9/11

(8.7) $\ln(1+x)$

$-1<x<1$

(IOC) Predict the IOC

$8+4+2+1=15$

Geometric series

Multiply by a common ratio

$r = r/2$ common ratio

$S_n = \frac{a(1-r^n)}{1-r}$ $\qquad S = (8-(1/2)^4) / [1-1/2]$

$\downarrow$ $\qquad = 15$

um of geometric series → very important.

Calculus deal with infinite.

Ex. $8+4+2+1+\ldots=$

Infinite Geometric series → Paradox: There is a finite sum

$S = \lim_{n\to\infty} \frac{8(1-(1/2)^n)}{1-1/2}$ → 0 vanishes → $S = \frac{a}{1-r}$ for $-1<r<1$

Therefore:

$8+4+2+1\ldots=16$ Converge to have a finite sum;

Geometric series converges to 16

3 If converge, find the sum

$\sum_{=0}^{\infty} 4(-1/2)^n$ $\quad r=-1/2$ Geometric

$S = a/1-r \quad S = 4/_{1+1/2} = 4/3/2 = 8/3$

(4) $\sum_{=0} \frac{1}{2}(2)^n$ Converge or diverge $\quad r>1$ Ratio is too big = Diverge

(5) $-1+1-1+1-1\ldots$

.) Diverge because $r=-1$

Diverge = multiple finite sum

(6) $(x) = 1/1-x$ Find a Maclaurin series.

$\sum_{n=}^{\infty} x^n$

Ma aurin series

$y_1 = 1/1-x$

$y_2 \quad 1+x+x^2+x^3+x^4+x^5+x^6 \}$ Geometric series

Ove p $-1<x<1$ $\qquad \downarrow$

Converges $-1<r<1$

$\boxed{-1<x<1}$, $x=1$ = No.

Tr scendental

$\rightarrow$ Polynomials

IOC

turn to

$\rightarrow$ Geometric series

6.) a in

$f(x) = 1/(1-x) = (1-x)^{-1}$

$a/1- = 1/1-x \quad a=1, \; r=x \;\Big|\; 1+x+x^2+x^3\ldots \quad r=x \quad -1<\underline{\underline{x}}<1$ overlap

7.) $f = 5/3+x$

Fin the Maclarin series and
+ interval of convergence
with + a calculator

$$\frac{\frac{5}{3}}{\frac{3}{3}+} = \frac{\frac{5}{3}}{1+\frac{x}{3}} \quad \sum_{n=0}^{\infty} ar^n$$

$-1<r<1$

$-1<-x/3<1$

$-3<-x<3$

$-3<x<3$

$$\sum_{n=0}^{\infty} \frac{5}{3}\left(\frac{-x}{3}\right)^n$$

9/12

1.) $\sum_{n=1}^{\infty} \quad 2^n$ Find IOC without calculator.

Ratio test

$r = \lim_{n\to\infty}$ (+1)th term / n^{th} term

$$r = \lim_{n\to\infty}\left[\frac{+1)^2}{n+1} \cdot \frac{2^n}{n^2}\right]$$ reciprocal of original

Converges

$-1<r<1$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{2} = \lim_{n\to\infty} \frac{n^2+2n+1}{2n^2} = \boxed{1/2}$$

(2.) $\sum_{n=1}^{\infty} n!\, 3^n$

$$r = \lim_{n\to\infty}\left[\frac{(\ \ +1)!}{n+1} \cdot \frac{\cancel{3^n}}{n!}\right]$$

$\cancel{\frac{5!}{4!}}$ $\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$

$$\frac{n}{3^1} = \lim_{n\to\infty}\left(\frac{n+1}{3}\right) = \infty$$

$-1 < r < 1$

(3) Ratio tes

$-1 < r <$	Converge
$r > 1$ or r 1	diverge
$r = 1$ or r -1	inconclusive

- Find the IOC

Maclaurin $\ln(1+x)$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+}}{n+1}$$

Calc

$-1 < x < 1$

$$r = \lim_{n\to\infty}\left[\frac{(-1)^{n+1}\ \ n+2}{n+2} \cdot \frac{n+1}{(-1)^n x^{n+1}}\right] = \left[(-1)x\, \frac{n+1}{n+2}\right] = -x$$

1

$r = -x$

$-1 < -x < 1$

$-1 < x < 1$

Interval of Convergence

$-1 < x < 1$

$-1 \leq x \leq 1$

$-1 < x \leq 1$

$-1 \leq x < 1$

*Webassign

Radius of convergence

$R = 1$

Radius

(4.) Find the r ius of convergence

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\lim_{n\to\infty}\left[\frac{(-1)^{n+1} x^{2n}}{(2n+3)!} \cdot \frac{(2n+1)!}{\cancel{(-1)^n x^{2n+1}}}\right] = \left[\frac{(-1)x^2}{(2n+3)(2n+2)}\right] = 0$$

$r = 0$

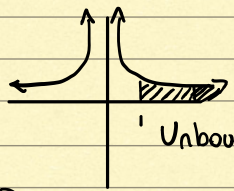
for every value of x,

we have convergence

9/18

8.3, 8.4, 8.5, 5.10 — Topics
1.) Integral + t
2.) Alternating ries test
3.) Interval of vergence with endpoint checking.

① Find the area under y $1/[x^2]$ for $x \geq 1$



Unbounded region

$\int_1^{\infty} x^{-2}\,dx = \lim_{b \to \infty} \int_1^b x^{-2}\,dx = \lim_{b \to \infty} (-1/x) \Big|_{x=1}^{x=b} \left[-\frac{1}{x} - \frac{-1}{1} \right]$ $0 = \boxed{1}$

Improper integral — anti-derivative

②. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

no ratio st $r = 1$ inconclusiv

$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots$

$\frac{1}{(n+1)^2} \quad \frac{n^2}{1}$

converge or diverges

$\int_1^{\infty} \frac{1}{x^2}\,dx = 1$ Area

$1(\frac{1}{4} + \frac{1}{9} + \ldots)$

$\frac{1}{4} + \frac{1}{9} + \frac{1}{16}$

$1 + 1 > 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} +$

$2 > \sum_{n=1}^{\infty} \frac{1}{n^2}$ || If onverges, so do $\sum$ — If $\int$ diverges then so does $\sum$.

↓

Integral test

3.) $\sum_{n=1}^{\infty} \frac{1}{n}$

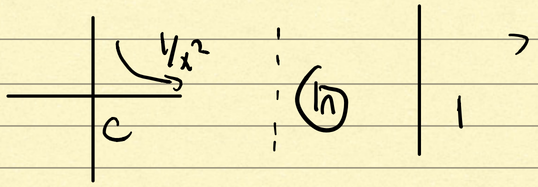
Ratio test
$r = 1$ inconclusive

converges or diverge

$\lim_{b \to \infty} \int_1^b \frac{1}{x}\,dx \quad \lim_{b \to \infty} (\ln x) \Big|_{x=1}^{x=b}$

$(\ln \infty - \ln 1) = \infty$ diverge

Harmonic



con. or ... con or die

P series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ $p>1$ converge, $p \leq 1$ diverge

$p > 0$

die? 5.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ P series

$p = 2/3$

$\frac{1}{n^{2/3}}$ $p \leq 1$ diverge

1.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}} = \frac{1}{n^{4/3}}$

P = series

$P = 4/3$

$P \leq 1$

diverge

$$\text{P series} = \sum \frac{1}{n^p}$$

6.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+7}}$

converge or diverge

- Geometric
- Ratio
- $\int$ test
- P series X

$$\lim_{x \to \infty} \int_1^t \frac{\cancel{dx}}{(n+7)^{1/2}} d = \lim_{x \to \infty} 2(x+7)^{1/2} \Big|_{x=1}^{x=\infty}$$

$$2(1+7)^{1/2} - 2(1+\infty)^{1/2}$$

$$\infty - 2$$

$$= \lim_{t \to \infty} \left[2\sqrt{t+7} - 2\sqrt{8} \right] = \infty \text{ div}$$

$$\int_{-1}^{1} \frac{1}{x} dx$$

discont. at 0

Improper

$$\int_{-1}^{0} \frac{1}{x} dx + \int_{0}^{1} \frac{1}{x} dx$$

$$= \lim_{t \to 0^+} \int_t^1 \frac{1}{x} dx = \lim_{t \to 0^+} (\ln x) \Big|_{x=1}^{x=1}$$

$$= \lim_{t \to 0^+} (\ln - \ln t) = \infty$$

9/20

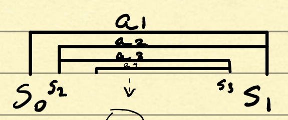
8.4/8.5

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Converge or dieverge $= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$ Al nerting harmonic

Ratio test
No!
No n in the experiment.

