BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

11. Novikov-Thorne Thin Disks

Edward Larrañaga

Outline for Part 1

Novikov-Thorne Thin Disks

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- Geometrically thin and optically thick accretion disk.

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- Has four parameters: BH mass, BH spin, mass accretion rate and viscosity parameter.

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- The inner edge of the disk is at the ISCO radius.

Modeling Viscosity

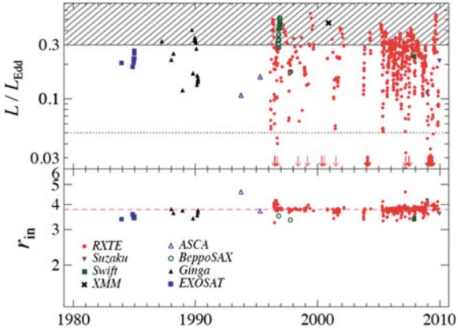

Fig. 6.1 *Top panel* Accretion disk luminosity in Eddington units versus time for 766 spectra of LMC X-3. The shaded region does not satisfy the thin disk selection criterion $L/L_{\rm Edd} < 0.3$, as well as the data below the dotted line, which marks $L/L_{\rm Edd} = 0.05$. *Bottom panel* fitted value of the inner disk radius of the 411 spectra in the *top panel* that can meet the thin disk selection criterion. See the text for more details. From [51]. © AAS. Reproduced with permission

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  Any physical quantity $\Psi(t, r, \theta, phi)$ will be averaged over t and ϕ,

$$\Psi(r, \theta) = \frac{1}{2\pi\Delta t} \int_0^{\Delta t} \int_0^{2\pi} \Psi(t, r, \theta, \phi) d\phi dt$$

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- Magnetic fields are ignored.

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- The effect of energy and angular momentum transport by photons emitted from the disk and returning to the disk due to strong light bending in the vicinity of the black hole (returning radiation) is neglected.

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- The description will be independent of the specific properties of the accretion.

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ρ: Time-averaged rest mass density

u^μ: Time-averaged 4-velocity of the fluid

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$$\gamma = -\left(\frac{g_{t\phi}^2}{g_{\phi\phi}} - g_{tt}\right) g_{rr} g_{\phi\phi}$$

$$\Sigma(r) = \int_{-h}^{h} \rho dz$$

Energy Flux and Torque

From the conservation of energy,

$$\nabla_\mu T^{t\mu} = 0$$

and angular momentum,

$$\nabla_\mu T^{\phi\mu} = 0$$

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ε: specific energy

ℓ_z: specific axial component of the angular momentum

Ω: Angular velocity for equatorial circular geodesics

Energy Flux and Torque

$$F(r) = -\frac{\partial_r \Omega}{(\varepsilon - \Omega \ell_z)^2} \frac{M^2}{\sqrt{-\gamma}} \int_{r_{in}}^{r} \left[(\varepsilon - \Omega \ell_z) \partial_x \ell_z \right] dx$$

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r_{in}: inner edge of the disk (presumably the ISCO radius)

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$$\eta_{NT} = \eta_r$$

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ε_{ISCO}: Specific energy of a test particle at the ISCO

ζ: Fraction of energy captured by the BH

Efficiency

$$\zeta = \frac{1}{\dot{M}} \int_{r_{ISCO}}^{\infty} \left[\int_{0}^{\pi/2} \int_{0}^{2\pi} \mathrm{C}\Upsilon(-n_t) \cos\theta \sin\theta d\phi d\theta \right] \mathscr{F}(r) 4rdr$$

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$$c = \begin{cases} 0 & \text{radiation escapes} \\ 1 & \text{radiation is captured} \end{cases}$$

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Υ: takes into account possible angular dependence of the emission

$$\begin{cases} \Upsilon = 1 & \text{isotropic emission} \\ \Upsilon \propto 1 + 2\cos\theta & \text{limb-darkened emission} \\ \dots & \end{cases}$$

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n^{μ}: normalized photon 4-momentum

$$n^{\mu} = \frac{k^{\mu}}{k_{(t)}}$$

k^{μ}: photon 4-momentum

$k_{(t)}$: photon energy in the rest frame of the emitter

Kerr Spacetime

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$$\varepsilon = \frac{r^{3/2} - 2Mr^{1/2} \pm aM^{1/2}}{r^{3/4}\sqrt{r^{3/2} - 3Mr^{1/2} \pm 2aM^{1/2}}}$$

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$$\sqrt{-\gamma} = r$$

Kerr Spacetime

$$
\begin{aligned}
F(x) = & \frac{3}{2}\frac{1}{x^4(x^3-3x+2a)}\left[x - x_0 - \frac{3}{2}a\ln\left(\frac{x}{x_0}\right)\right. \\
& - \frac{3(x_1-a)^2}{x_1(x_1-x_2)(x_1-x_3)}\ln\left(\frac{x-x_1}{x_0-x_1}\right) \\
& - \frac{3(x_2-a)^2}{x_2(x_2-x_1)(x_2-x_3)}\ln\left(\frac{x-x_2}{x_0-x_2}\right) \\
& \left. - \frac{3(x_3-a)^2}{x_3(x_3-x_1)(x_3-x_2)}\ln\left(\frac{x-x_3}{x_0-x_3}\right)\right]
\end{aligned}
$$

