Modal Substitution of Commuters Following Metro Service Disruptions

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Abstract

This paper analyzes the effect of a major metro service disruption in Washington, D.C. on demand for metro and bus service. For peak commuting hours, cross-elasticities are calculated for bus demand and own-elasticities are calculated for metro demand in regions near metro stations. Elasticities are calculated with respect to the total wage-dependent opportunity cost of using the metro, including access time, in-vehicle time, and fare. The study finds statistically significant estimates for cross-price elasticity, ranging from 0.47 to 1.03, for bus stops within 1000m of a metro station and depending on assumptions made about opportunity costs. An own-price elasticity for the metro of -0.80 is found, which is also robust to varying assumptions.

1 Introduction

The optimal provision of public transportation is a subject of frequent discussion in urban policy circles. Transit planners decide fare prices, schedules, and route placement, and these decisions affect demand via opportunity cost. Fares obviously place a cost on using public transit, but so do schedules and routing by affecting time costs. Less frequent service imposes costs on users via wait times, and routing affects costs via walking and in-vehicle times. Since transportation is a normal good, increases in these costs for a particular transit mode clearly have a negative effect on demand. But the effect of the metro disruption on other modes is less obvious, though equally important. If metro and bus serve as complements, then increases in metro costs would decrease bus demand, but the opposite would occur in the case that they are substitutes.

In this paper, variation in service characteristics is used to analyze the relationship between bus and metro service in Washington, D.C. by calculating cross-price elasticity of demand. Elasticities are used because they are the most fundamental and economically interpretable way to understand the relationships between products. Economists often look for exogenous sources of variation when faced with questions like this one. Such a source of variation is provided by a major service disruption to the Washington, D.C. metro system on October 12th, 2021, which serves as the impetus for this study. A train on the Blue line derailed, prompting the Washington Metro Area Transit Authority (WMATA) to sideline much of its fleet of trains starting the following week. The removal of these trains compelled WMATA to drastically reduce service to all metro stations, resulting in lower Metrorail ridership that is easily observable.

In this case, none of the transit fares for bus or metro changed during the time period studied. But the schedule adjustments increased wait times, affecting the opportunity cost of using the metro. Consumers value their time according to some function of their wage, and their commute is time they cannot devote to leisure or work. This means that when wait times at metro stations increase, consumers incur a time cost in addition to the transit fare. The change in this time cost can be used as the denominator in the elasticity formula. We might also consider the change in the total opportunity cost of the trip, which includes wait times, walking times, in-vehicle times, and fares. The elasticity value derived from this calculation tells us whether and how much bus and metro service are substitutes or complements.

2 Literature

This paper draws from a body of literature on public transit economics which concerns four main areas: User mode choice among a set of possible transportation options, substitution between transit modes through a competition economics framework, techniques for analyzing complex transit systems, and relevant characteristics of public transportation. Existing literature thoroughly discusses substitution between a variety of public transit modes, but there is not much study of the relationship between urban bus and metro service (Fearnley et al., 2018). There is also a large body of literature discussing own-price elasticity estimates for various aspects of transit service, including walk times, wait times, in-vehicle times, and fares (Lago et al., 1981). This paper aims to unite these two parts of the literature, filling in the gap that exists for bus and metro substitution.

Most of the extant transit competition literature studies longer-distance transit modes due to difficulties in comparing trips within complex urban transit networks. Fearnley et al. (2018) find that estimates of cross-price elasticity of bus demand with respect to light rail wait times average 0.05, and Paulley et al. (2006) note that own-price elasticities of the London Underground with respect to generalized cost (total opportunity cost) range from -0.4 to -1.85.

2.1 Mode choice and competition

Mode choice is an important part of the public transport economics literature. In a foundational paper in the field, McFadden (1974) employs a multinomial logistic regression model to analyze consumer mode choice decisions for shopping trips. The study uses survey data collected at the individual level, including demographics, car ownership, wage, and race, in addition to characteristics of the user's trip and characteristics of each mode. For transit, the walk time, wait plus transfer time, and in-vehicle time are collected for each trip, and for automobile travel, operating cost data are collected. The regression model estimates the probability of a given user choosing a particular transportation mode conditional on the aforementioned characteristics.

Using a multinomial logit model with a wide set of possible mode choices is the ideal framework for a study like this one—cross- and own-elasticities can be computed for a variety of different modes. However, collecting individual level data through surveys or otherwise is not always feasible. When working with aggregate data, different methods must be used, though many of the same principles apply. Gkritza et al. (2011) study mode choice between metro, bus, and trolley service over a twelve-year period in Athens, Greece. Data are collected at the city-level for each month over the period, and changes to fare levels and fare structure (monthly passes versus single-ticket fares) are tracked. They also estimate elasticities for multiple groups of users—those who purchase per-trip fares and those who purchase farecards with unlimited access over a given time period. The public transit system in Athens is somewhat unique in that the metro, bus, and trolley operators do not set fare and service levels in conjunction with each other, but rather they set them competitively. This creates a system in which we might expect competition between modes.

The study's time frame and granularity of data means that the model identifies the effects of fares on consumers' long-run decisions, rather than their short-run decisions. A fare change may have a short-run effect on demand in that consumers with comparable access to multiple modes may switch to a different one due to the price change. Such a change might occur over the course of a few days. That same fare change may have an additional long-run effect, inducing consumers to change their residences to somewhere with greater access to the newly preferred mode. This sort of change in demand might occur over months. These considerations induce the authors to consider monthly and one-year autocorrelative terms in their regressions. The authors find that for these data, assuming a higher-order autoregressive process yields a better fit, and for this particular city, the metro and bus systems appear to be competitive, with an elasticity of 0.2.