↓ No ratio

↓ Alternating series test (AST)



S converge

AST
1.) $\lim_{n\to\infty} a_n = 0$
ii.) a_n is decreasing

$\sum_{1}^{\infty} \frac{(-1)^{n+1}}{n}$ lternating series ✓

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

Converges by AST

2.) $\sum_{n=1}^{\infty} \frac{(-1)^n 2}{n^2+1}$ → Alternator

$\lim_{n\to\infty} \frac{2}{n^2+1} = 0$ $\frac{2}{n^2+}$ is dec. } Series converges by AST

AST Convergence tests

1.) Geometric
2.) Ratio
3.) P series
4.) Int test
5.) AST → conv.

3.) Find the interval of convergence

$\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ Power series

↓ $\ln(1+x)$

$\frac{(-1)^{n+1} x^{n+2}}{n+2} \cdot \frac{n+1}{\cancel{(-1)^n} x^{n+1}}$ $\cancel{(-1)^n}(-1)^1$

$r = \lim_{n\to\infty} (-1)(x) \cdot \frac{\cancel{n+1}}{\cancel{n+2}} = x$

$-1 < x < 1$ or
$R = 1$

IVT

↓

Test $x = 1$

Endpoint checking

Test $x=1$

$\sum_{n=0}^{\infty} \frac{(-1)^n (1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

AST
$\lim_{n\to\infty} \frac{1}{n+1} = 0$ ✓
$\frac{1}{n+1}$ is dec. ✓

~~$-1 < x < 1$~~
~~$-1 \le x \le 1$~~
$-1 < x \le 1$
~~$-1 \le x < 1$~~

$\sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{n+1}$ odd $= \sum_{n=0}^{\infty} \frac{(-1)}{n+1}$ $-\sum_{n=0}^{\infty} \frac{1}{n+1}$ $n=1$ -1 → $\sum_{n=1} \frac{1}{n}$

P series
$P = 1$ $P \le 1$
Diverges

$[-1 < x \le 1]$ IOC

$\ln(1+x) = \sum_{n=0}^{\infty} -1^n x^{n+1} / (n+1)$

for $-1 < x \leq 1$

$\boxed{\text{choose } x=1}$

$\ln(1+1) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

$\ln 2 = (-1/2 + \frac{1}{3} -)$

$\ln 2 = 1 - 1/2 + 1/3 - 1/4 + \ldots$

$\downarrow 0.693$

4.) Sample mid-term question

find the IOC

$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$ Ratio test $\rightarrow$ $\frac{(-2)^{n+1} x^{n+1}}{n^{1/4+1}} \cdot \frac{n^1}{(-2)^n} \rightarrow \frac{(-2)x^1}{n^1}$ $\lim_{n\to\infty} = -2x = r$

$-2x \sqrt[4]{\frac{n}{n+1}}$

$\cancel{-1 < 2x < 1}$

$\cancel{-1/2 < x < 1/2}$

$\cancel{-1/2 < x \leq 1/2}$

$\cancel{-1/2 \leq x < 1/2}$

$-1/2 \leq x \leq 1/2$

Substitute in power series for test.

Test $x = 1/2$

$\sum_{n=1}^{\infty} \frac{(-2)^n (1/2)^n}{\sqrt[4]{n}} = \frac{(-1)^n}{\sqrt[4]{n}}$ AST | $\lim_{n\to\infty} \frac{1}{\sqrt[4]{n}}$ 0

$\frac{1}{\sqrt[4]{n}}$ i dec.

Test $x = -1/2$

$\sum_{n=1}^{\infty} \frac{(-2)^n (-1/2)^n}{\sqrt[4]{n}}$

$= \sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$ P series

$p = 1/4$

$p \leq 1$ div

$(-1/2, 1/2]$

9/23

$\sum_{n=1}^{\infty} \frac{(-4)^n x^n}{\sqrt[7]{n}}$ $\frac{(-4)^{n+1} x^{n+1}}{(n)^{1/7+1}} \cdot \frac{n^{1/7}}{(-4)^n x^n}$

$\lim_{n\to\infty} \frac{-4x}{\cancel{n^1}}$ $-1 < -4x < 1$

r : $-1/4 < x < 1/4$

$\sum_{n=1}^{\infty} (-1)^n \frac{n^3 x^n}{7^n}$ $\frac{\cancel{(-1)^{n+1}} (n+1)^3 \cancel{x^{n+1}}}{\cancel{7^{n+1}}} \cdot \frac{\cancel{7^n}}{\cancel{(-1)^n} \cancel{(n)^3} \cancel{x^n}}$

$\lim_{n\to\infty} \left| \frac{\cancel{(-1)} (n+1)^3 x}{7 \frac{n^3}{n^3}} \right|$ $\frac{1}{7}x$ -1 $1/7 x < 1$

-7 $x < 7$

$$\sum_{n=0}^{\infty} \frac{(x-7)^n}{n^8+1} \qquad \frac{(x-7)^{n+1}}{(n+1)^8+1} \cdot \frac{n^8+1}{(x-7)^n} = \lim_{n\to\infty} (x-7 \; \frac{n^8+1}{1+(n+1)^8} \quad 0$$

R =

$$-1 < x-7 < 1$$
$$-6 < x < 8$$

Convergence or Divergence test

$\lim_{n\to\infty}$

$$\frac{7n^2}{6n^2}$$

$$\frac{7n^2}{\frac{6n}{n^2}} \qquad \frac{1}{n} - 0$$

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^3+6}}$$

$$\lim_{n\to\infty} \frac{n/n^3}{\sqrt{n^3/n^3 + 6/n^3}} = \lim_{n\to\infty} \frac{0}{\sqrt{1+0}} = 0$$

$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{3}{4+1} = \frac{3}{4} + 1$$

4 R 6

$x - 5$

9/25

a.) $\sum_{n=1}^{\infty} \frac{5^n}{3^{n+1}}$ Ratio test:

Geometric series $\sum_{n=1}^{\infty} \frac{5^n}{3^n 3} = 1/3 \sum_{n=1}^{\infty} (5/3)^n$

$r > 1$

$$\lim_{n \to \infty} \sum_{n=1}^{\infty} \frac{5^{n+1}}{3^{n+2}} \cdot \frac{3^{n+1}}{5^n} = \lim_{n \to \infty} \left| \frac{5}{3} \right| = 5/3 > 1$$

= diverge

b.) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ Ratio test:

$$r = \lim_{n \to \infty} \frac{3^{n+1}}{n+!} \cdot \frac{n!}{3^n} = \frac{3}{n+1} = |0| = 0 < 1$$

= converge

c.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ Ratio test:

$$\lim_{n \to \infty} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{1} = \lim_{n \to \infty} \frac{\sqrt{n+1}}{\sqrt{n+2}} = 1$$ inconclusiv

P series: $\frac{1}{(n+1)^{1/2}}$

$p > 1$ converges

$p \leq 1$ diverges

$1/2 \leq 1$ diverges

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}} = \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ Pseries $p = 1/2$ $p \leq 1$ diverge

!

d.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Alternating series test:

$\lim_{n \to \infty} \frac{1}{n} = 0$ ✓

$\frac{1}{n}$ is decreasing ✓

Converge

2.)

