Emergent Cosmic Structure from ECSM : Linear Growth Without Dark Matter or Expansion

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Abstract

We investigate whether large-scale cosmic structure can form at the observed rate in a non-expanding universe governed by a dynamical superfluid medium. Rather than modifying spacetime geometry or introducing unseen matter components, gravitational phenomena are treated as an emergent response of a physical medium to baryonic sources. We derive the linear perturbation equations governing density growth in this framework and show that scale-dependent amplification naturally arises without invoking dark matter or metric expansion. The resulting growth behavior is consistent with observed clustering amplitudes and avoids the standard horizon and coincidence problems. This work extends earlier galaxy-scale constraints on local and nonlocal response laws to the cosmological regime, providing a unified and testable alternative to ΛCDM structure formation.

1 Introduction

A central success of the standard cosmological model is its ability to predict the growth of structure from small initial perturbations. [1, 2] Many alternative gravity or medium-based frameworks fail at this stage, either by suppressing growth entirely or by introducing pathological scale dependence.

In previous work, we demonstrated that a response-based superfluid medium framework reproduces galaxy rotation curves, the radial acceleration relation, and cosmological distance indicators without invoking dark matter or physical expansion. The purpose of the present paper is to address the remaining consistency question: whether linear density perturbations grow in a controlled and observationally viable manner within this framework.

ECSM (Emergent Condensate Superfluid Medium) treats the vacuum as an effectively superfluid, condensate-like medium with dynamical fields whose gradients and defects carry stress, transport, and energy. In this view, phenomena usually attributed to spacetime curvature and unseen matter arise instead from the medium's local response laws (pressure-like stresses, solenoidal flow, and defect/flux-tube dynamics), with "geometry" emerging as an effective description of propagation and clock/ruler behaviour. The goal is not to draw a web by assumption, but to show that simple, conservative medium dynamics can self-organise into node–filament–void structure and reproduce the main cosmological observables through falsifiable, scale-bridging mechanisms.

2 Homogeneous Background Medium

We consider a homogeneous, isotropic background described by a time-dependent medium state variable $\chi(t)$. This variable encodes the effective response properties of the medium and replaces the role conventionally played by an expanding metric.

No assumption of physical expansion is made. Observable quantities such as redshift are treated as effective responses accumulated along null trajectories through the medium, rather than as clocks defining cosmic time.

3 Linear Density Perturbations

We introduce small density perturbations about the homogeneous background,

$$\rho(\mathbf{x}, t) = \rho_0(t)\,[1 + \delta(\mathbf{x}, t)], \tag{1}$$

with $|\delta| \ll 1$.

The gravitational response of the medium is governed by a response functional $\mathcal{R}$ relating the effective gravitational field $\mathbf{g}_{\text{eff}}$ to the baryonic source. Linearizing about the background state yields

$$\mathbf{g}_{\text{eff}} \simeq \mathcal{R}(\chi_0)\,\nabla\Phi + \mathcal{R}'(\chi_0)\,\delta\chi\,\nabla\Phi, \tag{2}$$

where $\chi_0(t)$ is the background medium state.

3.1 Linear perturbations and growth in a non-expanding background

We derive the linear growth equation for density perturbations in a static (non-metric-expanding) background, appropriate to a framework in which cosmological redshift and large-scale geometry arise from medium response rather than a global scale factor. We work in physical coordinates $\mathbf{x}$ and physical time t, with no $a(t)$ or $H(t)$.

Domain of validity. The analysis presented here is restricted to the linear and quasi-linear growth regime of cosmic structure at late times, where density contrasts remain sufficiently small that a perturbative treatment is appropriate. We focus explicitly on post-recombination evolution in a high-coherence background, assuming that the ECSM response operates in its locality-gated regime.

Nonlinear collapse, virialization, baryonic feedback processes, and strongly non-equilibrium phenomena are intentionally excluded from the present treatment and deferred to future numerical and simulation-based work. Within this domain, the growth equations are well-posed, stable, and predictive, and provide a controlled comparison with standard linear growth analyses in ΛCDM.

Operational dictionary (growth–lensing sector). Throughout this paper we distinguish between (i) the *matter response* of the ECSM medium that governs the evolution of density perturbations and inertial/gravitational acceleration, and (ii) the *propagation bookkeeping* used to map those perturbations to observables (e.g. weak lensing and time-delay integrals). Concretely: (i) the linear growth of $\delta(\mathbf{x}, t)$ is controlled by an effective, state- and scale-dependent coupling $G_{\rm eff}(k, t)$ (equivalently, a locality gate / response factor multiplying the Newtonian source term), and by the medium's dynamical response time through the absence of an expansion-induced friction term; (ii) the observable lensing and timing signals depend on the same underlying perturbations but may involve additional propagation suppression factors only when the line of sight traverses coherence-boundary or phase-transition environments. In the high-coherence, locality-gated regime relevant for late-time large-scale structure, ECSM reduces to a single-parameter family of departures encoded entirely in $G_{\rm eff}(k, t)$, with no separate "optical-sector" tuning introduced in this paper.

Fluid equations. Let $\rho(\mathbf{x}, t) = \bar{\rho}\,[1 + \delta(\mathbf{x}, t)]$ with $|\delta| \ll 1$, and let $\mathbf{v}(\mathbf{x}, t)$ denote the peculiar velocity field (relative to the static background). Mass conservation gives

$$\dot{\delta} + \nabla \cdot \mathbf{v} = 0, \tag{3}$$

where dots denote $\partial/\partial t$. The linearised Euler equation (allowing an effective sound speed c_s as a placeholder for dispersion/pressure support) is

$$\dot{\mathbf{v}} = \mathbf{g} - c_s^2 \nabla \delta, \tag{4}$$

with $\mathbf{g}(\mathbf{x}, t)$ the effective gravitational acceleration experienced by the matter sector. Taking a time derivative of Eq. (3) and substituting $\nabla \cdot \dot{\mathbf{v}}$ from Eq. (4) yields the general linear growth equation

$$\ddot{\delta} = -\nabla \cdot \mathbf{g} + c_s^2 \nabla^2 \delta. \tag{5}$$

Thus, linear structure growth closes once a response law is specified that relates $\nabla \cdot \mathbf{g}$ to the density perturbation.

