BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

09. Accretion in Binary Systems

Edward Larrañaga

Outline for Part 1

Accretion by Roche Lobe Overflow

Binary Systems

Binary System Geometry

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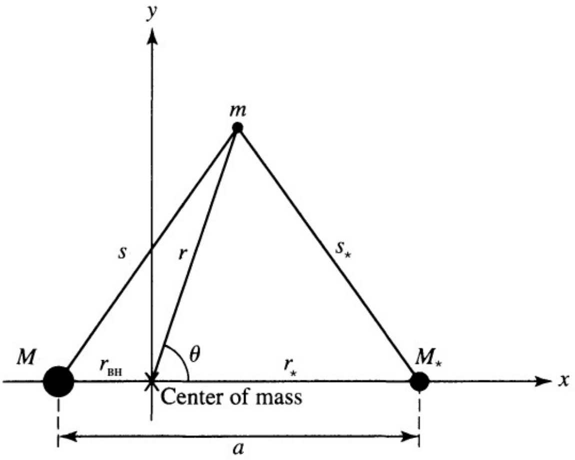
- This process may dissipate orbital and rotational energy until the system reaches the state of minimum energy for constant angular momentum: *synchronous rotation in circular orbits*

Binary System Geometry



Binary System Geometry

Center of mass:

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Angular velocity of the system:

$$\omega = \frac{v_*}{r_*} = \frac{v_{BH}}{r_{BH}}$$

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Using:

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$$s^2 = r^2 + r_{BH}^2 + 2rr_{BH}\cos\theta$$

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Kepler's third law

$$\omega^2 = \left(\frac{2\pi}{\tau}\right)^2 = \frac{G(M+M_*)}{a^3} = \frac{G(M+M_*)}{(r_{BH}+r_*)^3}$$

Lagrange Points

Lagrange Points:

$$\begin{cases} \frac{\partial \Phi}{\partial x} & = 0 \\ \frac{\partial \Phi}{\partial y} & = 0 \end{cases}$$

Lagrange Points

The location of the point L_1 from M_{BH} and from M_* are given by the approximations

$$b \approx a\left[0.500 - 0.227\log_1 0\,(q)\right]$$

$$b_* \approx a\left[0.500 + 0.227\log_1 0\,(q)\right]$$

B. Peterson. *An Introduction to Active galactic Nuclei*. Cambrdige University Press. (1997)
B. W. Carroll and D. A. Ostlie. *An Introduction to Modern Astrophysics*. Addison-Wesley (1996)

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- Since pressure is due to the weight of the overlaying layer of material, density has a constant value along the equipotential.

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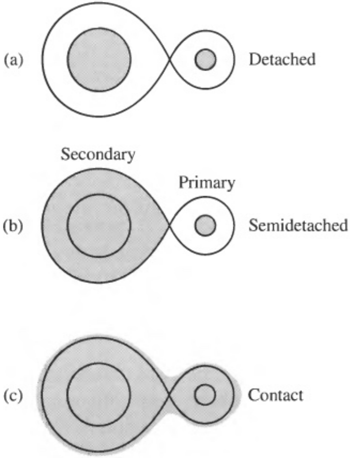

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$q = \frac{M_*}{M}$

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For $0.1 \leq q \leq 0.8$ there is the Paczynski approximation

$$\frac{R_{*R}}{a} \approx \frac{2}{3^{4/3}\left(\frac{q}{1+q}\right)^{1/3}}$$

$$\frac{R_{*R}}{a} \approx 0.462\left(\frac{M_*}{M + M_*}\right)^{1/3}$$

Self-sustainability of the Roche Overflow

$$\begin{aligned} \text{Roche overflow} &\longrightarrow \text{change } q \\ &\longrightarrow \text{change } a, \tau \\ &\longrightarrow \text{Roche lobe ?} \end{aligned}$$

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If Roche lobe shrinks $\longrightarrow$ overflow continues!

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$$a = \frac{J^2}{G} \frac{M + M_*}{M^2 M_*^2}$$

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For conservative systems: BH grows putting more mas near the CM and the star moves in a wider orbit, increasing a, in order to conserve J.

