05. Binary Systems. Accretion Basics

Disc Formation by Roche Lobe Overflow

The mass transfer due to Roche lobe overflow has as a consequence that the transferred material has a high specific angular momentum, and therefore the accretion is <u>not</u> a direct proccess.

As seen from the black hole, the mass transfer that occurs through the point L_1 is seen as if matter spreads from a nozzle rotating around. Except for very long period binaries, this nozzle rotates so rapidly that the gas stream appears to move orthogonal to the line joining the members of the system.

We will analyze the motion from a non-rotating frame. Let $v_{\parallel}$ and $v_{\perp}$ be the parallel and orthogonal components of the stream velocity with respect to the instantaneous line of centres. Then

$$v_{\parallel} \lesssim c_s$$

where c_s is the sound speed in the envelope of the star. This value is considered because we assume that the gas is pushed through L_1 by pressure forces. For a normal stellar envelope temperatures, $< 10^5 K$, we have $c_s \lesssim 10 \text{ km/s}$.

On the other hand

$$v_{\perp} \sim b\omega = \frac{b\, 2\pi}{P}$$

Using

$$b \approx a\left[0.500 - 0.227 \log_{10} q\right] \gtrsim 0.5a \quad \text{for small } q$$

$$a = 3.5\times10^{10}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{hr}^{2/3} \text{ cm}$$

we have

$$V_\perp \gtrsim 0.5\frac{a\,2\pi}{P} = 1.1\times10^{11}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3}\frac{P_{hr}^{2/3}}{P_{hr}} \text{ cm/hr}$$

$$V_\perp \gtrsim 1.1\times10^{6}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{hr}^{-1/3} \text{ Km/hr}$$

$$V_\perp \gtrsim 305{,}6\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{hr}^{-1/3} \text{ Km/s}$$

Using the period in days,

$$a = 2.9\times10^{11}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{days}^{2/3} \text{ cm}$$

we obtain instead

$$V_\perp \gtrsim 0.5\frac{a\,2\pi}{P} = 9\times10^{11}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3}\frac{P_{day}^{2/3}}{P_{day}} \text{ cm/day}$$

$$V_\perp \sim 9\times10^{6}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{day}^{-1/3} \text{ Km/day}$$

$$V_\perp \sim 104.2\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3} P_{hr}^{-1/3} \text{ Km/s}$$

These estimates show that $v_\perp \gg v_\parallel$. Hence, in order to study the dynamics of the stream we can consider initially a particle released from rest at L_1 (i.e. pressure effects are neglected). Thus, the particle will describe an elliptical trajectory around the black hole and the prescense of the star make the ellipse to precess slowly.

If we consider now a stream of particles, their orbits will intersect, resulting in dissipation of energy (via collisions). Since angular momentum is conserved, the particles will tend to go to the orbit with minimum energy for a given angular momentum; i.e. a circular orbit. The radius of this orbit, called circularization radius r_{circ} is such that the corresponding Kepler orbit has the same angular momentum.

The circular velocity of the gas is

$$r_{circ}\, \Omega_K^2(r_{circ}) = \frac{v_\phi^2(r_{circ})}{r_{circ}} = \frac{GM}{r_{circ}^2}$$

$$v_\phi(r_{circ}) = \sqrt{\frac{GM}{r_{circ}}}$$

and the conservation of angulam momentum states

$$r_{circ}\, v_\phi(r_{circ}) = b^2 \omega$$

which gives

$$r_{circ} \sqrt{\frac{GM}{r_{circ}}} = b^2 \frac{2\pi}{P}$$

$$r_{circ}\, GM = b^4 \frac{4\pi^2}{P^2}$$

$$\frac{r_{circ}}{a} = \frac{4\pi^2}{GMP^2} a^3 \left(\frac{b}{a}\right)^4$$

but Kepler's third law, $a^3 = \frac{G(M+M_*)}{4\pi^2} P^2$, gives

$$\frac{r_{circ}}{a} = \frac{M+M_*}{M} \left(\frac{b}{a}\right)^4$$

$$\frac{r_{circ}}{a} = (1+q)\left(\frac{b}{a}\right)^4$$

Finally, using the expression for b,

$$\frac{r_{circ}}{a} = (1+q)\left[0.500 - 0.227 \log_{10} q\right]^4$$

Tipically, the circularization radius is smaller than the radius of the Roche Lobe of the primary,

