Second Year/ Third Semester

Subject : Numerical Method **FM : 60**
Time : 3 hours **PM : 24**

Candidates are required to give their answers in their own words as for s practicable.
The figures in the margin indicate full marks.

Year: 2066

Attempt all the questions:

1. Define the fixed point iteration method. Given the function $f(x) = x^2-2x-3=0$, rearrange the function in such a way that the iteration method converses to its roots. (2+3+3)

2. What do you mean by interpolation problem? Define divided difference table & construct the table from the following data set. (2+2+4)

X	3.2	2.7	1.0	4.8	5.6
f	22.0	17.8	14.2	38.3	51.7

OR

Find the least squares line that fits the following data.

X	1	2	3	4	5	6
Y	5.04	8.12	10.64	13.18	16.20	20.04

What do you mean by least squares approximation?

3. Derive a composite formula of the trapezoidal rule with its geometrical figure. Evaluate $\int_0^1 e^{-x^2}dx$ using this rule with n=5, up to 6 decimal places. (4+4)

4. Solve the following system of algebraic linear equation using Jacobi or Gauss-seidal iterative method. (8)
$6x_1-2x_2+x_3=11$
$-2x_1+7x_2+2x_3=5$
$X_1+2x_2-5x_3=-1$

5. Write an algorithm & computer program to fit a curve y=ax2+bx+c for given sets of (x_1, y1, g, 0=1,……..,x) values by least square method. (4+8)

6. Derive a difference equation to represent Poisson's equation. Solve the Poisson's equation $\nabla^2 f=2x^2y^2$ over the square to main $0 \leq x \leq 3$, $0 \leq y \leq 3$ with f=0 on the boundary & h=1. (3+5)

7. Define Ordinary Differential Equation of the first order. What do you mean by initial value problem? Find by Taylor's series method, the values of y at x=0.1 & x=0.2 to fine places of decimal form. (2+6)
$\frac{dy}{dx} = x^2y-1$, y (0) =1

Year: 2067

Attempt all the questions:

1. Discuss methods of Half-Interval & Newton's f for solving the non-linear equation f(x) =0. Illustrate the methods by figures & compare them stating their advantages & disadvantages. (8)

2. Derive the equation for Lagrange's interpolating polynomial & find the value of f(x) at x = 1 for the following (4+4)

x	-1	-2	2	4

f(x)	-1	-9	11	69

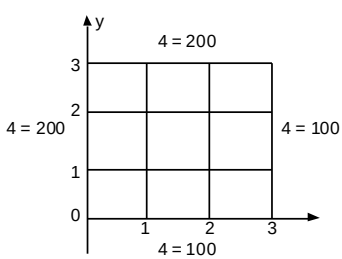
3. Write Newton-cotes integration formulas in basic form for x=1, 2, 3& give their composite rules. Evaluate $\int_0^1 e^{-x2}\,dx$ using the Gaussian integration three point formula. (4+4)
4. Solve the following system of algebraic linear equation using Gauss-Jordan lgorithm. (8)

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 \\ 4 & -3 & 0 & 1 \\ 6 & 1 & -6 & -5 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -7 \\ 6 \end{bmatrix}$$

5. Write an algorithm & computer program to solve system of linear equation using Gauss-Seidal iterative method. (4+8)
6. Explain the Picard's proves of successive approxiamtion. Obtain a solution upto the fifth approximation of the equation $\frac{dy}{dx}$ = y+x such that y=1 when x=0 using Picard's process of successive approximation. (3+5)
7. Derive a difference equation to represent a Laplace's equation. Solve the following Laplace equation. (3 + 5 = 8)

$\frac{\partial 2u}{\partial x2}+\frac{\partial 2u}{\partial y2}$= 0, within 0≤x≤3, 0≤y≤3

For the rectangular plate given as:



OR

Derive a difference equation to represent Poisson's equation. Solve the Poisson's equation $\nabla^2 f=2x^2y^2$ over the square to main 0≤x≤3, 0≤y≤3 with f=0 on the boundary & h=1. (3+5)

Year: 2068

Attempt all the questions:

1. Define the types of errors in numerical calculations. Derive the formula for secant method and illustrate the method by figure. (4+4)
2. Define the linear least squares approximations. Give the data set (x_i,y_i) as

(20.5, 765), (32.7, 826), (51.0, 873), (73.2, 942), (95.7, 1032)

Find the linear least square to fit given data. (2+6)

3. Evaluate $I = \int_0^1 e^{-x^2} dx$ using trapezodial rule with n=10. Also evaluate the same integral using Grossion 3 point formula and compare the result.
4. Solve the following system of linear equations using Gauss-elimination method (use partial pivoting if necessary): (8)

$2x_2 + x_4 = 0$

$2x_1+2x_2+3x_3+2x_4 = -2$

$4x_1-3x_2 +x_4 = -7$

$6x_1+ x_2- 6x_3- 5x4= 6$

OR

What do you mean by eigen-value, eigen-vector problem? Find the largest eigenvalue correct to two significant digits and corresponding eigen-vectors of the following matrix using power method. (2 + 6)

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 3 \end{pmatrix}$$

5. Write an algorithm and program to solve system of linear equations using Gauss-Jordan method. (4 + 8)
6. Apply Runge Kutta method of second order and 4th order to find an approximate value of when x=0.2 given that (8)
   $dy = x + y$ and $y(0)=1$.
7. How can you solve Laplace's equation? Explain. The steady-state two dimensional heat flow in a metal plate is defined by $\frac{d^2T}{dx^2} + \frac{d^2y}{dy^2} = 0$.
8. A steel plate of size 30*30 cm is given. Two adjacent sides are placed at 100°C and other sides are held at 0°C. Find the temperature at interior points, assuming the grid size of 10*10 cm. (3+5)