

# Memory Bounded Bidirectional Search

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## The Problem

- Unidirectional search goal test:

$$node == goal$$

**Memory required:** 1

- Naïve Bidirectional search goal test:

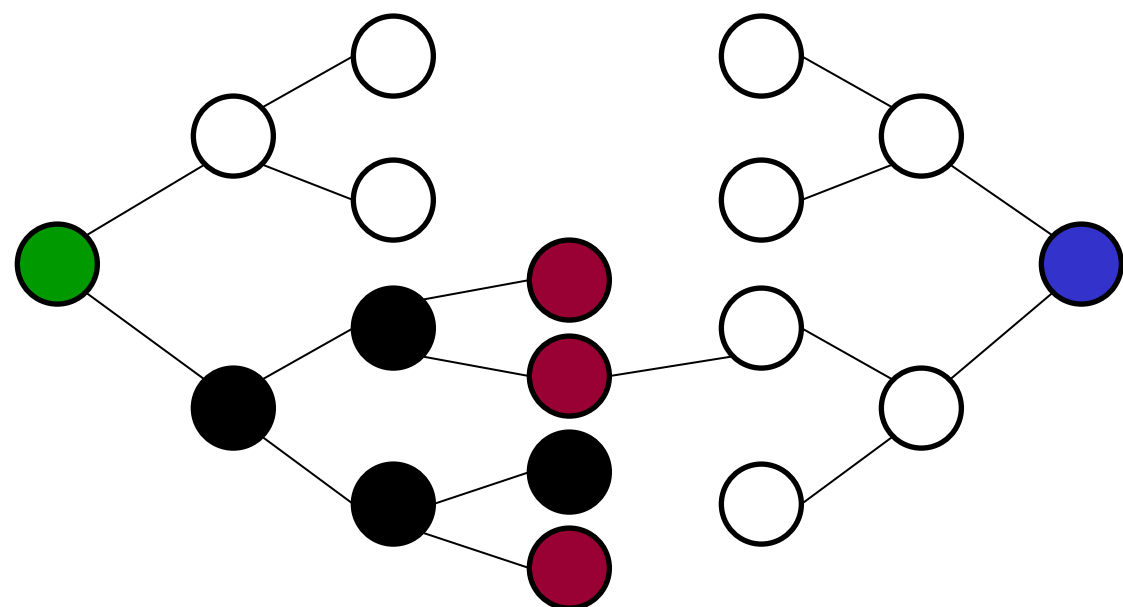
$$open_F \cup open_B == \emptyset$$

**Memory required:**  $\min(|open_F|, |open_B|)$

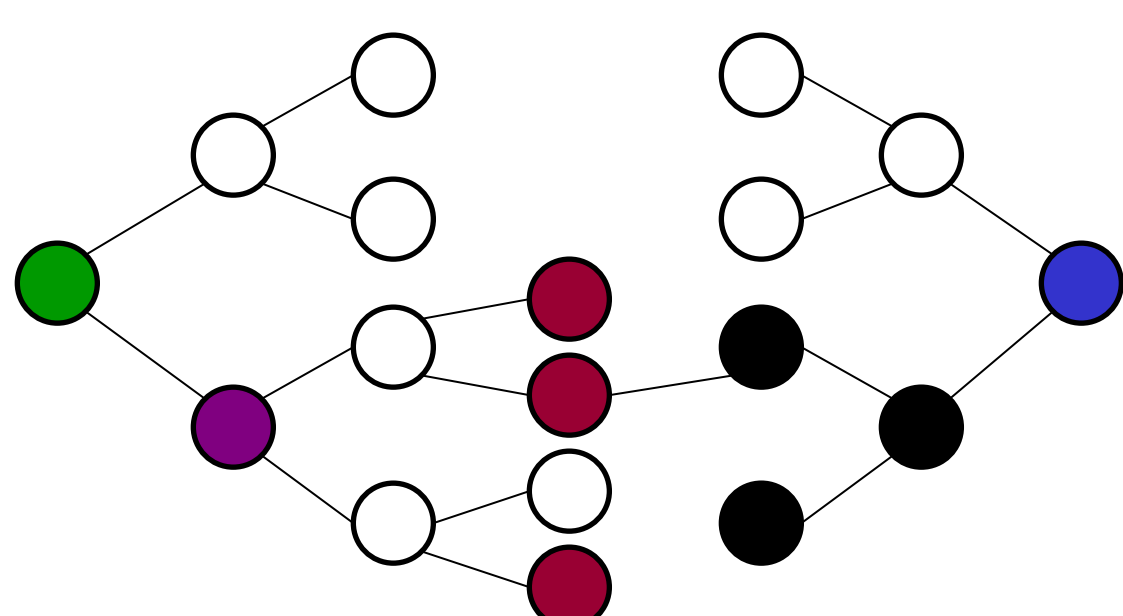
**Hard to bound!**

## Common Ancestor Search

**Step 1:** run IDA\* from **start** to **goal** and store **potential meeting points** in memory until it is full



**Step 2:** run A\* from **goal** to the **common ancestor of nodes in memory**



## Iterative Deepening Bidirectional Search

- A general framework for running Memory bounded bidirectional search.
