BLACK HOLES

OBSERVATORIO
ASTRONÓMICO
NACIONAL

Classical Black Holes

04. Schwarzschild’s Black hole Geometry

Edward Larrañaga

Outline for Part 1

Schwarzschild's Solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$$

Geometrical units: $G = c = 1$

Eddington-Finkelstein Coordinates

$$(t, r, \theta, \phi) \longrightarrow (v, u, \theta, \phi)$$

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Eddington-Finkelstein Coordinates

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While the original coordinate r takes values in the range $2M < r < \infty$, the new coordinates v and u take values in the ranges $-\infty < v < \infty$ and $-\infty < u < \infty$.

Eddington-Finkelstein Coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dvdu + r^2 d\Omega^2$$

r is defined implicitly by the relation

$$\frac{1}{2}(v - u) = r^* = r + 2M \ln\left|\frac{r - 2M}{2M}\right|$$

Kruskal Coordinates

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$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dV dU + r^2 d\Omega^2$$

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Kruskal Diagram

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Kruskal Diagram

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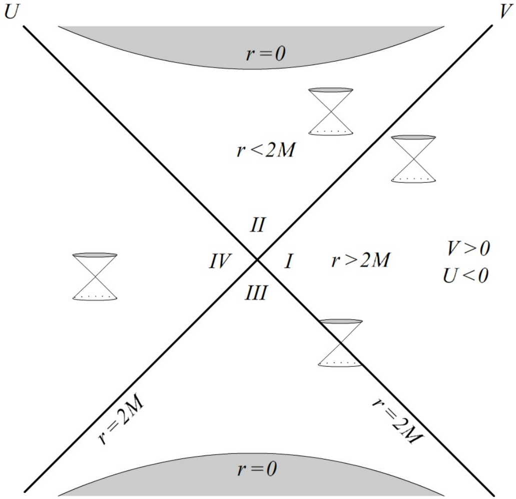
- $r = 2M$ is represented by the set $\{U = 0\} \cup \{V = 0\}$
- The exterior region $r > 2M$ is represented by the new coordinates in the range $U < 0$ and $V > 0$
- However, the metric tensor is regular at $r = 2M$ and therefore we can analytically extend the coordinates to include the regions with $U > 0$ and $V < 0$.

Kruskal Diagram



Kruskal Hypersurfaces

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(Hyperbolae)

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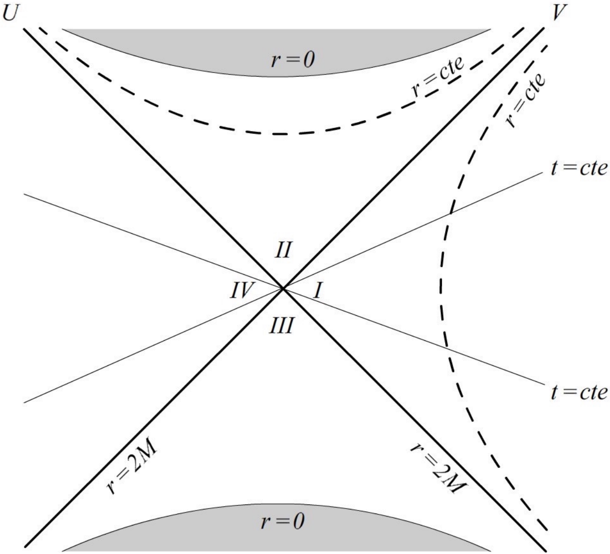
(Straight Lines)

Kruskal Diagram



Kruskal Hipersurfaces

Isometry of Kruskal Manifold:

$$(U, V) \longrightarrow (-U, -V)$$

i.e. region IV is indeed the time inverse of region I

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Isometry of Kruskal Manifold:

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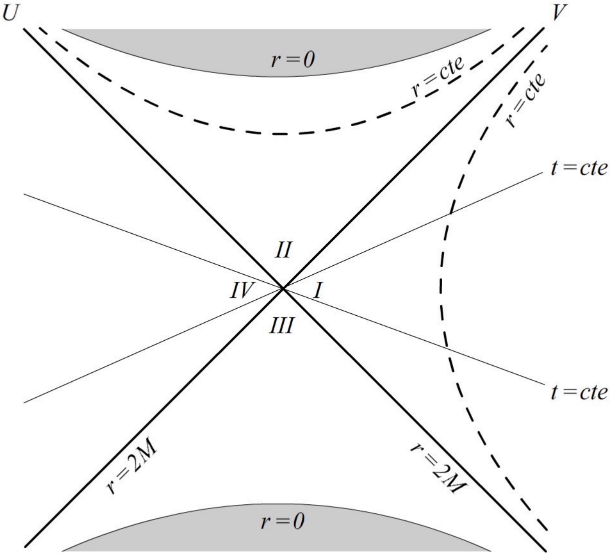
i.e. region IV is indeed the time inverse of region I

