Recursion

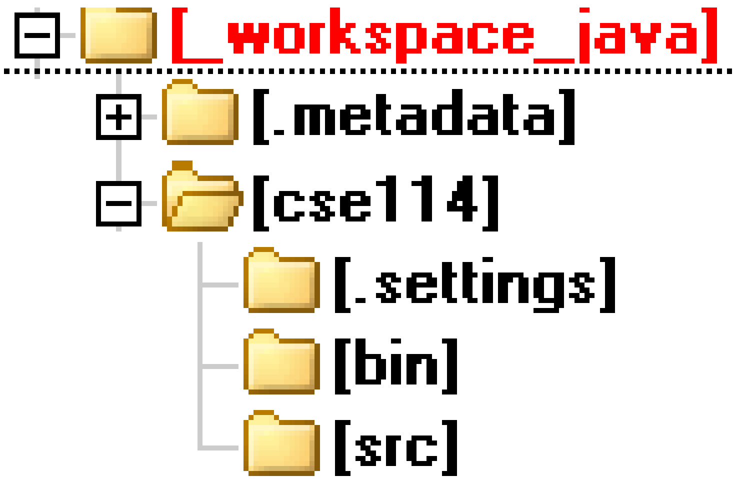
CSE260, Computer Science B: Honors

Stony Brook University

http://www.cs.stonybrook.edu/~cse260

Motivation

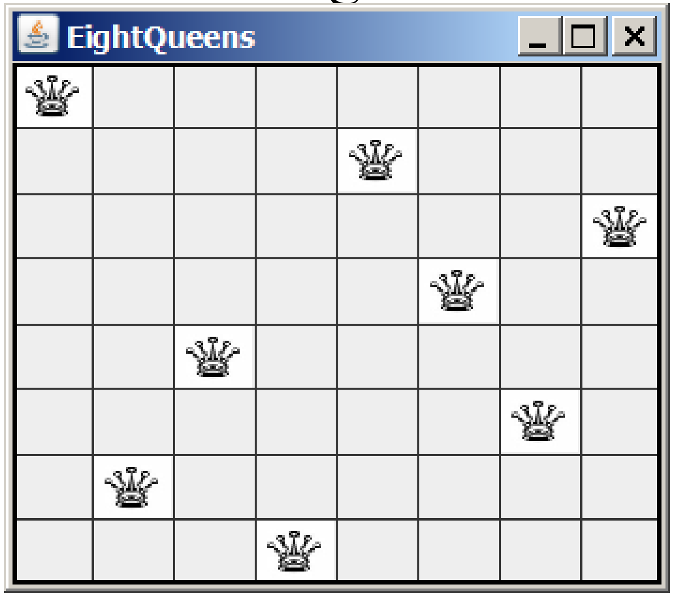
- Suppose you want to find all the files under a directory that contains a particular word.



  - The directory contains subdirectories that also contain subdirectories, and so on.
- The solution is to use recursion by searching the files in the subdirectories recursively.

Motivation

- The Eight Queens puzzle is to place eight queens on a chessboard such that no two queens are on the same row, same column, or same diagonal:



We solve this problem using recursion:

- we place the 8th queen after we placed 7 queens on the chessboard.
- we place the 7th queen after we placed 6 queens on the chessboard.

…

Computing Factorial

$n! = 1 * 2 * 3 * 4 * 5 * \ldots * (n-1) * n$

$(n-1)! \quad = 1 * 2 * 3 * 4 * 5 * \ldots * (n-1)$

So:

$$n! = n * (n-1)!$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1), for n>0

```java
import java.util.Scanner;
public class ComputeFactorial {
  public static void main(String[] args) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter a non-negative integer: ");
    int n = input.nextInt();
    // Display factorial
    System.out.println("Factorial of "+n+" is "+factorial(n));
  }
  /** Return the factorial for a specified number */
  public static int factorial(int n) {
    if (n == 0) // Base case
      return 1;
    else
      return n * factorial(n - 1); // Recursive call
  }
}
```

Computing Factorial

factorial(3) =

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

factorial(3) = 3 * factorial(2)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$\text{factorial}(3) = 3 * \text{factorial}(2)$$
$$= 3 * (2 * \text{factorial}(1))$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$
\begin{aligned}
\text{factorial}(3) &= 3 * \text{factorial}(2) \\
&= 3 * (2 * \text{factorial}(1)) \\
&= 3 * (2 * (1 * \text{factorial}(0)))
\end{aligned}
$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$\text{factorial}(3) = 3 * \text{factorial}(2)$$
$$= 3 * (2 * \text{factorial}(1))$$
$$= 3 * (2 * (1 * \text{factorial}(0)))$$
$$= 3 * (2 * (1 * 1)))$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$
\begin{aligned}
\text{factorial}(3) &= 3 * \text{factorial}(2) \\
&= 3 * (2 * \text{factorial}(1)) \\
&= 3 * (\ 2 * (1 * \text{factorial}(0))) \\
&= 3 * (\ 2 * (\ 1 * 1))) \\
&= 3 * (\ 2 * 1)
\end{aligned}
$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

Computing Factorial

$$
\begin{aligned}
factorial(3) &= 3 * factorial(2) \\
&= 3 * (2 * factorial(1)) \\
&= 3 * (2 * (1 * factorial(0))) \\
&= 3 * (2 * (1 * 1))) \\
&= 3 * (2 * 1) \\
&= 3 * 2
\end{aligned}
$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