Radial Structure Evolution

$x = \sqrt{\frac{r}{M}}$
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$$x_1 = 2\cos\left(\frac{1}{3}\arccos a - \frac{\pi}{3}\right)$$
$$x_2 = 2\cos\left(\frac{1}{3}\arccos a + \frac{\pi}{3}\right)$$
$$x_3 = -2\cos\left(\frac{1}{3}\arccos a\right)$$

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$$\eta_{NT}(a = 0) = 0.057$$

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$$\eta_{NT}(a = 1) = 0.038 \text{ (counter-rotating disk)}$$

Formation of the Accretion Disk

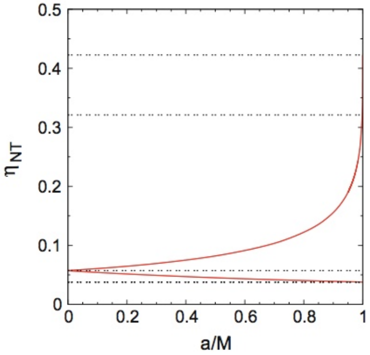

Fig. 6.2 Novikov-Thorne radiative efficiency $\eta_{NT} = 1 - E_{ISCO}$ as a function of the spin a in the Kerr metric. The *upper curve* is for corotating orbits, the *lower curve* for counterrotating orbits. The *dotted horizontal lines* correspond (from *top* to *bottom*) to the Novikov-Thorne radiative efficiency for $a/M = 1$ ($\eta_{NT} \approx 0.423$), $a/M = 0.998$ ($\eta_{NT} \approx 0.321$), $a/M = 0$ ($\eta_{NT} \approx 0.057$), and $a/M = 1$ in the case of a counterrotating disk ($\eta_{NT} \approx 0.038$)

Outline for Part 2

Transfer Function

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- The second part corresponds to describe the propagation of the radiation from the disk to observer

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  ν_e: emitted photon frequency (in the rest frame of the disk)
  r_e: emission radius
  ϑ_e: emission angle with respect to the normal to the disk

Formation of the Accretion Disk

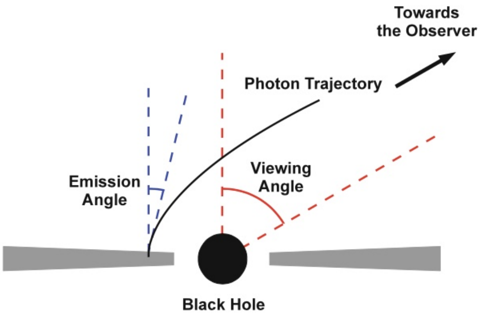

Fig. 6.3 The viewing angle i is the inclination angle of the disk, namely the angle between the axis normal to the disk and the line of sight of the distant observer; it is the same in Newtonian gravity and in general relativity. The emission angle ϑ_e is the angle, measured in the rest-frame of the gas and at a certain emission point, between the normal to the disk and the photon propagation direction. In Newtonian gravity, $i = \vartheta_e$, but in a curved spacetime the two angles are different in general

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The observed flux (in units of erg s^{-1} cm^{-2} Hz^{-1})

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$$g^* = \frac{g - g_{min}}{g_{max} - g_{min}}$$

$g_{max} = g_{max}(r_e, i)$: maximum value of the redshift factor
$g_{min} = g_{min}(r_e, i)$: minimum value of the redshift factor

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The transfer function

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$$f(g^*, r_e, i) = \frac{1}{\pi r_e} g \sqrt{g^*(1-g^*)} \left| \frac{\partial(X, Y)}{\partial(g^*, r_e)} \right|$$

Explicit form of the transfer function

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Explicit form of the transfer function

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- We need to relate the position of the received photon, (X, Y) with the emission point, r_e, and with the redshift factor ,g, and the emission angle, ϑ.
- This relation is obtained by evolving (numerically) the geodesic equations from the observation point to the emission point.

