$$g_{\mu\nu} = \eta_{\alpha\beta}\, e^{(\alpha)}_{\mu}\, e^{(\beta)}_{\nu} \qquad \eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1)$$

$$\boxed{\begin{aligned} e^{(0)}_{\mu} &= \sqrt{\frac{\Delta}{\Sigma}}\left(dt - a\sin^2\theta\, d\phi\right) \\ e^{(1)}_{\mu} &= \sqrt{\frac{\Sigma}{\Delta}}\, dr \\ e^{(2)}_{\mu} &= \sqrt{\Sigma}\, d\theta \\ e^{(3)}_{\mu} &= \frac{\sin\theta}{\sqrt{\Sigma}}\left[-a\,dt + (r^2+a^2)\, d\phi\right] \end{aligned}}$$

$$\boxed{\begin{aligned} \Delta &= r^2 + a^2 - 2G(r)Mr \\ \Sigma &= r^2 + a^2\cos^2\theta \\ G(r) &= \frac{G_0 r^3}{r^3 + \tilde{\omega} G_0 (r + \gamma G_0 M)} \end{aligned}}$$

PROOF:

$$g_{00} = \eta_{\alpha\beta}\, e^{(\alpha)}_{0} e^{(\beta)}_{0} = -e^{(0)}_{0} e^{(0)}_{0} + e^{(1)}_{0} e^{(1)}_{0} + e^{(2)}_{0} e^{(2)}_{0} + e^{(3)}_{0} e^{(3)}_{0}$$

$$g_{00} = -e^{(0)}_{0} e^{(0)}_{0} + e^{(3)}_{0} e^{(3)}_{0}$$

$$g_{00} = -\sqrt{\frac{\Delta}{\Sigma}}\sqrt{\frac{\Delta}{\Sigma}} + \left(-\frac{a\sin\theta}{\sqrt{\Sigma}}\right)\left(-\frac{a\sin\theta}{\sqrt{\Sigma}}\right)$$

$$g_{00} = -\frac{\Delta}{\Sigma} + \frac{a^2\sin^2\theta}{\Sigma}$$

$$g_{00} = -\frac{\Delta - a^2\sin^2\theta}{\Sigma}$$

$$g_{00} = -\frac{r^2 + a^2 - 2G(r)Mr - a^2\sin^2\theta}{\Sigma}$$

$$g_{00} = -\frac{r^2 + a^2(1-\sin^2\theta) - 2G(r)Mr}{\Sigma}$$

$$g_{00} = -\left(\frac{r^2+a^2\cos^2\theta}{\Sigma} - \frac{2G(r)Mr}{\Sigma}\right)$$

$$g_{00} = -\left(1 - \frac{2G(r)Mr}{\Sigma}\right) \checkmark$$

$$g_{01} = \eta_{\alpha\beta} e^{(\alpha)}_0 e^{(\beta)}_1 = -e^{(0)}_0 e^{(0)}_1 + e^{(1)}_0 e^{(1)}_1 + e^{(2)}_0 e^{(2)}_1 + e^{(3)}_0 e^{(3)}_1$$

$$g_{01} = 0 \quad \checkmark$$

$$g_{02} = \eta_{\alpha\beta} e^{(\alpha)}_0 e^{(\beta)}_2 = -e^{(0)}_0 e^{(0)}_2 + e^{(1)}_0 e^{(1)}_2 + e^{(2)}_0 e^{(2)}_2 + e^{(3)}_0 e^{(3)}_2$$

$$g_{02} = 0 \quad \checkmark$$

$$g_{03} = \eta_{\alpha\beta} e^{(\alpha)}_0 e^{(\beta)}_3 = -e^{(0)}_0 e^{(0)}_3 + e^{(1)}_0 e^{(1)}_3 + e^{(2)}_0 e^{(2)}_3 + e^{(3)}_0 e^{(3)}_3$$