$$r_{circ} < r_R .$$

For example for $q=1$ we have $r_{circ} = 0.125a$ and $r_R = 0.38a$

In terms of the period we can write

$$r_{circ} = (1+q)\left[0.500 - 0.227 \log_{10} q\right]^4 \times 2.9\times10^{11} \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3} P_{days}^{2/3} \text{ cm}$$

$$r_{circ} = (1+q)^{4/3}\left[0.500 - 0.227 \log_{10} q\right]^4 \left(\frac{M}{M_\odot}\right)^{1/3} P_{days}^{2/3} \times 2.9\times10^{9} \text{ m}$$

Considering the sun's radius

$$R_\odot \simeq 695\,000 \text{ Km} \sim 7 \times 10^8 \text{ m}$$

we obtain

$$r_{circ} = 4{,}1\,(1+q)^{4/3}\left[0.500 - 0.227 \log_{10} q\right]^4 \left(\frac{M}{M_\odot}\right)^{1/3} P_{days}^{2/3}\, R_\odot$$

Hence, the stream of gas moves in a ring at $r = r_{circ}$. In this ring we find collisions, shocks, viscous dissipation and other processes which will transform some of the potential energy into heat energy, which may be radiated. However, this release of energy needs the material to lose angular momentum. In absence of external torques the only possible process that would be useful to the description is the transfer of angular momentum outwards through the disk by internal torques.
If this is the case, the outer parts of the disk gain angular momentum, moving outwards, while the inner part spiral inwards; i.e. the original ring of matter spread to smaller and larger radii. [5]

Hence, the accretion disk extends from $r_{in} \geqslant r_{ms}$ to $r_{out} \leqslant r_R$.
The material moves in circular orbits with Keplerian angular velocity

$$\Omega_K(r) = \sqrt{\frac{GM}{r^3}}\,.$$

Viscous Torques

The Keplerian velocity $\Omega_K(r)$ depending on r implies that material at neighbouring radii moves with different velocity ; i.e. the disk has differential rotation and not solid body rotation.
Because of the thermal motion of the fluid molecules and the turbulent motion of the fluid, viscous stresses are generated. Here we will consider momentum transport in the radial direction; i.e. orthogonal to the direction of motion of the gas. The responsable process is known as shear viscosity, and it appears when there are internal distortions ; usualy local stresses are proportional to the local rate of strain. The coefficient of proportionality is the shear viscosity.

However, there are also non-local mechanisms for angular momentum transport, for example magnetic loops that couple fluid elements located at macroscopic distaces accross the disk, generating stresses that can not be modeled as proportional to local rate of strain.

Modelling Viscosity

Given a typical scale λ and speed $\tilde{v}$; it is possible to estimate the shear viscosity (see [1] Ch. 3)

As a first model, consider a uniform gas moving only in the x-direction with velocity $u(z)$. We will consider the x-momentum transport accross the plan $z = z_0$ due to the exchange of fluid elements between the levels $z_0 \pm \frac{\lambda}{2}$.

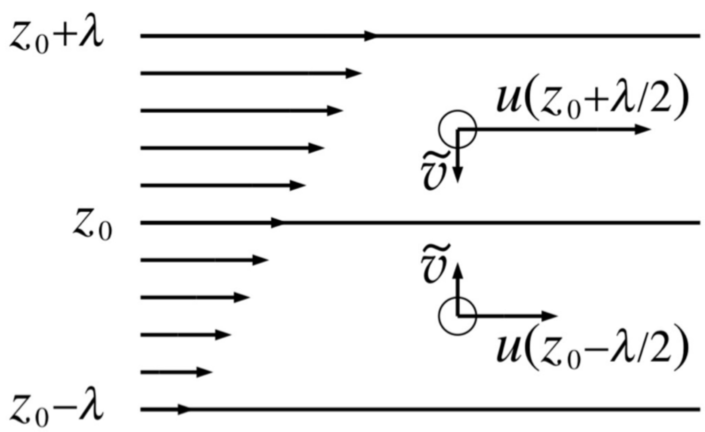

Fig. 4.7. Exchange of fluid blobs in a plane shearing flow.