This isometry has the *fixed point* $\{U = 0\} \cup \{V = 0\}$ or equivalently $r = 2M$

Kruskal Diagram



Timelike Killing Vector Field

- Schwarzschild's Coordinates:
  $\xi = \frac{\partial}{\partial t}$
  Corresponds to the invariance under $t \longrightarrow t + k$

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  $V \longrightarrow Ve^{\frac{k}{4M}}$
  $U \longrightarrow Ue^{-\frac{k}{4M}}$

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  Infinitesimal version of the transformation is

  $\delta U \longrightarrow -\frac{k}{4M} U$

  $\delta V \longrightarrow \frac{k}{4M} V$

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  Infinitesimal version of the transformation is

  $$\delta U \longrightarrow -\frac{k}{4M}U$$
  $$\delta V \longrightarrow \frac{k}{4M}V$$

  $$\xi = \frac{1}{4M}\left[V\frac{\partial}{\partial V} - U\frac{\partial}{\partial U}\right]$$

Timelike Killing Vector Field

$$\xi = \frac{1}{4M}\left[V\frac{\partial}{\partial V} - U\frac{\partial}{\partial U}\right]$$

$$\xi^2 = -\left(1 - \frac{2M}{r}\right)$$

- ξ is timelike in regions I and IV

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- ξ is timelike in regions I and IV
- ξ is spacelike in regions II and III

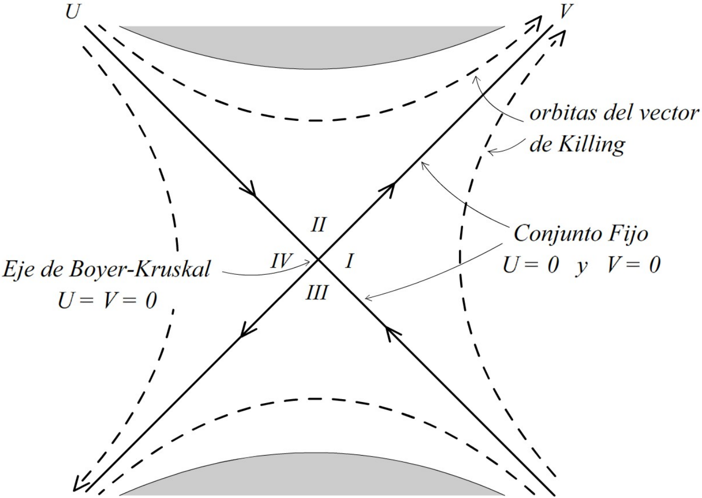
Timelike Killing Vector Field

$$\xi = \frac{1}{4M}\left[V\frac{\partial}{\partial V} - U\frac{\partial}{\partial U}\right]$$

$$\xi^2 = -\left(1 - \frac{2M}{r}\right)$$

- ξ is timelike in regions I and IV
- ξ is spacelike in regions II and III
- ξ is a null vector at the surface $r = 2M$

Kruskal Diagram



Back to Timelike and Spacelike Coordinates

$$(V, U, \theta, \phi) \longrightarrow (T, X, \theta, \phi)$$

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$$(V, U, \theta, \phi) \longrightarrow (T, X, \theta, \phi)$$

$$\begin{aligned} T &= \frac{1}{2}(V - U) \\ X &= \frac{1}{2}(V + U)\,, \end{aligned}$$

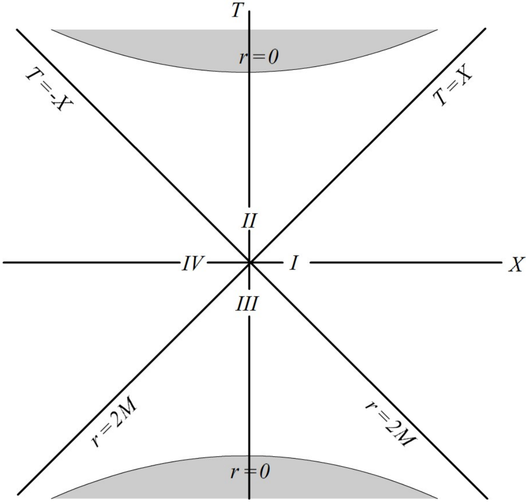
Back to Timelike and Spacelike Coordinates

