BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

# Classical Black Holes

## 11. Novikov-Thorne Thin Disks

Edward Larrañaga

# Outline for Part 1

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- Has four parameters: BH mass, BH spin, mass accretion rate and viscosity parameter.

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- The inner edge of the disk is at the ISCO radius.

# Modeling Viscosity

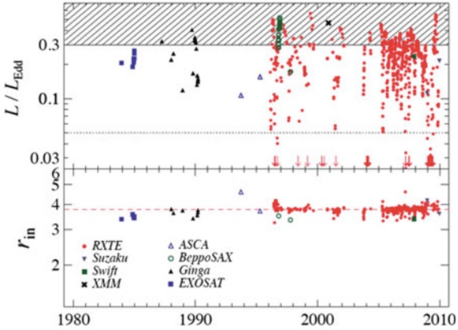


**Fig. 6.1** *Top panel* Accretion disk luminosity in Eddington units versus time for 766 spectra of LMC X-3. The shaded region does not satisfy the thin disk selection criterion $L/L_{\mathrm{Edd}} < 0.3$, as well as the data below the dotted line, which marks $L/L_{\mathrm{Edd}} = 0.05$. *Bottom panel* fitted value of the inner disk radius of the 411 spectra in the *top panel* that can meet the thin disk selection criterion. See the text for more details. From [51]. © AAS. Reproduced with permission

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  Any physical quantity $\Psi(t, r, \theta, phi)$ will be averaged over $t$ and $\phi$,

$$\Psi(r, \theta) = \frac{1}{2\pi\Delta t}\int_0^{\Delta t}\int_0^{2\pi}\Psi(t, r, \theta, \phi)d\phi dt$$

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- Magnetic fields are ignored.

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  This assumption let us neglect possible coherent superpositions of radiation reaction in nearby regions of the disk.
- The effect of energy and angular momentum transport by photons emitted from the disk and returning to the disk due to strong light bending in the vicinity of the black hole (returning radiation) is neglected.

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$\rho$: Time-averaged rest mass density

$u^\mu$: Time-averaged 4-velocity of the fluid

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$\gamma = -\left(\frac{g_{t\phi}^2}{g_{\phi\phi}} - g_{tt}\right) g_{rr} g_{\phi\phi}$

$\Sigma(r) = \int_{-h}^{h} \rho dz$

# Energy Flux and Torque

From the conservation of energy,

$$\nabla_\mu T^{t\mu} = 0$$

and angular momentum,

$$\nabla_\mu T^{\phi\mu} = 0$$

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$\varepsilon$: specific energy

$\ell_z$: specific axial component of the angular momentum

$\Omega$: Angular velocity for equatorial circular geodesics

## Energy Flux and Torque

$$F(r) = -\frac{\partial_r \Omega}{(\varepsilon - \Omega \ell_z)^2} \frac{M^2}{\sqrt{-\gamma}} \int_{r_{in}}^{r} \left[ (\varepsilon - \Omega \ell_z) \partial_x \ell_z \right] dx$$

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$r_{in}$: inner edge of the disk (presumably the ISCO radius)

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$\varepsilon_{ISCO}$: Specific energy of a test particle at the ISCO
$\zeta$: Fraction of energy captured by the BH

# Efficiency

$$\zeta = \frac{1}{\dot{M}} \int_{r_{ISCO}}^{\infty} \left[ \int_0^{\pi/2} \int_0^{2\pi} c\Upsilon(-n_t) \cos\theta \sin\theta d\phi d\theta \right] \mathscr{F}(r) 4rdr$$

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$$c = \begin{cases} 0 & \text{radiation escapes} \\ 1 & \text{radiation is captured} \end{cases}$$

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$\Upsilon$: takes into account possible angular dependence of the emission

$$\begin{cases} \Upsilon = 1 & \text{isotropic emission} \\ \Upsilon \propto 1 + 2\cos\theta & \text{limb-darkened emission} \\ \dots & \end{cases}$$

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$n^\mu$: normalized photon 4-momentum

$$n^\mu = \frac{k^\mu}{k_{(t)}}$$

$k^\mu$: photon 4-momentum

$k_{(t)}$: photon energy in the rest frame of the emitter

# Kerr Spacetime

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$$\sqrt{-\gamma} = r$$

## Kerr Spacetime

$$
\begin{aligned}
F(x) = & \frac{3}{2}\frac{1}{x^4(x^3-3x+2a)}\left[x-x_0-\frac{3}{2}a\ln\left(\frac{x}{x_0}\right)\right.\\
& -\frac{3(x_1-a)^2}{x_1(x_1-x_2)(x_1-x_3)}\ln\left(\frac{x-x_1}{x_0-x_1}\right)\\
& -\frac{3(x_2-a)^2}{x_2(x_2-x_1)(x_2-x_3)}\ln\left(\frac{x-x_2}{x_0-x_2}\right)\\
& \left.-\frac{3(x_3-a)^2}{x_3(x_3-x_1)(x_3-x_2)}\ln\left(\frac{x-x_3}{x_0-x_3}\right)\right]
\end{aligned}
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## Radial Structure Evolution

$x = \sqrt{\frac{r}{M}}$
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$$x_1 = 2\cos\left(\frac{1}{3}\arccos a - \frac{\pi}{3}\right)$$
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$$x_3 = -2\cos\left(\frac{1}{3}\arccos a\right)$$

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$$\eta_{NT}(a = 1) = 0.038 \text{ (counter-rotating disk)}$$