Still, it should not be taken as given that different public transit modes compete. One might expect that urban rail and bus systems are more often designed to complement each other instead of compete. Rail transport is generally faster and therefore better suited to long distances, while buses are more route-flexible, meaning they can serve a wider area for short trips (Fearnley et al., 2018). A look at Washington, D.C.'s metro and bus route map supports this intuition. It shows that metro lines are generally situated as spokes around the center city, providing fast transport from heavily-populated suburbs to the center city, while bus lines often (but not strictly) fill in the gaps in metro service. Some lines connect the ends of metro spokes, allowing users to travel from suburb-to-suburb without having to go through the center city, while others vaguely form spokes around major metro stations, feeding users into the rail system.

However, there are reasons to believe that this may not be how the system works in practice. For paths connecting the center city to neighborhoods within the district, buses can provide service that is not substantially slower than the metro—bus stops are more evenly distributed throughout the city, so for many users it is easier to access one. Furthermore, there are numerous bus lines closer to the city center which run alongside metro lines. The result is that there may be non-trivial substitution between modes.

It is also worth noting that a substantial review of the literature on mode choice and competition between public transit modes found no research studying public transit systems during 2021, when COVID significantly affected demand for public transportation. The more recent development of a large proportion of workers having the opportunity to forgo their commute entirely introduces a new problem in the study of mode choice.

2.2 General lessons

This paper concerns changes in public transit service, so it is important to establish some basic properties of transit service. The relationship between headways, or the time between vehicle arrivals, and actual wait times is at the core of this analysis. A common model assumes uniform user arrivals to a transit stop, where the average wait time is half of the headway (Mohring, 1972). However, when headways increase in a system with posted vehicle arrival times, actual user wait times do not increase linearly with headway. Ingvardson et al. (2018) hypothesized that some proportion of users arrive randomly and another proportion arrive in a manner that minimizes wait times. The authors found that as headways increase, wait times increase in a monotonically decreasing manner. Despite this, it is still not obvious that users who spend less time physically waiting at the transit stop are reclaiming that time for work or leisure. Therefore, this study employs the simpler and more common method in which expected wait time is half of headway.

Users also may value different components of their trip at different percentages of their wage. McFadden (1974) finds that users value access time (walk plus wait time) at over five times that of in-vehicle time. Others estimate that users value in-vehicle time between 25% and 50% of their wage, and access time at two to three times higher (Mohring, 1972). Lorenzo Varela et al. (2018) find via a study of public transit in Stockholm, Sweden that in-vehicle times on the bus and the metro are valued equally by users. This study uses a range of possible wage-rate multipliers based on reasonable assumptions from Mohring (1972).

3 Theory

Consider an agent who is commuting from home to work within a metropolitan area. She has multiple choices of how to get there—in this case bus or metro. This framework can be generalized to include more transit options. The agent leaves her home and walks to a transit stop, which we call the access time a. There, she waits for the vehicle for a period of length d, representing the wait or delay time. Once the bus or train arrives, she then incurs v, which is in-vehicle time. Finally, the agent leaves the vehicle and walks to her destination, taking egress time e.[1] In the case that she

[1]Because of the aggregate nature of the data used in this study, precise access and egress times are not available for particular consumers. Consequently, different values of a and e are considered as robustness checks. The nature of the elasticity formula means that increasing a and e strictly increases elasticity. This is because it raises the base cost of a trip, so a given gross increase in d

transfers between vehicles, she incurs switching time s.

In this model, d is a function of headway h, which is the time in between arriving vehicles. As discussed in the literature review, the model assumes a uniform distribution of users arriving to the transit stop, which gives us average wait time $d = \frac{h}{2}$. It is also important that d and v are generally known to the consumer prior to her decision to use a particular transit mode. Therefore, the values of d and v should be found in accordance with the expected trip time at the time that the consumer originates her trip.

In the city of interest, suppose there are i metro stations. Let each of these stations be surrounded by a region, defined by some radius around each metro station, within which all bus stops are considered. In a given region, $Q_{m,ijt}$ represents the quantity demanded of transit service, or the number of trips from region i to region j on day t using mode m. It is assumed that there is a consumer with a time-invariant wage who represents all consumers who originate trips at a given region. Let w_i be that wage.

To simplify the model, let x represent all of the spent walking or waiting, that is, $x_{m,ijt} = a_{m,i} + d_{m,ijt} + s_{m,ij} + e_{m,j}$. As described in the literature review, transit users generally value time spent walking and waiting comparably, and in-vehicle time much less (Mohring, 1972) (McFadden, 1974). Therefore, we consider the sum of the former category of trip time. Note that the different components of x have different subscripts—each value is differently depenent on origin, destination, and time.[2] Using this general setup, we can derive the price $P_{m,ijt}$ of a commute from i to j at time t using mode m:

$$P_{m,ijt} = f_{m,ij} + w_i(c_{am}x_{m,ijt} + c_{vm}v_{m,ijt}) \tag{1}$$

where $f_{m,ij}$ is the fare, and c_{xm} and c_{vm} are coefficients which represent the percentage of the agent's wage at which she values time spent walking and time spent in-vehicle, respectively. This formulation allows for standardization between

represents a smaller percent increase in the denominator of the formula resulting in a larger elasticity value.