$\sum_{n=1}^{\infty} \frac{(5x)^n}{\sqrt{n}}$ Ratio test:

$$r = \lim_{n \to \infty} \frac{(5x)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(5x)^n} = 5x \lim_{n \to \infty} \left| \frac{\sqrt{n}}{\sqrt{n+1}} \right| = 5x$$ inconclusive

$\to 1$

$-1 < 5x < 1$

$-1/5 < x < 1/5$

$R: 1/5$

test $x = 1/5$

$\sum_{n=1}^{\infty} \frac{(1)^n = 1}{(1/5)^{1/2}}$ Diverge

Pseries $p = 1/2$ diverge

test $x = -1/5$

$\sum_{n=1}^{\infty} \cancel{(-1/5)} \frac{(-1)^n}{(n)^{1/2}}$

AST $\frac{1}{\sqrt{n}} = 0$

$\frac{1}{\sqrt{n}}$ = decreasing

decreasing ..

Converge

3.) $f(x) = \ln(1-x)$

$\ln(1+x) \to \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

$\ln(1-x) \to \sum_{n=0}^{\infty} \frac{(-1)^n (-x)^{n+1}}{n+1}$

$$r = \lim_{n\to\infty} \left[\frac{(-1)^{n+1}(-x)^{n+2}}{n+2} \cdot \frac{n+1}{(-1)^n (x)^{n+1}} \right]$$

$$= \lim_{n\to\infty} \left[(-1)(-x)\left(\frac{n+1}{n+2}\right) \right] = x \qquad r = x$$

$\frac{n+1}{n+2} \to 1$

$-1 < x < 1$

$R = 1$

$x < 1$

Check $x = 1$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{n+1}}{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{2n+1}}{n+1} = (-1) \sum_{n=0}^{\infty} \frac{1}{n+1}$$

$$= -\sum_{n=1}^{\infty} \frac{1}{n}$$

Pseries test

$p = 1$

diverge

Check $x = -1$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ AST

$\lim_{n\to\infty} \frac{1}{n+1} = 0$

$\frac{1}{n+1}$ is dec.

Converge by AST

4.) $f(x) = e^{3x}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{3x} = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$$

$$r = \lim_{n\to\infty} \left[\frac{(3x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(3x)^n} \right]$$

$$= \lim_{n\to\infty} \left[\frac{3x}{n+1} \right] = 0$$

if $r = 0$
then $R = \infty$
$(-\infty, \infty)$

$S = \frac{a}{1-r}$

$4 = 3$

5.) $3 = a$
$4 = S$

$4 - 4r = 3$
$-4r = -1$
$r = 1/4$

$$\sum_{n=0}^{\infty} 3(1/4)^n$$

$$\frac{\sum_{n=0}^{\infty} ar^n}{\sum_{n=1}^{\infty} ar^{n-1}}$$

Always start with 0, unless is undefined.

9/25

- $1/(1-x) = 1 + x + x^2 + x^3 + x^4 + \dots = \sum_{n=0}^{\infty} x^n$
- $e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- $\cos x = 1 - x^2/2! + x^4/4! - x^6/6! = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n}$
- $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$ $(-1, 1]$ $| x - x^2/2 + x^3/3 - x^4/4 + x^5/5$
- $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

1.) $f(x) = \ln(1 + 4x)$
$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(4x)^n}{n}$

$R = 1/4$

2.) $\cos x = \sum_{n=0}^{\infty} (-1)^n x^{2n}/(2n)!$

$= \sum_{n=0}^{\infty} (-1)^n (5x)^{2n} / 4(2n!)$
$= 1/4 \sum_{n=0}^{\infty} (-1)^n (5x)^{2n} / (2n!)$
$r = \lim_{n \to \infty} \frac{\cancel{(-1)^{n+1}} \cancel{(5x)^{2n+1}}}{2n+2!} = \frac{(2n+1)}{\cancel{(-1)^n} \cancel{(5x)^{2n}}} \frac{(-1)5x}{2n!} = 0$

conv

3.) $e^x = \sum_{n=0}^{\infty} x^n / n!$
$4e^x = 4 \sum_{n=0}^{\infty} \frac{x^n}{n!}$
$e^{8x} = \sum_{n=0}^{\infty} \frac{(8x)^n}{n!} = \sum_{n=0}^{\infty} \frac{8^n x^n}{n!}$
$= 4 \sum_{n=0}^{\infty} \frac{x^n}{n!} + \frac{8^n x^n}{n!}$
$= \frac{4x^n + 8^n x^n}{n!}$
$= \frac{x^n(4 + 8^n)}{n!}$
$= \frac{(4 + 8^n)}{n!} x^n$

4.) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

$f(x) = 7x \cos(1/6 x^2)$
$= 7x \sum_{n=0}^{\infty} (-1)^n \frac{(1/6 x^2)^{2n}}{(2n)!}$
$\sum \quad (-1)^n \frac{(1/6)^{4n} x^{4n}}{(2n)!}$
$7(-1)^n \frac{x^{4n+1}}{6^{2n}(2n)!}$

5.) $\int \frac{e^x - 1}{x} dx$!

9/27

1. $\int x \cos(x^2) dx$
   1. $\cos x$
   2. $\cos(x^3)$
   3. $x \cos(x^3)$
   4. $C + \Sigma$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x)^{6k}}{(2k)!}$$

$$\int \sum_{k=0}^{\infty} \frac{(-1)^k (x)^{6k+1}}{(2k)!} dx$$

$$= C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+2}}{(6n+2)(2n)!}$$

No AST

x is not alternating

2. $r = \lim_{n \to \infty} \frac{(-1)^{n+1} x^{6n+8}}{(6n+8)(2n+2)!} \dots$ $r = 0$

$(-\infty, \infty)$

(3.) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converge or diverge

$$\lim_{l \to \infty} \int_2^l \frac{1}{x(\ln x)^2}\,dx = \lim_{l \to \infty}\left(\frac{1}{\ln l} + \frac{1}{\ln 2}\right) = \frac{1}{\ln 2}$$

(4.) Find a maclaurin series for $1/(1+x^2)$

and its IOC.