We will assume that the fluid elements do not feel forces during their free motion between collisions; and hence linear and angular momentum are conserved. Assuming also that there is no net mass motion across z_0, the average upward and downward mass fluxes are equal, $\rho\tilde{v}$.

The net upward x-momentum flux density is

$$\sim \rho\tilde{v}\left[u\left(z_0-\frac{\lambda}{2}\right) - u\left(z_0+\frac{\lambda}{2}\right)\right]$$

$$\left[\frac{\text{mass}\cdot\text{velocity}^2}{\text{volume}}\right] = \left[\frac{\text{momentum}}{\text{area}\cdot\text{time}}\right]$$

The only non-vanishing component of the stress is the x-component of the force per unit surface of the z_0-plane,

dynamical viscosity

adimensional coefficient ~ 1

$$\sigma_{xz} = -\eta \frac{\partial u}{\partial z} = -f \rho \tilde{v} \lambda \left.\frac{\partial u}{\partial z}\right|_{z_0}$$

velocity gradient

typical lenght that a gas element travels between collisions

The definiton of the Kinematical viscosity is

$$\nu = \frac{\eta}{\rho}$$

which gives in this case

$$\nu \sim \lambda \tilde{v}.$$

Molecular transport: λ: mean free path
$\tilde{v}$: thermal speed

Turbulent Motion: λ: spatial scale or characteristic wavelenght of the turbulence

$\tilde{v}$: typical velocity of the eddies.

Accretion Disk

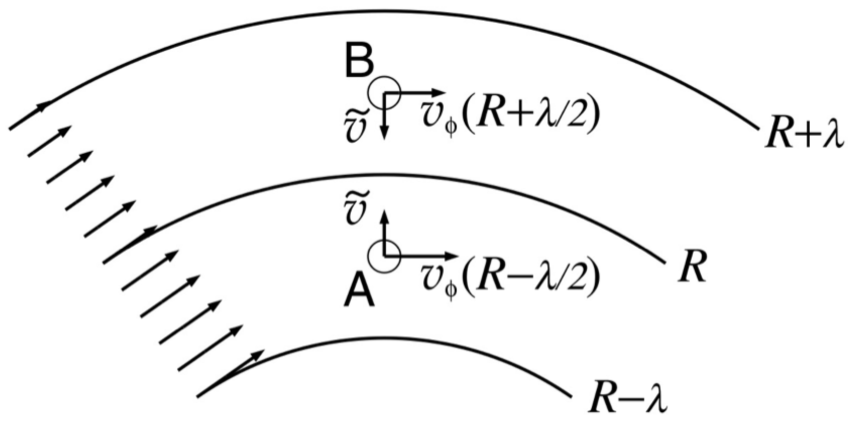

Fig. 4.8. Exchange of fluid blobs in differentially rotating flow.

The disk has azimuthal symmetry and lies between $z=0$ and $z=H$ in cylindrical coordinates. The speed of the gas is

$$V_\phi(r) = \Omega(r)\, r.$$

The elements A and B in the figure are exchanged through the surface $r=R$ with speeds $\tilde{v}$. They carry different amounts of angular momentum:

A $\rightarrow$ Angular momentum at $R-\frac{\lambda}{2}$

B $\rightarrow$ Angular momentum at $R+\frac{\lambda}{2}$

Conservation of linear momentum ?
Conservation of angular momentum ?

If no-interaction between fluid elements $\rightarrow$ only gravity $\downarrow$ angular momentum conserved

If we consider fluid eddis advected by the streaming fluid, then we need to consider pressure gradients.

→ In steady state the effects of rotation and pressure must cancel external force

→ Conservation of streamwise momentum.

The correct assumption is somewhere in between these two extremes.

We assume that the chaotic motion of the elements takes place in an equilibrium flow; and therefore there will be no net transfer of matter between the two rings located at $R \pm \frac{\lambda}{2}$. This means that the mass crossing rate through $r=R$ is the same in both directions,

$$\sim H \rho \tilde{v} \quad \text{(per unit arc lenght)}$$

where $\rho = \rho(r)$ is the mass density.