[2] Access time depends only on the agent's home's position relative to station i, and because she walks, it is time-invariant. Wait times depend on i, j, and t because (1) origin stations have varying train frequencies, (2) a user's wait time may differ between destinations due to wait time heterogeneity across metro lines, and (3) the schedules change across the observed period. Transfer times depend on i and j because various trips across i, j may or may not use a transfer. Similarly to access time, egress time depends only on the position of the final station relative to the agent's place of work.

Now that the price of a trip for each mode has been established, we can estimate the cross-price and own-price elasticity of demand for each mode. Letting $m = \{1, 2\}$, the standard equation for cross-price elasticity of demand ε_{21} is

$$\varepsilon_{21} = \frac{dQ_1}{dP_2}\frac{P_2}{Q_1} \tag{2}$$

which can be estimated using a regression of the log quantity of bus demand on the log price of a metro trip, where the coefficient on the log metro price term is the cross-price elasticity. The elasticity is interpreted as the percent change in the quantity of good 1 (bus service) in response to a one percent change in the price of good 2 (metro service). We can also estimate the own-price elasticity of demand using

$$\varepsilon_{11} = \frac{dQ_1}{dP_1}\frac{P_1}{Q_1} \tag{3}$$

It is important to note that elasticity depends not just on the gross change in price of the second good, but also the total price level. This motivates the paper's use of total opportunity cost and not only the opportunity cost presented by wait times. However, elasticities with respect to wait times are also useful and commonplace in the literature. Therefore, this paper also considers values for these elasticities as well. In the estimation of the coefficient, P_1 is replaced with d_1, where d is the wait time only.

4 Data

Because the setting for this research is the October 2021 metro derailment, the data used in this study are limited to September, October, and November of 2021. This time restriction is intended to limit contamination of the data by long-term consumer decisions, seasonal variation, and time trends. WMATA has provided ridership and schedule data for both bus and metro for the aforementioned time period.

4.1 Ridership

Metro ridership data are provided as the number of entries to and exits from a particular metro station in a given hour. An entry or exit is recorded when a user swipes their farecard at the entrance of a station. At a typical metro station, trains travel in two directions, so a user who swipes into the station could board either train, creating ambiguity about the travel cost associated with a station. It is also not possible to know which line a user boarded from a transfer station. These problems are partially mitigated by the study design. Limiting the study to morning-peak demand means that most users are commuting from the outside of the city to the center, alleviating the train direction problem. The time restriction also lessens the transfer station problem, since most transfer stations are in the center city where relatively fewer people originate trips during the morning peak.

The ridership data represent the measure of demand for the metro and bus systems. It is clear from a visual analysis that ridership changed significantly around the time that the derailment and subsequent schedule changes occurred. However, it is possible that it was not the service changes that affected ridership, but rather safety concerns. If this were the case, then the study would overestimate the impact of wait times and opportunity costs on bus or metro demand. However, metro ridership during the week of the derailment remained high—the incident happened on October 12th, and ridership remained constant for the 13th through the 15th. Only when the schedule changed on October 18th did ridership plummet, as is shown in 1. If safety concerns or fear were the main driver of the reduction in demand, then we would expect that ridership would decline during the days following the derailment.

The effect of the schedule changes on bus ridership are not clearly obvious from the charts. This is probably because the effect of the metro cost increases on bus ridership are low for bus trips that don't have a reasonable metro analogue. This paper restricts analysis of the bus demand to zones surrounding metro stations in order to address this concern.

4.2 Schedules

Bus and metro schedules are provided as data tables, with each observation representing an arrival or departure of a particular vehicle at a stop or station at a given time. Arrival and departure times are precise to the minute. It is well-known and admitted by WMATA that these times are not always accurate, but they are the best tool for calculating headways and travel times. The average daily peak-hour headway in seconds is calculated for each line at every station by dividing $(3,600 \times 4)$ seconds by the number of train arrivals for that line over the four hour peak period. These values (divided by two) form the waiting edges of the graph used to calculate the overall trip times between different station-regions.

For Metrorail, three schedule data tables are provided, one for each month in the study period. The "September" table represents the metro schedule that was in

