BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

10. Viscous Torques

Edward Larrañaga

Outline for Part 1

Viscous Torques

Accretion Disk

- If the only interaction between fluid elements is gravity, the angular momentum is conserved.

Accretion Disk

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- In the description of the accretion disk it is needed a force responsible for the redistribution of angular momentum.

Differential Rotation

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- Keplerian Velocity

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- Neighboring material at different radii moves with different velocity
- The disk is not a solid body. It moves with *differential rotation*.
- The thermal motion of the fluid molecules and the turbulent motion of the fluid produce *viscous stresses*.

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- It appears when there are internal distortions (usually local stresses that are proportional to the local rate of strain)
- Although the following is the simplest description, it can be used to describe other mechanisms of angular momentum transport such as magnetic loops that couple fluid elements at macroscopic distances across the disk.

Modeling Viscosity

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- $\tilde{v}$: typical speed in the accretion disk

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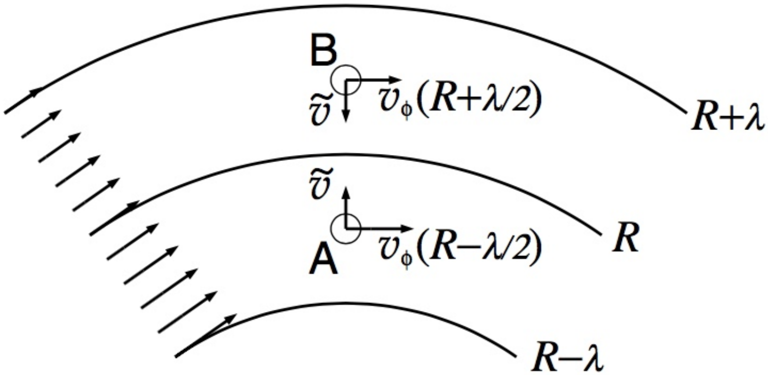
- As a first model, consider a uniform gas moving only in the tangential direction woth velocity $v_\phi(r)$.

Modeling Viscosity



Modeling Viscosity

The only non-vanishing component of the stress tensor is the ϕ-component of the force per unit surface of the $r =$constant surface,

$$\sigma_{r\phi} = -\eta \left. \frac{\partial v_\phi}{\partial r} \right|_R$$

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η: dynamical viscosity

$v_\phi(r) = \Omega(r)r$
$\frac{\partial v_\phi}{\partial r}$: velocity gradient

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H: Height of the disk

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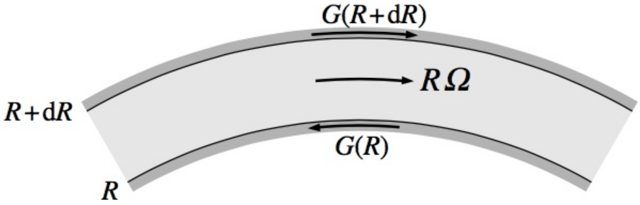
  $\longrightarrow$ The gas slowly spirals in!

Differential viscous torque



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and the associated power is

$$P = \frac{\partial G}{\partial r} \Omega dr = \left[\frac{\partial}{\partial r} (G(r)\Omega(r)) - G(r) \frac{\partial \Omega}{\partial r} \right] dr$$

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$\frac{\partial}{\partial r} \left(G(r)\Omega(r) \right) dr$: Rate of *convection* of the rotational energy through the gas by the torques.

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The zero corresponds to the rigid body, $\left(\frac{\partial\Omega}{\partial r} = 0\right)$

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$$\Omega = \Omega_k(r) = \sqrt{\frac{GM}{r^3}}$$

$$D(r) = \frac{9}{8}\nu\Sigma\frac{GM}{r^3}$$

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Surface mass density: $\Sigma = \Sigma(t, r)$

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$$r\frac{\partial}{\partial t}\left(\Sigma r^2 \Omega\right) + \frac{\partial}{\partial r}\left(r\Sigma v_r r^2 \Omega\right) = \frac{1}{2\pi}\frac{\partial G}{\partial r}$$

