BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

08. Accretion

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Outline for Part 1

Accretion Basics

Accretion Basics

Process of matter falling into the potential well of a gravitating object.

Accretion Regimes

1. Spherical Accretion
2. Cylindrical Accretion
3. Accretion Disk
4. Two-Stream Accretion

Spherical Accretion

- No (significant) angular momentum
- Determined by the relation between

  c_s: speed of sound in matter

  v_{rel}: relative velocity between accretor and matter
- $v_{rel} \ll c_s$
- If the accretor is a BH,

$$v_{rel} = v$$

  v: velocity of accreting matter (i.e. the BH doesn't move!)

Cylindrical Accretion

- Small angular momentum
- $v_{rel} \geq c_s$

Accretion Disk

- Angular momentum is high enough to form an accretion disk
- Matter spirals down into the accretor

Two-Stream Accretion

- Quasi-spherically symmetric inflow coexist with an accretion disk

Spherical Accretion

Spherical Accretion

First model of accretion. Smooth and time-steady accretion.

We assume a completely ionized hydrogen gas cloud as the accretion structure.

Gas moves very slowly. The description is made in terms of fluid mechanics equations.

First and simplest description: To avoid the disintegration of the accretion structure, the outward force due to radiation pressure must be counterbalanced by the gravitational force.

Radiation Pressure

Outward energy flux at distance r from the center

$$F = \frac{L}{4\pi r^2}$$

L: bolometric luminosity [$\mathrm{erg} \cdot \mathrm{s}^{-1}$]

For photons:

$$p^\mu = \left(\frac{E}{c}, \vec{p}\right) \qquad p^2 = \frac{E^2}{c^2} - |\vec{p}|^2 = 0$$

Then, the **outwards momentum flux** (or pressure) is

$$P_{rad} = \frac{F}{c} = \frac{L}{4\pi r^2 c}$$

Radiation Pressure

The radiation force on a single electron is

$$\vec{f}_{rad} = (\sigma_e P_{rad})\hat{r} = \sigma_e \frac{L}{4\pi r^2 c}\hat{r}$$

σ_e: Thomson cross-section

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-25}\ \mathrm{cm}^2$$

Interaction with protons is negligible because σ_p is lower by a factor of $\left(\frac{m_p}{m_e}\right)^2 \sim 3 \times 10^6$

Gravitational Force

The gravitational force between the central object M and one electron-proton pair is

$$\vec{f}_g = -\frac{GM(m_e + m_p)}{r^2}\hat{r} \sim -\frac{GMm_p}{r^2}\hat{r}$$

Spherical Accretion

To avoid the disintegration of the accretion structure,

$$|\vec{f}_{rad}| \leq |\vec{f}_g|$$

$$\sigma_e \frac{L}{4\pi r^2 c} \leq \frac{GMm_p}{r^2}$$

$$L \leq \frac{4\pi GMm_p c}{\sigma_e}$$

Eddington Luminosity

$$L \leq L_E$$

$$L_E = \frac{4\pi G M m_p c}{\sigma_e}$$

$$L_E = 6.31 \times 10^4 M \; [\mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{kg}^{-1}]$$

$$L_E = 1.26 \times 10^{38} \left(\frac{M}{M_\odot} \right) \left[\; \mathrm{erg} \cdot \mathrm{s}^{-1} \right]$$

Eddington Lumionisty: Maximum luminosity of a source M powered by spherical accretion.