$$(V, U, \theta, \phi) \longrightarrow (T, X, \theta, \phi)$$

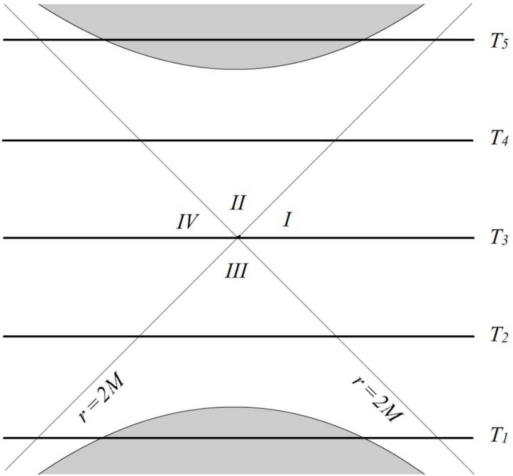
$$T = \frac{1}{2}(V - U)$$
$$X = \frac{1}{2}(V + U),$$

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \left[-dT^2 + dX^2\right] + r^2 d\Omega^2$$

Kruskal Diagram



Dynamical Behavior of the Manifold



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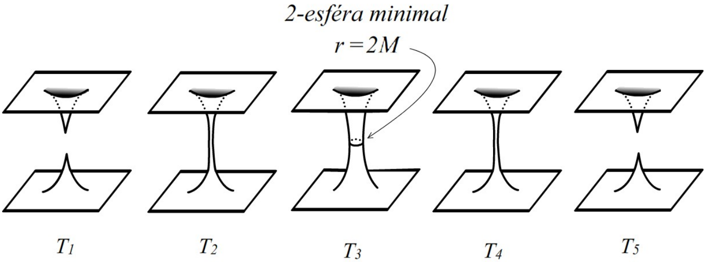

Isotropic Coordinates

$$(t, r, \theta, \phi) \longrightarrow (t, \rho, \theta, \phi)$$

such that the line element becomes conformally flat in its spatial part:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \omega^2(\rho)\left[d\rho^2 + \rho^2 d\Omega^2\right]$$

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This requires that

$$\omega^2(\rho)\,\rho^2 = r^2$$

$$\frac{dr^2}{\left(1 - \frac{2M}{r}\right)} = \omega^2 d\rho^2$$

$$\implies r = \left(1 + \frac{M}{2\rho}\right)^2 \rho$$

Isotropic Coordinates

$$ds^2 = -\frac{\left(1 - \frac{M}{2\rho}\right)^2}{\left(1 + \frac{M}{2\rho}\right)^2} dt^2 + \left(1 + \frac{M}{2\rho}\right)^4 \left[d\rho^2 + \rho^2 d\Omega^2\right]$$

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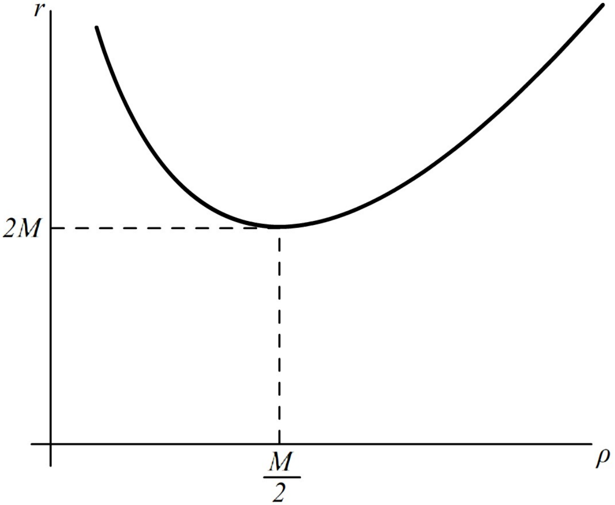
Fixed point at $\rho = \frac{M}{2}$ corresponding to the 2-sphere of radius $r = 2M$