Radial Structure Evolution

Equations determining the radial structure of the accretion disk:

$$r\frac{\partial\Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma v_r) = 0$$

$$r\frac{\partial}{\partial t}\left(\Sigma r^2\Omega\right) + \frac{\partial}{\partial r}\left(r\Sigma v_r r^2\Omega\right) = \frac{1}{2\pi}\frac{\partial G}{\partial r}$$

$$G(r,t) = 2\pi r\nu\Sigma r^2\frac{\partial\Omega}{\partial r}$$

Equation for the kinematical viscosity ν

Radial Structure Evolution

Combining the equations determining the radial structure of the accretion disk and assuming that $\frac{\partial \Omega}{\partial t} = 0$, we obtain

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Eliminating the velocity, v_r, we have

$$r\frac{\partial \Sigma}{\partial t} = -\frac{\partial}{\partial r}\left[\frac{1}{2\pi\frac{\partial(r^2\Omega)}{\partial r}}\frac{\partial G}{\partial r}\right]$$

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This is a *nonlinear diffusion equation* for Σ and one needs to know the function ν to solve it.

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The radial velocity becomes

$$v_r = -\frac{3}{\Sigma r^{1/2}}\frac{\partial}{\partial r}\left[\nu \Sigma r^{1/2}\right]$$

Example of Viscosity

Consider the particular case in which $\nu =$constant.

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$$\frac{\partial}{\partial t}\left(\Sigma r^{1/2}\right) = \frac{3\nu}{r}\left(r^{1/2}\frac{\partial}{\partial r}\right)^2\left(\Sigma r^{1/2}\right)$$

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T: Exponential function

S: Bessel function

Example of Viscosity

The solution of this equation considering the initial distribution

$$\Sigma(r, t = 0) = \frac{m}{2\pi r_0}\delta(r - r_0)$$

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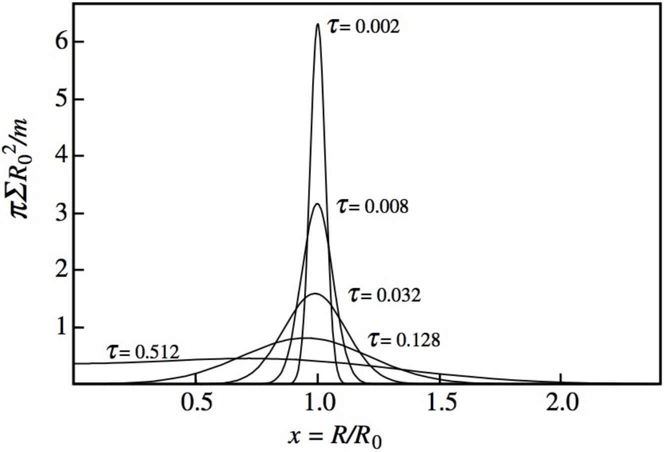
$I_{1/4}(z)$: Modified Bessel function
$x = \frac{r}{r_0}$
$\tau = \frac{12\nu t}{r_0^2}$

Formation of the Accretion Disk



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- The time scale of the spreading is (note the argument in the exponential function $(1 + x^2)\tau^{-1} \sim x^2\tau^{-1} \sim 1$)

$$t_{visc} \sim \frac{r^2}{\nu}$$

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- The time scale of the spreading can be written as

$$t_{visc} \sim \frac{r}{v_r}$$

This is called the *viscous* or *radial drift timescale*.

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Molecular transport:

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$\tilde{v}$: thermal speed

Turbulent motion:

λ: spatial scale (or characteristic wavelength) of the turbulence

$\tilde{v}$: typical velocity of the eddies

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$$Re = \frac{\text{inertia}}{\text{viscosity}} \sim \frac{\Omega^2 r}{\lambda \tilde{v} v_\phi / r^2}$$

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If $Re \gg 1$: Viscosity is unimportant.

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N: Gas density in cm^{-3}. $\left[N > 10^{15}\ \mathrm{cm}^{-3}\right]$
T_4: Temperature of the gas in $10^4\ \mathrm{K}$ units. $[T_4 \sim 1]$
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Molecular viscosity is too weak for the needed dissipation and angular momentum transport.