Estimation of the Central Mass from its Luminosity

$$L \leq L_E$$

$$M \geq \frac{L\sigma_e}{4\pi G m_p c}$$

$$M \geq \frac{L}{1.26 \times 10^{38}} \, [M_\odot]$$

$$M \geq 8 \times 10^5 L_{44} \, [M_\odot]$$

L_{44}: Central Source Luminosity in units of $10^{44}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$ (typical value for AGN's)
$M_E = 8 \times 10^5 L_{44} M_\odot$: *Eddington's Mass*. Minimum mass for a given luminosity

Estimation of the Central Mass from its Luminosity

AGN's have typically $L \sim 10^{43} - 10^{47}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

Estimation of the Central Mass from its Luminosity

AGN's have typically $L \sim 10^{43} - 10^{47}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

Black holes with $M \sim 10^5 - 10^9 M_\odot$

Estimation of the Central Mass from its Luminosity

Name	z	L_{bol}		M_{BH}	ref.	Type
3C 120	0.033	45.34	I	7.42	1	SY1
3C 390.3	0.056	44.88	I	8.55	1	SY1
Akn 120	0.032	44.91	I	8.27	1	SY1
F 9	0.047	45.23	F	7.91	1	SY1
IC 4329A	0.016	44.78	I	6.77	1	SY1
Mrk 79	0.022	44.57	I	7.86	1	SY1
Mrk 110	0.035	44.71	F	6.82	1	SY1
Mrk 335	0.026	44.69	I	6.69	1	SY1
Mrk 509	0.034	45.03	I	7.86	1	SY1
Mrk 590	0.026	44.63	I	7.20	1	SY1
Mrk 817	0.032	44.99	I	7.60	1	SY1
NGC 3227	0.004	43.86	I	7.64	1	SY1
NGC 3516	0.009	44.29	I	7.36	3	SY1
NGC 3783	0.010	44.41	I	6.94	2	SY1
NGC 4051	0.002	43.56	I	6.13	1	SY1
NGC 4151	0.003	43.73	I	7.13	1	SY1
NGC 4593	0.009	44.09	I	6.91	3	SY1
NGC 5548	0.017	44.83	I	8.03	1	SY1
NGC 7469	0.016	45.28	I	6.84	1	SY1
PG 0026+129	0.142	45.39	I	7.58	1	RQQ
PG 0052+251	0.155	45.93	F	8.41	1	RQQ
PG 0804+761	0.100	45.93	F	8.24	1	RQQ
PG 0844+349	0.064	45.36	F	7.38	1	RQQ
PG 0953+414	0.239	46.16	F	8.24	1	RQQ
PG 1211+143	0.085	45.81	F	7.49	1	RQQ
PG 1229+204	0.064	45.01	I	8.56	1	RQQ
PG 1307+085	0.155	45.83	F	7.90	1	RQQ
PG 1351+640	0.087	45.50	I	8.48	1	RQQ
PG 1411+442	0.089	45.58	F	7.57	1	RQQ
PG 1426+015	0.086	45.19	I	7.92	1	RQQ
PG 1613+658	0.129	45.66	I	8.62	1	RQQ
PG 1617+175	0.114	45.52	F	7.88	1	RQQ
PG 1700+518	0.292	46.56	F	8.31	1	RQQ
PG 2130+099	0.061	45.47	I	7.74	1	RQQ
PG 1226+023	0.158	47.35	I	7.22	1	RLQ
PG 1704+608	0.371	46.33	I	8.23	1	RLQ

[a] Column (1) Name, (2) redshift, (3) log of the bolometric luminosity (ergs s^{-1}), (4) method for bolometric luminosity estimation (I: flux integration; F: SED fitting), (5) black hole mass estimate from reverberation mapping (for Kaspi et al. (2000) sample, where black hole mass is log mean of rms FWHM and mean FWHM mass, in solar masses), (6) reference for black hole mass estimation, and (7) AGN type.

References. — (1) Kaspi et al. (2000), (2) Onken & Peterson (2002), (3) Ho (1999).