$$\sum_{n=0}^{\infty} (1)(-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

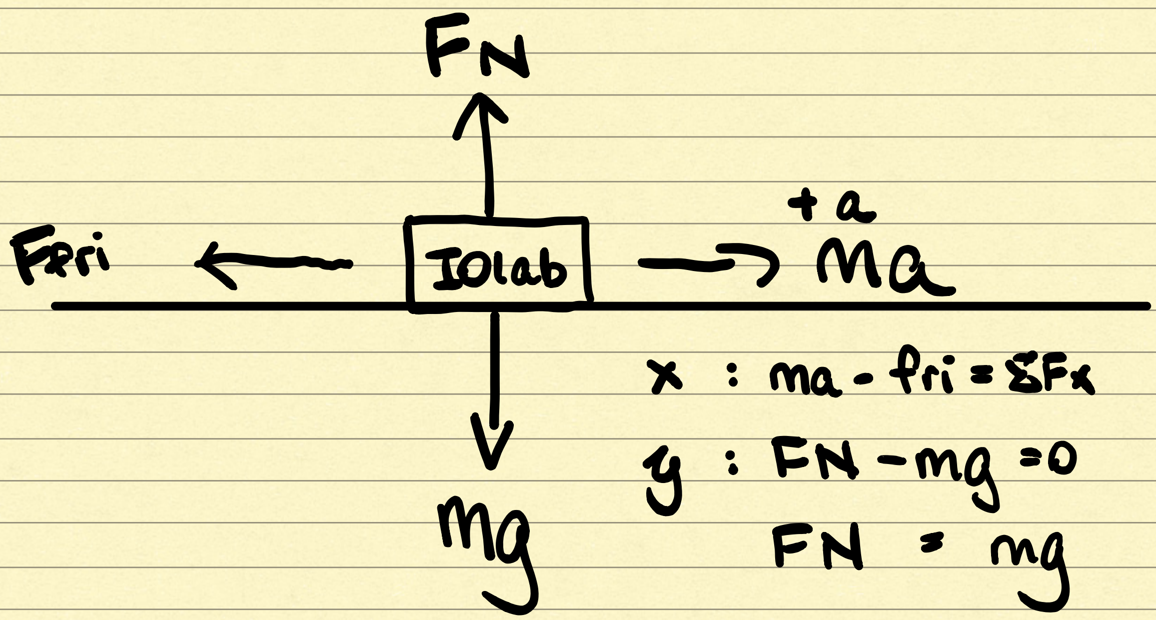
$-1 < -x^2 < 1$

$-1 < x^2 < 1$

(1, 1) Always open series

(5.) Find a maclaurin series of arctanx

$$C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad [-1, 1]$$



x : ma - fri = ΣFx

y : FN - mg = 0

FN = mg

$5 + 5x + 5x^2 + 5x^3$ ✔

5. 1/1 points | Previous Answers HHCalc6 10.2.022. My Notes Ask Your Teacher

Find an expression for the general term of the series and give the range of values for the index.

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$$

$\frac{x^{2k}}{k!}$ ✔ for $k \geq 0$

$e^x = 1 + x + \frac{x^2}{2!} \cdots \quad \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e^{x^2} = 1 + x^2 + x^4/2! \cdots \quad \frac{x^{2n}}{n!}$

6. 1/1 points | Previous Answers HHCalc6 10.2.023. My Notes Ask Your Teacher

Find an expression for the general term of the series and give the range of values for the index.

$$9x^2 \cos x^2 = 9x^2 - \frac{9x^6}{2!} + \frac{9x^{10}}{4!} - \frac{9x^{14}}{6!} + \cdots$$

$\frac{9(-1)^k x^{4k+2}}{(2k)!}$ ✔ for $k \geq 0$

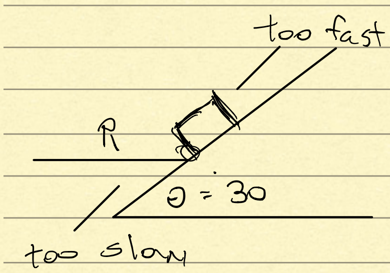
cosx

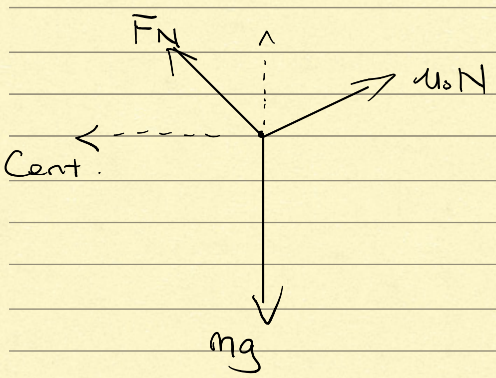
7. 1/1 points | Previous Answers HHCalc6 10.3.001. My Notes Ask Your Teacher

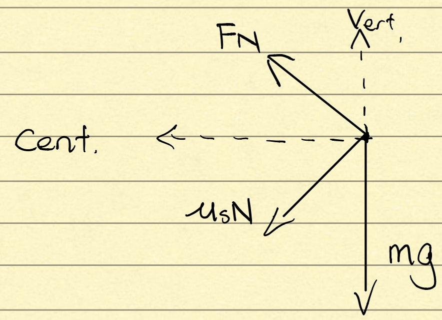
Using known Taylor series, find the first three nonzero terms of the Taylor series about 0 for the function.

e^{-x}

$1 - x + \frac{x^2}{2}$ ✔







$\boxed{10/3}$

Vert:

$N\cos 30 + \mu_s N \sin 30 - mg = 0$

Cent:

$N\sin 30 - \mu_s N\cos 30a = mv^2/r$

$N(\sin 30 - \mu_s(\cos 30a)) = mv^2/R$

$N(\cos 30 + \mu_s \sin 30) = mg$

$$\frac{\sin 30 - \mu_s \cos 30}{\cos 30 + \mu_s \sin 30} = \frac{v^2}{Rg}$$

Vert:

$N\cos 30 - \mu_s N\sin 30 - mg = 0$

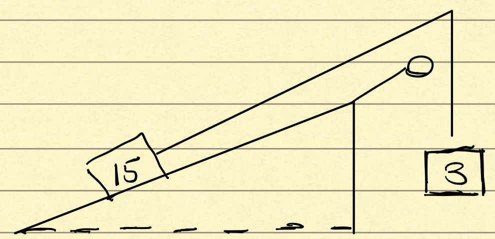
Cent:

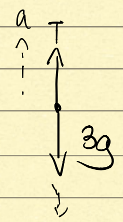
$N\sin 30 + \mu_s N\cos 30 = mv^2/R$

$N(\sin 30 + \mu_s(\cos 30)) = mv^2/R$

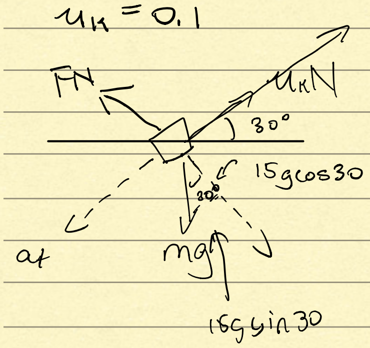
$N(\cos 30 - \mu_s \sin 30) = mg$

$$\frac{\sin 30 + \mu_s \cos 30}{\cos 30 - \mu_s \sin 30} = v^2/Rg$$





$\mu_k = 0.1$



y:

$T - 3g = 3a$

y:

$N - 15g\cos 30 = \emptyset$

$N = 15g\cos 30$

x:

$+15g\sin 30 - \mu_s N - T = 15a$

$15g\,\cancel{\sin 30 - \mu_k N - 3g} = 3$

10/10

Introduction to techiques of integration

- Finding anti-derivative is much more useful & hard.

① $\int xe^{x^2}\,dx$

$u = x^2$

$du/dx = 2x$

$du = 2x\,dx$

$\frac{1}{2}\int e^u\,du$

$= 1/2(e^u) + C$

$= (1/2)e^{x^2} + C$

② $\int \frac{x^2}{5-x^3}\,dx$

$u = 5-x^3$
$du/dx = -3x^2$
$du = -3x^2\,dx$
$-1/3\,du = x^2\,dx$

$-1/3\int \frac{1}{u}\,du = -1/3 \ln|5-x^3| + C$

③ $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$-\int \frac{1}{u}\,du = -\ln|u| + C$
$= -\ln|\cos x| + C$

④ $\int x(3x-5)^{99}\,dx$

$u = 3x-5$ $\quad \frac{u+5}{3} = x$

$du/dx = 3$

$\frac{1}{9}\int u^{99}(u+5)\,du$ $\quad \frac{1}{3}du = dx$

$= 1/9\int (u^{100} + 5u^{99})\,du = \frac{1}{9}\left(\frac{(3x-5)^{101}}{100} + \frac{(3x-5)^{100}}{20}\right) + C$

