FACULTY
OF ARTS
Masaryk University

Classical Black Holes

02. Black holes Introduction

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Outline for Part 1

1. Schwarzschild's Solution
    1.1 General Relativity Field Equations
    1.2 Physical Conditions to obtain Schwarzschild's Metric

Einstein Field Equations

Einstein field equations are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

Riemann Tensor

Riemann tensor is defined as

$$R^{\lambda}{}_{\mu\rho\nu} = \partial_\rho \Gamma^{\lambda}_{\nu\mu} - \partial_\nu \Gamma^{\lambda}_{\rho\mu} + \Gamma^{\lambda}_{\rho\alpha}\Gamma^{\alpha}_{\nu\mu} - \Gamma^{\lambda}_{\nu\alpha}\Gamma^{\alpha}_{\rho\mu}$$

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$$\Gamma^{\nu}_{\mu\sigma} = \frac{1}{2} g^{\nu\alpha}\left[\partial_{\mu} g_{\alpha\sigma} + \partial_{\sigma} g_{\mu\alpha} - \partial_{\alpha} g_{\mu\sigma}\right]$$

Ricci Tensor and Curvature Scalar

$$R_{\mu\nu} = g^{\sigma\rho} R_{\sigma\mu\rho\nu}$$

$$R = R^{\sigma}_{\sigma} = g^{\sigma\tau} R_{\sigma\tau}.$$

Ricci Tensor and Curvature Scalar

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\nu\mu} - \partial_\nu \Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\rho\alpha}\Gamma^\alpha_{\nu\mu} - \Gamma^\rho_{\nu\alpha}\Gamma^\alpha_{\rho\mu}$$

$$R = g^{\sigma\tau}\partial_\rho \Gamma^\rho_{\tau\sigma} - g^{\sigma\tau}\partial_\tau \Gamma^\rho_{\rho\sigma} + g^{\sigma\tau}\Gamma^\rho_{\rho\alpha}\Gamma^\alpha_{\tau\sigma} - g^{\sigma\tau}\Gamma^\rho_{\tau\alpha}\Gamma^\alpha_{\rho\sigma}$$

Einstein Field Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

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$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$\partial_\rho \Gamma^\rho_{\nu\mu} - \partial_\nu \Gamma^\rho_{\rho\mu} + \Gamma^\rho_{\rho\alpha}\Gamma^\alpha_{\nu\mu} - \Gamma^\rho_{\nu\alpha}\Gamma^\alpha_{\rho\mu} - \frac{1}{2} g_{\mu\nu}\left[g^{\sigma\tau}\partial_\rho\Gamma^\rho_{\tau\sigma} - g^{\sigma\tau}\partial_\tau\Gamma^\rho_{\rho\sigma} + g^{\sigma\tau}\Gamma^\rho_{\rho\alpha}\Gamma^\alpha_{\tau\sigma} - g^{\sigma\tau}\Gamma^\rho_{\tau\alpha}\Gamma^\alpha_{\rho\sigma}\right] = 8\pi G T_{\mu\nu}$$

Physical Conditions to obtain Schwarzschild's Metric

- Spherical Symmetry

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- Asymptotically flat spacetime
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Spherical Coordinates: (t, r, θ, ϕ)

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Spherical Coordinates: (t, r, θ, ϕ)

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

Physical Conditions to obtain Schwarzschild's Metric

- Spherical Symmetry

Spherical Coordinates: (t, r, θ, ϕ)

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$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2$$

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- Spherical Symmetry

Spherical Coordinates: (t, r, θ, ϕ)

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$g_{tt} = g_{tt}(t, r)$$
$$g_{rr} = g_{rr}(t, r)$$

Physical Conditions to obtain Schwarzschild's Metric

- Asymptotically flat

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- Asymptotically flat

$$\lim_{r \to \infty} g_{tt} \longrightarrow 1$$
$$\lim_{r \to \infty} g_{rr} \longrightarrow 1$$

Physical Conditions to obtain Schwarzschild's Metric

- Static Metric

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$$g_{tt} = g_{tt}(r)$$
$$g_{rr} = g_{rr}(r)$$

Physical Conditions to obtain Schwarzschild's Metric

- Static Metric

$$g_{tt} = g_{tt}(r)$$
$$g_{rr} = g_{rr}(r)$$

"Birkhoff Theorem"

Physical Conditions to obtain Schwarzschild's Metric

- Empty Spacetime

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$$T_{\mu\nu} = 0$$

Physical Conditions to obtain Schwarzschild's Metric

- Empty Spacetime

$$T_{\mu\nu} = 0$$

Field Equations:

$$R_{\mu\nu} = 0$$

Spherically Symmetric Solution in Empty Space

$$ds^2 = -\left(1 - \frac{K}{r}\right) dt^2 + \left(1 - \frac{K}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

K: Integration constant to be determined

Next Lecture

03. Schwarzschild Solution. Properties.