However the mass fluxes carry different angular momenta and hence there is a viscous torque (on the outer stream due to the inner stream and similarly on the inner due to the outer)

As in the first model (linear flow), we consider the ϕ-momentum flux per unit arc lenght through $r=R$ outwards as

$$\rho \tilde{v} H \left(R + \frac{\lambda}{2}\right) V_\phi\left(R - \frac{\lambda}{2}\right)$$

and inwards as

$$\rho \tilde{v} H \left(R - \frac{\lambda}{2}\right) V_\phi\left(R + \frac{\lambda}{2}\right).$$

The net torque exerted on the outer ring is given by the net outward angular momentum flux:

$$\sim \rho \tilde{v} H \left[\left(R+\frac{\lambda}{2}\right) V_\phi\left(R-\frac{\lambda}{2}\right) - \left(R-\frac{\lambda}{2}\right) V_\phi\left(R+\frac{\lambda}{2}\right) \right]$$

$$\sim \rho \tilde{v} H \left[\left(R+\frac{\lambda}{2}\right)\left(R-\frac{\lambda}{2}\right) \Omega\left(R-\frac{\lambda}{2}\right) - \left(R-\frac{\lambda}{2}\right)\left(R+\frac{\lambda}{2}\right) \Omega\left(R+\frac{\lambda}{2}\right) \right]$$

Up to first order in λ we have

$$\sim \rho \tilde{v} H \left[R^2 \Omega\left(R-\frac{\lambda}{2}\right) - R^2 \Omega\left(R+\frac{\lambda}{2}\right) \right]$$

$$\sim -\rho \tilde{v} H R^2 \lambda \left[\frac{\Omega\left(R+\frac{\lambda}{2}\right) - \Omega\left(R-\frac{\lambda}{2}\right)}{\lambda} \right]$$

Since λ is small $(\lambda \to 0)$ we write

$$\frac{\text{Torque}}{\text{per unit arc lenght}} \sim -\rho \tilde{v} H R^2 \lambda \left.\frac{d\Omega}{dr}\right|_R$$

Therefore, the non-vanishing component of the stress is the force in the ϕ-direction per unit area:

$$\sigma_{r\phi} = -\eta R \left.\frac{d\Omega}{dr}\right|_R \sim -\rho \tilde{v} \lambda R \left.\frac{d\Omega}{dr}\right|_R$$

which gives the kinematic viscosity

$$\nu = \frac{\eta}{\rho} \sim \lambda \tilde{v}$$

Now we can write

$$\underset{\text{per unit arc lenght}}{\text{Torque}} \sim -\rho H R^2 \nu \left.\frac{d\Omega}{dr}\right|_R$$

In order to obtain the total torque, we multiply by the total arc lenght $2\pi R$

$$\text{Torque} \sim -2\pi R \rho H R^2 \nu \left.\frac{d\Omega}{dr}\right|_R$$

and we may use the surface density

$$\Sigma = \int_0^H \rho \, dz = \rho H$$

assuming $\rho = \rho(r)$

to obtain

$$\text{Torque} \sim -2\pi R \Sigma R^2 \nu \left.\frac{d\Omega}{dr}\right|_R$$

Torque on the outer ring due to the inner

Now we will define the function $G(r)$ as the torque exerted by the outer ring on the inner ring as function of r;

$$G(r) = 2\pi r \nu \Sigma r^2 \frac{d\Omega}{dr}$$

This function satisfies

i) If $\frac{d\Omega}{dr} = 0$ (Rigid body rotation) $\longrightarrow$ No torque.

ii) If $\Omega(r)$ decreases outwards, $\frac{d\Omega}{dr} < 0$ and

$G(r) < 0$ → angular momentum goes from the inner circles to the outer rings.

→ The gas slowly spirals in.

Now we will consider the net torque on a ring between r and $r+dr$. This gives

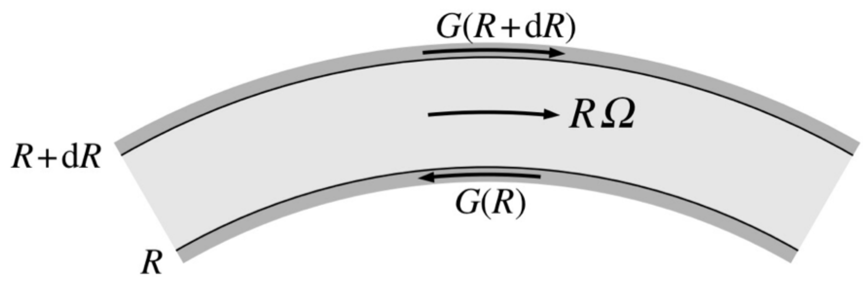

Fig. 4.9. Differential viscous torque.