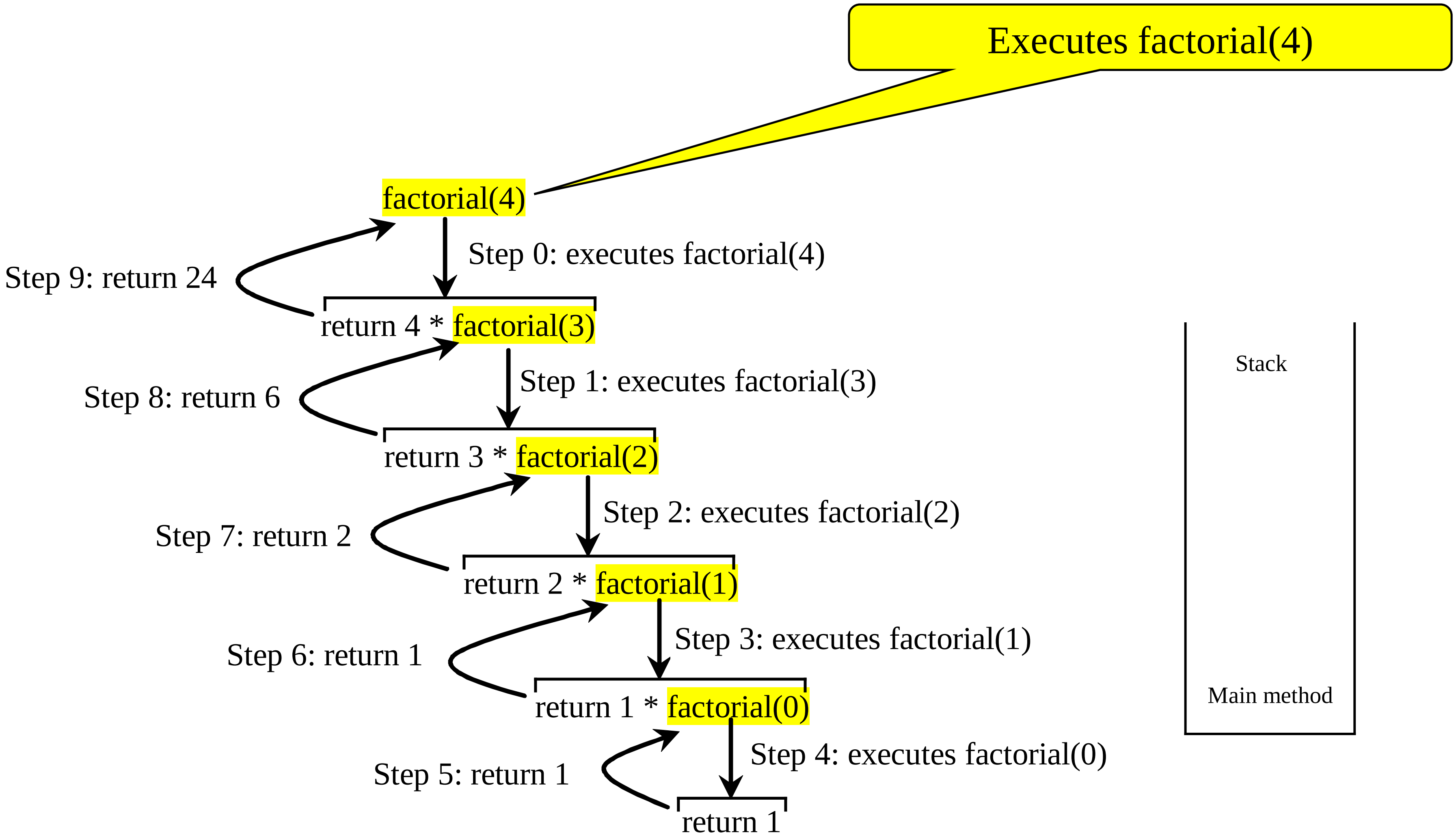
Computing Factorial

$$
\begin{aligned}
\text{factorial}(3) &= 3 * \text{factorial}(2) \\
&= 3 * (2 * \text{factorial}(1)) \\
&= 3 * (2 * (1 * \text{factorial}(0))) \\
&= 3 * (2 * (1 * 1))) \\
&= 3 * (2 * 1) \\
&= 3 * 2 \\
&= 6
\end{aligned}
$$

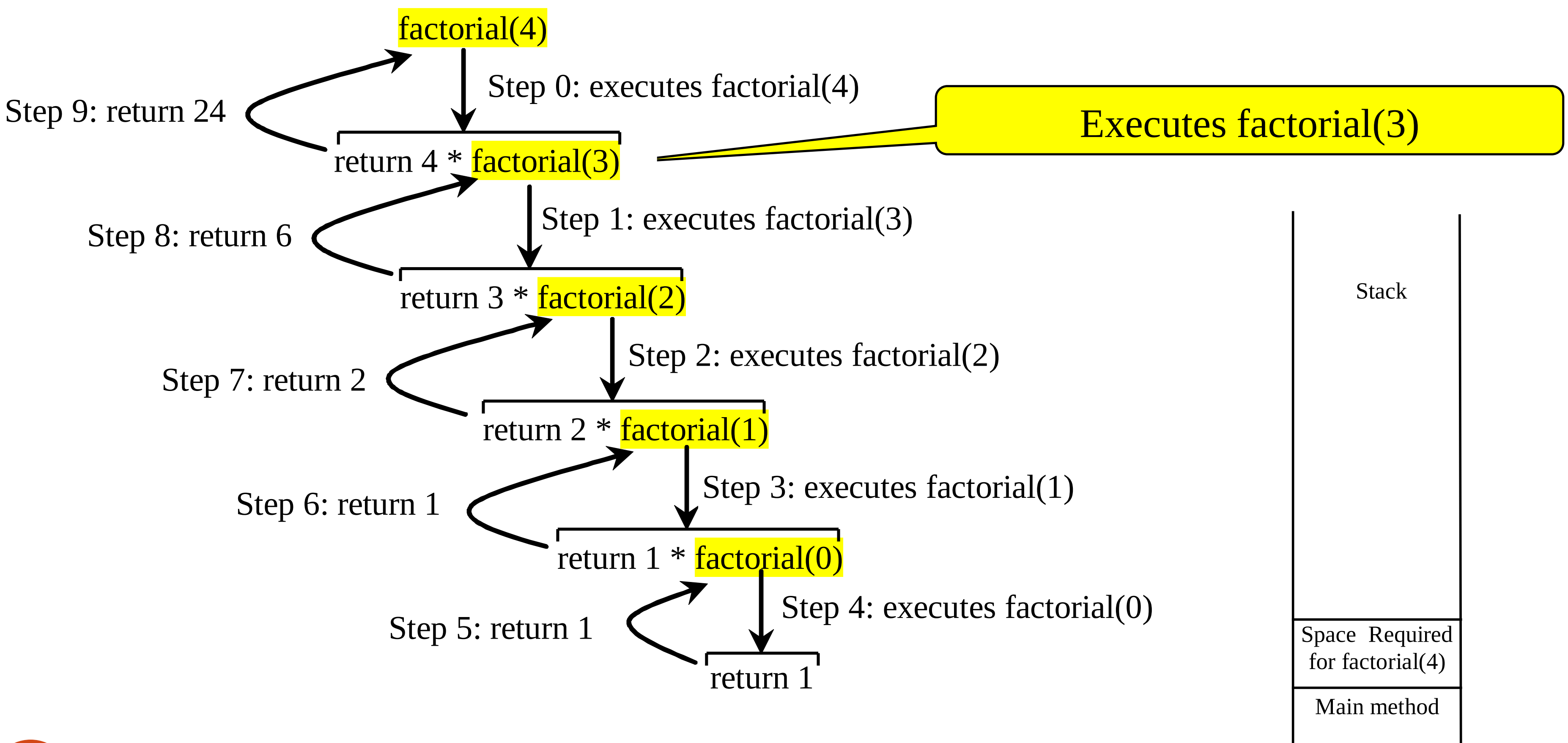
factorial(0) = 1;