Newtonian baseline. For reference, standard Newtonian gravity is $\mathbf{g}_N = -\nabla \Phi_N$ with

$$\nabla^2 \Phi_N = 4\pi G\,\delta\rho = 4\pi G\,\bar{\rho}\,\delta, \tag{6}$$

so $\nabla \cdot \mathbf{g}_N = -4\pi G \bar{\rho}\,\delta$ and Eq. (5) becomes

$$\ddot{\delta} = 4\pi G \bar{\rho}\,\delta + c_s^2 \nabla^2 \delta. \tag{7}$$

In Fourier space ($\nabla^2 \to -k^2$) this reads

$$\ddot{\delta}_{\mathbf{k}} = 4\pi G\bar{\rho}\,\delta_{\mathbf{k}} - c_s^2 k^2 \delta_{\mathbf{k}}. \tag{8}$$

Medium-response closure (QUMOND form). We adopt a QUMOND-type constitutive relation as an effective description of the medium response:

$$\nabla^2 \Phi \;=\; \nabla \cdot \left[\nu\left(\frac{|\nabla \Phi_N|}{a_0}\right) \nabla \Phi_N \right], \qquad \mathbf{g} = -\nabla \Phi, \tag{9}$$

where a_0 is the characteristic acceleration scale and $\nu(y)$ interpolates between the high-acceleration regime ($\nu \to 1$ for $y \gg 1$) and the deep regime ($\nu \gg 1$ for $y \ll 1$). This form is convenient because Φ_N remains sourced directly by baryons via Eq. (6), while Φ encodes the medium-amplified response.

Linearisation and the role of an external field (EFE). A subtlety is that ν depends on $|\nabla \Phi_N|$; linearising about a perfectly homogeneous background with $\nabla \Phi_N = 0$ is ill-defined. The physically relevant procedure is to linearise about a patch permeated by a nonzero approximately uniform background acceleration $\mathbf{g}_{\text{ext}}$ (an "external field"), and consider perturbations ϕ_N about it:

$$\nabla \Phi_N \;=\; -\,\mathbf{g}_{\text{ext}} + \nabla \phi_N, \qquad |\nabla \phi_N| \ll |\mathbf{g}_{\text{ext}}|. \tag{10}$$

To leading order the interpolation factor is constant across the patch,

$$\nu\left(\frac{|\nabla \Phi_N|}{a_0}\right) \simeq \nu_e \equiv \nu\left(\frac{g_{\text{ext}}}{a_0}\right), \tag{11}$$

so Eq. (9) reduces to $\nabla^2 \Phi \simeq \nu_e \nabla^2 \Phi_N$ and hence

$$\nabla \cdot \mathbf{g} \;\simeq\; -\,\nu_e\, 4\pi G \bar{\rho}\, \delta. \tag{12}$$

Substitution into Eq. (5) gives a renormalised linear growth law,

$$\ddot{\delta} = 4\pi G_{\text{eff}} \bar{\rho}\, \delta + c_s^2 \nabla^2 \delta, \qquad G_{\text{eff}} \equiv \nu_e\, G, \tag{13}$$

Here G_{eff} denotes an *effective* gravitational coupling induced by the linear response of the medium in the presence of an external field, rather than a modification of the fundamental Newtonian constant. [3]

or in Fourier space,

$$\ddot{\delta}_{\mathbf{k}} = 4\pi \nu_e G \bar{\rho}\, \delta_{\mathbf{k}} - c_s^2 k^2 \delta_{\mathbf{k}}. \tag{14}$$

Therefore, in linear theory the medium-response law behaves as an environment-dependent effective gravitational coupling controlled by the external-field parameter g_{ext}/a_0.

Optional nonlocal response (coherence length). A minimal nonlocal generalisation is to replace the local Newtonian field entering ν by a smoothed field $\nabla\Phi_{N,\sigma}$ obtained by convolution with a normalised kernel W_σ:

$$\nabla\Phi_{N,\sigma}(\mathbf{x}) \equiv \int d^3x'\, W_\sigma(|\mathbf{x}-\mathbf{x}'|)\, \nabla\Phi_N(\mathbf{x}'). \tag{15}$$

Linearising as above in the presence of $\mathbf{g}_{\rm ext}$ yields a scale-dependent effective coupling in Fourier space,

$$\ddot{\delta}_{\mathbf{k}} = 4\pi G\, \nu_e\, \tilde{W}_\sigma(k)\, \bar{\rho}\, \delta_{\mathbf{k}} - c_s^2 k^2 \delta_{\mathbf{k}}, \tag{16}$$

where $\tilde{W}_\sigma(k)$ is the Fourier transform of W_σ (e.g. $\tilde{W}_\sigma = e^{-(k\sigma)^2/2}$ for a Gaussian kernel). This provides a falsifiable route to scale-dependent growth without invoking non-baryonic matter components.

3.2 Constitutive medium-response law and its limits

We now define the constitutive relation that links baryonic sources to the emergent gravitational response of the medium, prior to any linearisation or perturbative expansion.

We model the gravitational response as an emergent *constitutive* property of the superfluid medium rather than as a fundamental modification of geometry. Operationally, the medium maps a Newtonian (baryonic) acceleration field g_N to an effective acceleration $g_{\rm eff}$ via a dimensionless response function,

$$g_{\rm eff} \;=\; \nu(y)\; g_N, \qquad y \;\equiv\; \frac{g_N}{a_0(\chi)}. \tag{17}$$

Here $a_0(\chi)$ is a *state-dependent response scale* set by the local medium state χ (e.g. phase fraction, coherence length, compressibility, or an equivalent scalar order parameter that characterizes the local/nonlocal medium environment). The response function $\nu(y)$ is taken to be universal across systems, while environment enters only through $a_0(\chi)$.

It is convenient to package the response as an effective coupling,

$$G_{\rm eff}(\chi; g_N) \;\equiv\; \nu\!\left(\frac{g_N}{a_0(\chi)}\right) G, \qquad g_{\rm eff} \;=\; \frac{G_{\rm eff}}{G}\, g_N, \tag{18}$$

so that departures from Newtonian behaviour are interpreted as a medium-induced renormalization of the gravitational response rather than additional unseen matter.

High-acceleration (Newtonian) limit. To preserve Solar-System and laboratory constraints, the response must recover Newtonian dynamics for $y \gg 1$:

$$\nu(y) \to 1 \quad (y \gg 1), \qquad \Rightarrow \qquad g_{\rm eff} \to g_N. \tag{19}$$

Deep-response (low-acceleration) limit and BTFR. In the deep-response regime $y \ll 1$, observations imply the scaling $g_{\rm eff} \propto \sqrt{a_0 g_N}$, which yields asymptotically flat rotation curves and the baryonic Tully–Fisher relation. We therefore require

$$\nu(y) \;\simeq\; y^{-1/2} \quad (y \ll 1), \qquad \Rightarrow \qquad g_{\rm eff} \;\simeq\; \sqrt{a_0(\chi)\, g_N}. \tag{20}$$

For a circular orbit with speed $v(r)$, $g_{\rm eff} = v^2/r$ and $g_N = GM_b(<r)/r^2$. In the deep regime and for finite enclosed baryonic mass at large r, Eq. (20) gives

$$v^4 \;\simeq\; G\, M_b\, a_0(\chi), \tag{21}$$

so that the BTFR emerges directly from the constitutive response.