⑤ $\int \frac{x}{1+x^4}\,dx = \int \frac{x}{1+(x^2)^2}\,dx$

$\frac{1}{2}\int \frac{1}{1+u^2}\,du = 1/2\arctan(x^2) + C$

⑥ $\int x\sqrt{2x+1}\,dx = \int x(2x+1)^{1/2}\,dx$

$u = 2x+1$ $\quad (u-1)/2 = x$
$\frac{1}{2}du = x\,dx$ $\quad du = 2\,dx$
$du/2 = dx$

$1/2\int (u-1)/2\,(u^{1/2})\,du = 1/4\int (u^{3/2} - u^{1/2})\,du$
$= 1/4\,(2/5\,u^{5/2} - 2/3\,u^{3/2}) + C$

⑦ $\int \sin^3 x\,dx$
$-\cos^4 x/4$ $\quad = \int \sin^2 x \sin x\,dx$
$\int \frac{(1-\cos^2 x)\sin x\,dx}{\cos^2 x}$

$u = \cos x$
$du = -\sin x\,dx$ $\quad = -\int (1-u^2)\,du$
$-du = \sin$ $\quad = -(u - u^3/3) + C$

10/15

5.6 Integration by parts
(Backwards product rule)

- Backward chain rule
- Backward product rule

1. $\int x\cos(3x)\,dx$

$\int u\,dv$

$u = x$

$dv = \cos(3x)\,dx$

Integration by parts
Backwards product rule

$\int u\,dv = uv - \int v\,du$

$u = x$	$v = \frac{1}{3}\sin(3x)$
$du = 1dx$	$dv = \cos(3x)dx$

$= \frac{1}{3}x\sin(3x) - \frac{1}{3}\int \sin(3x)\,dx$

$= \frac{1}{3}x\sin(3x) - \frac{1}{3}\cdot\frac{1}{3}\cdot -\cos 3x + C$

$= \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos 3x + C$

2. $\int x^2\ln x\,dx$

$\int u\,dv = uv - \int v\,du$

$\int \underbrace{\ln x}_{u}\; x^2\underbrace{dx}_{dv}$

$u = \ln x$	$v = \frac{1}{3}x^3$
$du = \frac{1}{x}dx$	$dv = x^2dx$

L	i	a	t	e
n	n	l	r	e^x
	v	g	i	
	e	e	g	
	r	b		
	s	r		
	e	i		
	t	c		
	r			
	i			
	g			

$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3\frac{1}{x}\,dx$

$= \frac{1}{3}x^3\ln x - \int x^2/3\,dx$

$= x^3/3\ln x - x^3/9 + C$

3. $\int x^2e^{2x}$

$u = x^2$ $\quad v = \frac{1}{2}e^{2x}$

$du = 2x\,dx$ $\quad dv = e^{2x}\,dx$

$\frac{x^2e^{2x}}{2} - \frac{1}{2}\int xe^{2x}\,dx$

$u = x$ $\quad v = 1/2\,e^{2x}$

$du = dx$ $\quad dv = e^{2x}$

$uv - \int v\,du$

$= 1/2\,x^2e^{2x} - 1/2\,xe^{2x} + 1/4\,e^{2x} + C$

4. $\int \ln x\,dx$

$u = \ln x$ $\quad v = x$

$du = 1/x\,dx$ $\quad dv = dx$

$x\ln x - \int 1dx$

$x\ln x - x + C$

5. $\int \underbrace{\text{Arctan}x}_{u}\,\underbrace{dx}_{dv}$

$u = \arctan x$ $\quad v = x$ $\qquad uv - \int v\,du$

$du = \frac{1}{1+x^2}\,dx$ $\quad dv = dx$

$x\arctan x - \int \frac{x}{1+x^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x\arctan x - \frac{1}{2}\int \frac{1}{t}\,du$

$x\arctan x - \frac{1}{2}\ln(1+x^2) + C$

(6) $\int e^x \sin x\,dx$ $\quad u = e^x \quad v = -\cos x \quad uv - \int v\,du$

$\quad u \quad dv \quad du = e^x dx \quad dv = \sin x\,dx$

$-e^x\cos x + \int \cos x\, e^x dx \quad u = e^x \quad v = \sin x$

$du = e^x dx \quad dv = \cos x\,dx$

$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$

$= \frac{-e^x\cos x + e^x \sin x}{2} + C$

5.4 The FTC

6.2 Volume

7.3 Differential equations

Review & M2

Exam return

Review for finale

The FTC

$$\int_a^b f(x)dx = F(b) - F(a)$$

Evaluation Theorem

(1.) $f(x) = \sin(x^2)$

$F(x) = \int_a^x \sin(t^2)\,dt$

$f(x) = a$

$F(x) = x^2/2 + C$

$$\int_a^x t\,dt = \frac{t^2}{2}\Big|_{t=a}^{t=x}$$

$$= \frac{x^2}{2} - \frac{a^2}{2}$$

(2.) $f(x) = \sqrt{x^3+1}$

$F(x) = \int_a^x \sqrt{t^3+1}\,dt$

Fundamental theorem

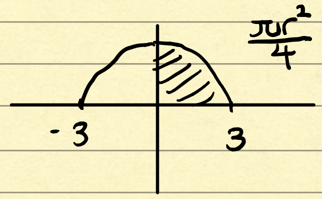
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

(4.) $\frac{d}{dx}\int_1^x \cos(t^3)\,dt = \cos x^3$

10/24

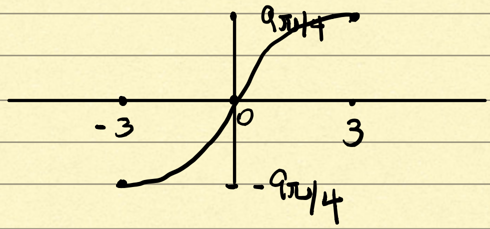
5.4 The fundamental theorem of Calculus.

1. $F(x) = \int_0^x \sqrt{9-t^2}\, dt$



x	F(x)
0	0
3	$9\pi/4$
-3	$-9\pi/4$

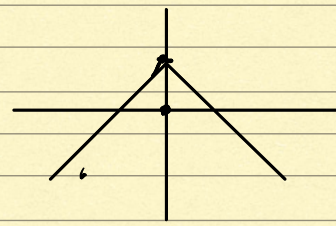
Graph of anti derivative



$F'(x) = \sqrt{9-x^2}$

2. Graph

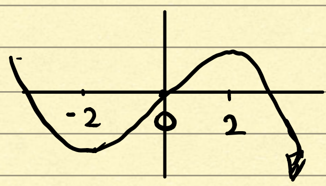
$F(x) = \int_0^x (2-|t|)\,dt$



x	F(x)
0	0
+2	2
-2	-2

$F'(x) = 2-|x|$

Graph of anti



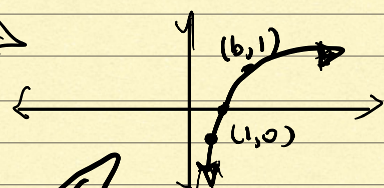
$F(x) = \int_0^x (2-|t|)dt$

2 parabolas

not cubic! is piece-wise

3. $F(x) = \int_1^x 1/t\, dt$

x	F(x)
1	0
2	Small positive
1/2	Large negative



historically

$\int \to \ln \to e$

It looks like logarithmatic.

$F(x) = \int_1^x 1/t\, dt = \log_b x$

. . .

