BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

10. Viscous Torques

Edward Larrañaga

Outline for Part 1

Viscous Torques

Accretion Disk

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- In the description of the aaccretion disk it is needed a force responsible for the redistribution of angular momentum.

Differential Rotation

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- The disk is not a solid body. It moves with *differential rotation*.
- The thermal motion of the fluid molecules and the turbulent motion of the fluid produce *viscous stresses*.

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- It appears when there are internal distortions (usually local stresses that are proportional to the local rate of strain)
- Although the following is the simplest description, it can be used to describe other mechanisms of angular momentum transport such as magnetic loops that couple fluid elements at macroscopic distances across the disk.

Modeling Viscosity

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- As a first model, consider a uniform gas moving only in the tangential direction woth velocity $v_\phi(r)$.

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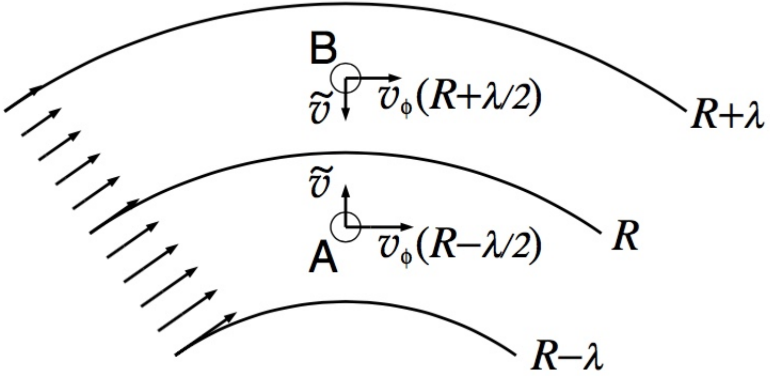

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η: dynamical viscosity

$v_\phi(r) = \Omega_k(r) r$
$\frac{\partial v_\phi}{\partial r}$: velocity gradient

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Turbulent motion:
λ: spatial scale (or characteristic wavelength) of the turbulence
$\tilde{v}$: typical velocity of the eddies

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H: Height of the disk

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  $\longrightarrow$ angular momentum goes from inner circles to outer circles.
  $\longrightarrow$ The gas slowly spirals in!

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For $0.1 \leq q \leq 0.8$ there is the Paczynski approximation

$$\frac{R_{*R}}{a} \approx \frac{2}{3^{4/3}\left(\frac{q}{1+q}\right)^{1/3}}$$

$$\frac{R_{*R}}{a} \approx 0.462\left(\frac{M_*}{M + M_*}\right)^{1/3}$$

Self-sustainability of the Roche Overflow

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\begin{aligned}
\text{Roche overflow} &\longrightarrow \text{change } q \\
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If Roche lobe grows $\longrightarrow$ overflow stops

If Roche lobe shrinks $\longrightarrow$ overflow continues!

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$$a = \frac{J^2}{G} \frac{M + M_*}{M^2 M_*^2}$$

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For conservative systems: BH grows putting more mas near the CM and the star moves in a wider orbit, increasing a, in order to conserve J.

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  The Roche overflow is rapid and violent but stops when q is smaller than $\frac{5}{6}$.

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The transfer of mass continues if

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  - Tidal forces on the star
  - Wind (magnetically linked to the star)

Outline for Part 2

1. Viscous Torques
    1.1 Differential Rotation
    1.2 Roche Lobe Overflow
2. Formation of the Accretion Disk

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$v_{||}$: parallel component of the velocity with respect to the line of centers.
$v_{\perp}$: perpendicular component of the velocity with respect to the line of centers.

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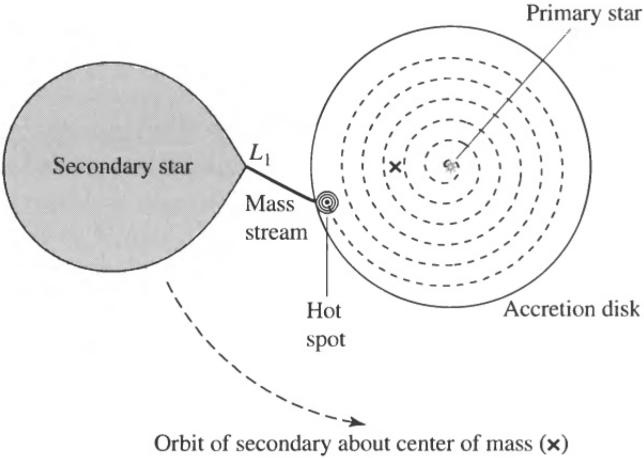

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Using

$$b \approx a\left[0.500 - 0.227\log_1 0\left(q\right)\right]$$

$$a = 3.5 \times 10^{10}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3}\,\tau_{hr}^{2/3}\ [\mathrm{cm}]$$

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we get

$$v_\perp \gtrsim 305.6 \left(\frac{M}{M_\odot}\right)^{1/3} (1+q)^{1/3}\,\tau_{hr}^{-1/3}\ [\mathrm{km/s}]$$

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- The particle will describe an elliptical trajectory around the BH but the presence of the star will make this ellipse to precess slowly.

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- The radius of the resulting circular orbit is called *circularization radius*, r_{circ}.

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$$v_\phi(r_{circ}) = \sqrt{\frac{GM}{r_{circ}}}$$

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Formation of the Accretion Disk

$$r_{circ} = (1+q)^{4/3}\left[0.500 - 0.227\log_{10} q\right]^4 \left(\frac{M}{M_\odot}\right)^{1/3} \tau_{days}^{2/3} \; [R_\odot]$$

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- The redistribution of angular momentum makes particles in the outer parts move outwards (gaining angular momentum) and the particles in the inner particles spiral inwards.

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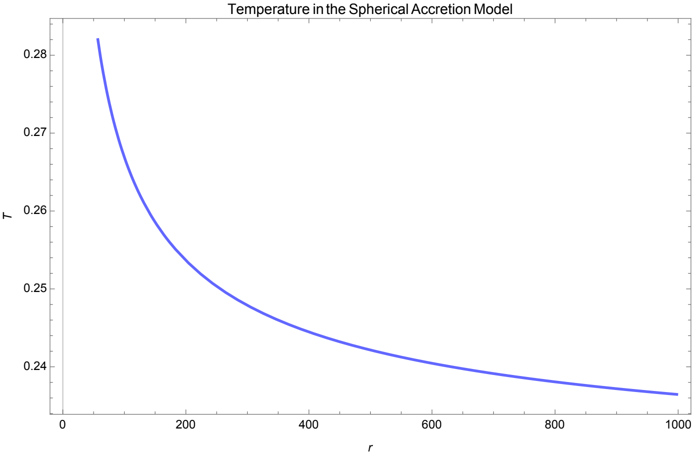
Temperature of the gas in the accretion structure



Next Lecture

10. Accretion Disks. Detailed Description