- Uses two thresholds –  $th_F$  for the forward search and  $th_B$  for the backward search
- Every iteration uses an external search procedure to look for a node distanced  $th_F$  from the start and  $th_B$  from the goal.
- If the search found no solution, either  $th_F$  or  $th_B$  is increased according to a predetermined policy.

Examples:

Search	Policy	Result
DFS	Always $th_F$	IDA* [1]
BFS	Always $th_F$	BFIDA* [2]
BFS	Alternate	Iterative version of MM [3]

## Type Systems

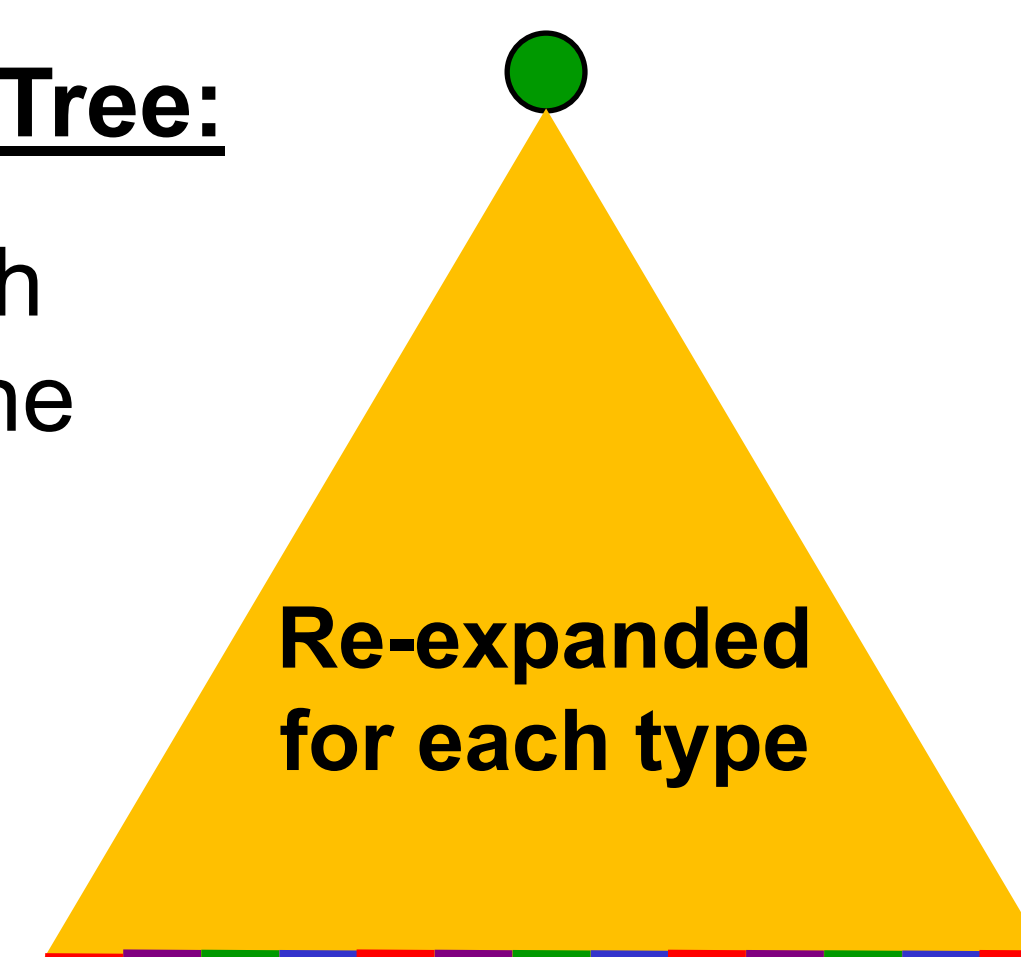
- Partition  $open_F$  into  $k$  disjoint types  $T_1, T_2, \dots, T_k$ , such that every type fits in memory.
- Run the search algorithm  $k$  times, each time only looking for meeting points in a single type.