factorial(n) = n*factorial(n-1);

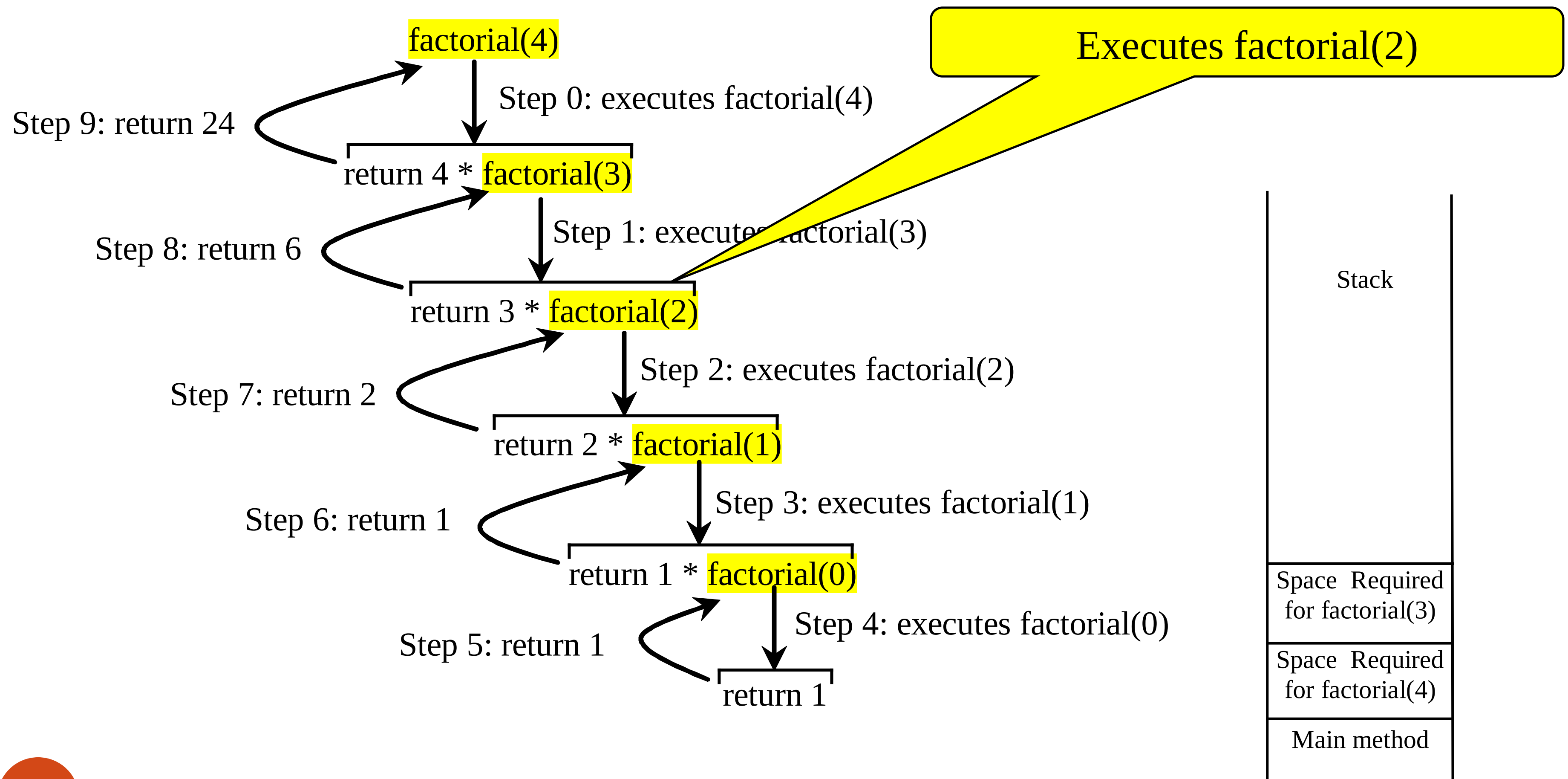
Trace Recursive factorial