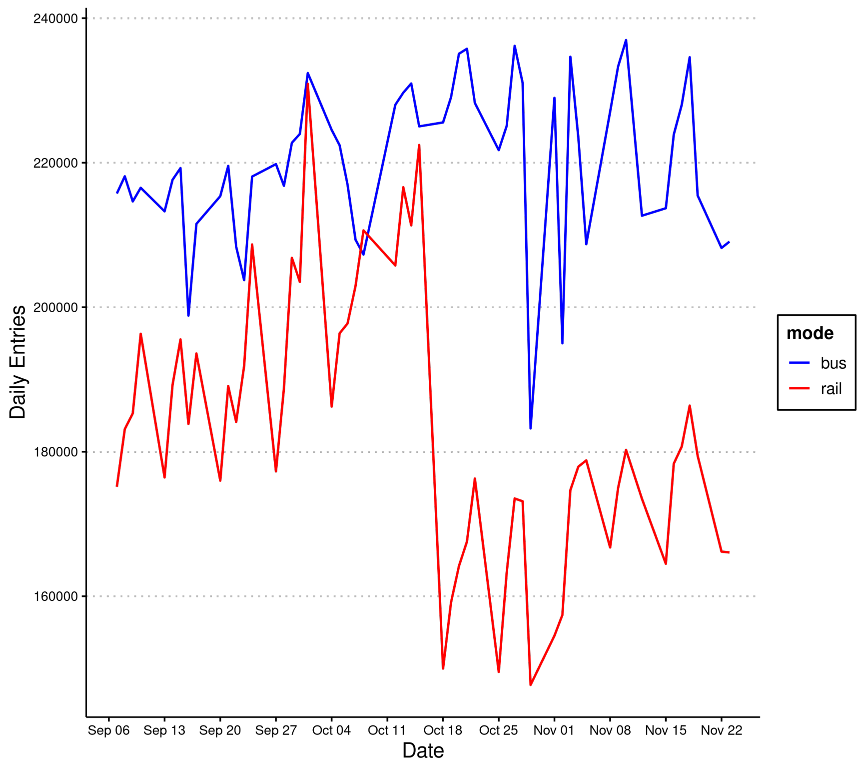

Figure 1: Daily MetroRail and MetroBus entries from September 07 to November 23, 2021.

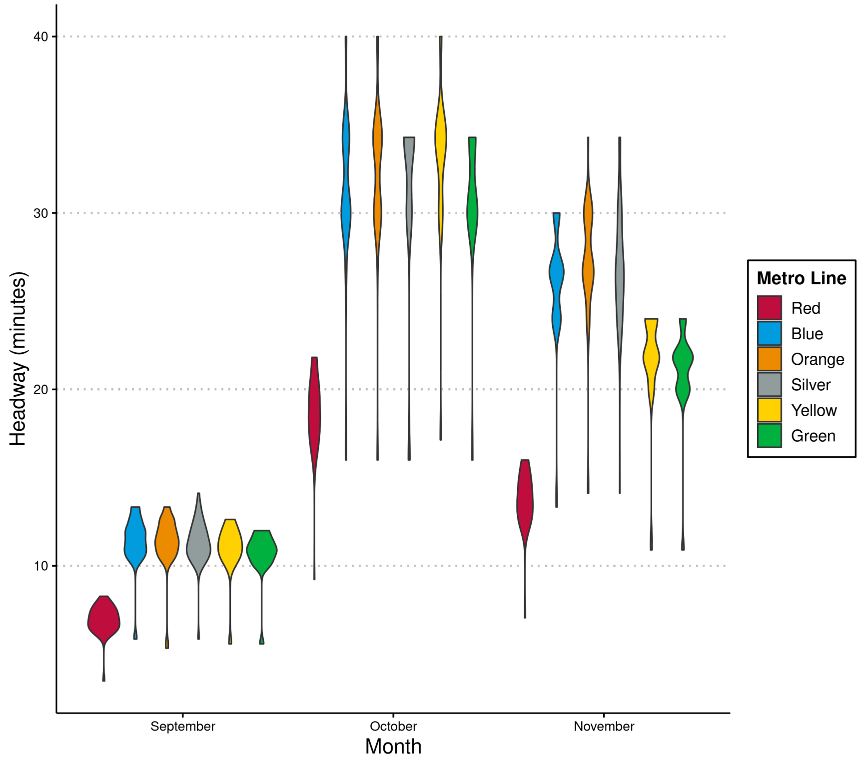

Figure 2: Metrorail headways for each provided schedule, denoted by the month in which that schedule begins.

effect from before the study period began until October 15th, the "October" table represents the schedule from October 18th until November 8th, and the "November" table represents the schedule from November 11th until the end of the study period.

4.3 Wages

For wages, American Community Survey (ACS) 5-year estimates of earnings by census block group are used. There were a few options from the ACS for determining the wage of a representative consumer from each region. Household income, individual earnings, and individual earnings for those working full time were all available variables. Household income could create problems, since we theorize that consumers make their commute decision based on their individual wage. Earnings for full-time workers would be the most ideal data for approximating the wage, since earnings could be divided by 2,000 to approximate the wage. However, earnings for full-time workers are only available at the census tract level, which is not geographically precise enough to capture necessary variation between station-regions within the central parts of the city. Therefore, census block group estimates of overall earnings were used, and divided by 2,000 to approximate the hourly wage.

Wage estimates are included as part of the calculation of rail time costs, not as a coregressor. Wages are static across the observed time period, meaning that adding them to the model along with cross-sectional fixed effects would not be useful. The wages serve as a conversion factor between minutes and dollars which varies across space. Accurate wage data are not available for regions around a few metro stations in the center city, so for some regressions these units are dropped. This should not present a serious problem, since these particular stations are in nonresidential areas such as Arlington Cemetery or parts of the CBD in which some people live but do not use the public transit system in large numbers.

4.4 Panel construction

Each cross-sectional unit i is defined as a radius around each metro station which also includes some bus stops. The units are created by matching each bus stop to its nearest metro stop, and then filtering the data for bus stops that are within a particular distance of their corresponding metro station. All entries to buses from stops within a given unit are aggregated, producing the variable for bus quantity, and all entries to that unit's metro station become the variable for metro quantity. Only entries between 5:00AM and 9:00AM are used, so that the period studied falls entirely within the 5:00AM to 9:30AM peak-fare time for the metro system. This method provides a one-to-one comparison of transit demand for each area. Each metro station thus defines a "station-region." It is reasonable to assume that different radii might give different results, so results are computed for different radii as a robustness check.