$b = 2.718 = e$ $= \log_e x$

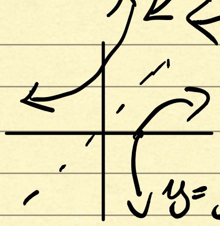
so when $\int_1^x dt = 1$

$= \ln x$

e^x

inverse

inverse



$y = \int_1^2 1/t\,dt$

$x = \int_1^y 1/t\,dt$

$y = \int_1^x \frac{1}{t}\,dt = \ln x$

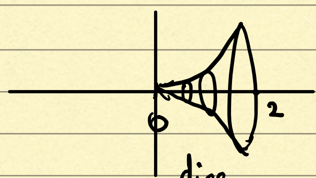
$d/dx(x) = d/dx \int_1^y 1/t\,dt$

$1 = 1/y \; dy/dx$

$y = \frac{dy}{dx}$

6.2 Volume

1. Find the volume of $y=x^2$ when it is revolved around the x-axis from $x=0$ to $x=2$



disc

$\pi (x^2)^2\,dx$

$\rightarrow \int_a^b \pi \left(\substack{\text{radius} \\ f(x)}\right)^2 dx$

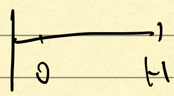
discs

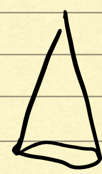
$$\pi \int_0^2 (x^2)^2\,dx = \pi \int_0^2 x^4\,dx$$
$$= \pi\left(\frac{x^5}{5}\right)\Big|_{x=0}^{x=2}$$
$$= \frac{32\pi}{5} - 0 = \boxed{\frac{32\pi}{5}}$$

2. Derive the volume of the cone.

$V = 1/3\,\pi r^2 h$

$y = r/h\,x$





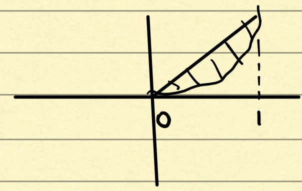
3. $\sqrt{r^2 - x^2}$ Integral volume of hemisphere

$$\pi \int_0^r (\sqrt{r^2 - x^2})^2\,dx = \pi\left(r^2 x - x^3/3\right)\Big|_{x=0}^{x=r}$$

$V = 4/3\,\pi r^2$

More 6.2

① Find the volume when the region enclosed by $y=x^2$ and $y=x$ is revolved the x axis



*Big radius - Short radius

Set up integral and do not compute

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x=0 \quad x=1$

② Find the volume when region enclosed by $y = 1/4x^2$ and $y = 5-x^2$ is revolved around the x axis

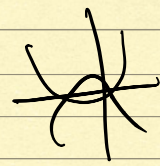
$$\pi \int_{-2}^{2} \left[(5-x^2)^2 - (1/4x^2) \right] dx$$

$5 - x^2 = 1/4\, x^2$

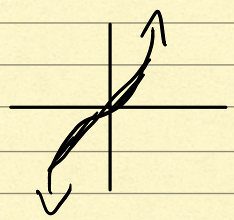
$20 - 4x^2 = x^2$

$20 = 5x^2$

$4 = x^2$



③ Find the volume when the region enclosed by $y=x^3$ and $y=x$ is revolved around $y=5$.



$x^3 = x$

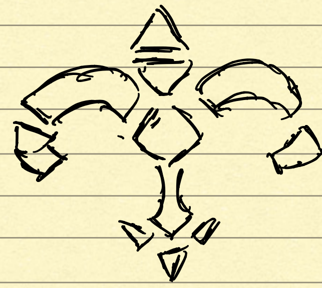
$x^3 - x = 0$

$x = 0, -1, 1$

$$\pi \int_{-1}^{0} (5-x)^2 - (5-x^3)^2 dx + \pi \int_{0}^{1} (5-x^3)^2 - (5-x)^2 dx$$

④ Find the volume when the region enclosed by $y=x^2$ and $y=x$ is revolved around $y=-4$.

$$\pi \int_0^1 (x+4)^2 + (x^2+4)^2 dx$$



⑤ Find the volume when $x-y=1$ and $y=x^2-4x+3$ is revolved around $y=3$.

$y = 1-x$

$y = -1+x$

$y = x-1$

$x^2 - 4x + 3 = x-1$

$x^2 - 5x + 4 = 0$

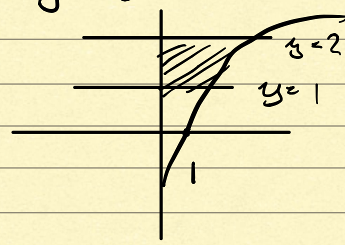
$(x-1)(x-4) = 0$

$x=1, x=4$

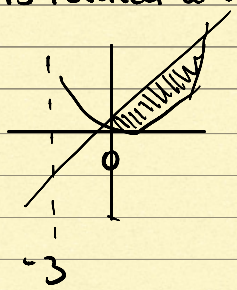
$$\pi \int_1^4 \left[3-(x^2-5x+4)\right]^2 - \left[3-(x-1)\right]^2 \, dx$$

6) Find the volume when the region enclosed by $y=\ln x$, $y=1$, $y=2$, $x=0$ is revolved around the y axis.

$$\pi \int_1^2 (e^y)^2 \, dy$$



7) find the volume when the region enclosed by $y=x^2$ and $y=2x$ is revolved around $x=-3$



$$\pi \int_0^4 (\sqrt{y}+3)^2 - \left(\frac{y}{2}+3\right)^2 \, dy$$

7/15

7.3 Introduction to differential equations

1. $dy/dx = x^2$ "solve" Find a general explicit solution

first order D.E. $\xrightarrow{\text{integ. anti-deri.}}$ $= \frac{x^3}{3} + C$ explicitly a function of x

general

2. $\frac{dy}{dx} = \sin x$; $\underbrace{y(0)=2}_{\text{initial condition}}$

Find particular explicit solution.

$y = -\cos x + C$
$2 = -\cos(0) + C$
$2 = -1 + C$
$C = 3$

$\boxed{y = -\cos x + 3}$

3. $dy/dt = -9.8$; $y(0) = V_0$

Find an explicit particular solution

$y = -9.8t + C$
$V_0 = -9.8(0) + C$
$C = V_0$

$\boxed{y = -9.8t + V_0}$

4. $dy/dt = -9.8t + V_0$; $y(0) = S_I$

Find an explicit particular solution

$y = \frac{-9.8t^2}{2} + V_0 t + C$

$\boxed{y = -4.9t^2 + V_0 t + S_I}$

5. $\frac{dy}{dx} = y$ Find an explicity

Separation of variable

$$\frac{\cancel{y}\,dx}{\cancel{y}} = \frac{dy}{y}$$

$$\int dx = \int \frac{1}{y}\,dy$$

$$x + C = \ln|y|$$
$$e^{x+C} = |y|$$
$$y = \pm e^{x+C}$$

$y = -9.8t + V_0$

$y = \pm e^{C} e^{x}$

$y = Ae^{x}$

6. Predict population of a city

$y(0) = P_0$
Initial

· The more people there are, the faster the people grow

The rate of change equals to the population

$$dy/dt = ky$$

· $\frac{k\cancel{y}\,dt}{\cancel{y}} = dy/y$

· $k\,dt = dy/y$

· $kt + C = \ln y$ Population positive

· $kt + C = \ln y$
$k(0) + C = \ln P$
$C = \ln P$

· $e^{kt + \ln P} = e^{\ln y}$
$Pe^{kt} = y$

$$\boxed{y = Pe^{kt}}$$

1118

7.3 More D.E

1. $dy/dx = -x/y$
Find an implicit general solution.
do not solve for y

$$\int y\,dy = \int -x\,dx$$

$$y^2/2 = -x^2/2 + C$$

$$\frac{x^2}{2} + y^2/2 = C$$

$\frac{x^2}{2} + \frac{y^2}{2} = C$

$x^2 + y^2 = 2C$

(2.) Suppose the rate of change of the population of town is directly proportional to the population find an explicit particular formula for the population if the initial population is 10,000.

y is population

$dy/dt = k\sqrt{y}$

$dy/dt = ky^{1/2},$
$y(0) = 10{,}000;$

$ky^{1/2}dt = dy$

$\int k\,dt = \int y^{-1/2}dy$

$kt + C = 2y^{1/2}$

$k(0) + C = 2y^{1/2}$

$k(0) + C = 2(10000)^{1/2}$

$C = 200$

$kt + 200 = 2\sqrt{y}$

$\frac{kt + 200}{2} = \sqrt{y}$

$y = \left(\frac{kt + 200}{2}\right)^2$

(3.) Kill Bill

Find time of death

temp of the body : 58°f
temp of the surrounding area: 43°f
temp at death : 99°f
temp again later

Newton's law of Cooling

$dy/dt = -k(y - R),\ y(0) = P$

y = body temp, R = surrounding temp

Find an explicit particular solution.

