff}^{vis} = \left[\frac{3GM\dot{M}}{8\pi\sigma r^3} \left\{ 1 - \left(\frac{r_{in}}{r} \right)^{1/2} \right\} \right]^{1/4}$$

where r_{in} defines the inner edge of the accretion disk, $r_H < r_{isco} \leq r_{in}$.

If $r \gg r_{in}$ we approximate

$$T_{eff}^{vis} = \left[\frac{3GM\dot{M}}{8\pi\sigma r_{in}^3} \frac{r_{in}^3}{r^3} \left\{ 1 - \left(\frac{r_{in}}{r} \right)^{1/2} \right\} \right]^{1/4}$$

$$T_{eff}^{vis} = \left[\frac{3GM\dot{M}}{8\pi\sigma r_{in}^3} \left\{ \frac{r_{in}^3}{r^3} - \frac{r_{in}^{7/2}}{r^{7/2}} \right\} \right]^{1/4}$$

$$T_{eff}^{vis} \simeq \left[\frac{3GM\dot{M}}{8\pi\sigma r_{in}^3} \right]^{1/4} \left\{ \frac{r_{in}^3}{r^3} \right\}^{1/4}$$

$$T_{eff}^{vis} \simeq \left[\frac{3GM\dot{M}}{8\pi\sigma r_{in}^3} \right]^{1/4} \left\{ \frac{r}{r_{in}} \right\}^{-3/4}$$

Considering r_{in} as the Innermost Stable Circular Orbit (ISCO) we have for Schwarzschild

$$r_{in} = r_{isco} = 3 r_H$$

and then

$$T_{eff}^{vis} \simeq \left[\frac{3GM}{8\pi\sigma \; 27 r_H^3} \right]^{1/4} \dot{M}^{1/4} \left\{ \frac{r}{3 r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3GM}{8\pi\sigma r_H^3} \right]^{1/4} \dot{M}^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^6}{64\pi\sigma G^2 M^2} \right]^{1/4} \dot{M}^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^6}{64\pi\sigma G^2} \right]^{1/4} M^{-1/2} \dot{M}^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^6}{64\pi\sigma G^2} \right]^{1/4} \left(\frac{L_E}{c^2 \eta} \right)^{1/4} M^{-1/2} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^4}{64\pi\sigma G^2 \eta} \; 1.51 \times 10^{38} \frac{M}{M_\odot} \right]^{1/4} M^{-1/2} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^4}{64\pi\sigma G^2 \eta} \; \frac{1.51 \times 10^{38}}{M_\odot} \right]^{1/4} M^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3c^4}{64\pi\sigma G^2 \eta} \; \frac{1.51 \times 10^{38}}{M_\odot^2} \right]^{1/4} \left(\frac{M}{M_\odot} \right)^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx \left[\frac{3}{16\pi\sigma \eta} \; \frac{1.51 \times 10^{38}}{R_{s\odot}^2} \right]^{1/4} \left(\frac{M}{M_\odot} \right)^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx 1.12 \times 10^8 \left(\frac{M}{M_\odot} \right)^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx 1.12 \times 10^8 \times (10^8)^{-1/4} M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4}$$

$$T_{eff}^{vis} \approx 1.12 \times 10^6 \, M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left\{ \frac{r}{r_H} \right\}^{-3/4} \quad \text{K}$$

* Consider a black hole with $M = 10^8 M_\odot$ and an Eddington accretion rate, $\dot{M} = \dot{M}_E$. The emission near the inner edge of the disk, $r \sim r_{in} \sim r_H$ have an effective temperature of

$$T_{eff}^{vis} \sim 1.12 \times 10^6 \text{ K}.$$

This radiation has a maximum at the frequency

$$\nu_{max} \sim 5.88 \times 10^{10} \text{ Hz} \cdot \text{K}^{-1} \, T_{eff}^{vis} \sim 6.59 \times 10^{16} \text{ Hz}$$

This frequency corresponds to the wavelenght

$$\lambda_{max} \sim 4.6 \text{ nm}$$

and it is classified as Extreme UV or soft X-rays with $E \sim 289$ eV.

