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**State Director Contacted About Submission: No**
**Pilot Test Scheduled: Harvard Undergraduate Science Olympiad, Winter 2021**

## Algorithms

**PROPOSED EVENT CATEGORY: Inquiry & Problem Solving**

**PROPOSED EVENT DIVISION: Division C**

**DESCRIPTION:** Teams will solve problems by demonstrating efficient algorithmic thinking and analysis.

**A TEAM OF UP TO:** 2

**EYE PROTECTION:** None

**IMPOUND:** None

**EVENT TIME:** 50 minutes.

1. **EVENT PARAMETERS:** Participants will bring writing utensils. No other resources are allowed. The event supervisor will provide the exam.
2. **THE COMPETITION:**
   a. Each team will have the same amount of time to solve as many problems as possible.
   b. The exam will consist of at least one question from each of the following areas:
      i. Fundamentals: Proof Techniques, Recurrences, Growth of Functions, Sorting, Divide and Conquer
      ii. Graph Algorithms: Minimum Spanning Trees, Shortest Path Algorithms
      iii. Advanced Algorithms: Dynamic Programming, Greedy Algorithms, Linear Programming
      iv. Random Processes: Randomized Algorithms, Hashing, Random Variables, Basics of Counting, Tail Bounds, Markov Chains, Probability Distributions
      v. Data Structures: Stacks and Queues, Linked Lists, Binary Search Trees, Heaps, Disjoint Sets
   c. When giving an algorithm, competitors must prove the correctness of the algorithm and establish the best upper bound for its running time. Competitors should write a clear, informal description of their algorithm in English. They may also write pseudocode if they feel that their informal explanation requires more precision and detail. Competitors should try and make their answers as clear and concise as possible.
3. **SAMPLE QUESTIONS:**
   a. T/F:  $(\log \log(n))^{\log(n)} = O(n^c)$  for some $c > 0$.
   b. T/F: In a Depth First Search of a directed graph $G$, the set of vertices reachable from the vertex with the lowest post-order is a strongly connected component of $G$.
   c. Given an array of integers, design an algorithm that finds number of un-ordered pairs in the array.
   d. Given three strings $X$, $Y$, and $Z$, design an algorithm that determines whether $Z$ contains all of the characters of $X$ and $Y$ in their original order.
   e. You are given a graph $G = (V, E)$ and a minimum spanning tree $T$. The weight of one of the edges in $T$ is increased. Give a linear time algorithm that determines the new minimum spanning tree.
4. **SCORING:** Questions will be worth a predetermined number of points. High score wins. Selected questions will break ties.

**RECOMMENDED RESOURCES:** *Introduction to Algorithms* by CLRS, *Algorithms* by S. Dasgupta, *Introduction to Probability* by Joe Blitzstein, *Probability and Computing: Randomized Algorithms and Probabilistic Analysis* by Michael Mitzenmacher.