$$e^{(0)}_\mu = \sqrt{\frac{\Delta}{\Sigma}}\left(dt - a\sin^2\theta\, d\phi\right)$$

$$e^{(1)}_\mu = \sqrt{\frac{\Sigma}{\Delta}}\, dr$$

$$e^{(2)}_\mu = \sqrt{\Sigma}\, d\theta$$

$$e^{(3)}_\mu = \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,dt + (r^2+a^2)\,d\phi\right]$$

$$\Delta = r^2 + a^2 - 2G(r)Mr$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$G(r) = \frac{G_0 r^3}{r^3 + \tilde{\omega} G_0 (r + \gamma G_0 M)}$$

Killing Vectors

$$\xi = \frac{\partial}{\partial t} \qquad \xi^\mu = (1,0,0,0)$$

$$\xi^{(\alpha)} = e^{(\alpha)}{}_\mu\, \xi^\mu$$

$$\xi^{(\alpha)} = e^{(\alpha)}{}_0$$

$$\boxed{\begin{aligned}
\xi^{(0)} &= e^{(0)}{}_0 = \sqrt{\frac{\Delta}{\Sigma}}\\
\xi^{(1)} &= e^{(1)}{}_0 = 0\\
\xi^{(2)} &= e^{(2)}{}_0 = 0\\
\xi^{(3)} &= e^{(3)}{}_0 = -\frac{a\sin\theta}{\sqrt{\Sigma}}
\end{aligned}}$$

$$\xi_{(0)} = \eta_{(0)(\alpha)}\,\xi^{(\alpha)} = \eta_{(0)(0)}\,\xi^{(0)} = -\sqrt{\frac{\Delta}{\Sigma}}$$

$$\xi_{(1)} = \eta_{(1)(\alpha)}\,\xi^{(\alpha)} = \eta_{(1)(1)}\,\xi^{(1)} = 0$$

$$\xi_{(2)} = \eta_{(2)(\alpha)}\,\xi^{(\alpha)} = \eta_{(2)(2)}\,\xi^{(2)} = 0$$

$$\xi_{(3)} = \eta_{(3)(\alpha)}\,\xi^{(\alpha)} = \eta_{(3)(3)}\,\xi^{(3)} = -\frac{a\sin\theta}{\sqrt{\Sigma}}$$

$$\varphi = \frac{\partial}{\partial\phi} \qquad \varphi^{\mu} = (0,0,0,1)$$

$$\varphi^{(\alpha)} = e^{(\alpha)}{}_{\mu}\,\varphi^{\mu}$$

$$\varphi^{(\alpha)} = e^{(\alpha)}{}_{3}$$

$$\varphi^{(0)} = e^{(0)}{}_{3} = -\sqrt{\frac{\Delta}{\Sigma}}\,a\sin^2\theta$$

$$\varphi^{(1)} = e^{(1)}{}_{3} = 0$$

$$\varphi^{(2)} = e^{(2)}{}_{3} = 0$$

$$\varphi^{(3)} = e^{(3)}{}_{3} = (r^2+a^2)\frac{\sin\theta}{\sqrt{\Sigma}}$$

$$\varphi_{(0)} = \eta_{(0)(\alpha)}\,\varphi^{(\alpha)} = \eta_{(0)(0)}\,\varphi^{(0)} = \sqrt{\frac{\Delta}{\Sigma}}\,a\sin^2\theta$$

$$\varphi_{(1)} = \eta_{(1)(\alpha)}\,\varphi^{(\alpha)} = \eta_{(1)(1)}\,\varphi^{(1)} = 0$$

$$\varphi_{(2)} = \eta_{(2)(\alpha)}\,\varphi^{(\alpha)} = \eta_{(2)(2)}\,\varphi^{(2)} = 0$$

$$\varphi_{(3)} = \eta_{(3)(\alpha)}\,\varphi^{(\alpha)} = \eta_{(3)(3)}\,\varphi^{(3)} = (r^2+a^2)\frac{\sin\theta}{\sqrt{\Sigma}}$$