$$G(r+dr) - G(r) = \frac{\partial G}{\partial r} dr$$

The work done by this torque is

$$\Omega(r) \frac{\partial G}{\partial r} dr = \left[\frac{\partial}{\partial r}\left[\Omega(r) G(r)\right] - G(r) \frac{d\Omega}{dr} \right] dr.$$

The first term, $\frac{\partial}{\partial r}\left[\Omega(r) G(r)\right] dr$, is the rate of "convection" of the rotational energy through the gas by the torques. When integrating this expression over the whole disk, we obtain

$$\int_{r_{in}}^{r_{out}} \frac{\partial}{\partial r}\left[\Omega(r) G(r)\right] dr = \Omega(r) G(r) \Big|_{r_{in}}^{r_{out}}$$

so its contribution is determined solely by boundary conditions.
On the other hand, the second term, $-G(r)\frac{d\Omega}{dr}dr$, corresponds to the local rate of loss of mechanical energy, which is transformed into heat (i.e. internal energy).

Viscous torques $\longrightarrow$ viscous dissipation at rate

$$G\frac{d\Omega}{dr}dr \quad \left(\text{per ring of width } dr\right)$$

Since each ring of width dr has two sides of area $2\pi r\,dr$, we can define the dissipation rate per unit plane surface area as:

$$D(r) = \frac{G(r)\frac{d\Omega}{dr}dr}{2 \cdot 2\pi r\,dr}$$

$$D(r) = \frac{2\pi r \nu \Sigma r^2 \frac{d\Omega}{dr}\frac{d\Omega}{dr}}{4\pi r}$$

$$\boxed{D(r) = \frac{1}{2}\nu\Sigma\left(r\frac{d\Omega}{dr}\right)^2}$$

Note that $D(r) \geq 0$; i.e. it always dissipate. The zero dissipation corresponds to $\frac{d\Omega}{dr} = 0$ (rigid body).

Using the Keplerian angular velocity,

$$\Omega(r) = \Omega_K(r) = \sqrt{\frac{GM}{r^3}}$$

we obtain

$$\frac{d\Omega_K}{dr} = \sqrt{GM}\left(-\frac{3}{2}\, r^{-5/2}\right) = -\frac{3}{2}\frac{\sqrt{GM}}{r^2\, r^{1/2}}$$

and thus

$$D(r) = \frac{1}{2}\nu\Sigma\left(-r\,\frac{3}{2}\frac{\sqrt{GM}}{r^2 r^{1/2}}\right)^2$$

$$D(r) = \frac{9}{8}\nu\Sigma\sqrt{\frac{GM}{r^3}}\,.$$

THE α-PRESCRIPTION FOR VISCOSITY

The force density produced by the shear viscosity is

$$f_v \sim \rho \lambda \tilde{v} \frac{\partial^2 v_\phi}{\partial r^2} \sim \rho \lambda \tilde{v} \frac{v_\phi}{r^2}$$

and points in the ϕ-direction.
In order to estimate the importance of viscosity, it is defined the Reynolds number:

$$Re = \frac{\text{inertia}}{\text{viscous}} \sim \frac{\Omega^2 r}{\lambda \tilde{v} \frac{v_\phi}{r^2}} = \frac{v_\phi^2 / r}{\lambda \tilde{v} v_\phi / r}$$

$$Re \sim \frac{r v_\phi}{\lambda \tilde{v}}$$

If $Re \ll 1 \rightarrow$ viscous force dominates the flow

$Re \gg 1 \rightarrow$ viscosity associated with λ and $\tilde{v}$ is unimportant.

For "molecular" viscosity one obtains (see [1] Ch 3&4)

$$Re_{mol} \sim 0.2 \; N \left(\frac{M}{M_\odot}\right)^{1/2} R_{10}^{1/2} \; T_4^{-5/2}$$

N: gas density in cm^{-3}
T_4: Temperature of the gas in $10^4 K$ units
R_{10}: Distance from the primary in units of 10^{10} cm

In a typical accretion disk,

$$\left(\frac{M}{M_\odot}\right)^{1/2} \sim 10$$

$$R_{10}^{1/2} \sim T_4^{-5/2} \sim 1$$

$$N > 10^{15}\ \mathrm{cm}^{-3}$$

Thus, $\quad Re_{mol} > 10^{15}$

This means that molecular viscosity is too weak for the needed dissipation and momentum transport.