J.-H. Woo, C. M. Urry. *AGN Black Hole Masses and Bolometric Luminosities*. *Astrophys. J.* 579 (2002) 530-544

Accretion Rate

The luminosity is just a fraction of the relativistic energy of the accreting mass, $E = mc^2$. The other fraction goes into the BH making it grow.

$$L \propto \frac{dE}{dt} = \frac{dm}{dt}c^2 = \frac{dM}{dt}c^2$$

$$L = \eta \dot{M} c^2$$

η: Efficiency of the process

Accretion Rate

Accretion produces radiation by conversion of gravitational potential.

$$U = \frac{GMm}{r}$$

$$L \sim \frac{dU}{dt} = \frac{GM}{r}\frac{dm}{dt} = \frac{GM}{r}\dot{M}$$

$$\eta = \frac{GM}{rc^2}$$

Accretion Rate

In order to estimate the efficiency, consider the ISCO for Schwarzchild,

$$r_{ISCO} = 3r_S = \frac{6GM}{c^2}$$

Supposing that a particle falling from this orbit into the BH looses all its energy as radiation gives

$$\eta \sim \frac{GM}{r_{ISCO}c^2} = \frac{1}{6}$$

Hence

$$\eta \sim 0.1 - 0.2$$

Accretion Rate

In some books consider the accretion of particles falling from $r = 5r_S$, because it gives most of the optical/UV continuum radiation.
Using this point, one obtains an efficiency of $\eta \sim 0.1$

Accretion Rate

$$\dot{M} = \frac{L}{\eta c^2} = 1.11 \times 10^{23} \frac{L_{44}}{\eta} \left[\frac{\mathrm{gr}}{\mathrm{s}}\right]$$

$$\dot{M} = 1.77 \times 10^{-2} L_{44} \left(\frac{\eta}{0.1}\right)^{-1} \left[\frac{M_\odot}{\mathrm{yr}}\right]$$

Accretion Rate

For a typical AGN, $L \sim 10^{47}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$.

- If $\eta \sim 0.007$ as in the Hydrogen burning process (Nuclear fusion), it gives

$$\dot{M} = \frac{L}{\eta c^2} \sim 250\, M_\odot \cdot \mathrm{yr}^{-1}$$

- If $\eta \sim 0.1$ we obtain a more realistic value of

$$\dot{M} = \frac{L}{\eta c^2} \sim 18\, M_\odot \cdot \mathrm{yr}^{-1}$$

Eddington Accretion Rate

$$\dot{M}_E = \frac{L_E}{\eta c^2} = \frac{4\pi G M m_p}{\sigma_e \eta c}$$

$$\dot{M}_E = 2.67 \times 10^{-8} \left(\frac{\eta}{0.1}\right)^{-1} \frac{M}{M_\odot} \left[\frac{M_\odot}{\mathrm{yr}}\right]$$

$$\dot{M}_E = 3 \left(\frac{\eta}{0.1}\right)^{-1} M_8 \left[\frac{M_\odot}{\mathrm{yr}}\right]$$

$\dot{M}_E$: Maximum possible accretion rate for a mass $M_8 = \frac{M}{10^8 M_\odot}$.

Eddington Accretion Rate

May $\dot{M} > \dot{M}_E$?

1. It depends on a careful determination of η. e.g. if $\eta < 0.1$, the outwards flux is diminished.
2. $\dot{M}_E$ can be exceeded with non-spherical models.

Growth Time

$$\dot{M} = \frac{dM}{dt} = \frac{L}{\eta c^2}$$

$$\dot{M} = \frac{dM}{dt} = \frac{L}{L_E} \frac{4\pi G M m_p}{\sigma_e \eta c}$$

$$\int \frac{dM}{M} = \frac{L}{L_E} \frac{4\pi G m_p}{\sigma_e \eta c} \int dt$$

$$M(t) = M_0 \exp\left(\frac{t}{t_{growth}}\right)$$

$$t_{growth} = \frac{\sigma_e \eta c}{4\pi G m_p} \left(\frac{L_E}{L}\right)$$

Growth Time

$$t_{growth} = \frac{\sigma_e \eta c}{4\pi G m_p} \left(\frac{L_E}{L} \right)$$

$$t_{growth} = 3.7 \times 10^8 \eta \left(\frac{L_E}{L} \right) [\mathrm{yr}]$$

For $L \sim L_E$, the BH grows exponentially on time scales of the order $\sim 10^8\ \mathrm{yr}$.