5 Methodology

5.1 Regression

This study employs a two-way fixed effects model to estimate the elasticities. The natural log of bus ridership is regressed against the natural log of metro price, with fixed effects for station-region and date. Therefore, the regression to determine the cross-price elasticity of demand for bus ridership with respect to metro price is as follows:

$$\log Q_{1,it} = \alpha_i + \delta_t + \varepsilon_{11} P_{1,it} + \varepsilon_{12} P_{2,it} + u_{it} \tag{4}$$

where Q_1 represents bus demand, P_1 is the bus cost, P_2 is the metro cost, α encompasses unit fixed effects, δ encompasses time fixed effects, and u is the error term.

In the case that the own-price (P_1) is constant across time, it is absorbed in the unit fixed-effect term α_i, so it is unnecessary to include it in the regression. The bus schedule did not change over the three month time period studied, so it is reasonable to assume that there was no variance across time and space which affected the opportunity cost of using the bus.[3] Thus, we can drop the bus opportunity cost variable:

$$\log Q_{1,it} = \alpha_i + \delta_t + \varepsilon_{12} \log P_{2,it} + u_{it} \tag{5}$$

Similarly, one might consider the prices of other substitute goods, such as commuting by walk, automobile, or bicycle. However, these prices, like that of the bus, can be assumed constant either across time or cross-sectional unit. For example, gas prices vary across different parts of the city, but the extent of this variation would not change across three months. One can also expect any macro-trends in gas prices to affect all parts of the Washington metro area equally. Similarly, the ease of walking or cycling to work is dependent largely on a person's proclivity to do so, which is captured by unit-fixed effects, or the weather, which is captured by date fixed effects.

For estimating the own-price elasticity of bus demand, what about using a simple linear model to avoid issues with fixed effects? Even in the case that there is variation across time in the own-price, an instrument would be necessary because price and quantity are in equilibrium (Angrist and Krueger, 2001). Some justification for the bus cost changes being exogenous is necessary because no suitable instruments are available. Because we do have exogenous variation in metro opportunity cost, we can calculate the own-price elasticity of demand for the metro, using the following equation:

$$\log Q_{2,it} = \alpha_i + \delta_t + \varepsilon_{22} \log P_{2,it} + u_{it} \tag{6}$$

[3] This is a fairly strong assumption which depends on a few characteristics of the bus system and some other observations. First, Metrobus has never been perfectly consistent in its arrivals. Buses stop only to pick up or drop off users, and so their arrivals at particular stops carry much more error than do Metrorail arrivals. Furthermore, buses can be stuck in traffic, which further decreases the extent to which specific arrival and departure times can be relied on. A related challenge is that possible switching from Metrorail to personal automobiles could increase traffic for buses. This is certainly plausible, but a separate investigation of metro-car substitution did not yield significant results, implying that there was likely not enough additional congestion to slow down the buses more than usual.

where instead of ε_{12}, we have the own-price elasticity of demand, or ε_{11}. Additionally, a more flexible yet less interpretable model for cross-elasticity is considered. Instead of regressing bus or metro demand on the total opportunity cost of the metro, one might consider only average wait times:

$$\log Q_{1,it} = \alpha_i + \delta_t + \varepsilon_{12} \log a_{2,it} + u_{it} \tag{7}$$

$$\log Q_{1,it} = \alpha_i + wday_t + \varepsilon_{12} \log a_{2,it} + u_{it} \tag{8}$$

where $a_{2,it}$ is the access time (in this case, equivalent to wait time) for the metro at station i and day t. This specification is less constrained, since it makes no assumptions about weights given to access time and in-vehicle time. However, it is less interpretable because it is not a price-elasticity; rather, it is an elasticity with respect to service quality. Still, a positive and significant coefficient from this regression would indicate that the desired effect is present. Two similar specifications are considered: one with two-way fixed effects, and another with unit fixed effects and weekday dummy variables. The latter specification is meant as a test to see whether the very demanding two-way fixed effects specification is absorbing too much variation. The result of this sort of regression is more commonly found in the literature, since generalized cost elastcity estimates are less common.

5.2 Travel costs

To compute this regression, we must determine the travel times. For this, the metro and bus schedules are each used to create directed, weighted graphs. These graphs contain vertices representing bus stops, metro stations, metro platforms, and "waiting nodes" for each bus stop and metro platform. The edges between vertices on the graph represent walking times (from station entrances to the platform or along transfer edges), wait times (from a waiting node to a platform or stop) and in-vehicle times (between stops or nodes, representing paths between stops on the bus or train). Dijkstra's algorithm is used to calculate the walk, wait, and in-vehicle times from the center of each station-region to the center of every other station-region. Dijkstra's algorithm minimizes the total time of the trip, not accounting for fares. It is assumed that when deciding to take the bus or the metro, a commuter is deciding between each mode's best possible option in terms of time-cost. Fares are not accounted for in Dijkstra's algorithm because fares vary only for the metro and are constant for a given station. This methodology does mean that a regression cannot determine the wage multipliers for wait time, walk time, and in-vehicle time, since these must be presupposed by the model in order to choose the route.