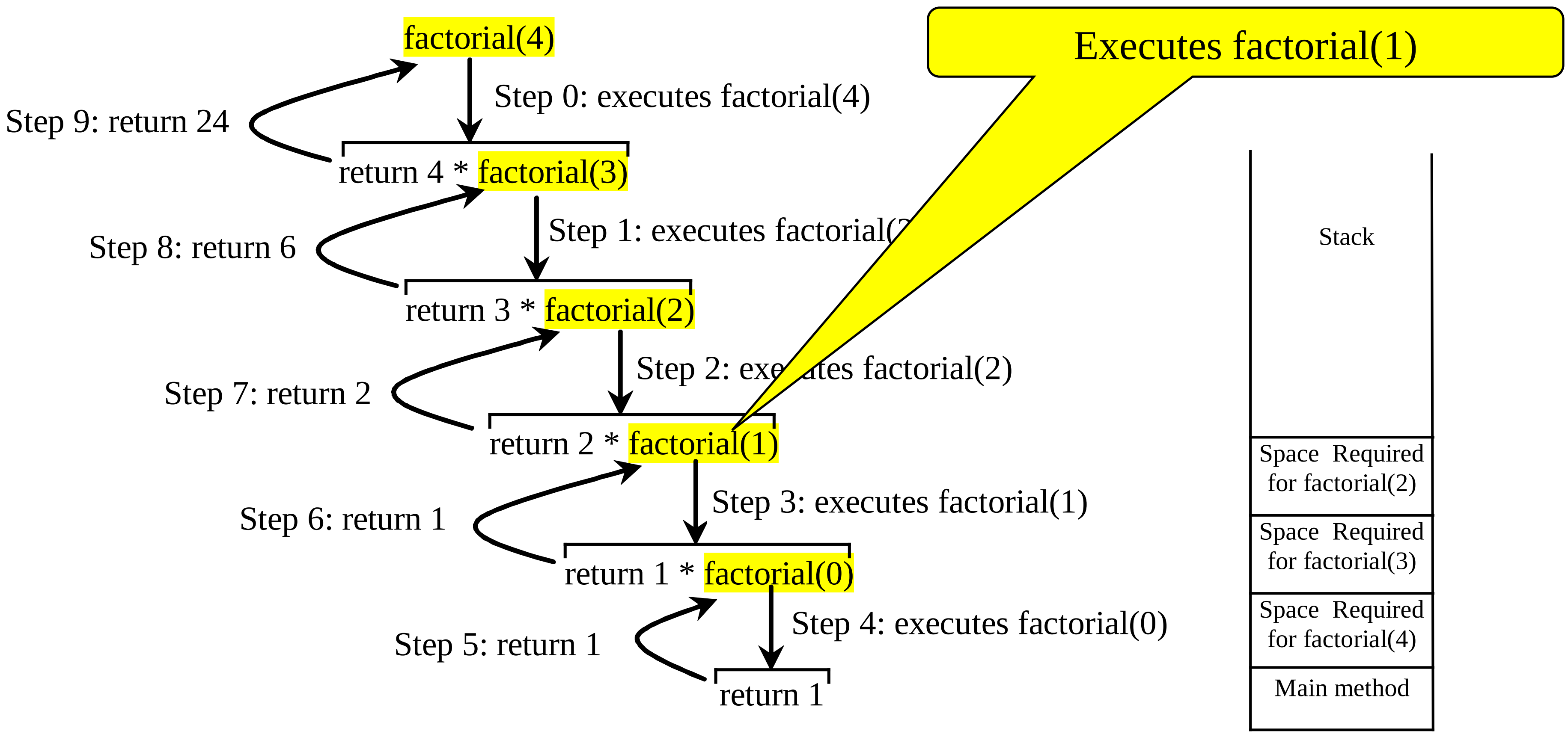
Trace Recursive factorial



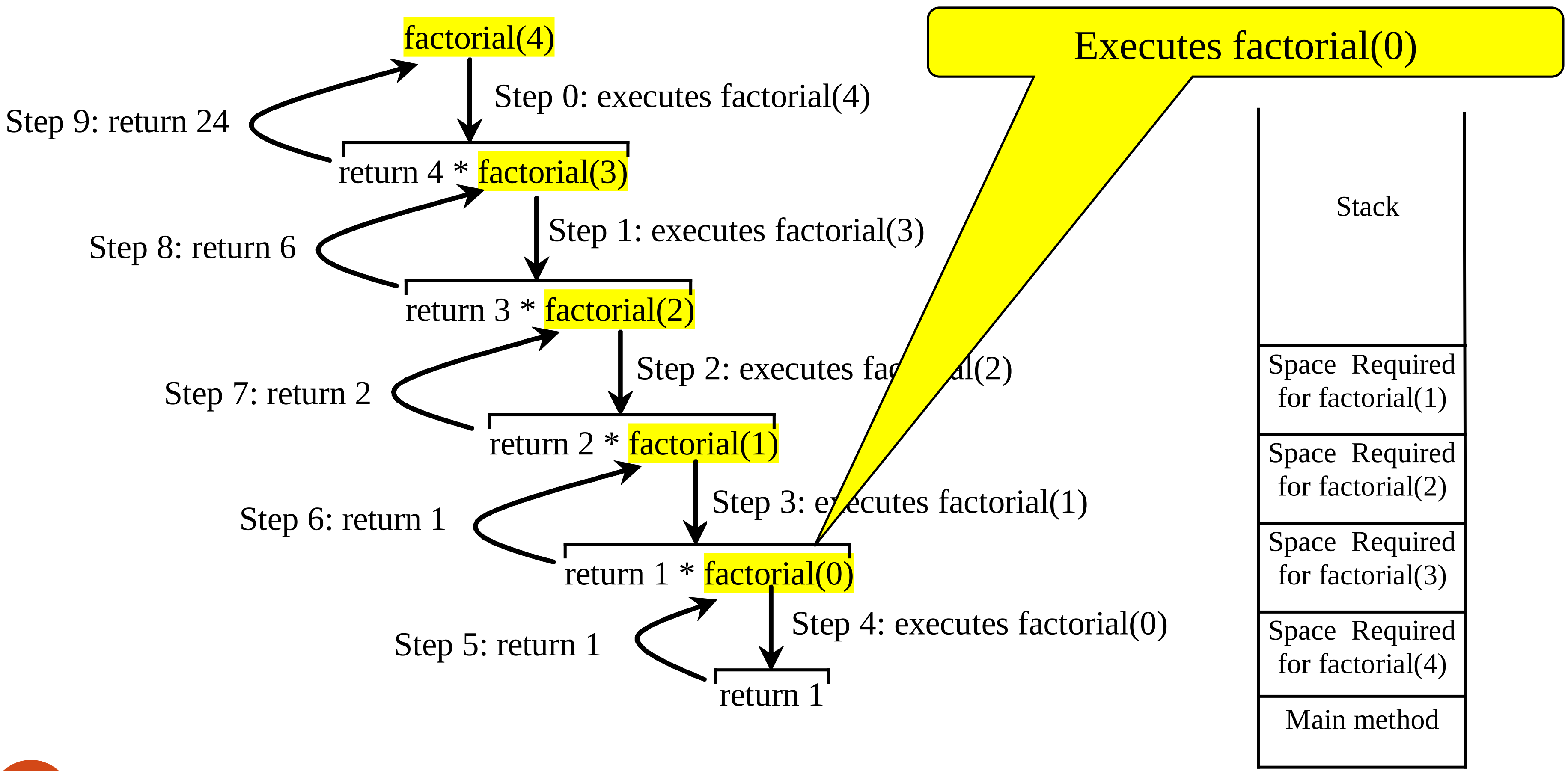