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But, how to model turbulence viscosity?

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The velocity is unlikely to be supersonic,

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α is a constant or a function that includes all our ignorance about viscosity.

Outline for Part 2

Novikov-Thorne Thin Disks

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- Has four parameters: BH mass, BH spin, mass accretion rate and viscosity parameter.

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- The accretion disk is geometrically thin, namely the disk opening angle is $h/r \ll 1$, where $H(r)$ is the semi-thickness of the disk at the radial coordinate r .

Novikov-Thorne Thin Disks

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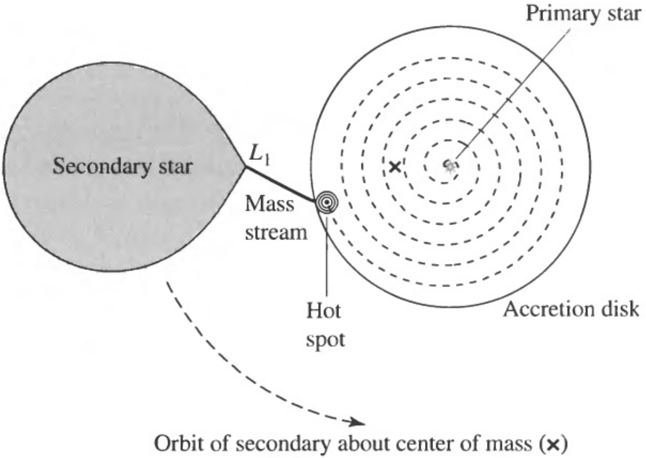
- The effect of energy and angular momentum transport by photons emitted from the disk and returning to the disk due to strong light bending in the vicinity of the black hole (returning radiation) is neglected.

Formation of the Accretion Disk



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Using

$$b \approx a\left[0.500 - 0.227\log_1 0\left(q\right)\right]$$

$$a = 3.5 \times 10^{10}\left(\frac{M}{M_\odot}\right)^{1/3}(1+q)^{1/3}\,\tau_{hr}^{2/3}\ [\mathrm{cm}]$$

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- The particle will describe an elliptical trajectory around the BH but the presence of the star will make this ellipse to precess slowly.

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- The radius of the resulting circular orbit is called *circularization radius*, r_{circ}.

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$$r_{circ} = (1+q)^{4/3}\left[0.500 - 0.227\log_{10} q\right]^4 \left(\frac{M}{M_\odot}\right)^{1/3} \tau_{days}^{2/3}\ [R_\odot]$$

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- In the absence of external torques, the only possible process is a *transfer of angular momentum* from inner regions outwards by internal torques.
- The redistribution of angular momentum makes particles in the outer parts move outwards (gaining angular momentum) and the particles in the inner particles spiral inwards.

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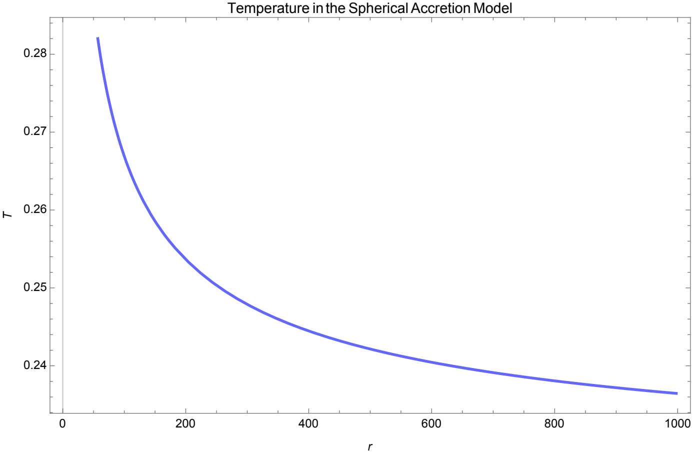
Temperature of the gas in the accretion structure



Next Lecture

10. Accretion Disks. Detailed Description