The bus and metro price variables are computed according to equation (1), taking into account the fare, travel times, and wage for routes originating at each station-region. The travel time operation using Dijkstra's algorithm calculates the times between every possible pair of station-regions, but the regression requires one value of wait time or generalized cost per origin point. Some representative value per origin station must be determined. As discussed, this study concerns morning commuters, so it employs a method from Liao et al. (2020) which, for each origin station, finds the average of travel costs to all destination stations, weighted by the number of users who exit each destination station during peak hours. That is, the average travel cost P_{it} from station i on day t is

$$P_{it} = \frac{\sum_{j=1}^{N} f_j P_{ij}}{\sum_{j=1}^{N} P_{ij}} \tag{9}$$

where f_j is station j's proportion of all station exits, over all t. This method gives the best estimation for the conditions the representative user faces when commuting from station i on day t, and approximates the general model of users commuting from outside the center city into the CBD. Liao et al. (2020) observe the frequency of geotagged tweets and use these data to determine where people are going at a given time. They use this because it is a more exogenous measure of how popular a place is as a destination. This paper does not use any external measures like that because it aims to model commutes to the city, and we also do not expect that consumer behavior would change significantly over the three-month period studied.

6 Results

6.1 Wait times

The first model considered was the simple wait-time effect model, based on service characteristic elasticity research from Lago et al. (1981), the results of which are included in Table 2. The model faces a significant challenge—the denominator of the elasticity term is too large. The wait time is only one component of the total opportunity cost of a trip, so any nominal increase in wait time is a larger percentage of only wait time than of opportunity cost. Therefore, if opportunity cost represents the true causal mechanism, the coefficient for the elasticity would be too small. If there is already limited variation to capture, the standard error might be too large to confidently estimate the elasticity.

To test whether this is the case, consider the model where instead of date fixed effects, we use controls for each day of the work week. It might be the case that the date fixed effects are absorbing a significant amount of variation, diluting the strength of the elasticity coefficient. Much of the expected variation across days is by day of the week, so such a model is reasonable. The results of this regression are shown in Table 3. All of the point estimates are smaller but they are statistically significant at the 1% level, supporting the hypothesis that the two-way fixed effects model may suffer from a dilution problem. These challenges arise only in the cross-price elasticity estimates. Even in the more demanding two-way fixed effects specification for wait times, the own-price estimate was significant, with a value of -0.22. This is in line with some of the literature on the subject.

6.2 Generalized cost

6.2.1 Cross-price elasticity

The core model presented in this paper is the cross-price elasticity model with respect to generalized opportunity cost. As discussed, the Metro cost variable is constructed using the components of travel time, weighted by pre-determined multipliers, and the fares. This model is more constrained than a model using only wait times, but it more closely resembles the actual choice faced by the consumer. We don't expect a consumer to choose her travel mode using only the wait time; we expect her to choose based on the total time cost, so any changes to that opportunity cost should reflect the total price of the trip. Table 1 shows the output for such a regression. Metro opportunity cost is calculated according to the formula for $P_{2,it}$, with c_a set to 100% of the wage, and c_v set to 50% of the wage. Access and egress time combined is assumed to be 10 minutes. Sensitivities for different assumed levels of c_a and c_v are included in the Appendix in Table 4.

The results of this regression are reasonable and in line with elasticity estimations for similar mode choices in the literature. A cross-price elasticity of 0.88 is quite high, meaning that for consumers living near metro stations, urban bus and metro service are very substitutable, provided that the bus and metro stops in the choice set are within 500 meters of one another. For stops within 1km and 2km away, the elasticities fall to 0.75 and 0.57, respectively. The results here follow intuition—bus stops that are further away constitute a less cohesive geographic market than stops that are closer to the metro station. The results here also imply that a relatively large fraction of metro users live near the station they primarily use to commute—if this were not the case, then there would not be such a large effect on demand for buses at stops very close to the metro station.

6.2.2 *Own-price elasticity*

Next, the own-price elasticity of metro demand is calculated. Using the same independent variable representing the opportunity cost of using the metro, elasticities are calculated using a log-log regression, with station and date fixed effects. As with the cross-elasticity regression, it is assumed that any effects which might meaningfully impact the relationship here are time-invariant or unit-invariant, meaning no other controls are necessary.

Table 1: Estimates for the cross-price elasticity of bus demand, using total metro opportunity cost, with both station and date fixed effects. Time-cost multipliers of 50% and 100% are used for in-vehicle time and access time, respectively. A reasonable assumption of walking 5 minutes to the origin station and 5 minutes from the destination station are assumed.

Dependent Variable:	Entries (log)			
	Rail	Bus 5000m	Bus 1km	Bus 2km
Model:	(1)	(2)	(3)	(4)
Variables				
Metro opportunity cost (log)	-0.8050***	0.8754***	0.7483***	0.5675***
	(0.2039)	(0.2252)	(0.2012)	(0.1856)
Fixed-effects				
Station	Yes	Yes	Yes	Yes
Date	Yes	Yes	Yes	Yes
Fit statistics				
Observations	4,626	4,630	4,634	4,635
R^2	0.95966	0.97128	0.98024	0.98706
Within R^2	0.03326	0.01524	0.01569	0.01589

Clustered (Station) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

The results of this regression are also noteworthy. The coefficient of the natural log of metro opportunity cost is estimated at -0.81, which has the correct sign and is statistically significant at the one-percent level. Such an elasticity means that demand for the metro is inelastic, but not terribly so. Such a result aligns with intuition—WMATA is the only provider of metro service, meaning that people are unable to find the same product from a different firm, but there are still other ways to get to work (such as the bus) which people do employ when metro costs rise significantly. However, when varying the access and egress times, there are fairly wide swings in the elasticity values. These robustness regressions are shown in Appendix B. This follows from the intuitions about elasticity. Varying the walk time increases the base price of the trip, meaning that the elasticity will rise. These sensitivities are included in Table 5.