$-k\,dt = \int (1/y-R)\,dy$

$-kt + C = \ln(y - R)$

$C = \ln(P - R)$

$-kt + \ln(P - R) = \ln(y - R)$

$y - R = e^{-kt + \ln(P - R)}$

$y = R + e^{\ln(P-R)} e^{-kt}$

$y = R + (P-R)e^{-kt}$

11/12

1.) $\int x\cos(x^2)dx = \frac{1}{2}\int \cos u\, du$

$u = x^2$ $\quad = 1/2 \sin u + C$

$\frac{du}{2} = x\,dx$ $\quad = 1/2 \sin x^2 + C$

2.) Find $y(1)$

b.) $\boxed{y(0) = 2}$ $\rightarrow$ $\frac{dy}{dt} = ky^2$

$\frac{ky^2 dt}{y^2} = \frac{dy}{y^2}$

$\int k\,dt = \int y^{-2} dy$

$kt + C = -y^{-1}$

$kt + C = -1/y$

$k(0) + C = -1/2$

$C = -1/2$

$kt - 1/2 = -1/y$

$y = 1/kt + 1/2$

$y(1) = -1/-k(1) + 1/2$

$= 2/-kt + 1$

c.) $F(2)$ $|t|$

$F(x) = \int_0^x |t+1|\,dt$

$f(2) = \int_0^2 |t+1|dt$

$A = 1/2\, h(b_1 + b_2)$
trapazoid

$A = 1/2(2)(1+3)$
$= 4$

d.) $\int \underset{u}{x}\underbrace{e^x dx}_{dv}$

$u = x$	$v = e^x$
$du = dx$	$dv = e^x$

$uv - \int v\,du$

$xe^x - \int e^x dx$

$= \boxed{xe^x - e^x + C}$

e.) $\int \underset{u}{\text{Arctan}(1-x)} \underset{dv}{dx}$ $\quad$ $u = \arctan(1-x)$ | $v = x$
$du = 1/1+(1-x)^2$ | $dv = dx$

$uv - \int v\,du$

$x\arctan(1-x) + \int \frac{x}{1+(1-x)^2} dx$

$\int \frac{x}{1+(1-x)^2} dx$ $\quad \begin{cases} u = 1-x \\ du = -dx \end{cases}$ $\quad x = 1-u$

$-\int \frac{1-u}{1+u^2} du = -\int \frac{1}{1+u^2} + \int \frac{u}{1+u^2} dx$

$\int \frac{u}{1+u^2} du = 1/2 \int \frac{1}{w} = dw$

$w = 1+u^2$

$\frac{dw}{2} = \frac{2u\,du}{2}$ $\quad$ $1/2 \ln|w| = 1/2 \ln|1+u^2| = 1/2 \ln|1+(1-x)^2|$

$$\boxed{x\arctan x(1-x) - \arctan(1-x) + 1/2 \ln|1+(1-x^2) + C|}$$

f.) $y = 3x$ $\quad$ y=axis $\quad$ $3x = x^2$ $\quad$ $x = y/3$
$y = x^2$ $\quad$ $x^2 - 3x = 0$ $\quad$ $x = \sqrt{y}$
$x(x-3) = 0$
$x = 0, x = 3$

$= \pi \int_0^9 \left[(\sqrt{y})^2 - (y/3)^2\right] dy$

$= \pi \int_0^9 (y - y^2/9)\,dy$

$= \pi\left(y^2/2 - y^3/27\right)\Big|_{y=0}^{y=9}$

$= \frac{27\pi}{2}$

g.) $\int x^2 \sqrt{8x+2}\,dx$

$\int x(8x+2)^{1/3} dx$

$u = 8x+2$

$\frac{du}{8} = \frac{8dx}{8}$

$x = \frac{u-2}{8}$ $\quad$ $1/8 \int \frac{u-2}{8} (u)^{1/3} du$

$= 1/64 \int (4-2)u^{1/3} + du$

$= 1/64 \left(u^{4/3} - 2u^{1/3}\right) du$

$= 1/64 \left(\frac{3}{7} u^{7/3} - 2\left(\frac{3}{4}\right) u^{4/3}\right) + C$

$$= \boxed{1/64 \left(\frac{3}{7}(8x+2)^{7/3} - \frac{3}{2}(8x+2)^{4/3}\right) + C}$$

h.) $G'(6)$

$G(x) = \int_2^x [\sin(t^2) - \cos(t^3)\, dt$

$G'(x) = \sin(x^2) - \cos(x^3)$

$G'(6) = \sin(36) - \cos(216)$

I) $\frac{dy}{dx} = \frac{y}{x^2}$

$x^2 dy = y\, dx$

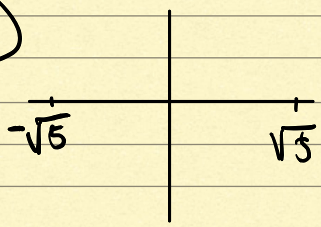
$\int \frac{1}{y} dy = \int x^{-2} dx$

$\ln|y| = -x^{-1} + C$

$|y| = e^{-x^{-1} + C}$

$y = \pm e^{-x^{-1}} + C$

J.



$2\pi \int_0^{\sqrt{5}} \left(\sqrt{5-x^2}\right)^2 dx$

$2\pi \int_0^{\sqrt{5}} (5 - x^2) dx$

$2\pi \left(5x - x^3/3\right) \Big|_{x=0}^{x=\sqrt{5}}$

$= 2\pi\left(5\sqrt{5} - 5\sqrt{5}/3\right.$

11/26

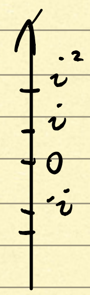
$x^2 = -1$

$x = \sqrt{-1}$

$x = i$

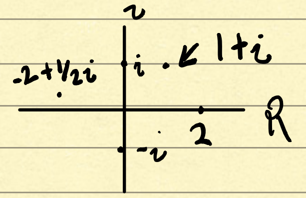
$\rightarrow$ i

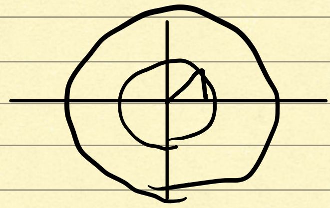
$i^2 = -1$



Pure imaginary num.