Trace Recursive factorial



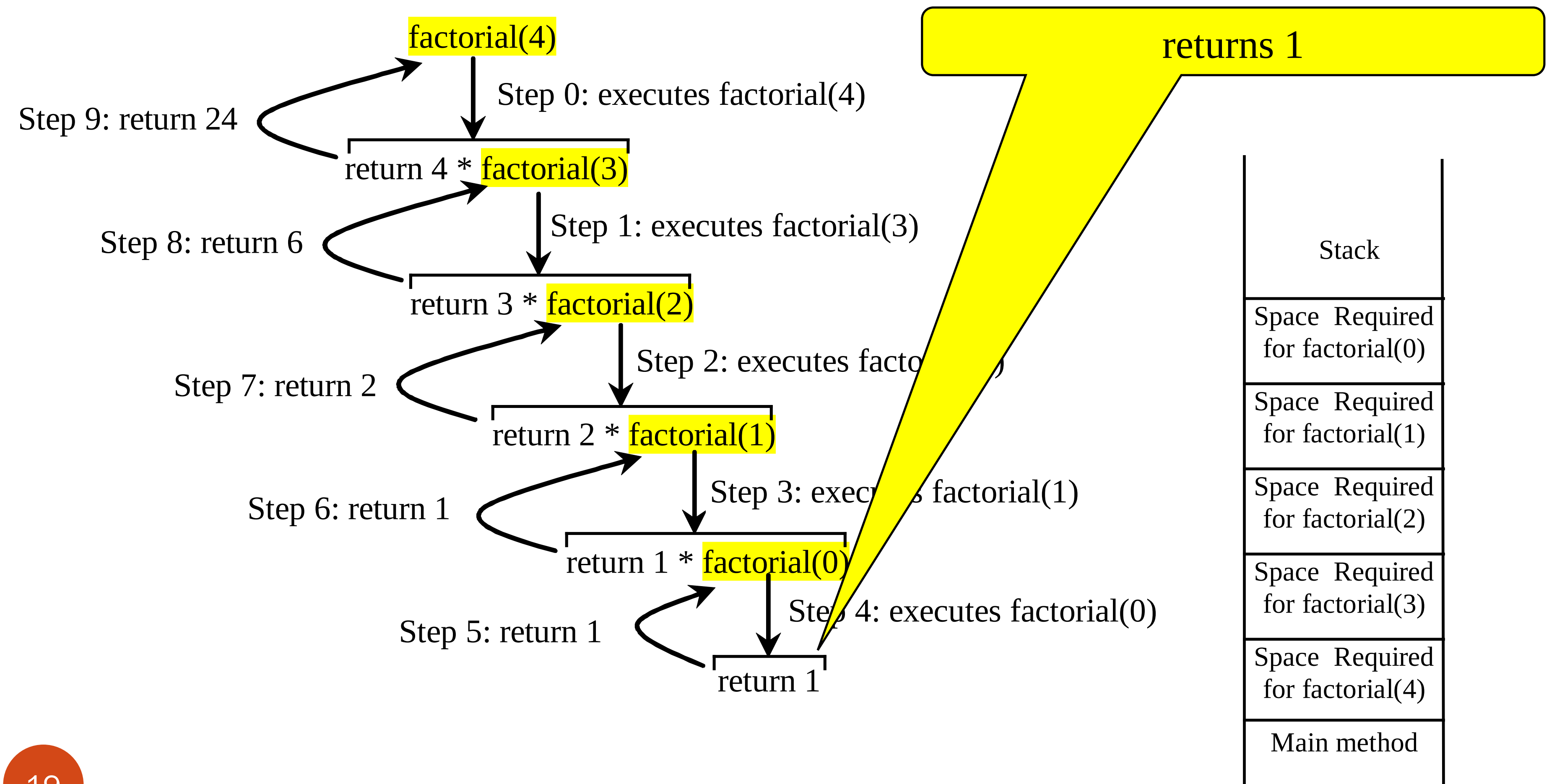
Trace Recursive factorial