The correct viscosity to explain accretion disks is due to turbulence.

Laboratory $\rightarrow$ if $Re_{mol} > Re_{mol\,crit} \sim 10\text{-}10^3$
$\rightarrow$ Turbulence appears!

Thus, it is expected turbulence in accretion disks (although there is no proof!)

Turbulence $\rightarrow$ the fluid velocity begins to exhibit large and chaotic variations

$$\nu_T \sim \tilde{v}_T\, \lambda_T$$

? ?

No complete description of turbulence.
But we can estimate some limits for λ_T and $\tilde{v}_T$

The largest eddies can not exceed the disk thickness

$\lambda_T \lesssim H$

It is unlikely that the velocity is supersonic,

$\tilde{v} \lesssim c_s$

Then, Shakura & Sunyaev [2] proposed the α-prescription

$$\nu_T = \alpha c_s H$$

α includes all our ignorance about viscosity

α may be constant or a function
It is expected $\alpha \lesssim 1$
α may be evaluated from observation?

Some theoretical considerations expect
$\alpha \sim 0.1$ for Cataclysmic Variables (CV)

$\alpha \lesssim 1$ for an α-modelled viscosity due to the winding-up of chaotic magnetic fields in the disk.

ORIGIN OF THE TURBULENCE

Mechanisms

- **Thermally driven convection.**
  The estimates considering the angular momentum transport by convection gives a value of α smaller than required by observation.
  Viscosity by convection is importan around protostars and young stars.
  Modern analytical and numerical thechiques show that the viscosity (magnitude and sign) produced by convection depends on the particular model.

  * Local normal mode analysis → negative viscosity
  * Based on Boltzmam equation → particles or blobs as in planetary rings

- **Pure hydrodynamic instabilities**
  Accretion disks are linearly stable to pure hydrodynamic perturbations. Maybe finite disturbances produce the turbulence? yet to be proben!
  Some theoretical arguments and simulations show that pure local hydrodynamic turbulences can not be self-sustained in accretion disks [3]. Some authors propoose that some extra effects such as tides or spiralwaves may help.

- **Magnetohydrodynamic (MHD) Turbulence**
  Promising mechanism, right viscosity (magnitude & sign)
  $\alpha \sim 0.01 - 0.1$
  The turbulence may be self-sustained and even grow.
  Active field for research! (see Sec. 11.2 of [4])

ACCRETION BY STELLAR WIND

This mechanism is relevant when the companion is an early type (O-B) star.

Cen X-3
SMC X-1
Vela X-1

} Neutron Star spinning → X-ray pulses

Cyg X-1 : Black hole candidate

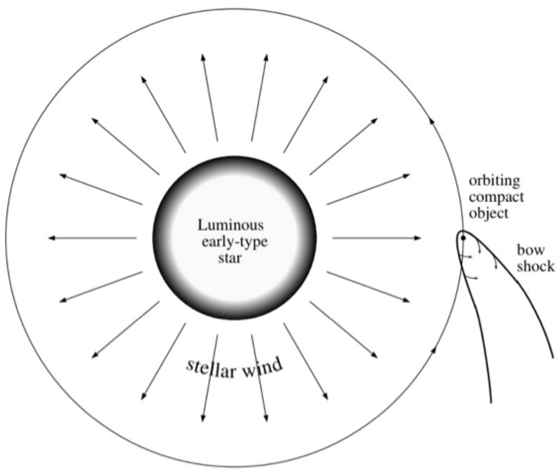

Fig. 4.10. Compact star accreting from the stellar wind of an early-type companion.