Radiation Temperature

The *continuum spectrum* of the emitted radiation is characterized by a temperature

$$T_{rad} = \frac{h\bar{\nu}}{k_B}$$

$\bar{\nu}$: frequency of a typical (average) photon

Black Body Temperature

For a source with accretion luminosity L and radius r, it is defined the *blackbody temperature* through

$$F = \sigma T_{eff}^4$$

$\sigma = 5.6 \times 10^{-5} \frac{\mathrm{erg}}{\mathrm{cm^2 \cdot s \cdot K^4 \cdot sr}}$
Steffan-Boltzman Constant

$$T_{eff} = \left(\frac{L}{4\pi r^2 \sigma} \right)^{1/4}$$

Black Body Temperature

Using $L = \frac{GM}{r}\dot{M}$,

$$T_{eff} = \left(\frac{GM\dot{M}}{4\pi r^3 \sigma} \right)^{1/4}$$

$$T_{eff} = 1.01 \times 10^6 M_8^{-1/4} \left(\frac{\dot{M}}{\dot{M}_E} \right)^{1/4} \left(\frac{r}{r_S} \right)^{-3/4}$$

Compactness

One way to estimate the compactness of a source is using the luminosity and the effective surface temperature.

$$r_{BB} = \sqrt{\frac{L}{4\pi\sigma T_{eff}^4}}$$

Example

Consider a system in our galaxy with $L = 10^{37}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

From the Eddington limit, the mass of the central object must be

$$M \geq \frac{L}{1.26 \times 10^{38}} M_\odot \sim \frac{10^{37}}{10^{38}} M_\odot \sim 0.1 M_\odot$$

The luminosity of the binary Cygnus X-1 is $L \sim 10^5 L_\odot \sim 10^{37}\ \mathrm{erg} \cdot \mathrm{s}^{-1}$

Example

If the radiation is in the optical-UV,
$\nu_{max} \sim 10^{15}\mathrm{Hz}$

$$T_{eff} \sim T_c = \frac{10^{15}}{5.88 \times 10^{10}} \sim 10^5\mathrm{K}$$

$$r_{bb} \sim 10^{12}\mathrm{cm} \sim 10^7\mathrm{km}$$

Typical size of a Star!

https://rechneronline.de/spectrum

Example

If the radiation is in soft X-rays at 1 keV,
$\nu_{max} \sim 10^{17}\mathrm{Hz}$

$$T_{eff} \sim T_c = \frac{10^{17}}{5.88 \times 10^{10}} \sim 10^7\mathrm{K}$$

$$r_{bb} \sim 10^6\mathrm{cm} \sim 10\mathrm{km}$$

Typical size of a neutron star or a BH!

https://rechneronline.de/spectrum

Virial Temperature

T_{th}: Temperature reached by the accreted material if all the gravitational energy is transformed into thermal energy.
$2\langle K\rangle + \langle U\rangle = 0$: Virial Theorem

$$\langle U\rangle = \frac{GM(m_p + m_e)}{r} \sim \frac{GMm_p}{r}$$

$$2\langle K\rangle \sim 2 \times \frac{3}{2} k_B T_{th}$$

$$T_{th} = \frac{GMm_p}{3k_B r} = T_{vir}$$

Temperatures

If the accretion energy is converted directly into radiation escaping without interaction,

$$T_{rad} \sim T_{th}$$

If the accretion flow is optically thick, the radiation reaches thermal equilibrium with the accreted material before escaping out to the observer,

$$T_{rad} \sim T_{eff}$$

In general,

$$T_{eff} \lesssim T_{rad} \lesssim T_{th}$$

Outline for Part 2

Hydrodynamics Description of Spherical Accretion

Hydrodynamics Description of Spherical Accretion

Assumptions:

- No viscosity
- No angular momentum
- No electromagnetic fields

We are looking for

- Steady accretion rate $\dot{M}$ in terms of the asymptotic density and temperature of the gas, ρ_∞ and T_∞.
- Size of region where gas is influenced by the gravity of the BH
- Local velocity of the gas and local speed of sound
- Spectrum of the emitted radiation by the accretion structure