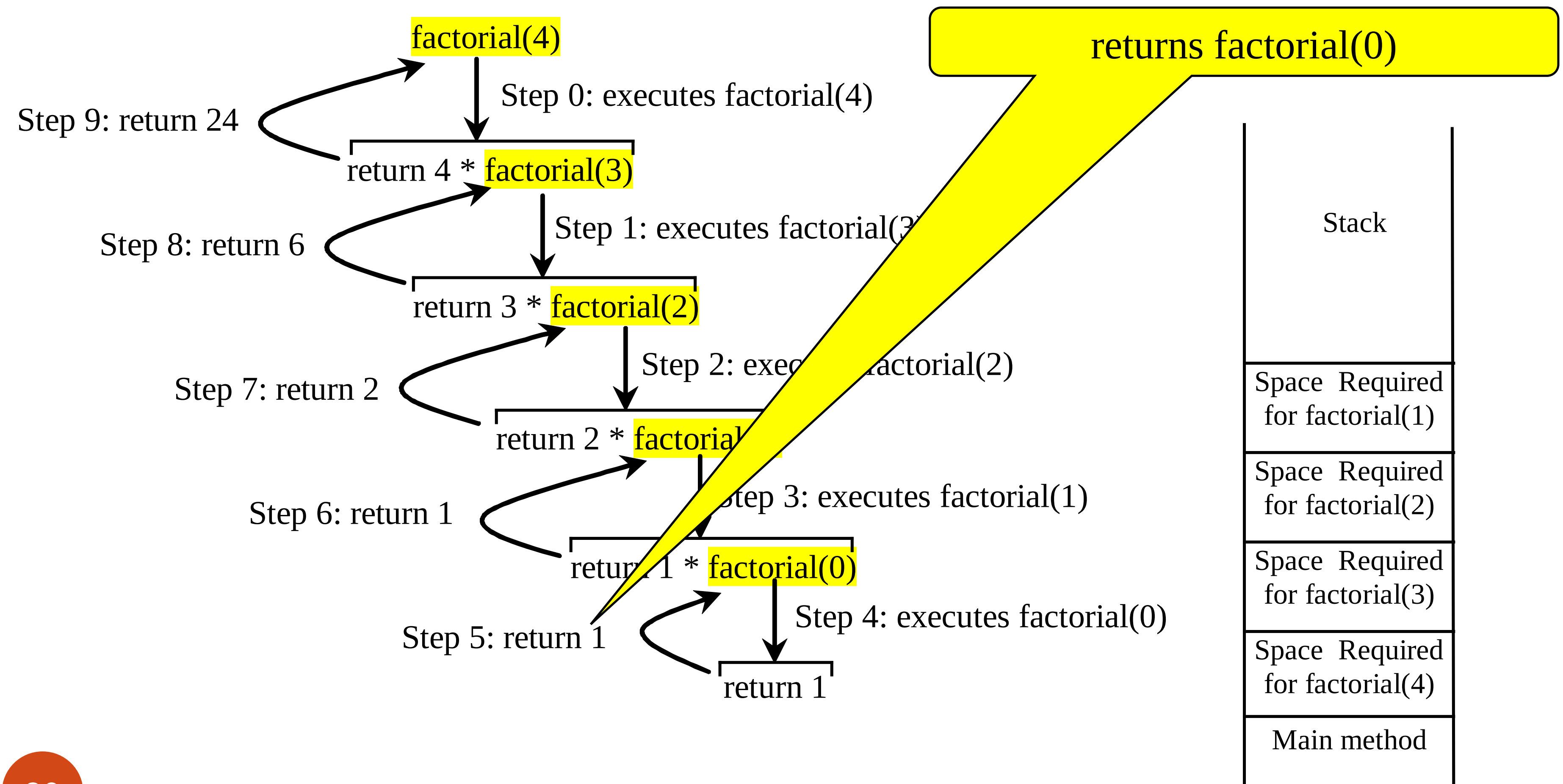
Trace Recursive factorial



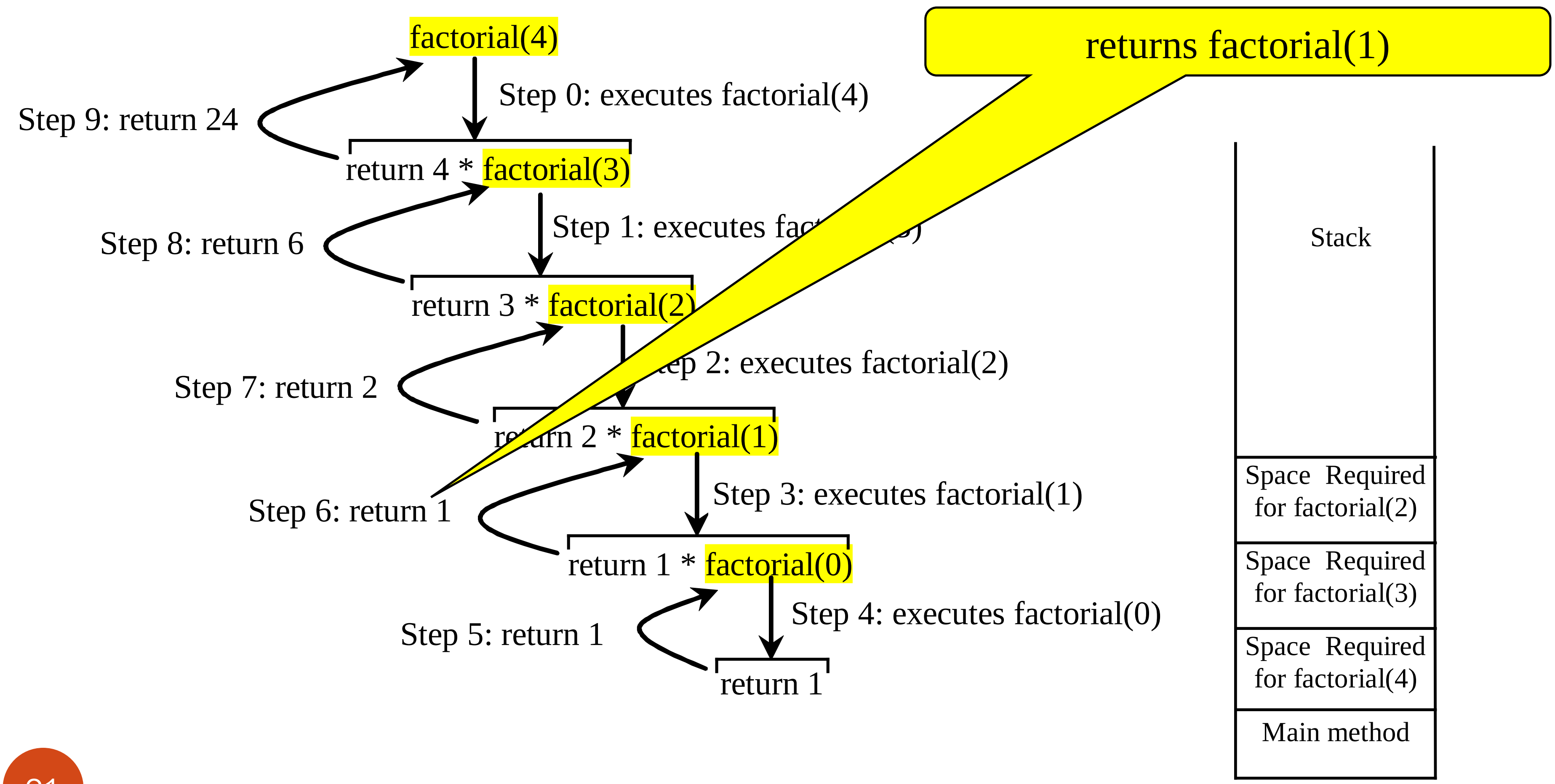
Trace Recursive factorial