Isotropic Coordinates Isometry



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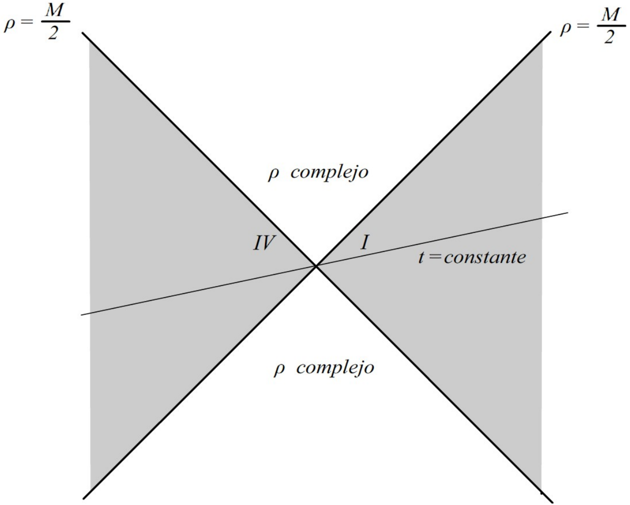

Cartesian Coordinates

$$ds^2 = -\frac{\left(1 - \frac{M}{2\rho}\right)^2}{\left(1 + \frac{M}{2\rho}\right)^2} dt^2 + \left(1 + \frac{M}{2\rho}\right)^4 \left[dx^2 + dy^2 + dz^2\right]$$

$$r = \sqrt{x^2 + y^2 + z^2} = \rho \left(1 + \frac{M}{2\rho}\right)^2$$

Killing Vectors in Cartesian Coordinates

$$\xi = \frac{\partial}{\partial t}$$

$$\zeta_1 = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$$

$$\zeta_2 = y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}$$

$$\zeta_3 = z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}$$

Carter-Penrose Diagrams

Compactification

$$ds^2 \longrightarrow d\tilde{s}^2 = \omega^2\left(x^\mu\right) ds^2$$

Compactification

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$$\omega \longrightarrow 0 \text{ when } x^i \longrightarrow \infty$$

Minkowski's Spacetime

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$

Range of t and r

$$-\infty < t < \infty$$

$$0 \leq r < \infty$$

Minkowski's Spacetime in Null Coordinates

Null coordinates:

$$v = t + r$$
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Extra Condition:

$$r = \frac{1}{2}(v-u) \geq 0 \Rightarrow u \leq v$$

.

Compactification of Minkowski's Spacetime

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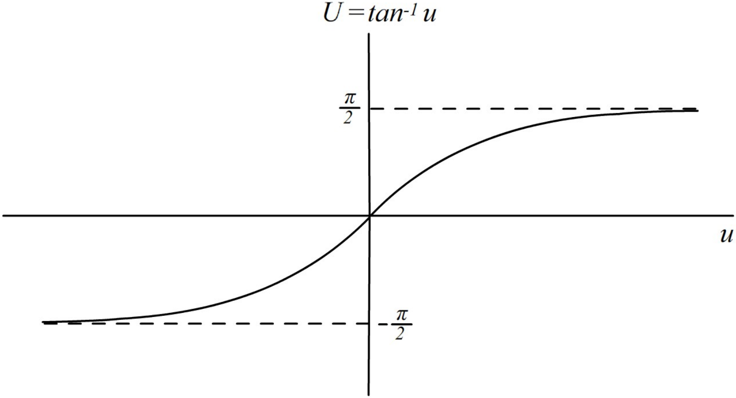

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$$-\frac{\pi}{2} < V < \frac{\pi}{2}$$

$$-\frac{\pi}{2} < U < \frac{\pi}{2}$$

Together with the condition $U \leq V$

Compactification of Minkowski's Spacetime

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$$\omega = 2\cos V \cos U$$

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It satisfies $\omega \longrightarrow 0$ when $|V| \longrightarrow \frac{\pi}{2}$ or when $|U| \longrightarrow \frac{\pi}{2}$

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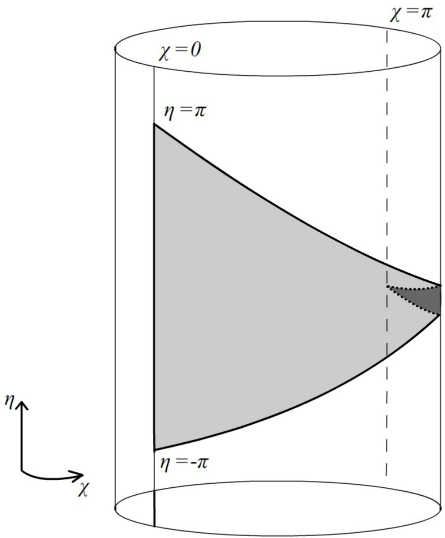
$$-\pi < \eta < \pi$$