Hydrodynamics Equations

Hydrodynamics Equations

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Continuity Equation

ρ: Mass density

$\vec{v}$: Velocity of the gas

Hydrodynamics Equations

$$\rho\left[\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right]=-\vec{\nabla}P+\rho\vec{g}+\vec{\nabla}\cdot\vec{\sigma}$$

Conservation of Momentum (ignoring radiation pressure)

P: Pressure

$\vec{g}$: Acceleration due to gravity

If self-gravity of the accretion structure is negligible,

$$\vec{g}=-\vec{\nabla}\Phi=-\frac{GM}{r^2}\hat{r}$$

Hydrodynamics Equations

$$\rho\left[\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right]=-\vec{\nabla}P+\rho\vec{g}+\vec{\nabla}\cdot\vec{\sigma}$$

Conservation of Momentum (ignoring radiation pressure)

σ_{ij}: Viscosity Stress Tensor

$$\sigma_{ij}=2\eta\tau_{ij}$$

$$\tau_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x^j}+\frac{\partial v_j}{\partial x^i}-\frac{2}{3}\frac{\partial v_k}{\partial x^k}\delta_{ij}\right)$$

Hydrodynamics Equations

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P + \rho \vec{g} + \vec{\nabla} \cdot \vec{\sigma}$$

Conservation of Momentum (ignoring radiation pressure)

$$\sigma_{ij} = 2\eta \tau_{ij}$$

$$\eta = \rho \nu$$

η: dynamic viscosity

ν: kinematic viscosity

(we will assume $\nu = 0$ for spherical accretion!)

* No-viscosity: Euler equation

* Viscosity: Navier-Stokes equation

Hydrodynamics Equations

$$P = P(\rho)$$

Equation of state

Usually a polytropic: $P \propto \rho^\gamma$

$1 \leq \gamma \leq \frac{5}{3}$
$\gamma = 1$: Iosthermic flow
$\gamma = \frac{5}{3}$: Adiabatic flow

Hydrodynamics Equations

$$\rho \frac{d\varepsilon}{dt} = -P\vec{\nabla} \cdot \vec{v} + 2\eta \left[S_{ij}S_{ij} - \frac{1}{3}(\vec{\nabla} \cdot \vec{v})^2 \right] + Q$$

Energy Balance

ε: Internal energy per unit mass of the fluid

Q: Net heat exchanged by an element of the fluid per unit time per unit volume

$$S_{ij} = \tau_{ij} - \frac{1}{3}(\vec{\nabla} \cdot \vec{v})\delta_{ij}$$

Hydrodynamics Equations

$$T = \frac{\mu m_H}{k_B} \frac{P}{\rho}$$

Perfect gas temperature

$m_H \sim m_p$: Hydrogen mass

μ: Mean molecular weight

$\mu = 1$ for neutral Hydrogen

$\mu = \frac{1}{2}$ for fully ionized Hydrogen

Spherical Accretion Hydrodynamics

Spherical Accretion Hydrodynamics

Spherical symmetry and steady state:

$$\rho = \rho(r)$$

$$\vec{v} = v(r)\hat{r}$$

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \rho v \right) = 0$$

Integrating,

$$\dot{M} = 4\pi r^2 \rho v$$

Conservation of Momentum

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} \right] = -\vec{\nabla} P + \rho \vec{g}$$

$$v \frac{dv}{dr} + \frac{1}{\rho} \frac{\partial P}{\partial r} + \frac{GM}{r^2} = 0$$