7 Conclusions

The results of this paper are in line with intuitive understandings of substitution between transit modes and add to a body of literature lacking specific research on public transit competition between urban bus and metro systems. The bus and metro systems in Washington, DC appear to act as significant competitors within relatively small radii around metro stations. Significant cross-price elasticity estimates ranging from 0.47 to 1.03 show that competition not only exists, but moreover users readily switch modes based on the opportunity cost of each. Own price elasticity estimates for metro demand ranging from -0.50 to -1.06 show that metro demand is relatively inelastic with respect to total opportunity cost. Still, the metro is not extremely inelastic, meaning that the transit authority, namely WMATA, must work to keep service quality high lest opportunity costs rise and consumers switch to a different mode.

Furthermore, this paper is one of the first to study public transit competition in the post-COVID period. Even now, ridership on Washington, DC's metro and bus systems have not fully recovered to pre-pandemic levels. Even with the new mode choice of "nothing" now available for many workers with the proliferation of work-from-home, this paper was still able to identify a significant effect of metro service changes on bus demand. This does not mean that a significant number of users did not choose to work from home; rather, a significant number still chose to switch to the bus despite more having the option to work from home than in years past.

Lastly, the paper employs a strategy for calculating total opportunity cost using routing algorithms on a weighted, directed graph constructed from GTFS (General Transit Feed Specification) data. This means that the method used in this paper can be flexibly ported to the transit systems of any city whose transit authority conforms to GTFS. Using this method unites research from public transit economics and urban geography (Liao et al., 2020). Despite relative success in identifying elasticities, research in this area would greatly benefit from access to records from individual farecards which show each user's trip for any given time. Tracking thousands of users' choices over long periods of time would give transit authorities important information about mode choice without the constraints of aggregate models like this one.

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Appendix A

The graph representing the transit network is a crucial part of this paper, but understanding it is not necessary to understand the economics. It's importance to calculating the travel times and subsequently the metro price variable means that it deserves additional explanation.

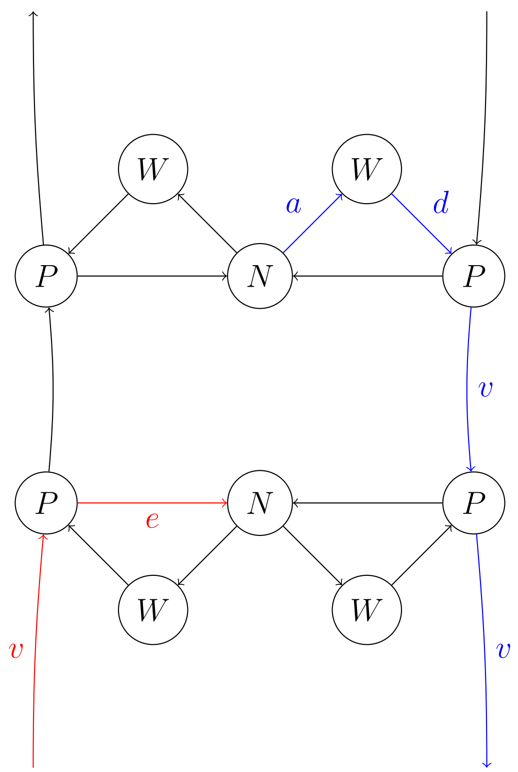

Figure 3: A weighted, directed graph indicating the connections between two metro stations. The path highlighted in red represents an arrival to the below station, and the path highlighted in blue represents a departure from the above.

A graph is a mathematical structure which consists of vertices and edges, in which each vertex is connected to at least one other vertex by an edge. In this graph, the edges are weighted, meaning they have an associated distance, and they are also directed. Graphs are the objects upon which shortest-path algorithms like Dijkstra's algorithm operate. In order to find the shortest path between two vertices, every possible time cost along the route must be expressed through the graph. This means that the naïve graph in which metro stations are vertices and the tracks are edges is not sufficient, because it does not account for wait times and transfers. Therefore, the more convoluted structures shown in figures 3 and 4 must be used. These figures are small selections of the graph which represents the entire metro network.

In figures 3 and 4, N denotes a station, W denotes a "waiting vertex," and P denotes a platform. The edge labels correspond to the time cost referred to in the Theory section. The waiting vertices seem superfluous, but they allow an extra edge between the station and the platform which represents the wait time.