$$g_{03} = -e^{(0)}_0 e^{(0)}_3 + e^{(3)}_0 e^{(3)}_3$$

$$g_{03} = -\sqrt{\frac{\Delta}{\Sigma}}\left(-\sqrt{\frac{\Delta}{\Sigma}}\, a\sin^2\theta\right) + \left(-\frac{a\sin\theta}{\sqrt{\Sigma}}\right)\left(\frac{\sin\theta}{\sqrt{\Sigma}}(r^2+a^2)\right)$$

$$g_{03} = \frac{\Delta}{\Sigma} a\sin^2\theta - \frac{a\sin^2\theta}{\Sigma}(r^2+a^2)$$

$$g_{03} = \frac{a\sin^2\theta}{\Sigma}(\Delta - r^2 - a^2)$$

$$g_{03} = \frac{a\sin^2\theta}{\Sigma}(r^2+a^2-2G(r)Mr - r^2 - a^2)$$

$$g_{03} = -\frac{2G(r)Mr}{\Sigma} a\sin^2\theta \quad \checkmark$$

$$g_{11} = \eta_{\alpha\beta} e_1^{(\alpha)} e_1^{(\beta)} = -e_1^{(0)} e_1^{(0)} + e_1^{(1)} e_1^{(1)} + e_1^{(2)} e_1^{(2)} + e_1^{(3)} e_1^{(3)}$$

$$g_{11} = e_1^{(1)} e_1^{(1)}$$

$$g_{11} = \frac{\Sigma}{\Delta} \quad \checkmark$$

$$g_{22} = \eta_{\alpha\beta} e_2^{(\alpha)} e_2^{(\beta)} = -e_2^{(0)} e_2^{(0)} + e_2^{(1)} e_2^{(1)} + e_2^{(2)} e_2^{(2)} + e_2^{(3)} e_2^{(3)}$$

$$g_{22} = e_2^{(2)} e_2^{(2)}$$

$$g_{22} = \Sigma \quad \checkmark$$

$$g_{33} = \eta_{\alpha\beta} e_3^{(\alpha)} e_3^{(\beta)} = -e_3^{(0)} e_3^{(0)} + e_3^{(1)} e_3^{(1)} + e_3^{(2)} e_3^{(2)} + e_3^{(3)} e_3^{(3)}$$

$$g_{33} = -e_3^{(0)} e_3^{(0)} + e_3^{(3)} e_3^{(3)}$$

$$g_{33} = -\left(-\sqrt{\frac{\Delta}{\Sigma}}\, a \sin^2\theta\right)^2 + \left(\frac{\sin\theta}{\sqrt{\Sigma}}(r^2+a^2)\right)^2$$

$$g_{33} = -\frac{\Delta}{\Sigma} a^2 \sin^4\theta + \frac{\sin^2\theta}{\Sigma}(r^2+a^2)^2$$

$$g_{33} = \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]$$

$$g_{33} = \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)^2 - a^2(r^2+a^2-2G(r)Mr)\sin^2\theta\right]$$

$$g_{33} = \frac{\sin^2\theta}{\Sigma}\left[\underbrace{(r^2+a^2)^2 - r^2a^2\sin^2\theta - a^4\sin^2\theta}\, + 2G(r)Mra^2\sin^2\theta\right]$$

$$= r^4 + 2r^2a^2 + a^4 - r^2a^2\sin^2\theta - a^4\sin^2\theta$$

$$= r^4 + r^2a^2 + r^2a^2\cos^2\theta + a^4\cos^2\theta$$

$$= (r^2+a^2)(r^2+a^2\cos^2\theta)$$

$$g_{33} = \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)(r^2+a^2\cos^2\theta) + 2G(r)Mra^2\sin^2\theta\right]$$

$$g_{33} = \sin^2\theta \left[\frac{(r^2+a^2)(r^2+a^2\cos^2\theta)}{\Sigma} + \frac{2G(r)Mr}{\Sigma} a^2 \sin^2\theta \right]$$

$$g_{33} = \sin^2\theta \left(r^2 + a^2 + \frac{2G(r)Mr}{\Sigma} a^2 \sin^2\theta \right) \quad \checkmark$$