External field effect as a background-state shift. A distinctive consequence of a state-dependent response scale is that a nonzero background acceleration $g_{\rm ext}$ can shift systems out of the deep-response regime. A minimal way to encode this is to replace g_N by an effective argument that includes the background,

$$y \;=\; \frac{\sqrt{g_N^2 + g_{\rm ext}^2}}{a_0(\chi)}, \tag{22}$$

(or an equivalent smooth combination), which ensures that sufficiently large $g_{\rm ext}$ drives $\nu \to 1$ and restores Newtonian-like behaviour even when g_N alone would be small. In this framework the EFE is interpreted as a medium-state effect: the environment modifies the effective depth parameter y.

Why this is not an arbitrary fit function. Equations (17)–(22) are restrictive: the same $\nu(y)$ must describe all galaxies and must satisfy the Newtonian and deep-response limits (19)–(20). Environmental dependence is confined to the single scalar scale $a_0(\chi)$, which we treat as slowly varying and bounded over cosmological environments. In later sections we show that allowing only modest bounded variation in a_0 maps to modest variation in the implied $\nu_e \equiv G_{\rm eff}/G$, providing a controlled handle on structure growth while preserving galaxy-scale regularities.

Relation to MOND language. If one chooses a fixed constant a_0 and a specific $\nu(y)$, Eq. (17) reproduces the usual MOND-like algebraic radial-acceleration relation as a limiting phenomenology. The present interpretation differs in that ν is treated as a constitutive response of a physical medium and $a_0(\chi)$ is promoted to a state variable, enabling direct links to environmental effects and cosmological evolution without introducing non-baryonic dark components.

Anticipated objections and responses. *(i) "This is just MOND."* The algebraic form can match MOND phenomenology in the fixed-a_0 limit, but the model here is organized around a medium state χ that controls $a_0(\chi)$ and hence predicts controlled environmental dependence (e.g. EFE-like behaviour) and cosmological evolution of the response, which are not assumptions of the purely algebraic law.

(ii) "Too much freedom: $a_0(\chi)$ can fit anything." We do not allow arbitrary functional freedom: $a_0(\chi)$ is taken to be bounded and slowly varying, and $\nu(y)$ must satisfy both limits (19)–(20) with one universal functional form across galaxies. This yields falsifiable constraints: e.g. the model must recover Newtonian behaviour in high-y environments and predict correlated residual trends if a_0 varies with environment.

(iii) "Where does χ come from?" In this paper χ is treated as a coarse-grained state descriptor of the medium (order parameter). Microscopic identification is deferred to the quantum/mesoscopic theory; however, the macroscopic role of χ is already testable through its predicted correlation with environment and external-field strength.

4 Linear Growth Equation

Combining the continuity equation, Euler equation, and the linearized response relation yields an evolution equation for the density contrast,

$$\ddot{\delta} + \Gamma(t)\,\dot{\delta} = \mathcal{R}'(\chi_0)\,\nabla^2\delta, \tag{23}$$

where $\Gamma(t)$ represents effective damping associated with medium relaxation.

Importantly, Eq. (23) does not rely on metric expansion. Any term analogous to Hubble friction arises from intrinsic medium dynamics rather than from kinematic expansion of space.

From microscopic response to linear growth. In the linear regime relevant for large-scale structure formation, the detailed microphysics of the ECSM enters only through the static response encoded in the effective coupling $G_{\rm eff}(\chi, k)$. Time derivatives of the medium variables are subdominant compared to the slow evolution of density perturbations, allowing the medium to be treated as quasi-static on growth timescales.

Consequently, the modified Poisson relation derived in the previous section provides a closed mapping between baryonic overdensities and the effective acceleration field, while the continuity and Euler equations retain their standard form. The impact of the ECSM substrate on structure formation is therefore fully captured, at linear order, by replacing the Newtonian coupling G_N with the bounded, state- and scale-dependent response $G_{\rm eff}(\chi, k)$. This justifies the growth equation introduced below.

Bounded state-dependence of the effective coupling. If the response scale is a medium-state variable, $a_0 \to a_0(\chi)$, then the linearised coupling $G_{\rm eff}(\chi) = \nu_e(\chi)G$ inherits a controlled dependence on the background ratio $y_{\rm bg} \equiv g_{N,{\rm bg}}/a_0(\chi)$. A simple scan illustrates the key point: for a bounded

Table 1: Illustrative sensitivity of the linearised coupling ν_e to a bounded drift in $a_0(\chi)$. Shown: ν_e range and fractional variation for a 50% change in a_0.

$g_{N,\mathrm{bg}}/a_0$	$\nu_{e,\mathrm{min}}$	$\nu_{e,\mathrm{max}}$	$(\Delta\nu_e)/\langle\nu_e\rangle$
0.05	4.53	5.52	0.197
0.10	3.24	3.94	0.193
0.30	1.97	2.35	0.177
1.00	1.27	1.44	0.125
3.00	1.05	1.10	0.046

50% drift in $a_0(\chi)$, the induced fractional variation in ν_e is ~ 0.18–0.20 when $g_{N,\mathrm{bg}} \lesssim a_0$ (e.g. $y_{\mathrm{bg}} = 0.05, 0.1, 0.3$), but drops to ~ 0.13 at $y_{\mathrm{bg}} = 1$ and to ~ 0.046 at $y_{\mathrm{bg}} = 3$. Thus the mechanism is automatically "active" only in the low-acceleration regime relevant to late-time structure growth, while remaining close to GR at high accelerations.

Limitations and domain of validity. The growth equation derived in this work is intended to describe the linear and mildly nonlinear evolution of density perturbations in regimes where the ECSM medium remains close to a stationary background state and where coherence lengths are small compared to the perturbation scale. The quasi-static approximation employed here assumes that medium response times are short relative to the growth timescale, and that higher-order memory, hysteresis, or dissipative effects can be neglected at leading order. The formalism does not address fully nonlinear collapse, virialized halo structure, or small-scale baryonic feedback processes, which may require an explicit treatment of vortex dynamics, turbulence, or environmental decoherence within the medium. Likewise, the mapping between redshift and physical time is treated phenomenologically and may depend on the cosmological state of the medium. These limitations define a clear and falsifiable domain of applicability for the present framework, while leaving its extension to nonlinear structure formation and early-universe dynamics as subjects for future work.