Complex plane



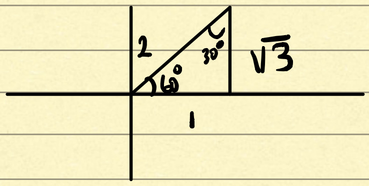


$\boxed{r(\cos\theta + i\sin\theta)}$

$\sqrt{2}\left(\cos \pi/4 + \sin \pi/4\right)$

$\sqrt{2}$ C is $\pi/4$

1. Convert $1+i\sqrt{3}$ to polar form



r cis θ

$\boxed{2 \text{ cis } \pi/3}$

r(modulus)

θ(argument)

$\boxed{|1-\sqrt{3}|} = 2$

asking for modulus

$\arg(1-i\sqrt{3}) = \pi/3$

$e^x = 1 + x + x^2/2! + x^3/3! + x^4/4! \ldots$

$\cos x = 1 - x^2/2! + x^4/4! \ldots$

$\sin x = x - x^3/3! + x^5/5! \ldots$

• $re^{i\theta} = r\left(1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} \ldots\right)$

$re^{i\theta} = r\left(\boxed{1} + i\theta \boxed{-\theta^2/2!} - i\theta^3/3! + \boxed{\theta^4/4!} \ldots\right)$

sin θ

cos θ

$re^{i\theta} = r(\cos\theta + i\sin\theta)$

$re^{i\theta} = r \text{ cis } \theta$

• $re^{i\theta}$ =

Exponential

$1+i\sqrt{3}$	Complex II
$2 \text{ cis } \pi/3$	polar
$2e^{i\frac{\pi}{3}}$	Exponential

• $(1+i)(1+i\sqrt{3}) = 1 + i\sqrt{3} + i + i^2\sqrt{3}$

$= (1-\sqrt{3}) + i(\sqrt{3}+1)$

$$\left(\sqrt{2}e^{i\pi/4}\right)\left(2e^{i\pi/3}\right) = 2\sqrt{2}e^{i\frac{7\pi}{12}}$$

$2\sqrt{2}e^{i7\pi/12} = 2\sqrt{2} \text{ cis } 7\pi/12$

$= 2\sqrt{2}\left(\cos 7\pi/12 + i\sin 7\pi/12\right)$

$= 2\sqrt{2}(-0.258 + 0.965i)$

$= -0.7297 + 2.7294i$

- $(3e^{i\pi/6})^4 = 243e^{i2\pi/3}$
- What is $(1+i)^8$

Exponential

$(\sqrt{2}e^{i\pi/4})^8 = 16e^{2\pi}$

$= 16 \text{ cis } 2\pi$

$= 16(\cos 2\pi + i\sin 2\pi)$

$= 16(1+0)$

$= 16$

- $f(z) = (1+i)z$

$x+yi$

$z = x+yi$

$2+3i$ input

Complex numbers continued

① $f(z) = (1+i)z$

$x+yi$

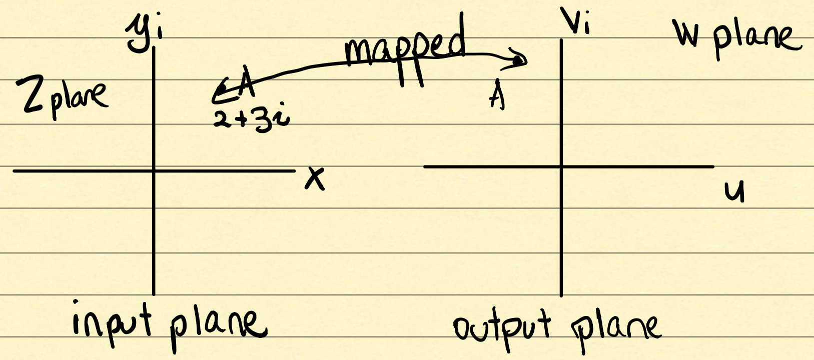
$f(2+3i) = (1+i)(2+3i)$

input

$= 2+5i+3i^2$

$= -1+5i$

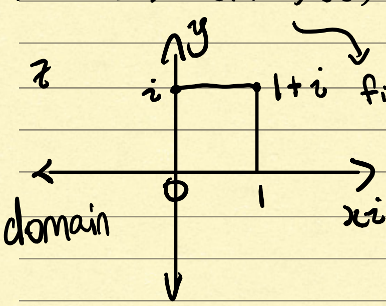
output

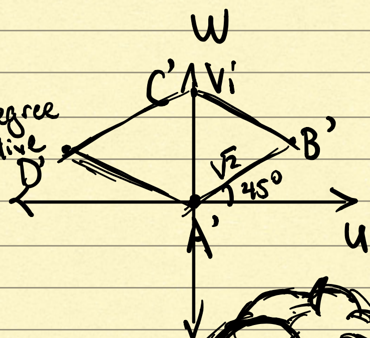


How do we depict this function?

$W = f(z) = (1+i)(z)$

first degree derivative



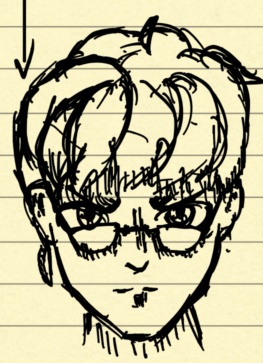


z	W
0	0
i	$-1+i$
1	$1+i$
$1+i$	$2i$

$f'(z) = 1+i$

$|1+i| = \sqrt{2}$

$\arg(1+i) = \pi/4$

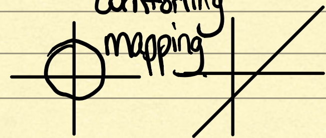


② Graph $f(z) = -2z+1$

adding real

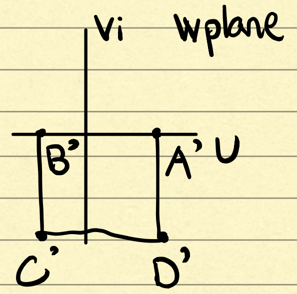
$1/z$ z^2

Conforting mapping

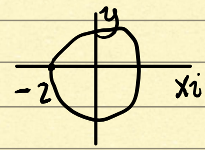


a. Graph $w = f(z)$ the image of ABCD in the w plane
b. Describe the transformations of the square
c. How do these transformations relate to $f'(z)$?

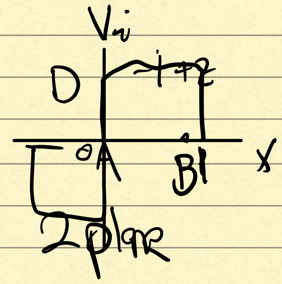
z	W
0	1
1	-1
$1+i$	$-1-2i$
i	$1-2i$



$r = 2$
$\theta =$
$-2 = -2 + 0i$



$f'(z) = -2$



$z = 2\sqrt{3} - 2i$; $w = -1 + i$

$\sqrt{(2\sqrt{3})^2 + (-2)^2} = 4$ $\quad -2/(2\sqrt{3}) = -\sqrt{3}/3 = \tan\theta \quad 4 \text{ cis} -\pi/6$

$\sqrt{(-1)^2 + (1)^2} = \sqrt{2}$ $\quad 1/(-1) = -1 \quad \sqrt{2} \text{ cis} -1/4\pi$

$(4)(-1) \text{ cis} -\pi/6 - 1/4\pi$

zw: $-4 \text{ cis} -5/12\pi$
z/w: $-4 \text{ cis } 1/12\pi$
$1/z$: $1/4 \text{ cis } \pi/6$

$(1 - \sqrt{3}i)^5$

$\sqrt{(1)^2 + (-\sqrt{3})^2} = 2 \quad r = 2$

$(-\sqrt{3}/1)^{-1} = \tan = \boxed{-1/3\pi}$

$(1-\sqrt{3}i)^5 = (2 \text{ cis} -1/3\pi)^5$

$(2^5) \text{ cis} -5/3\pi$

$2^5 \left[1/2 + i\frac{\sqrt{3}}{2}\right]$

$2^5 \cdot \left[1/2 + i\sqrt{3}/2\right]$

$(1+i)^{20}$

$\sqrt{(1)^2 + (1)^2} = \sqrt{2} \quad r = \sqrt{2}$

$(1/1 \quad) = \boxed{1/4\pi}$

$= \sqrt{2} \text{ cis } (1/4\pi)^{20}$

$1024 \text{ cis}(5\pi)$

$1024(-1+0)$

-1024

$\cancel{8 + i8\sqrt{3}}$

$16 + 16\sqrt{3}\, i$