In early-type stars, the wind may produce mass loss rates $\sim 10^{-6} - 10^{-5}\ M_\odot\ yr^{-1}$

The velocity of the wind is highly supersonic,

$$v_w(r) \sim v_{esc}(R_*) = \sqrt{\frac{2GM_*}{R_*}} \sim 10^3\ km/s$$

while $c_s \sim 10\ km/s$

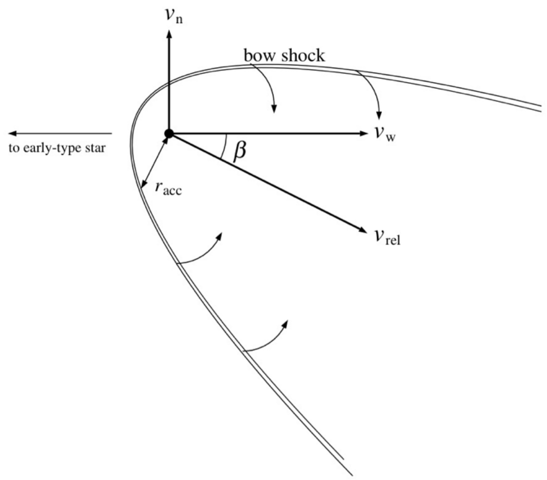

Fig. 4.11. Bow shock and accretion radius in stellar wind accretion.

V_n: velocity of the compact object (B.H or neutron star) relative to the star

V_w: Velocity of the wind

The wind sweeps past the neutron star at an angle

$\beta = \mathrm{Tan}^{-1}\left(\frac{V_n}{V_w}\right)$ with respect to the line of centres

The relative speed with respect to the compact object is

$$V_{rel} = \sqrt{V_n^2 + V_w^2}.$$

Since the wind is supersonic → - neglect gas presion
- the flow is a collection of particles

Particles moving close enough will be captured by the compact object (if its Kinetic energy is less than the gravitational potential).

Region of capture: cylinder with axis along the relative wind direction ($\vec{V}_{rel}$) and radius

$$r_{acc} \sim \frac{2GM}{V_{rel}^2}$$

Consider the approximation $v_w \gg v_n$
Then $\beta \approx 0$

$$v_{rel} \approx v_w$$

The fraction of the stellar wind captured by the compact object can be estimated by comparing the mass flux into the accretion cylinder,

$$\dot{M} \sim \pi r_{acc}^2 \, v_w(a),$$ (a is the binary separation)

with the total mass loss rate,

$$-\dot{M}_w \sim 4\pi a^2 \, v_w(a).$$

This gives

$$\frac{\dot{M}}{-\dot{M}_w} \sim \frac{\pi r_{acc}^2 \, v_w(a)}{4\pi a^2 \, v_w(a)} = \frac{r_{acc}^2}{4a^2}$$

$$\frac{\dot{M}}{-\dot{M}_w} \sim \frac{G^2 M^2}{a^2 v_{rel}^4} = \frac{G^2 M^2}{a^2 v_w^4(a)}$$

Using the escape velocity,

$$\frac{\dot{M}}{-\dot{M}_w} \sim \frac{G^2 M^2}{a^2} \left(\frac{R_*}{2GM_*}\right)^2$$

$$\frac{\dot{M}}{-\dot{M}_w} \sim \frac{1}{4} \left(\frac{M}{M_*}\right)^2 \left(\frac{R_*}{a}\right)^2$$

Takin $M_* \sim 5M$ and $R_* \sim 0.5\,a$

$$\frac{\dot{M}}{-\dot{M}_w} \sim \frac{1}{4}\left(\frac{1}{5}\right)^2 \left(0.5\right)^2 \sim 10^{-3}$$

In fact, typical parameters for X-ray binaries give values of $\dot{M} \sim 10^{-4} - 10^{-3}(-\dot{M}_w)$
Thus, accretion by stellar wind is very inefficient (just a small mass is accreted while in the Roche Lobe overflow almost all matter is accreted).
However, since mass-loss is large $(-\dot{M}_w \sim 10^{-6} - 10^{-5}\, M_\odot\, yr^{-1})$ we have

$$\dot{M} \sim 10^{-10} - 10^{-8}\, M_\odot\, yr^{-1}$$

which is comparable with the value for the Roche Lobe overflow and average observed accretion rates $\sim 10^{15} - 10^{18}\, gr\, s^{-1}$ $(\sim 10^{-11} - 10^{-8}\, M_\odot\, yr^{-1})$ [1]