### Blind

- Example:** Partition according to an arbitrary hash function on the state
- Performs the entire search every iteration
- Domain agnostic, requires no special properties
- Saves a factor of  $k$  in space for a cost of  $k$  in running time

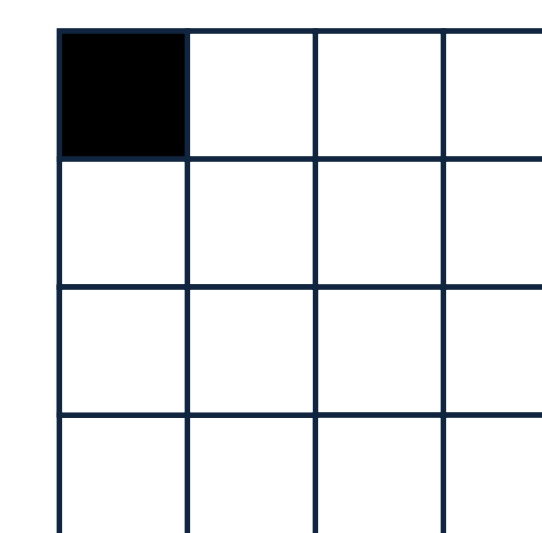
#### Forward Search Tree:

Types marked with colored lines on the search front



### Informed

- Example:** In the sliding tile puzzle, partition according to the location of the blank.
- Performs only part of the search every iteration
- Prunes states with no descendants in the current type.
- Saves a factor of  $k$  in space for a significantly smaller cost in running time