Explicit form of the transfer function. Redshift

- Numerically solving the geodesic equations give the position of the emitted photon in the accretion disk, r_e

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$u^{\mu}_{obs} = u^{t}_{e}(1, 0, 0, \Omega)$: 4-velocity of the observer
$u^{\nu}_{e} = (1, 0, 0, 0)$: 4-velocity of the gas particles

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$\lambda = -\frac{k_{\phi}}{k_t}$
$k_t = -\varepsilon$: energy of the photon (constant of motion)

Explicit form of the transfer function. Emission angle

- Normal to the disk

$$n^\mu = \left(0, 0, \sqrt{g^{\theta\theta}}, 0\right)\Big|_{r_e, \theta_e = \pi/2}$$

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$$\cos\vartheta_e = \pm \left.\frac{n^\mu k_\mu}{u_e^\nu k_\nu}\right|_e$$

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Explicit form of the transfer function. Jacobian

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$$
\begin{aligned}
r_e &= r_e(X, Y) \\
g &= g(X, Y) \\
\vartheta &= \vartheta(X, Y)
\end{aligned}
$$

Explicit form of the transfer function. Jacobian

- At the end of the numerical integration of the geodesic equations one has

$$r_e = r_e(X, Y)$$
$$g = g(X, Y)$$
$$\vartheta = \vartheta(X, Y)$$

- From the first two relations we may calculate the Jacobian as

$$\left|\frac{\partial(X, Y)}{\partial(g^*, r_e)}\right| = (g_{max} - g_{min})\left|\frac{\partial X}{\partial g}\frac{\partial Y}{\partial r_e} - \frac{\partial X}{\partial r_e}\frac{\partial Y}{\partial g}\right|$$

Continuum-Fitting Method

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- In the Novikov-Thorne model and assuming that the spacetime is described by the Kerr metric and that the inner edge of the accretion disk is th ISCO radius, one can calculate the thermal spectrum which will be function only on the spin parameter a.

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- In the Novikov-Thorne model and assuming that the spacetime is described by the Kerr metric and that the inner edge of the accretion disk is th ISCO radius, one can calculate the thermal spectrum which will be function only on the spin parameter a.
- In the Continuum-Fitting method, the observed spectrum is matched with an appropriate theoretical spectrum to identify the BH spin parameter.
- However, to correctly estimate a we need an independent measurement of the mass M, the distance D and the inclination angle i !!!

Thermal Spectrum

- Assuming that the Novikov-Thorne disk in in local thermal equilibrium, it will have a black-body emission with an effective temperature T_{eff} related with the time averaged energy flux $\mathscr{F}$ by

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$$\mathscr{F} = \sigma T_{eff}^4$$

Temperature of the gas in the accretion structure

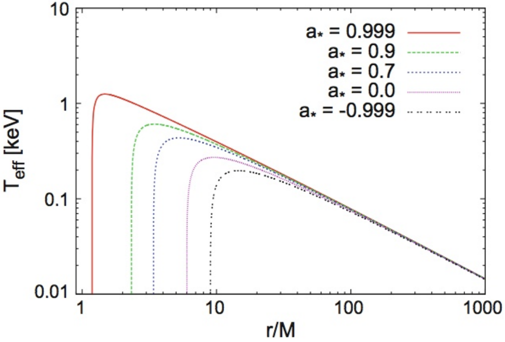
Table 7.1 Summary of the continuum-fitting and iron line measurements of the spin parameter of stellar-mass black holes under the assumption of the Kerr background. In some cases, only upper or lower bounds have been obtained. See the references in the last column for more details

BH Binary	a_* (Continuum)	a_* (Iron)	Principal references
GRS 1915-105	>0.98	0.98 ± 0.01	[28, 32]
Cyg X-1	>0.98	$0.97^{+0.014}_{-0.02}$	[14, 18, 19, 34, 48, 52]
GS 1354-645	—	>0.98	[13]
LMC X-1	0.92 ± 0.06	$0.97^{+0.02}_{-0.25}$	[16, 47]
GX 339-4	<0.9	0.95 ± 0.03	[15, 20, 35, 36]
MAXI J1836-194	—	0.88 ± 0.03	[38]
M33 X-7	0.84 ± 0.05	—	[27]
4U 1543-47	0.80 ± 0.10[a]	—	[41]
IC10 X-1	$\gtrsim 0.7$	—	[43]
Swift J1753.5	—	$0.76^{+0.11}_{-0.15}$	[37]
XTE J1650-500	—	$0.84 \sim 0.98$	[51]
GRO J1655-40	0.70 ± 0.10[a]	>0.9	[37, 41]
GS 1124-683	$0.63^{+0.16}_{-0.19}$	—	[4, 33]
XTE J1752-223	—	0.52 ± 0.11	[39]
XTE J1652-453	—	<0.5	[6]
XTE J1550-564	0.34 ± 0.28	$0.55^{+0.15}_{-0.22}$	[46]
LMC X-3	0.25 ± 0.15	—	[45]
H1743-322	0.2 ± 0.3	—	[44]
A0620-00	0.12 ± 0.19	—	[17]
XMMU J004243.6	<-0.2	—	[31]

[a] These sources were studied in an early work of the continuum-fitting method, within a more simple model, and therefore the published 1-σ error estimates are doubled following [29]

Temperature of the gas in the accretion structure

Fig. 7.1 Radial profile of the effective temperature T_{eff} of a Novikov–Thorne accretion disk in Kerr spacetime for different values of the spin parameter a_*. Here $M = 10\,M_\odot$ and $\dot{M} = 10^{18}\,\text{g}\,\text{s}^{-1}$



Next Lecture

10. Accretion Disks. Detailed Description