Now lets consider the angular momentum of the material

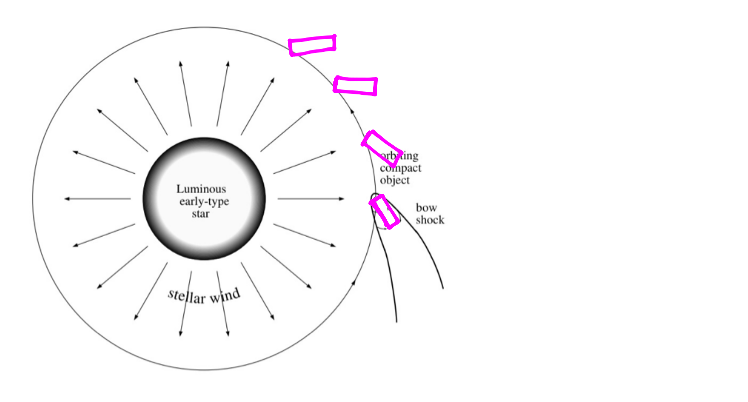

Fig. 4.10. Compact star accreting from the stellar wind of an early-type companion.

The accretion cylinder rotates about the companion with $\omega = \frac{V_n}{a}$

The angular momentum is that of a flat circular disk of radius r_{acc} rotating rigidly about a diameter

$$l \sim \frac{1}{4} r_{acc}^2\, \omega$$

Remember that for the Roche Lobe overflow mechanism we obtained $\sim b^2 \omega \sim 0.500\, a^2 \omega$ (for small q)
Then, it is clear that ℓ is smaller than the corresponding value for Roche Lobe overflow by a factor $\sim \left(\frac{r_{acc}}{a}\right)^2$ so chances of disk formation are smaller.

Let us define the circularization radius as before,

$$V_{circ} = \frac{\ell}{r_{circ}} = \sqrt{\frac{GM}{r_{circ}}}$$

$$\ell \sim \frac{1}{4} r_{acc}^2 \omega = \sqrt{GM r_{circ}}$$

$$\frac{1}{4}\left(\frac{2GM}{V_{rel}^2}\right)^2 \omega = \sqrt{GM r_{circ}}$$

$$\left[\frac{G^2 M^2}{V_{rel}^4} \omega\right]^2 = GM\, r_{circ}$$

$$r_{circ} = \frac{G^3 M^3 \omega^2}{V_{rel}^8} \sim \frac{G^3 M^3 \omega^2}{V_w^8}$$

Considering $V_w^2 = \lambda(r)\, V_{esc}^2(R_*)$ with $\lambda(r) \sim 1$ we have

$$r_{circ} = \frac{G^3 M^3 \omega^2}{\lambda^4(a)\, V_{esc}^8(R_*)} = \frac{G^3 M^3}{\lambda^4(a)\, V_{esc}^8(R_*)} \frac{4\pi^2}{P^2}$$

and from Kepler's third law, $4\pi^2 a^3 = G(M + M_*) P^2$, we have

$$r_{circ} = \frac{G^3 M^3}{\lambda^4(a)\, V_{esc}^8(R_*)} \frac{G(M+M_*)}{a^3}$$

$$r_{circ} = \frac{M^3(M+M_*)}{\lambda^4(a)} \frac{G^4}{V_{esc}^8(R_*)\, a^3}$$

$$r_{circ} = \frac{M^3(M+M_*)}{\lambda^4(a)} \frac{R_*^4}{16 M_*^4 a^3}$$

$$\frac{r_{circ}}{a} = \frac{M^3(M+M_*)}{16\, \lambda^4(a)\, M_*^4} \left(\frac{R_*}{a}\right)^4$$

Wind law $\lambda(r)$ is unknown and because it appears with a 4-th power, a small uncertainty affects strongly the estimate of r_{circ}.

Accretion radius r_{acc} is just an approximation because the highly supersonic speed of the wind produces a strong bow shock that may change the description.

OTHER METHODS OF MASS TRANSFER IN BINARY SYSTEMS

* **Eccentric orbits** (i.e. before circularization)
  If the periastron separation of the system is small enough, some matter can be pulled off the envelope of the star.
  There are bursts of accretion (and emission) each time the star reaches the periastron. (see Sec. 6.3 [6])
  This type of mass transfer is a short-lived phenomenon (it ends once circularization occurs)

* Asynchronous Rotation (i.e. before synchronization)
  Similar as above situation. An asynchronous rotating star may loose mass without filling its Roche Lobe.
  This type of mass transfer is also a short-lived phenomenon (it ends once synchronization occurs)

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