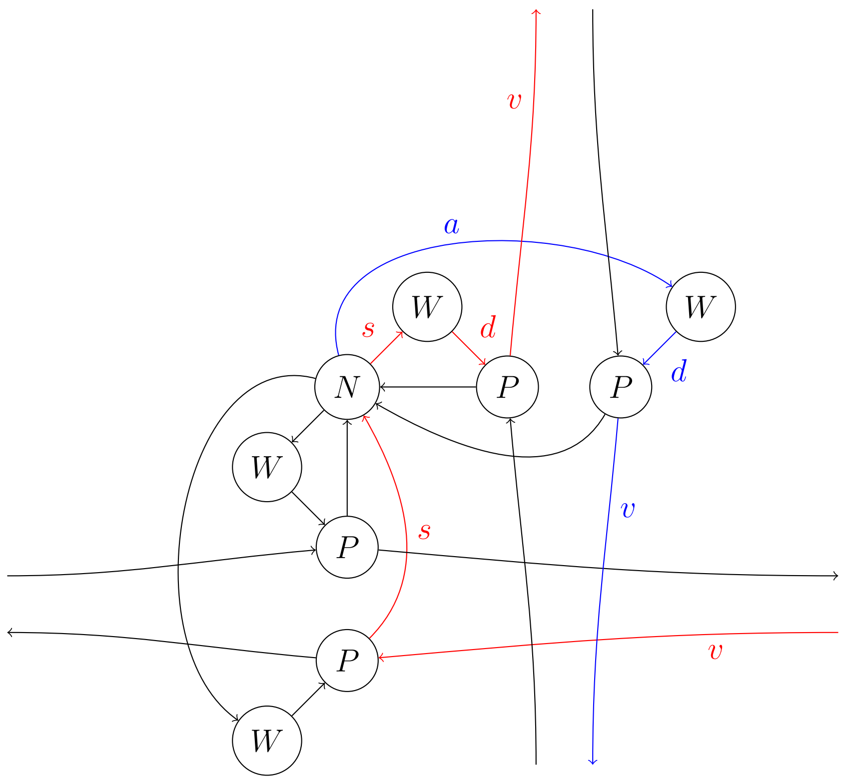

Figure 4: An example of a graph representing a transfer metro station. A trip originating at a different station and transferring to a different line through this station is labeled in red. The edges labeled s represent transfer edges where the user incurs walking time cost s. A trip originating from this station and going to a different one is labeled in blue. The weight assigned to each of the highlighted edges is depicted, with other weights omitted for clarity.

Appendix B

Table 2: Cross-price and own-price elasticity estimates with respect to metro wait times (not generalized cost). This table comes from a model which uses two-way fixed effects.

Dependent Variable:	Entries (log)			
	Metro	Bus 500m	Bus 1km	Bus 2km
Model:	(1)	(2)	(3)	(4)
Variables				
Metro wait time (log)	-0.2264**	0.1504	0.0993	0.0657
	(0.0989)	(0.1481)	(0.1426)	(0.1375)
Fixed-effects				
Station	Yes	Yes	Yes	Yes
Date	Yes	Yes	Yes	Yes
Fit statistics				
Observations	4,931	4,931	4,935	4,936
R^2	0.96727	0.97076	0.97930	0.98581
Within R^2	0.00549	0.00090	0.00053	0.00037

Clustered (Station) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 3: Cross-price and own-price elasticity estimates with respect to metro wait times (not generalized cost). This table comes from a model which uses unit fixed effects and weekday dummies.

Dependent Variable:	Entries (log)			
	Metro	Bus 500m	Bus 1km	Bus 2km
Model:	(1)	(2)	(3)	(4)
Variables				
Metro wait time (log)	-0.1892***	0.0737***	0.0609***	0.0381***
	(0.0125)	(0.0185)	(0.0157)	(0.0116)
Tuesday	0.0710***	0.0065	0.0141	0.0183**
	(0.0056)	(0.0114)	(0.0103)	(0.0079)
Wednesday	0.0328***	0.0006	0.0042	0.0005
	(0.0050)	(0.0116)	(0.0095)	(0.0079)
Thursday	0.0328***	-0.0030	0.0050	0.0030
	(0.0050)	(0.0121)	(0.0102)	(0.0081)
Friday	-0.2252***	-0.1423***	-0.1400***	-0.1391***
	(0.0093)	(0.0139)	(0.0128)	(0.0095)
Fixed-effects				
Station	Yes	Yes	Yes	Yes
Fit statistics				
Observations	4,931	4,931	4,935	4,936
R^2	0.92542	0.96332	0.97214	0.97763
Within R^2	0.30204	0.06773	0.07980	0.09454

Clustered (Station) standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 4: Cross-price elasticity of bus demand estimates for a variety of access time and in-vehicle time multiplier assumptions, using a 1km catchment radius for bus entries. Different assumptions for users' time walking to and from the metro stations are also tested, which result in much larger differences in the elasticity values.

In-vehicle wage rate	Access wage rate	Walk time (min)	Estimate	Std. Error
25%	50%	0	0.5823045	0.0634052
25%	50%	10	0.8195016	0.0905137
25%	50%	20	1.0288740	0.1171403
25%	75%	0	0.5106260	0.0574787
25%	75%	10	0.7445934	0.0863343
25%	75%	20	0.9336425	0.1135554
50%	100%	0	0.5200544	0.0599384
50%	100%	10	0.7483337	0.0883950
50%	100%	20	0.9329998	0.1151388
50%	150%	0	0.4714756	0.0566819
50%	150%	10	0.6861341	0.0863150
50%	150%	20	0.8434148	0.1128313

Table 5: Own-price elasticity of metro demand estimates for a variety of access time and in-vehicle time multiplier assumptions. Different assumptions for users' time walking to and from the metro stations are also tested, which result in much larger differences in the elasticity values.

In-vehicle wage rate	Access wage rate	Walk time (min)	Estimate	Std. Error
25%	50%	0	-0.5406446	0.0466489
25%	50%	10	-0.8195559	0.0664418
25%	50%	20	-1.0830919	0.0858640
25%	75%	0	-0.4990944	0.0422496
25%	75%	10	-0.7956526	0.0632877
25%	75%	20	-1.0553858	0.0831306
50%	100%	0	-0.5166555	0.0440301
50%	100%	10	-0.8050170	0.0647776
50%	100%	20	-1.0556173	0.0842800
50%	150%	0	-0.4958435	0.0415996
50%	150%	10	-0.7907854	0.0631830
50%	150%	20	-1.0242888	0.0825248