Equations Governing Spherical Accretion

$$\dot{M} = 4\pi r^2 \rho v$$

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{GM}{r^2} = 0$$

$$P \propto \rho^\gamma$$

Local Speed of Sound

$$c_s^2 = \gamma \frac{P}{\rho}$$

Sonic radius: The gas moves with the speed of sound,

$$r_s = \frac{GM}{2c_s^2}$$

Behavior of the gas in the accretion structure

For $r \gg r_s$

$$c_s \approx c_\infty \left[1 - \frac{\gamma - 1}{4} \frac{r_{acc}}{r}\right] \approx c_\infty$$

$$v \approx \frac{c_\infty}{16} \left(\frac{2}{5 - 3\gamma}\right)^{\frac{5-3\gamma}{2(\gamma-1)}} \left(\frac{r_{acc}}{r}\right)^2 \left[1 - \frac{1}{2} \frac{r_{acc}}{r}\right] \approx 0$$

$$\rho \approx \rho_\infty \left[1 - \frac{1}{2} \frac{r_{acc}}{r}\right] \approx \rho_\infty$$

$r_{acc} = \frac{2GM}{c_\infty^2}$: Accretion radius

Behavior of the gas in the accretion structure

For $r \ll r_s$

$$v \approx \sqrt{\frac{2GM}{r}} = v_{ff}$$

$$\rho \approx \rho(r_s) \left(\frac{r_s}{r}\right)^{\frac{3}{2}}$$

Temperature of the gas in the accretion structure

$$T = \frac{\mu m_H}{k_B} \frac{P}{\rho}$$

$$T = T(r_s) \left(\frac{r_s}{r} \right)^{\frac{3}{2}(\gamma - 1)}$$

Temperature of the gas in the accretion structure

First law of thermodynamics:

$$d\varepsilon = dQ - PdV$$

$$d\varepsilon = dQ + \frac{k_B}{\mu m_H} \frac{T}{\rho} d\rho$$

Bremsstrahlung (free-free) radiation

$\frac{dQ}{dt} = -\alpha_{ff} T^{1/2} \rho$

$\alpha_{ff} \approx 5 \times 10^{20}\ \mathrm{erg\ cm^3\ g^{-2}\ s^{-1}\ K^{-1/2}}$ for Hydrogen.

Temperature of the gas in the accretion structure

$$\frac{dT}{dr} = -\frac{T}{r} - \alpha_{ff}\rho(r_s)\sqrt{\frac{r_s}{2GM}}\frac{T^{1/2}}{r}\left(\frac{2\mu m_H}{3k_B}r_s\right) + \frac{2\mu m_H}{3k_B}\frac{dQ}{dr}$$

Temperature of the gas in the accretion structure

If there is only Bremsstrahlung radiation,

$$\frac{dT}{dr} = -\frac{T}{r} - \alpha_{ff}\rho(r_s)\sqrt{\frac{r_s}{2GM}}\frac{T^{1/2}}{r}\left(\frac{2\mu m_H}{3k_B}r_s\right)$$

Therefore,

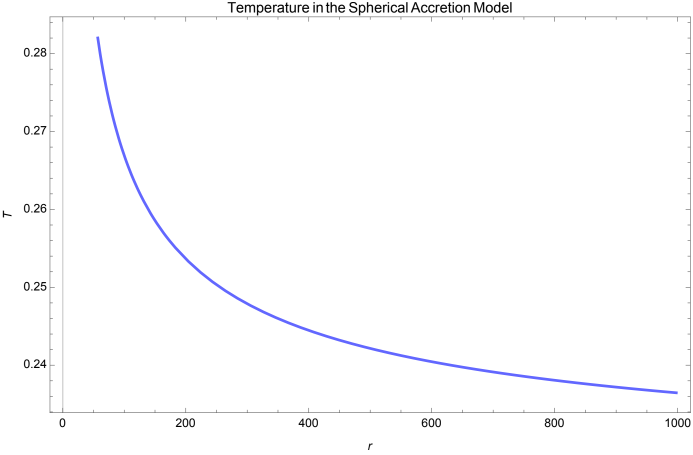
$$\frac{dT}{dr} < 0$$

The temperature of the flow decreases as the gas approaches the BH (*cooling flow*).

Temperature of the gas in the accretion structure

$$T = \left[-\frac{4}{K} + \sqrt{\frac{16}{K^2} + \frac{4}{K\sqrt{r}} + C} \right]^2$$

Temperature of the gas in the accretion structure



Next Lecture

09. Accretion in Binary Systems