Inverse Metric

$$g^{00} = -\frac{1}{\Delta}\left[r^2 + a^2 + \frac{2G(r)M\,r}{\Sigma}\,a^2\sin^2\theta\right] = -\frac{(r^2+a^2)^2 - \Delta\, a^2\sin^2\theta}{\Delta\Sigma}$$

$$g^{03} = -\frac{2G(r)M\,r}{\Sigma\Delta}\,a \qquad g^{11} = \frac{\Delta}{\Sigma} \qquad g^{22} = \frac{1}{\Sigma}$$

$$g^{33} = \frac{\Delta - a^2\sin^2\theta}{\Sigma\Delta\,\sin^2\theta}$$

Inverse Tetrad

$$e^{\mu}{}_{(\alpha)} = \eta_{(\alpha)(\beta)}\, e^{(\beta)}{}_{\nu}\, g^{\nu\mu} \qquad \eta_{(\alpha)(\beta)} = \mathrm{diag}(-1,1,1,1)$$

$$e^{(0)}_{\mu} = \sqrt{\frac{\Delta}{\Sigma}}\left(dt - a\sin^2\theta\, d\phi\right)$$

$$e^{(1)}_{\mu} = \sqrt{\frac{\Sigma}{\Delta}}\, dr$$

$$e^{(2)}_{\mu} = \sqrt{\Sigma}\, d\theta$$

$$e^{(3)}_{\mu} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,dt + (r^2 + a^2)\, d\phi\right]$$

$$\Delta = r^2 + a^2 - 2G(r)M\,r$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$G(r) = \frac{G_0\, r^3}{r^3 + \tilde{\omega}\, G_0\,(r + \gamma G_0 M)}$$

$$e^{\mu}{}_{(0)} = \eta_{(0)(\beta)}\, e^{(\beta)}{}_{\nu}\, g^{\nu\mu}$$

$$e^{\mu}{}_{(0)} = \eta_{(0)(0)}\, e^{(0)}{}_{\nu}\, g^{\nu\mu} = -e^{(0)}{}_{0}\, g^{0\mu} - e^{(0)}{}_{3}\, g^{3\mu}$$

$$e^{\mu}{}_{(0)} = -\sqrt{\frac{\Delta}{\Sigma}}\, g^{0\mu} + \sqrt{\frac{\Delta}{\Sigma}}\, a\sin^2\theta\, g^{3\mu}$$

$$e^{\mu}{}_{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\left(-g^{0\mu} + a\sin^2\theta\, g^{3\mu}\right)$$

$$e^{0}{}_{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\left(-g^{00} + a\sin^2\theta\, g^{30}\right)$$

$$e^{0}{}_{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\left[\frac{1}{\Delta}\left(r^2 + a^2 + \frac{2G(r)Mr}{\Sigma}\, a^2\sin^2\theta\right) - a\sin^2\theta\, \frac{2G(r)Mr}{\Sigma\Delta}\, a\right]$$

$$e^{0}{}_{(0)} = \frac{1}{\sqrt{\Delta\Sigma}}\left[r^2 + a^2 + \frac{2G(r)Mr}{\Sigma}\, a^2\sin^2\theta - \frac{2G(r)Mr}{\Sigma}\, a^2\sin^2\theta\right]$$

$$e^{0}{}_{(0)} = \frac{1}{\sqrt{\Delta\Sigma}}\left(r^2 + a^2\right)$$

$$e^{3}{}_{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\left(-g^{03} + a\sin^2\theta\, g^{33}\right)$$

$$e^{3}{}_{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\left[\frac{2G(r)Mr}{\Sigma\Delta}\, a + a\sin^2\theta\, \frac{\Delta - a^2\sin^2\theta}{\Sigma\Delta\,\sin^2\theta}\right]$$

$$e^{3}{}_{(0)} = \frac{a}{\sqrt{\Sigma^5\Delta}}\left[2G(r)Mr + \Delta - a^2\sin^2\theta\right]$$

$$e^{3}{}_{(0)} = \frac{a}{\sqrt{\Sigma^5\Delta}}\left(2G(r)Mr + r^2 + a^2 - 2G(r)Mr - a^2\sin^2\theta\right)$$

$$e^{3}{}_{(0)} = \frac{a}{\sqrt{\Sigma^5\Delta}}\left(r^2 + a^2\cos^2\theta\right)$$

$$e^{3}{}_{(0)} = \frac{a}{\sqrt{\Sigma\Delta}}$$

$$e^{\mu}{}_{(0)} = \frac{(r^2+a^2)}{\sqrt{\Sigma\Delta}}\, dt + \frac{a}{\sqrt{\Sigma\Delta}}\, d\phi$$

$$e^{\mu}{}_{(1)} = \eta_{(1)(\beta)}\, e^{(\beta)}{}_{\nu}\, g^{\nu\mu}$$

$$e^{\mu}{}_{(1)} = \eta_{(1)(1)}\, e^{(1)}{}_{\nu}\, g^{\nu\mu}$$

$$e^{\mu}{}_{(1)} = e^{(1)}{}_{1}\, g^{1\mu}$$

$$e^{\mu}{}_{(1)} = \sqrt{\frac{\Sigma}{\Delta}}\, g^{1\mu}$$

$$e^{\mu}{}_{(1)} = \sqrt{\frac{\Sigma}{\Delta}}\, g^{11}\, dr$$

$$e^{\mu}{}_{(1)} = \sqrt{\frac{\Sigma}{\Delta}}\, \frac{\Delta}{\Sigma}\, dr$$

$$e^{\mu}{}_{(1)} = \sqrt{\frac{\Delta}{\Sigma}}\, dr$$

$$e^{\mu}{}_{(2)} = \eta_{(2)(\beta)}\, e^{(\beta)}{}_{\nu}\, g^{\nu\mu}$$

$$e^{\mu}{}_{(2)} = \eta_{(2)(2)}\, e^{(2)}{}_{\nu}\, g^{\nu\mu} = e^{(2)}{}_{2}\, g^{2\mu}$$