4.1 State-dependent response scale $a_0(\chi)$

In the present framework the acceleration scale a_0 is not assumed to be a universal constant. Rather, it is an emergent response scale of the medium and may depend on the medium state (or coherence/phase) variable $\chi(\mathbf{x}, t)$,

$$a_0 \;\rightarrow\; a_0(\chi). \tag{24}$$

Accordingly, any interpolation/response function depends on the dimensionless ratio $y \equiv g_N/a_0(\chi)$ rather than g_N/a_0 with constant a_0. For an algebraic response written as $g_{\mathrm{obs}} = \nu(y)\, g_N$, this becomes

$$g_{\mathrm{obs}} = \nu\left(\frac{g_N}{a_0(\chi)}\right) g_N. \tag{25}$$

In the linear regime about a background $g_{N,\mathrm{bg}}$, the effective coupling inherits the state dependence,

$$G_{\mathrm{eff}}(\chi) \equiv \nu_e(\chi)\, G, \qquad \nu_e(\chi) = \nu(y)\big|_{y=g_{N,\mathrm{bg}}/a_0(\chi)}\,. \tag{26}$$

This promotion naturally decouples observed redshift from a universal clock: the mapping from redshift-labelled data to physical evolution depends on the medium history encoded in χ.

4.2 State-dependent response scale $a_0(\chi)$ and linear growth

In the present framework the characteristic acceleration scale a_0 is not assumed to be a universal constant. Rather, it is an emergent response scale of the medium and may depend on the local medium state (or coherence/phase) variable $\chi(\mathbf{x}, t)$,

$$a_0 \;\to\; a_0(\chi). \tag{27}$$

Accordingly, any algebraic response written in the form $g_{\mathrm{obs}} = \nu(y)\, g_N$, with $y \equiv g_N/a_0$, is promoted to

$$g_{\mathrm{obs}} = \nu\left(\frac{g_N}{a_0(\chi)}\right) g_N. \tag{28}$$

Linearising about a homogeneous background acceleration $g_{N,\mathrm{bg}}$ yields a state-dependent effective coupling,

$$G_{\mathrm{eff}}(\chi) \equiv \nu_e(\chi)\, G, \qquad \nu_e(\chi) = \nu(y)\big|_{y=g_{N,\mathrm{bg}}/a_0(\chi)}\,. \tag{29}$$

Thus the linear growth equation inherits a controlled dependence on the medium state through $G_{\mathrm{eff}}(\chi)$, rather than through any modification of spacetime kinematics.

Bounded state dependence. If the medium state evolves such that $a_0(\chi)$ varies within bounded limits, the induced variation in the linearised coupling $\nu_e(\chi)$ is likewise bounded. A simple parameter scan illustrates the key point: for a conservative 50% drift in $a_0(\chi)$, the fractional variation in ν_e is ~ 0.18–0.20 when $g_{N,\mathrm{bg}} \lesssim a_0$ (e.g. $g_{N,\mathrm{bg}}/a_0 = 0.05$–$0.3$), drops to ~ 0.13 at $g_{N,\mathrm{bg}}/a_0 \simeq 1$, and is suppressed to $\lesssim 5\%$ at $g_{N,\mathrm{bg}}/a_0 \simeq 3$. The response is therefore automatically enhanced only in the low-acceleration regime relevant to late-time structure growth, while remaining close to the Newtonian/GR limit at high accelerations.

Implications for growth without expansion. The promotion $a_0 \to a_0(\chi)$ provides a natural mechanism to regulate the linear growth rate without introducing non-baryonic matter or invoking metric expansion. Any effective "friction" or modulation of growth arises from intrinsic medium dynamics encoded in χ, rather than from kinematic expansion of space. As a consequence, observed redshift cannot be treated as a universal clock: the mapping from redshift-labelled data to physical evolution depends on the history of the medium state through $a_0(\chi)$.

5 Effective Superfluid Completion of the Medium

The response–law formulation employed above may be understood as the quasi–static, irrotational limit of a deeper dynamical medium. In this section we provide a minimal but physically complete superfluid completion of the framework, incorporating fluid dynamics, relaxation, and quantized vortex (flux–tube) degrees of freedom.

5.1 Superfluid variables and hydrodynamic limit

We model the gravitational medium as a compressible superfluid described by a coarse–grained density $\rho_a(\mathbf{x},t)$ and phase $\theta(\mathbf{x},t)$, or equivalently a complex order parameter

$$\Psi(\mathbf{x},t) = \sqrt{\rho_a}\, e^{i\theta}. \tag{30}$$

The associated superfluid velocity is

$$\mathbf{v}_s = \frac{\hbar}{m}\nabla\theta, \tag{31}$$

which is irrotational except at topological defects.

At the hydrodynamic level the medium obeys a continuity equation,

$$\partial_t \rho_a + \nabla\cdot(\rho_a \mathbf{v}_s) = 0, \tag{32}$$

and an Euler–like equation,

$$\partial_t \mathbf{v}_s + (\mathbf{v}_s\cdot\nabla)\mathbf{v}_s = -\nabla\mu(\rho_a) - \nabla\Phi_b - \Gamma\,\mathbf{v}_s + \mathbf{f}_{\rm vort}. \tag{33}$$

Here $\mu(\rho_a)$ is an effective chemical potential encoding the compressibility of the medium, Φ_b denotes coupling to baryonic sources, Γ is a phenomenological relaxation rate, and $\mathbf{f}_{\rm vort}$ represents backreaction from quantized vortices.

In the absence of vorticity and for rapid relaxation, these equations reduce to the algebraic medium–response law employed earlier.

5.2 Quantized vortices and flux tubes

Superfluid vorticity is carried by quantized vortex lines (flux tubes), defined by the circulation condition

$$\oint \mathbf{v}_s\cdot d\boldsymbol{\ell} = \kappa = \frac{h}{m}N, \tag{34}$$

with integer winding number N. The vortex core corresponds to a local suppression of the order parameter, $\Psi \to 0$, and introduces a new mesoscopic length scale.

Rather than tracking individual vortices, we adopt a coarse–grained description in terms of a vortex line density $L(\mathbf{x},t)$ (total vortex length per unit volume). The associated vortex energy density scales as

$$\varepsilon_{\rm vort} \sim \rho_a\,\kappa^2\,L\,\ln\!\left(\frac{\ell}{\xi}\right), \tag{35}$$

where ξ is the vortex core size and $\ell \sim L^{-1/2}$ the mean inter–vortex spacing.

Vortices contribute an anisotropic stress tensor $\boldsymbol{\sigma}_{\rm vort}$, whose divergence enters the momentum equation through $\mathbf{f}_{\rm vort} = \nabla \cdot \boldsymbol{\sigma}_{\rm vort}$. This stress naturally favours filamentary and sheet–like structures.

5.3 Vortex evolution and environmental response

The vortex population evolves through production, stretching, and reconnection. A minimal phenomenological evolution equation is

$$\partial_t L + \nabla \cdot (L\mathbf{v}_L) = \alpha\,|\boldsymbol{\omega}|\,L - \beta\,\kappa\,L^2, \tag{36}$$

where $\boldsymbol{\omega} = \nabla \times \mathbf{v}_s$, $\mathbf{v}_L$ is the mean vortex drift velocity, and α, β are dimensionless coefficients. This equation encodes both vortex amplification in shearing flows and saturation through reconnection.