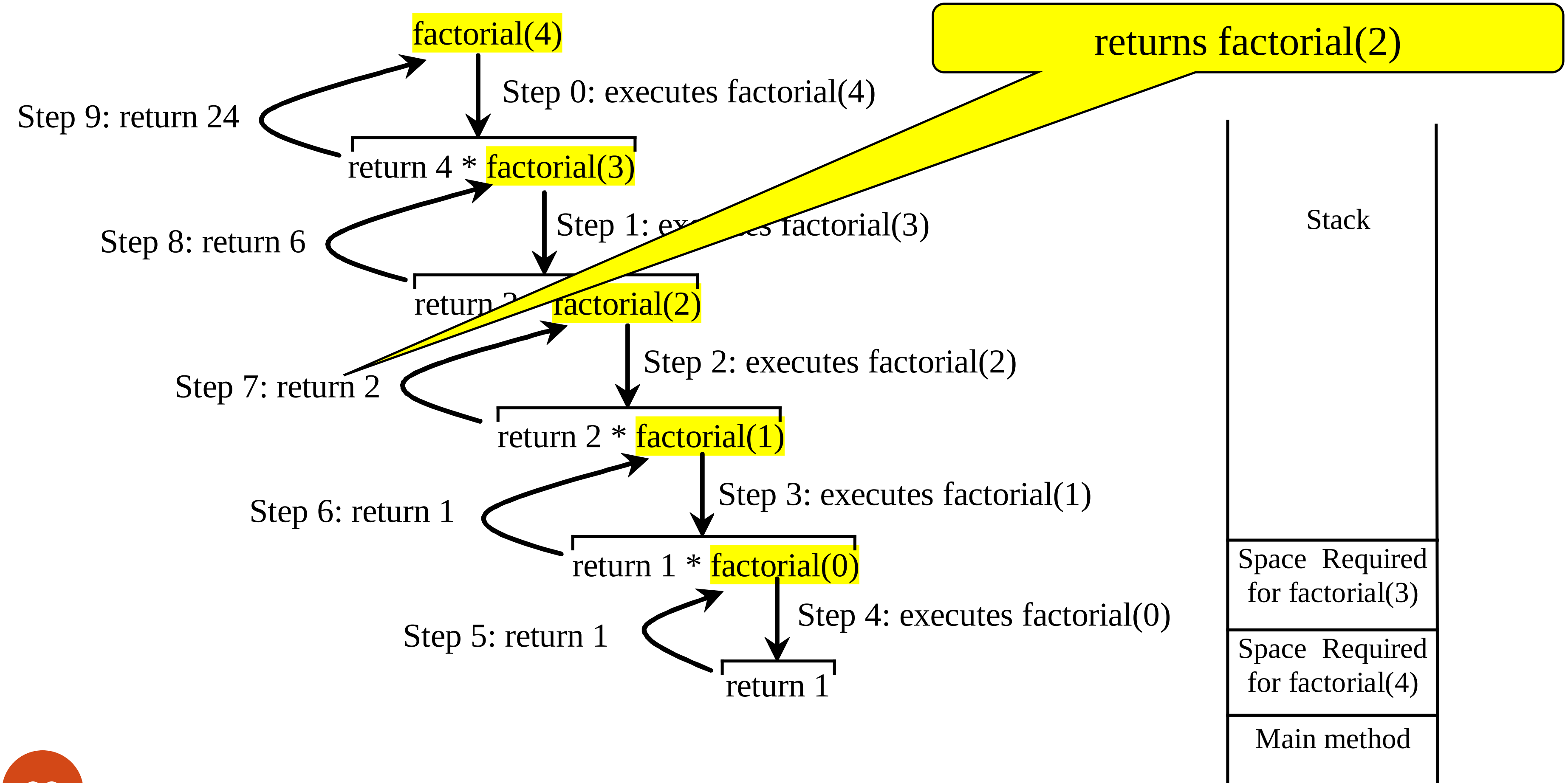
Trace Recursive factorial



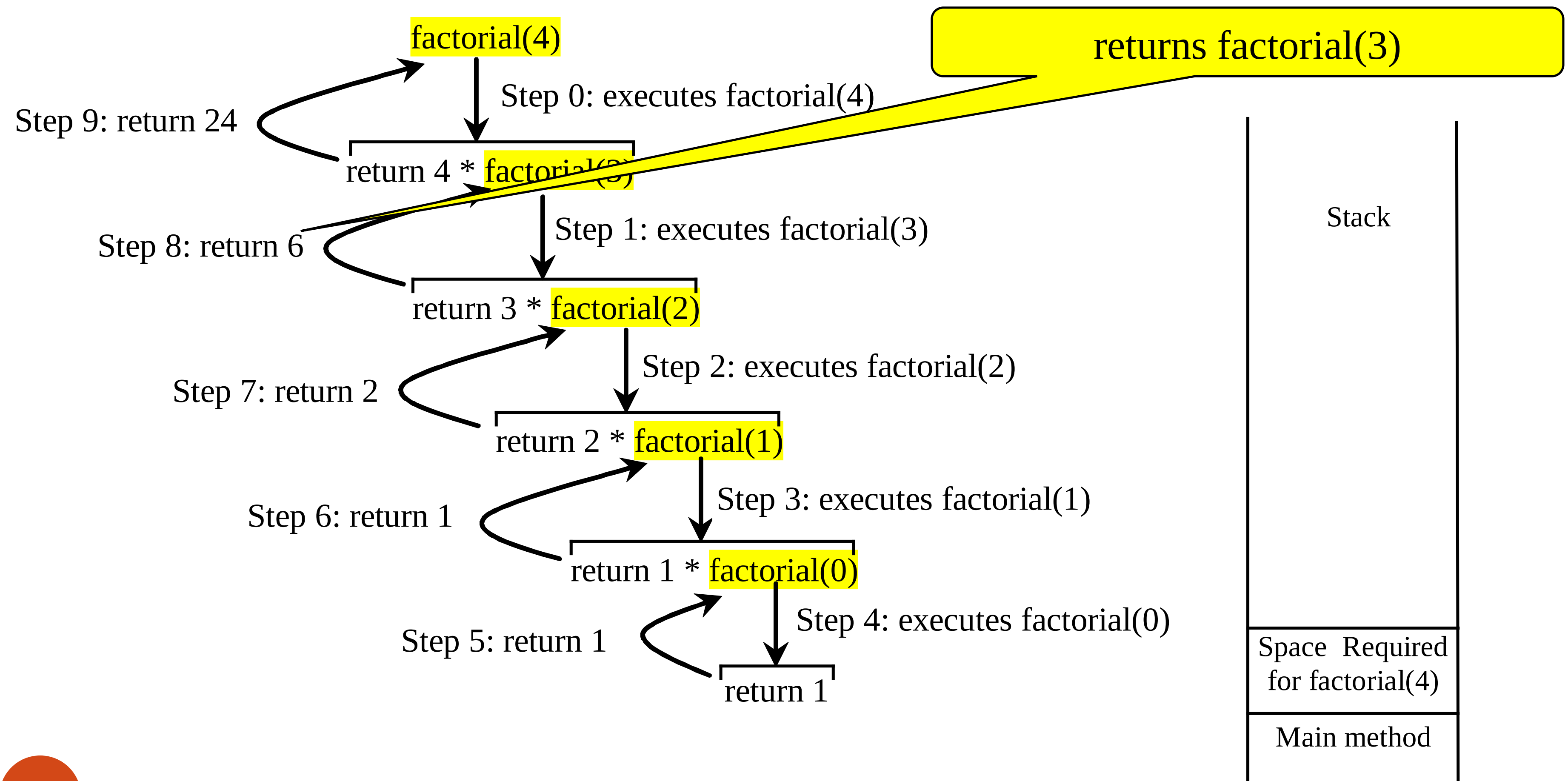