$$e^{\mu}{}_{(2)} = \sqrt{\Sigma}\, g^{2\mu}$$

$$e^{\mu}{}_{(2)} = \sqrt{\Sigma}\, g^{22}\, d\theta$$

$$e^{\mu}_{(2)} = \sqrt{\Sigma}\,\frac{1}{\Sigma}\,d\theta$$

$$e^{\mu}_{(2)} = \frac{1}{\sqrt{\Sigma}}\,d\theta$$

$$e^{\mu}_{(3)} = \eta_{(3)(\beta)}\,e^{(\beta)}{}_{\nu}\,g^{\nu\mu}$$

$$e^{\mu}_{(3)} = \eta_{(3)(3)}\,e^{(3)}{}_{\nu}\,g^{\nu\mu} = e^{(3)}{}_{0}\,g^{0\mu} + e^{(3)}{}_{3}\,g^{3\mu}$$

$$e^{\mu}_{(3)} = -\frac{a\sin\theta}{\sqrt{\Sigma}}\,g^{0\mu} + \frac{(r^2+a^2)}{\sqrt{\Sigma}}\sin\theta\,g^{3\mu}$$

$$e^{\mu}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,g^{0\mu} + (r^2+a^2)\,g^{3\mu}\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,g^{00} + (r^2+a^2)\,g^{30}\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[\frac{a}{\Delta}\left(r^2+a^2+\frac{2G(r)Mr}{\Sigma}a^2\sin^2\theta\right) - (r^2+a^2)\frac{2G(r)Mr}{\Sigma\Delta}a\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\frac{a}{\Delta}\left[r^2+a^2+\frac{2G(r)Mr}{\Sigma}a^2\sin^2\theta - (r^2+a^2)\frac{2G(r)Mr}{\Sigma}\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\frac{a}{\Delta}\left[r^2+a^2+\frac{2G(r)Mr}{\Sigma}\left(a^2\sin^2\theta - r^2 - a^2\right)\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\frac{a}{\Delta}\left[r^2+a^2-\frac{2G(r)Mr}{\Sigma}\left(r^2+a^2\cos^2\theta\right)\right]$$

$$e^{0}_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\frac{a}{\Delta}\left[r^2+a^2-2G(r)Mr\right]$$

$$e^{0}_{(3)} = \frac{a\sin\theta}{\sqrt{\Sigma}}$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a g^{03} + (r^2+a^2)\, g^{33}\right]$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[a\,\frac{2G(r)Mr}{\Sigma\Delta}\,a + (r^2+a^2)\,\frac{\Delta - a^2\sin^2\theta}{\Sigma\Delta\,\sin^2\theta}\right]$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\,\frac{1}{\Sigma\Delta}\left[2G(r)Mr\,a^2 + \frac{r^2+a^2}{\sin^2\theta}\,\Delta - a^2(r^2+a^2)\right]$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\,\frac{1}{\Sigma\Delta}\left[-a^2(r^2+a^2-2G(r)Mr) + \frac{r^2+a^2}{\sin^2\theta}\,\Delta\right]$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\,\frac{\Delta}{\Sigma\Delta}\left[-a^2 + \frac{r^2+a^2}{\sin^2\theta}\right]$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\,\frac{1}{\Sigma}\,\frac{r^2+a^2-a^2\sin^2\theta}{\sin^2\theta}$$

$$e^3_{(3)} = \frac{\sin\theta}{\sqrt{\Sigma}}\,\frac{1}{\Sigma}\,\frac{r^2+a^2\cos^2\theta}{\sin^2\theta}$$

$$e^3_{(3)} = \frac{1}{\sqrt{\Sigma}\,\sin\theta}$$

$$e^{\mu}_{(3)} = \frac{a\sin\theta}{\sqrt{\Sigma}}\,dt + \frac{1}{\sqrt{\Sigma}\,\sin\theta}\,d\phi$$

RESULTS:

$$e^{\mu}_{(0)} = \frac{1}{\sqrt{\Sigma\Delta}}\left[(r^2+a^2)\,dt + a\,d\phi\right]$$

$$e^{\mu}_{(1)} = \sqrt{\frac{\Delta}{\Sigma}}\,dr$$

$$e^{\mu}_{(2)} = \frac{1}{\sqrt{\Sigma}}\,d\theta$$

$$e^{\mu}_{(3)} = \frac{1}{\sqrt{\Sigma}\sin\theta}\left(a\sin^2\theta\,dt + d\phi\right)$$

$$e^{(0)}_{\mu} = \sqrt{\frac{\Delta}{\Sigma}}\left(dt - a\sin^2\theta\,d\phi\right)$$

$$e^{(1)}_{\mu} = \sqrt{\frac{\Sigma}{\Delta}}\,dr$$

$$e^{(2)}_{\mu} = \sqrt{\Sigma}\,d\theta$$

$$e^{(3)}_{\mu} = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,dt + (r^2+a^2)\,d\phi\right]$$

$$\Delta = r^2 + a^2 - 2G(r)Mr$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$G(r) = \frac{G_0 r^3}{r^3 + \tilde{\omega} G_0 (r + \gamma G_0 M)}$$