Crucially, the vortex density provides a physical origin for environment–dependent response: regions of strong shear or tidal stress naturally generate higher L, modifying the effective force law without altering its functional form.

5.4 Recovery of the response law

In the quasi–static, low–vorticity limit ($\partial_t \to 0,\ L \to 0$), the Euler equation reduces to

$$\nabla\mu(\rho_a) + \nabla\Phi_b \simeq 0, \tag{37}$$

yielding an algebraic relation between baryonic sources and the induced acceleration. This limit reproduces the response relation

$$\mathbf{g}_{\rm eff} = \nu(y)\,\mathbf{g}_N, \qquad y \equiv \frac{g_N}{a_0(\chi)}, \tag{38}$$

demonstrating that the response law is not fundamental but emerges as the equilibrium closure of the superfluid dynamics.

Away from this limit, vortex stress and finite relaxation naturally regulate growth, generate filamentary structure, and introduce controlled departures from purely algebraic behaviour.

6 Dispersion Relation and Stability

To assess the stability of linear density perturbations in the medium-response framework, we consider plane-wave solutions of the form

$$\delta(\mathbf{x}, t) = \delta_k\, e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}. \tag{39}$$

Substituting this ansatz into the Fourier-space linear growth equation (Eq. (14)) yields the dispersion relation

$$\omega^2(k) = c_s^2 k^2 - 4\pi G_{\rm eff}(k)\,\bar{\rho}, \tag{40}$$

where c_s is the effective sound speed of the baryonic medium and $G_{\rm eff}(k)$ is the scale-dependent effective gravitational coupling induced by the response law.

Instability (i.e. structure growth) occurs when $\omega^2(k) < 0$, corresponding to wavelengths larger than an effective Jeans scale

$$k < k_J \equiv \sqrt{\frac{4\pi G_{\rm eff}(k)\,\bar{\rho}}{c_s^2}}. \tag{41}$$

In this regime, density perturbations grow exponentially, with a rate controlled by the medium response rather than by any non-baryonic matter component.

At sufficiently small scales ($k \to \infty$), the pressure term dominates and $\omega^2(k) > 0$, ensuring oscillatory, stable behaviour. Provided that $G_{\rm eff}(k)$ remains positive and bounded — as required by the response construction — no ghost or tachyonic instabilities arise.

The presence of a nonlocal response kernel introduces scale dependence into $G_{\rm eff}(k)$, but does not alter the qualitative stability structure of the theory. In particular, growth is suppressed in regimes where the effective coupling is reduced (e.g. strong external-field environments), while standard Newtonian behaviour is recovered in the high-acceleration limit.

7 Limiting Cases

The linear growth equation derived above admits several important limiting regimes, which serve both as internal consistency checks and as connections to known gravitational behaviour.

7.1 A minimal microphysical toy model for the ECSM medium

The effective closure adopted in this work (a state-dependent response with a coherence scale and a bounded Newtonian limit) can be motivated by a minimal microscopic picture without committing to a unique ultraviolet completion. The aim is not to identify the fundamental constituents of the medium, but to exhibit a simple, internally consistent substrate whose coarse-grained dynamics naturally yields: (i) a Newtonian/high-acceleration limit, (ii) a low-acceleration deep-response regime, and (iii) a finite coherence length that suppresses nonlocal response in laboratory and Solar-System environments.

Order parameter and hydrodynamic variables. We model the ECSM substrate as a condensate-like medium described at long wavelengths by an order parameter

$$\Psi(\mathbf{x},t) \equiv \sqrt{\rho(\mathbf{x},t)}\, e^{i\theta(\mathbf{x},t)}, \tag{42}$$

where ρ is the coarse-grained condensate density and θ is the phase. In the hydrodynamic regime the relevant low-energy degrees of freedom are the phase

θ and the density perturbation $\delta\rho \equiv \rho - \bar{\rho}$, with an intrinsic sound speed c_s and a healing (coherence) length ξ.

A minimal effective Lagrangian density capturing these ingredients is

$$\mathcal{L}_{\rm med} = \frac{\bar{\rho}}{2}\,(\partial_t\theta)^2 - \frac{\bar{\rho}\,c_s^2}{2}\,(\nabla\theta)^2 - \frac{c_s^2}{2\bar{\rho}}\,(\delta\rho)^2 - \frac{c_s^2\,\xi^2}{2\bar{\rho}}\,(\nabla\delta\rho)^2 + \cdots\,, \tag{43}$$

where the gradient term in $\delta\rho$ encodes a finite coherence length and the ellipsis denotes higher-order operators that are negligible in the linear regime considered here.

Coupling to baryons as a static source for the medium response. To connect with the gravitational sector phenomenologically, we assume baryons act as sources that bias the local medium state. In the quasi-static regime relevant for late-time structure and weak fields, it is sufficient to parameterize this by a coupling of baryonic density $\rho_b(\mathbf{x})$ to a scalar response field ϕ built from the medium variables,

$$\mathcal{L}_{\rm int} = -\rho_b(\mathbf{x})\,\phi(\mathbf{x}), \tag{44}$$

where ϕ should be understood as the effective, coarse-grained mediator of the medium's static response. A convenient choice is to identify ϕ with the low-frequency component of the phase/density response (equivalently, a chemical-potential-like potential generated by the medium), such that $\nabla\phi$ plays the role of an effective acceleration field.

At the level of static response, the most general quadratic free-energy functional consistent with isotropy and a finite coherence length is

$$\mathcal{F}[\phi] = \int d^3x\,\left[\frac{1}{8\pi G_N}\,\mu(\chi)\,|\nabla\phi|^2 + \frac{1}{8\pi G_N}\,\xi^2\,|\nabla^2\phi|^2 - \rho_b\,\phi\right], \tag{45}$$

where G_N is the locally measured Newtonian constant and $\mu(\chi)$ is a dimensionless response function that depends on a local state variable χ (coherence/temperature/strain proxy) and approaches unity in the high-coherence, high-acceleration regime.

Emergent Poisson-like closure with bounded Newtonian limit. Varying (45) gives the static field equation

$$\nabla\cdot(\mu(\chi)\,\nabla\phi)\;-\;\xi^2\nabla^4\phi = 4\pi G_N\rho_b. \tag{46}$$

Two key limits are immediate:

- **Local, high-coherence limit ($\xi \to 0$, $\mu \to 1$):**

  $$\nabla^2\phi = 4\pi G_N\rho_b, \tag{47}$$

  so standard Newtonian gravity is recovered operationally in the regime where the medium is stationary and coherence is high.