**Current type:**  
Blank in top left corner

1	2	3	4
5	6	7	8
9	10	11	
13	14	15	12

This node can be pruned if  $g > th_F - 5$

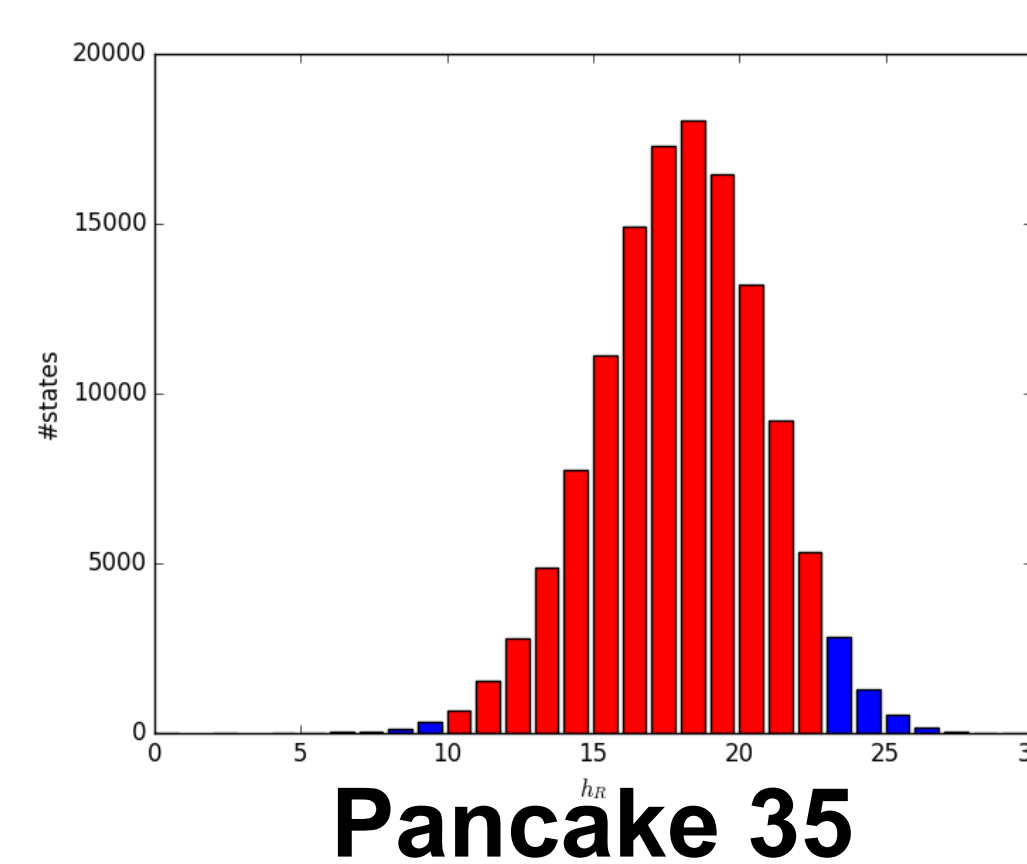
## Reference Point Search

- An informed type system where states are partitioned into types according to the heuristic value from some state  $R$
- Assuming a consistent heuristic, we can prune node  $n$  in iteration  $i$  if

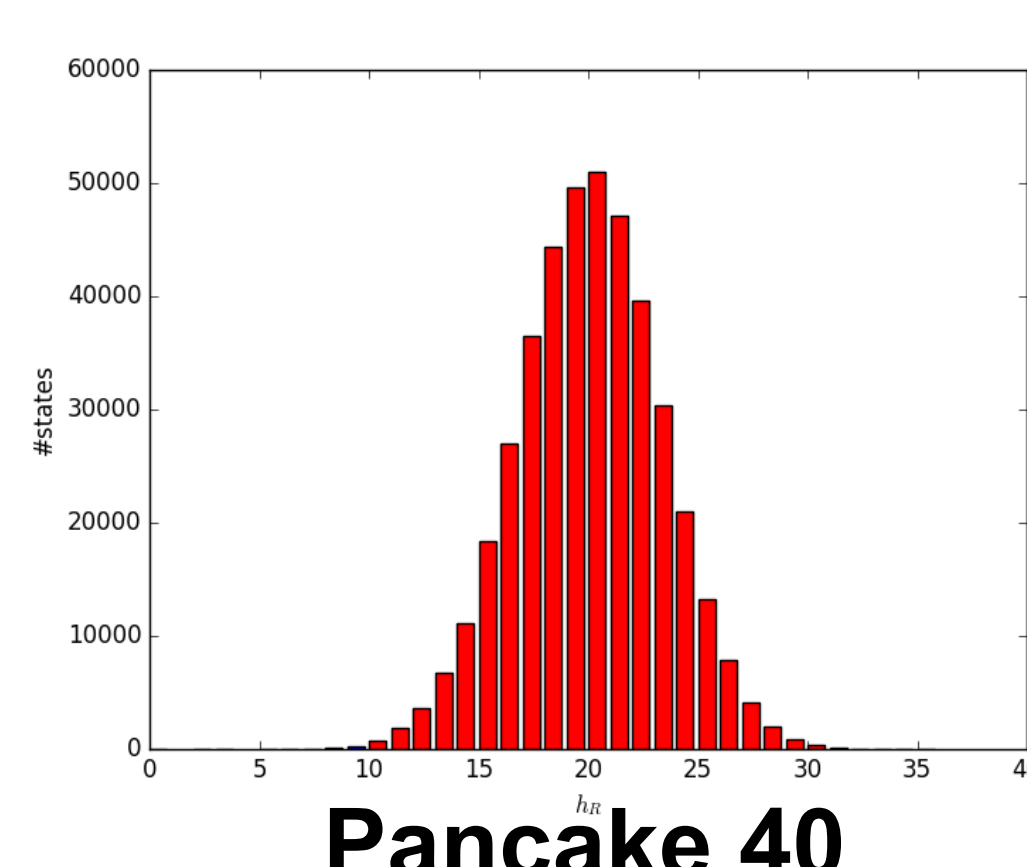
$$|h(r, n) - i| > th - g(n)$$

- Runtime cost for finding a solution is less than half the full traversal of the search tree and memory requirements are orders of magnitude smaller