$$0 \leq \chi < \pi$$

infinite	(t, r)	(v, u)	(V, U)	(η, χ)
i_0 : spatial	$\begin{cases} t & \text{finite} \\ r & \to +\infty \end{cases}$	$\begin{cases} v & \to +\infty \\ u & \to -\infty \end{cases}$	$\begin{cases} V & = \frac{\pi}{2} \\ U & = -\frac{\pi}{2} \end{cases}$	$\begin{cases} \eta & = 0 \\ \chi & = \pi \end{cases}$
i_+ : temporal future	$\begin{cases} t & \to \infty \\ r & \text{finite} \end{cases}$	$\begin{cases} v & \to +\infty \\ u & \to +\infty \end{cases}$	$\begin{cases} V & = \frac{\pi}{2} \\ U & = \frac{\pi}{2} \end{cases}$	$\begin{cases} \eta & = \pi \\ \chi & = 0 \end{cases}$
i_- : temporal past	$\begin{cases} t & \to -\infty \\ r & \text{finite} \end{cases}$	$\begin{cases} v & \to -\infty \\ u & \to -\infty \end{cases}$	$\begin{cases} V & = -\frac{\pi}{2} \\ U & = -\frac{\pi}{2} \end{cases}$	$\begin{cases} \eta & = -\pi \\ \chi & = 0 \end{cases}$
$\mathscr{I}_+$: null future	$\begin{cases} t \to +\infty \\ r \to +\infty \\ t - r \text{ finite} \end{cases}$	$\begin{cases} v & \to +\infty \\ u & \text{finite} \end{cases}$	$\begin{cases} V = \frac{\pi}{2} \\ -\frac{\pi}{2} < U < \frac{\pi}{2} \end{cases}$	$\begin{cases} \eta = \pi - \chi \\ 0 < \chi < \pi \end{cases}$
$\mathscr{I}_-$: null past	$\begin{cases} t \to -\infty \\ r \to +\infty \\ t + r \text{ finite} \end{cases}$	$\begin{cases} v & \text{finite} \\ u & \to -\infty \end{cases}$	$\begin{cases} -\frac{\pi}{2} < V < \frac{\pi}{2} \\ U = -\frac{\pi}{2} \end{cases}$	$\begin{cases} \eta = -\pi + \chi \\ 0 < \chi < \pi \end{cases}$

Carter-Penrose Diagram for Minkowski's spacetime



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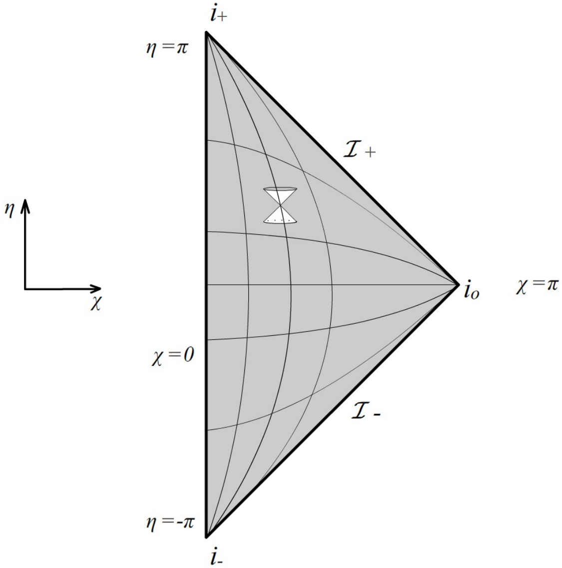

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Compactification of Kruskal Spacetime

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Compactification of Kruskal Spacetime

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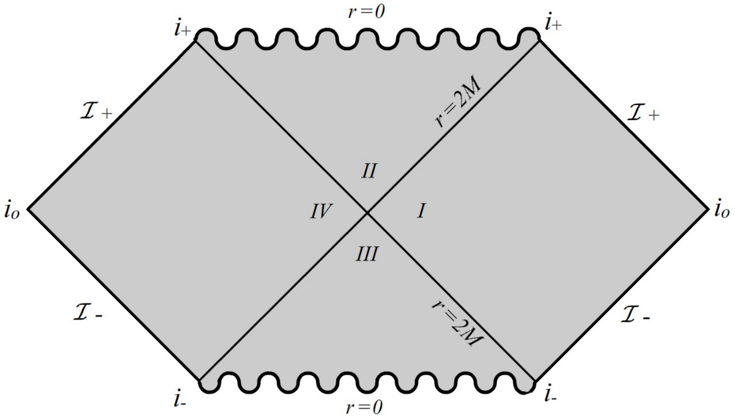
Carter-Penrose Diagram for Kruskal Spacetime



Next Lecture

05. Hypersurfaces. Horizons and Black Holes.