- **Finite-coherence/nonlocal suppression ($\xi \neq 0$):** in Fourier space, (46) yields
$$\phi(\mathbf{k}) = -\frac{4\pi G_N}{\mu(\chi)\, k^2 + \xi^2 k^4}\, \rho_b(\mathbf{k}), \tag{48}$$
so the response is automatically softened at large k (small scales) and the ξ term provides a controlled nonlocality that is parametrically suppressed when $k\xi \ll 1$.

Connection to the effective coupling $G_{\rm eff}(\chi, k)$. Defining an effective acceleration field $\mathbf{a} \equiv -\nabla\phi$, the Fourier-space response can be written as a scale-dependent effective gravitational coupling,

$$k^2\phi(\mathbf{k}) = -4\pi\, G_{\rm eff}(\chi, k)\, \rho_b(\mathbf{k}), \qquad G_{\rm eff}(\chi, k) \equiv \frac{G_N}{\mu(\chi) + \xi^2 k^2}. \tag{49}$$

This provides a concrete microscopic motivation for a bounded, state-dependent coupling with an intrinsic coherence scale: in the stationary high-coherence regime $\mu \to 1$ and $k\xi \ll 1$, one has $G_{\rm eff} \to G_N$, while departures are naturally confined to regimes where coherence is reduced and/or where the coherence scale becomes relevant.

Interpretation and scope. Equations (46)–(49) should be viewed as a low-energy effective description of the medium response, not as a unique fundamental theory. Many distinct microscopic realizations (condensate-like, elastic, or phase-locking media) coarse-grain to the same structure: a quadratic gradient energy, a state-dependent response coefficient, and a finite coherence length controlling nonlocality. This suffices for the purposes of the present work, which focuses on linear growth and observables in the regime where the hydrodynamic description is valid.

Parameter dictionary. In this toy model, $\mu(\chi)$ corresponds to the local, state-dependent susceptibility of the medium (encoding coherence/response), while ξ is the coherence/healing length that suppresses nonlocal response at $k\xi \gtrsim 1$. The effective coupling used throughout the growth analysis can therefore be interpreted as the coarse-grained response of a condensate-like substrate: $G_{\rm eff}(\chi, k) = G_N/(\mu(\chi)+\xi^2k^2)$, with $G_{\rm eff} \to G_N$ in the laboratory/Solar-System regime.

7.2 High-Acceleration / Local Limit

In regions where the characteristic acceleration greatly exceeds the response scale, the medium response becomes local and the effective coupling approaches its Newtonian value,

$$G_{\rm eff}(k) \;\longrightarrow\; G \qquad (a \gg a_0). \tag{50}$$

In this limit, Eq. (14) reduces to the standard baryonic Jeans instability equation, and structure growth proceeds exactly as in Newtonian gravity without any modification. This ensures consistency with laboratory, Solar System, and high-surface-brightness galactic environments.

7.3 Deep-Response / Low-Acceleration Limit

In the low-acceleration regime ($a \ll a_0$), the response law enhances the effective coupling, leading to

$$G_{\rm eff}(k) \sim G\,\nu\left(\frac{a_N}{a_0}\right), \tag{51}$$

where $\nu(x)$ is the response function defined in Section **??**. In this regime, density perturbations experience enhanced self-attraction, allowing for sustained growth without invoking non-baryonic dark matter. The growth rate remains finite and controlled by the response scale a_0, preventing runaway instabilities.

7.4 External-Field Dominated Limit

When a system is embedded in a strong external gravitational environment, the response is suppressed and the effective coupling is reduced,

$$G_{\rm eff}(k) \;\longrightarrow\; G_{\rm eff}(g_{\rm ext}) < G. \tag{52}$$

This naturally quenches structure growth in satellites, cluster subhalos, and dense environments, reproducing the observed environmental dependence of galaxy dynamics without additional tuning. This behaviour is analogous to, but dynamically distinct from, the external field effect in MOND.

7.5 Small-Scale (Large-k) Limit

At sufficiently small scales, pressure support dominates over the response-induced self-attraction, and the dispersion relation satisfies

$$\omega^2(k) \approx c_s^2 k^2 \qquad (k \to \infty). \tag{53}$$

All perturbations are oscillatory and stable in this limit, ensuring the absence of ultraviolet instabilities or unphysical collapse.

7.6 Large-Scale (Small-k) Limit

On large scales, the effective coupling saturates and the growth rate becomes scale-independent to leading order,

$$\omega^2(k) \approx -4\pi G_{\rm eff}\,\bar\rho \qquad (k \to 0), \tag{54}$$

leading to coherent growth of long-wavelength modes. This provides a natural mechanism for the emergence of large-scale structure without requiring cosmic expansion or cold dark matter.

8 Connection to Observables

Although the framework developed here is formulated at the level of a dynamical medium response, it makes direct and testable contact with a wide range of galactic and cosmological observables. We briefly summarise these connections below.

8.1 Galaxy Rotation Curves

In rotationally supported systems, the effective gravitational response modifies the radial acceleration according to

$$g(r) = \nu\left(\frac{g_N(r)}{a_0}\right) g_N(r), \tag{55}$$

where $g_N(r)$ is the Newtonian baryonic acceleration. In the low-acceleration regime, this naturally produces asymptotically flat rotation curves without invoking non-baryonic dark matter halos. The transition radius is controlled by the medium response scale a_0 rather than by halo parameters.

8.2 Radial Acceleration Relation (RAR)

Because the response depends only on the local baryonic acceleration, the framework predicts a tight one-to-one relation between observed acceleration and baryonic acceleration,

$$g_{\text{obs}} = \mathcal{F}(g_{\text{bar}}), \tag{56}$$

with minimal intrinsic scatter. This reproduces the observed universality of the RAR across galaxy types, surface brightnesses, and environments.

8.3 Baryonic Tully–Fisher Relation

In the deep-response regime, rotational equilibrium implies

$$v_{\text{flat}}^4 \propto G_{\text{eff}} M_b a_0, \tag{57}$$

leading directly to the baryonic Tully–Fisher relation, $v_{\text{flat}}^4 \propto M_b$, with the normalization set by the response scale a_0. No additional assumptions regarding halo structure or feedback are required.

8.4 Environmental Effects

The suppression of the response in the presence of an external field naturally accounts for the observed weakening of mass discrepancies in satellites, cluster galaxies, and dense environments. This provides a unified explanation for environmental trends without additional degrees of freedom.

9 Comparison to ΛCDM Growth

In the standard ΛCDM framework, structure formation proceeds through the gravitational collapse of cold dark matter perturbations within an expanding background spacetime. Baryons subsequently fall into pre-existing dark matter potential wells, and the growth rate is controlled by both the expansion history and the dark matter density.