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  The Roche overflow is rapid and violent but stops when q is smaller than $\frac{5}{6}$.

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The transfer of mass continues if

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  - Tidal forces on the star
  - Wind (magnetically linked to the star)

Outline for Part 2

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$v_{||}$: parallel component of the velocity with respect to the line of centers.
$v_{\perp}$: perpendicular component of the velocity with respect to the line of centers.

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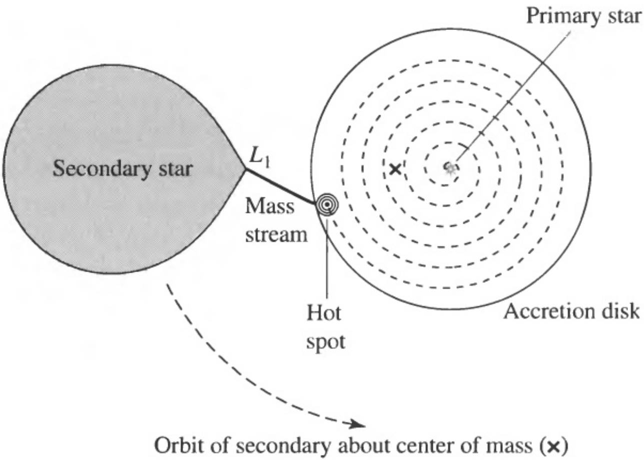

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we get

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- Because of the motion of the star, this particle is moving with velocity $v_\perp$ as seen from the BH.
- The particle will describe an elliptical trajectory around the BH but the presence of the star will make this ellipse to precess slowly.

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- However, angular momentum will be conserved in this collisions.
- Hence, particles will go into the trajectories with minimum energy for a given angular momentum, i.e. circular orbits!
- This process is called *circularization*.
- The radius of the resulting circular orbit is called *circularization radius*, r_{circ}.

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$$v_\phi\left(r_{circ}\right) = \sqrt{\frac{GM}{r_{circ}}}$$

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and using Kepler's third law,

$$\frac{r_{circ}}{a} = (1+q) \left(\frac{b}{a}\right)^4$$

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$$\frac{r_{circ}}{a} = \frac{4\pi^2}{GM\tau} a^3 \left(\frac{b}{a}\right)^4$$

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$$\frac{r_{circ}}{a} = (1+q)\left(\frac{b}{a}\right)^4$$

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Formation of the Accretion Disk

$$r_{circ} = (1+q)^{4/3}\left[0.500 - 0.227\log_{10} q\right]^4 \left(\frac{M}{M_\odot}\right)^{1/3} \tau_{days}^{2/3}\ [R_\odot]$$

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- In the absence of external torques, the only possible process is a *transfer of angular momentum* from inner regions outwards by internal torques.
- The redistribution of angular momentum makes particles in the outer parts move outwards (gaining angular momentum) and the particles in the inner particles spiral inwards.

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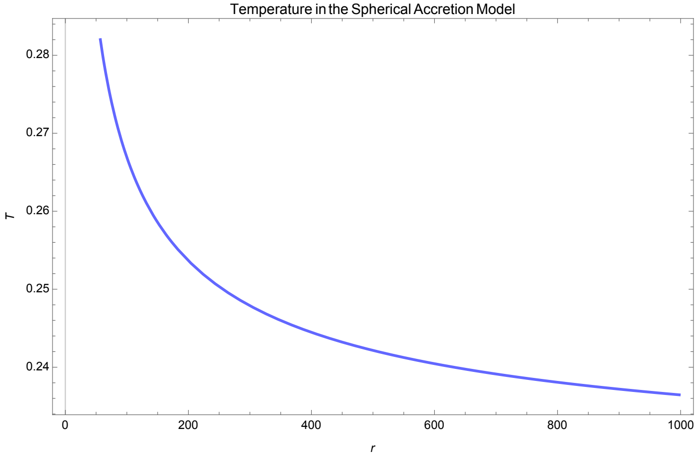
Temperature of the gas in the accretion structure



Next Lecture

10. Accretion Disks. Detailed Description