Domain	Time ratio	Memory ratio
Pancake 35	0.44	1/98.7
Pancake 40	0.47	1/124.2
Pancake 45	0.41	1/170.5

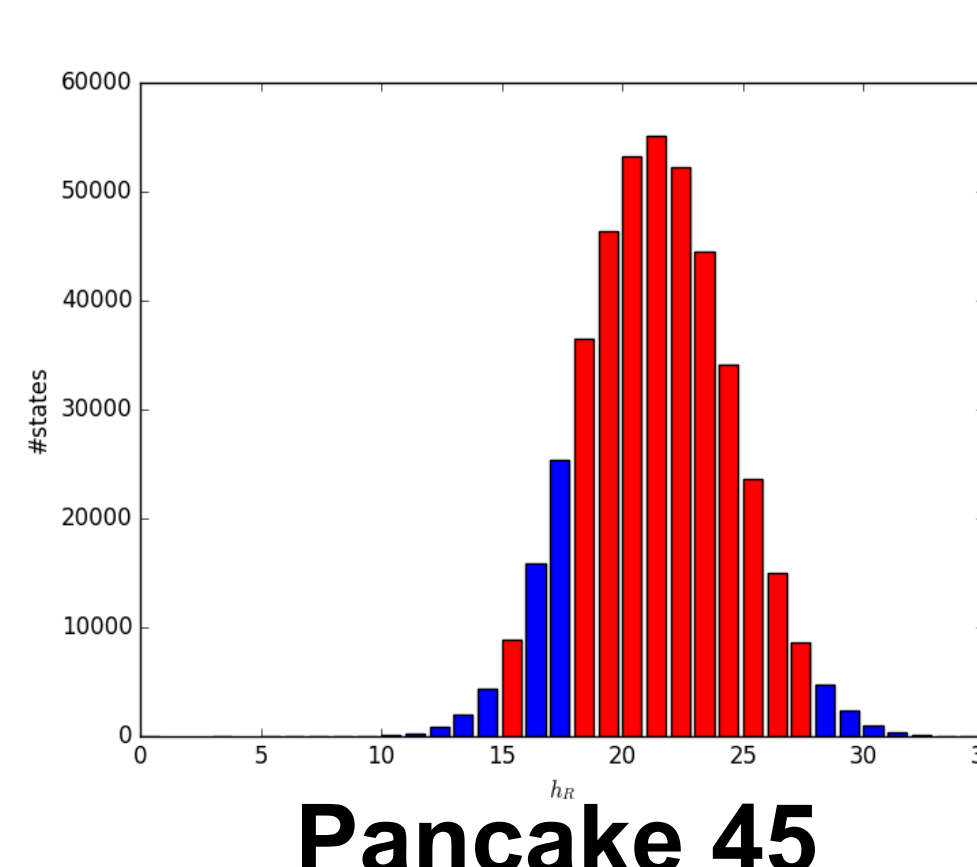


**Pancake 35**



**Pancake 40**

**Distribution of states in each type (h-value)**  
**For sample instances**  
**Red:** types containing a solution  
**Blue:** types without a solution



**Pancake 45**

## Bloom Filters and Results

- An orthogonal approach of storing the open list in Bloom Filters was examined.
- Experiments applying Iterative Deepening Bidirectional Search with Reference Point Search and Bloom Filters managed to optimally solve the 86-pancake puzzle in a few hours on a workstation.
- A memory bound of approx.  $8 \times 10^5$  states was used and in total, more than  $4 \times 10^{10}$  nodes were generated.
- This is the largest Pancake puzzle ever solved.

[1] Korf R. E. Artificial intelligence 27(1):97–109 (1985).  
[2] Rong Zhou and Eric A. Hansen, ICAPS 2004.  
[3] R. Holte et al., AAAI 2016.