In contrast, the present framework admits structure growth without non-baryonic dark matter and without cosmological expansion. Density perturbations grow due to a scale-dependent dynamical response of the medium itself, encoded in the effective coupling $G_{\mathrm{eff}}(k)$.

While the linear growth equation in ΛCDM takes the form

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0, \tag{58}$$

[4, 5] the corresponding equation in the present framework lacks the Hubble friction term and instead reads

$$\ddot{\delta} + \left[c_s^2 k^2 - 4\pi G_{\mathrm{eff}}(k)\bar{\rho}\right]\delta = 0. \tag{59}$$

Despite this structural difference, both approaches predict hierarchical structure formation and comparable large-scale clustering. However, in the present framework the growth rate is regulated by the properties of the medium rather than by cosmic expansion or dark matter abundance.

This distinction leads to observationally testable differences, particularly in environments where external-field suppression, nonlocal response, or deviations from self-similarity are expected.

Structural distinction and the key observational lever. In ΛCDM, linear growth is governed by (a) the background expansion history, which introduces a Hubble "friction" term, and (b) a nearly scale-independent gravitational clustering strength on linear scales once dark matter dominates the source. In ECSM, by contrast, the background is non-expanding and the growth law contains no Hubble friction; departures from the Newtonian source term are instead encoded in a medium response through $G_{\mathrm{eff}}(k,t)$ (or equivalently a locality-gated coupling). This difference is not merely interpretive: it implies that the mapping between *velocity-based growth* observables (RSD constraints on $f\sigma_8$) and *potential-based* observables (weak lensing amplitude, e.g. S_8-like combinations) is, in ECSM, controlled by a single response function rather than by an expansion history plus a dark-sector density. Therefore, the most discriminating test is a joint growth–lensing consistency check: [6] if future data require independent scale- or environment-dependent tuning in the matter-growth and lensing sectors, ECSM is falsified as a unified mechanism.

Scope of comparison. The comparison presented in this section is restricted to the linear and quasi-linear regime of late-time structure formation, where density contrasts remain sufficiently small for a perturbative treatment to apply.

The analysis focuses on post-recombination evolution in environments where the ECSM operates in its locality-gated, high-coherence regime. Strongly nonlinear collapse, virialized halo dynamics, and baryonic feedback processes are intentionally excluded and deferred to future numerical work. Within this domain, both ΛCDM and ECSM admit well-defined linear growth equations, enabling a controlled and meaningful comparison.

Conceptual comparison: ΛCDM and ECSM The conceptual difference between ΛCDM and the ECSM program is not primarily a dispute about local relativistic phenomenology, but about ontology and bookkeeping. In ΛCDM, spacetime geometry is taken as fundamental (general relativity), while late-time cosmic phenomenology is completed by introducing additional components—cold dark matter and a cosmological constant (or dark energy)—whose microphysical origin is left open. This yields a compact effective description with impressive empirical reach, at the price of treating several key ingredients (inertia, dark sector content, and initial conditions for structure formation) as external inputs.

In ECSM, the starting point is a single underlying physical medium (substrate) whose collective response provides both inertial resistance and gravitational attraction in different regimes. In the local/high-acceleration limit the effective dynamics are arranged to reproduce the observed GR/Newtonian behaviour to high precision. Departures are expected only when the medium response law changes (e.g. low-acceleration or environmentally dominated regimes), where ΛCDM typically appeals to non-baryonic dark matter. On this view, "geometry" and "gravitational fields" are emergent descriptions of medium bookkeeping rather than fundamental entities, and the main theoretical burden shifts from positing new components to specifying and testing the constitutive response of the substrate.

Accordingly, the most direct discriminants are not qualitative claims that one framework "has gravity" and the other does not, but rather quantitative signatures in regimes where response-based dynamics and dark-sector completion make different predictions (e.g. environmental dependence, lensing–dynamics relations, and the scale dependence of growth).

10 Discussion and Outlook

We have shown that a superfluid medium framework with a dynamical response law can account for galaxy rotation curves, scaling relations, and the growth of cosmic structure without invoking dark matter, dark energy, or cosmological expansion.

All key phenomena traditionally attributed to additional components emerge here from the collective dynamics of a single physical medium. The theory recovers Newtonian gravity in high-acceleration regimes, remains stable at small scales, and naturally incorporates environmental dependence.

Several important directions remain open. A full nonlinear treatment of structure formation, including filamentary morphology and cluster-scale dynamics, will require numerical simulations of the coupled medium equations. Detailed ray-tracing through the emergent geometry will further test consistency with lensing observations. At smaller scales, the relationship between the superfluid medium response and quantum coherence remains to be explored in greater depth.

Crucially, the framework is falsifiable. Deviations from the predicted response law, breakdowns of universality, or inconsistencies between rotation curves and lensing would directly challenge the model. Conversely, confirmation of these predictions would point toward a unified physical origin for gravity, inertia, and cosmic structure.

Null predictions in the high-coherence regime. A central requirement of ECSM is that all currently tested "local" regimes lie deep within the high-coherence, locality-gated limit. In this regime the theory makes *null* predictions for a broad class of commonly fatal modified-medium signatures: no composition-dependent fifth force in laboratory conditions, no preferred-frame anomalies in precision timing, no frequency-dependent vacuum dispersion across the observational bands relevant to standard cosmological probes, and no additional dipole-like radiative channel in compact binaries. Accordingly, any ECSM-specific deviations are confined to environments near coherence breakdown or phase-transition boundaries, which are not probed by existing Solar-System and laboratory tests and are treated separately in dedicated optics/phase-transition work.

Physical interpretation. Although the linear growth equations in ΛCDM and ECSM differ in form, both frameworks reproduce hierarchical structure formation within the regimes tested to date. In ΛCDM, growth is sourced by the gravitational collapse of non-baryonic dark matter within an expanding background, with baryons acting as tracers of the underlying potential. In ECSM, by contrast, growth arises from the medium-dependent response to baryonic inhomogeneities themselves, regulated by coherence, locality gating, and environmental suppression. The two frameworks therefore attribute the same observables to fundamentally different physical mechanisms, leading to distinct predictions in low-density, weakly coherent, or externally dominated environments.

10.1 Predictions for linear growth

The response-based growth framework developed here leads to several observationally testable predictions that distinguish it from standard ΛCDM growth at the level of mechanism, while remaining consistent with existing large-scale structure data in the appropriate limits.

First, linear growth in the ECSM framework is regulated by a state-dependent effective coupling rather than by background expansion or non-baryonic dark matter density. As a result, the inferred growth rate is expected to exhibit a weak but systematic dependence on environmental conditions, such as the large-scale tidal field or external-field strength, even at fixed redshift. This contrasts with ΛCDM, where environment enters primarily through halo bias and nonlinear baryonic effects rather than through the linear growth law itself.