# Formation of the Accretion Disk

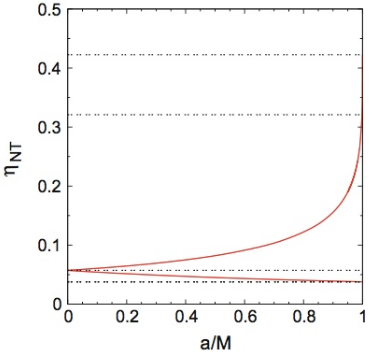


**Fig. 6.2** Novikov-Thorne radiative efficiency $\eta_{NT} = 1 - E_{ISCO}$ as a function of the spin $a$ in the Kerr metric. The *upper curve* is for corotating orbits, the *lower curve* for counterrotating orbits. The *dotted horizontal lines* correspond (from *top* to *bottom*) to the Novikov-Thorne radiative efficiency for $a/M = 1$ ($\eta_{NT} \approx 0.423$), $a/M = 0.998$ ($\eta_{NT} \approx 0.321$), $a/M = 0$ ($\eta_{NT} \approx 0.057$), and $a/M = 1$ in the case of a counterrotating disk ($\eta_{NT} \approx 0.038$)

## Outline for Part 2

# Transfer Function

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  $\nu_e$: emitted photon frequency (in the rest frame of the disk)
  $r_e$: emission radius
  $\vartheta_e$: emission angle with respect to the normal to the disk

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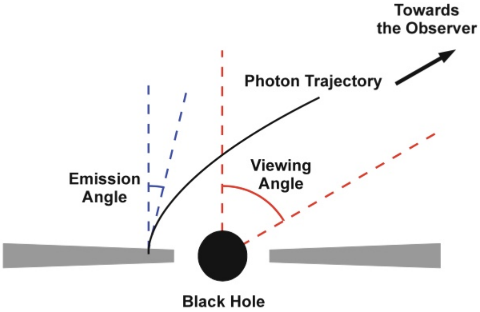


**Fig. 6.3** The viewing angle $i$ is the inclination angle of the disk, namely the angle between the axis normal to the disk and the line of sight of the distant observer; it is the same in Newtonian gravity and in general relativity. The emission angle $\vartheta_e$ is the angle, measured in the rest-frame of the gas and at a certain emission point, between the normal to the disk and the photon propagation direction. In Newtonian gravity, $i = \vartheta_e$, but in a curved spacetime the two angles are different in general

# Observed Spectrum

The observed flux (in units of erg $s^{-1}$ $cm^{-2}$ $Hz^{-1}$ )

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$$g^* = \frac{g - g_{min}}{g_{max} - g_{min}}$$

$g_{max} = g_{max}(r_e, i)$: maximum value of the redshift factor
$g_{min} = g_{min}(r_e, i)$: minimum value of the redshift factor

# The transfer function

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$$f(g^*, r_e, i) = \frac{1}{\pi r_e} g \sqrt{g^*(1-g^*)} \left| \frac{\partial(X, Y)}{\partial(g^*, r_e)} \right|$$

## Explicit form of the transfer function

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- We need to relate the position of the received photon, $(X, Y)$ with the emission point, $r_e$, and with the redshift factor ,$g$, and the emission angle, $\vartheta$.
- This relation is obtained by evolving (numerically) the geodesic equations from the observation point to the emission point.

# Explicit form of the transfer function. Redshift

- Numerically solving the geodesic equations give the position of the emitted photon in the accretion disk, $r_e$

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$u^{\mu}_{obs} = u^t_e(1, 0, 0, \Omega)$: 4-velocity of the observer
$u^{\nu}_e = (1, 0, 0, 0)$: 4-velocity of the gas particles

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$\lambda = -\frac{k_\phi}{k_t}$
$k_t = -\varepsilon$: energy of the photon (constant of motion)

# Explicit form of the transfer function. Emission angle

- Normal to the disk

$$n^{\mu} = \left(0, 0, \sqrt{g^{\theta\theta}}, 0\right)\Big|_{r_e, \theta_e = \pi/2}$$

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$$\cos\vartheta_e = \pm \left.\frac{n^\mu k_\mu}{u_e^\nu k_\nu}\right|_e$$

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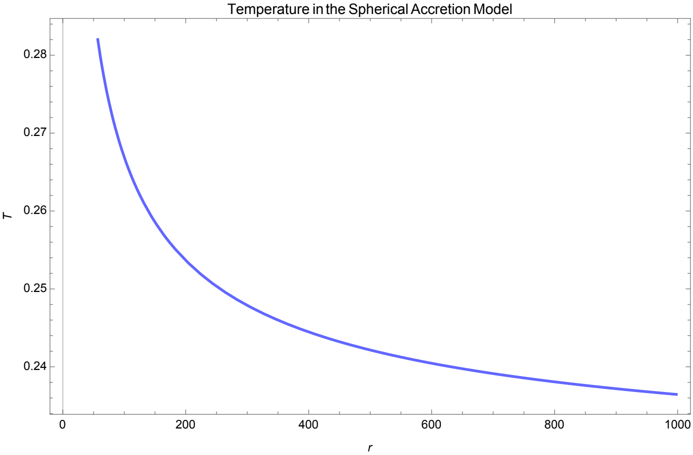
- From the first two relations we may calculate the Jacobian as

$$\left|\frac{\partial(X, Y)}{\partial(g^*, r_e)}\right| = (g_{max} - g_{min})\left|\frac{\partial X}{\partial g}\frac{\partial Y}{\partial r_e} - \frac{\partial X}{\partial r_e}\frac{\partial Y}{\partial g}\right|$$

# Temperature of the gas in the accretion structure



## Next Lecture

10. Accretion Disks. Detailed Description