* If we consider now $M \sim 1 M_\odot$ the effective temperature is

$$T_{eff}^{vis} \sim 1.12 \times 10^8 \text{ K}$$

and the maximum is located at

$$\nu_{max} \sim 6.59 \times 10^{18} \text{ Hz}$$

This frequency corresponds to the wavelenght

$$\lambda_{max} \sim 4.3 \text{ pm}$$

and it is classified as Hard X-rays with $E \sim 29$ KeV.

The temperature that the accreted material would reach if the gravitational energy is turned completely into thermal energy is T_{th}. From the virial theorem

$$2K + U = 0.$$

For each pair p^+-e^- accreted, the potential gravitational energy is

$$U = \frac{GM(m_p + m_e)}{r} \approx \frac{GMm_p}{r}$$

while the corresponding thermal energy is

$$2K = 2 \times \frac{3}{2} kT.$$

Therefore,

$$2 \times \frac{3}{2} kT_{th} = \frac{GMm_p}{r}$$

$$T_{th} = \frac{GMm_p}{3kr} = T_{vir}$$

If the accretion energy is converted directly into radiation which escapes without interaction,

$$T_{rad} \sim T_{th}$$

If the accretion flow is optically thick, the radiation reaches thermal equilibrium with the accreted material before leaking out to the observer. Thus

$$T_{rad} \sim T_{eff}$$

In general $$T_{eff} \lesssim T_{rad} \lesssim T_{th}$$

Structure of the Accretion Disk

The value of the accretion rate $\dot{M}$ and the value of the opacity of the accreting material determines the basic structure of the disk.

Ⓘ Low accretion rates $\frac{\dot{M}}{\dot{M}_E} \lesssim 1$ and high opacities $\tau \gg 1$ give a "thin accretion disk". This have a height < diameter. The emission is has high efficiency, $\eta \simeq 0.1$.
The advection inwards is negligible compared with the radiation in the vertical direction. It has a continuum spectrum with UV-X-rays from the inner region and visible from the outer region.

Ⓘ With high accretion rates $\frac{\dot{M}}{\dot{M}_E} \gg 1$, part of the radiation is trapped by the disk, expanding it into a "radiation torus" or "thick accretion disk". The efficiency is small $\eta \sim \left(\frac{\dot{M}}{\dot{M}_E}\right)^{-1} \ll 1$.

In these disks the inwards advection is so strong that the radiation can not cool it efficiently. The heat transport is high in the radial direction, and thus the spectrum is almost that of a blackbody with a single temperature $\sim 10^4\,K$.

Ⓘ At very low accretion rates $\frac{\dot{M}}{\dot{M}_E} \lesssim 0.1$ the disk becomes optically thin. The structure corresponds to a stable, two temperature structure called an "Ion Torus".

The ion temperature reaches the virial value,

$$T_{vir} = T_{th} = \frac{GMm_p}{3kr}$$

$$T_{vir} = \frac{m_p c^2}{6K} \frac{r_H}{r}$$

$$T_{vir} = 2 \times 10^{12} \left(\frac{r}{r_H} \right)^{-1} \quad K$$

If the black hole has a magnetic field, it "freezes" in the ion torus creating a rapidly rotating field with its axis along the angular momentum vector of the disk. This strong field may collimate the outflow of charged particles, producing jets.

Finally, it is important to stress that the theoretical models of accretion disks have many free parameters to define its structure and thus its radiation spectrum. Some of these parameters are: magnetic field strenght, disk inclination, etc.

REFERENCIAS

[1] Bradley Peterson. An Introduction to Active Galactic Nuclei. Cambidge University Press. (1997)

[2] https://rechneronline.de/spectrum/