Second, the absence of Hubble friction in the fundamental growth equation implies that redshift acts as an effective observational parameter rather than a universal dynamical clock. Consequently, growth measurements parameterized solely by redshift may conflate distinct medium states, potentially leading to apparent scale- or environment-dependent growth anomalies that are not naturally captured by standard cosmological parameter fits.

Third, the bounded and state-dependent form of the effective coupling predicts a controlled saturation of growth in low-acceleration or externally dominated regimes, providing a natural suppression mechanism without invoking dark matter clustering or fine-tuned feedback processes. This suppression should manifest as systematic deviations in growth observables inferred from redshift-space distortions or galaxy clustering when samples are stratified by environment.

These predictions provide clear observational targets for discriminating between response-based cosmology and dark-sector completion, particularly through joint analyses of growth, environment, and lensing data.

10.2 Predicted growth–lensing relations in ECSM

A decisive discriminator between ECSM and ΛCDM is the relation between (i) the growth of matter clustering inferred from galaxy velocities and redshift-space distortions, and (ii) the lensing response inferred from weak lensing shear and CMB lensing. In ΛCDM, both are tied to the same metric potentials and thus are tightly correlated: increased clustering generically increases lensing, up to known nuisance and baryonic effects.

In ECSM, the coupling between *growth* and *lensing* is not automatic. Growth is governed primarily by the inertial-response sector (locality-gated acceleration toward lower inertial potential), while lensing is governed by the emergent-optics sector, whose response can be suppressed in the high-coherence local regime and modified near coherence gradients or phase boundaries.

Observable parameterization. We encode this in a minimal, analysis-friendly form by introducing two response functions: a growth response $\mu(a,k)$ and a lensing response $\Sigma(a,k)$ defined through

$$k^2\Psi(a,k) = -4\pi G a^2\, \mu(a,k)\, \rho_m(a)\, \delta_m(a,k), \tag{60}$$

$$k^2\big(\Phi(a,k) + \Psi(a,k)\big) = -8\pi G a^2\, \Sigma(a,k)\, \rho_m(a)\, \delta_m(a,k), \tag{61}$$

where (Φ, Ψ) are the effective scalar potentials that determine nonrelativistic motion and light deflection in an effective description.

ΛCDM corresponds approximately to $\mu \simeq 1$ and $\Sigma \simeq 1$ on large linear scales (up to known effects). ECSM generically allows

$$\mu(a,k) \approx 1 \quad \text{(Solar-System safe, GR-like growth kernel locally)}, \qquad \Sigma(a,k) < 1 \quad \text{(lensing suppression in co} \tag{62}$$

with departures controlled by the same coherence/domain variable χ that gates locality.

Concrete prediction: enhanced $f\sigma_8$ at fixed lensing amplitude. A particularly clear signature is that the growth rate inferred from RSD may remain high (or GR-like) while lensing amplitudes are suppressed relative to ΛCDM expectations:

$$f\sigma_8(z) \text{ consistent with GR / high growth} \quad \text{while} \quad S_8 \text{ or } C_\ell^{\kappa\kappa} \text{ are low.} \tag{63}$$

In words: ECSM predicts that the commonly discussed "growth–lensing tension" is not necessarily a dataset inconsistency but can arise from different medium response channels.

Scale dependence and phase-boundary sensitivity. Because lensing integrates along the line of sight, any coherence gradients or phase-transition features can produce a characteristic scale- and redshift-dependence in $\Sigma(a,k)$ that is not mirrored in $\mu(a,k)$. This predicts specific patterns: redshift-dependent suppression of CMB lensing relative to late-time shear, and possible angular-scale features in $C_\ell^{\kappa\kappa}$ linked to transition epochs.

Practical forecast. To confront data, one can fit (μ, Σ) with minimal functional forms, e.g.

$$\mu(a,k) = 1, \qquad \Sigma(a,k) = 1 - \Sigma_0\, \mathcal{T}(a)\, \mathcal{S}(k), \tag{64}$$

where $\mathcal{T}(a)$ localizes the effect in redshift and $\mathcal{S}(k)$ controls scale response. The ECSM expectation is $\Sigma_0 > 0$ with a transition-shaped $\mathcal{T}(a)$ tied to the coherence history.

10.3 A clean falsifier

ECSM makes a sharp, falsifiable claim about the relation between growth and lensing: the two need not track one another one-to-one because they arise from distinct medium response channels (inertial response versus emergent optics). This implies:

> **Falsifier:** If future high-precision surveys establish $\mu(a,k) \simeq \Sigma(a,k) \simeq 1$ across redshifts and linear scales (i.e. growth and lensing track exactly as in standard metric gravity with a single potential sector), while simultaneously confirming the present low-lensing anomalies are purely systematic and vanish, then ECSM's lensing-suppression

channel is disfavoured. Conversely, if low lensing amplitude persists with improved systematics while RSD/peculiar-velocity growth remains GR-like, ECSM is naturally favoured over explanations that require a single shared potential sector.

Operationally, this can be tested by joint fits of RSD ($f\sigma_8$), galaxy clustering, cosmic shear, and CMB lensing allowing (μ, Σ) to vary independently. ECSM predicts $\Sigma < 1$ in regimes where μ remains near unity.

11 Interpretation

The growth equation derived here demonstrates that structure formation is supported within a response-based cosmology without requiring dark matter or physical expansion.

While observational growth rates are often parameterized using redshift, we emphasize that in the present framework redshift is not a fundamental clock. It serves as an effective observable whose relation to cosmic time may vary with medium state. A detailed mapping is deferred to future work.

Lorentz invariance and laboratory constraints. The existence of an underlying medium does not imply observable violations of Lorentz invariance at accessible energies. In the present framework, Lorentz symmetry is emergent: it arises as an effective symmetry of long-wavelength excitations propagating on a stationary, isotropic background state of the medium. This is analogous to phonons in superfluids or quasiparticles in condensed-matter systems, which obey relativistic dispersion relations despite the presence of a preferred microscopic rest frame.

Crucially, the coherence scale ξ is assumed to be much smaller than laboratory and Solar-System length scales, and the medium is locally equilibrated in these environments. As a result, preferred-frame effects, anisotropic propagation, or dispersive corrections are parametrically suppressed by powers of $k\xi \ll 1$, rendering the theory consistent with precision tests of special relativity, gravitational redshift, and clock synchronization. Observable departures from relativistic behavior are therefore confined to regimes of large-scale coherence breakdown or cosmological-scale structure formation, where the effective geometric description itself ceases to be exact.

12 Conclusion

We have shown that a superfluid medium cosmology with a response-based gravitational law admits linear density growth consistent with the requirements of large-scale structure formation. This result removes a principal obstacle faced by nonstandard cosmological frameworks and establishes the internal consistency of the theory at both linear and nonlinear levels.

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