Trace Recursive factorial



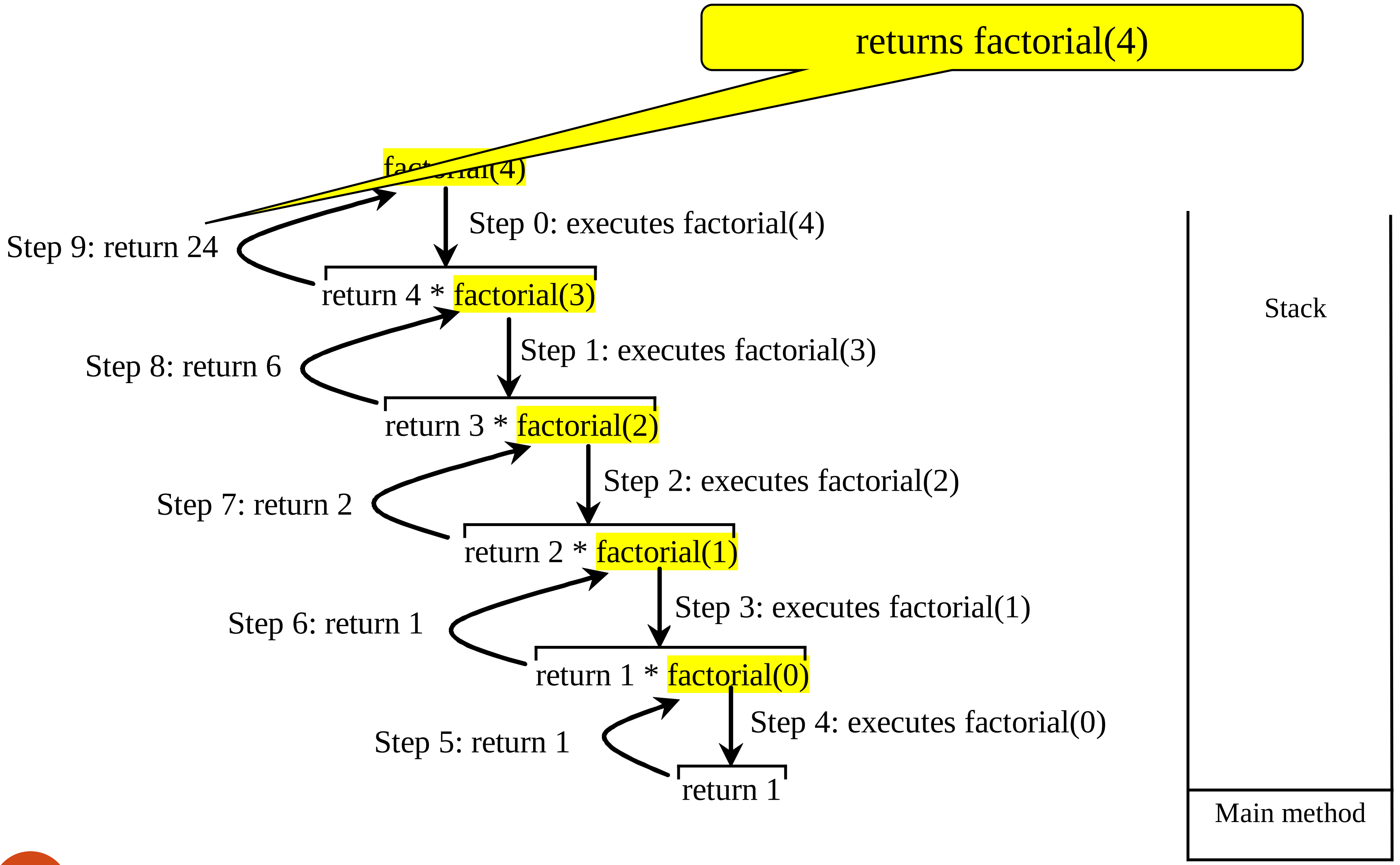
Trace Recursive factorial



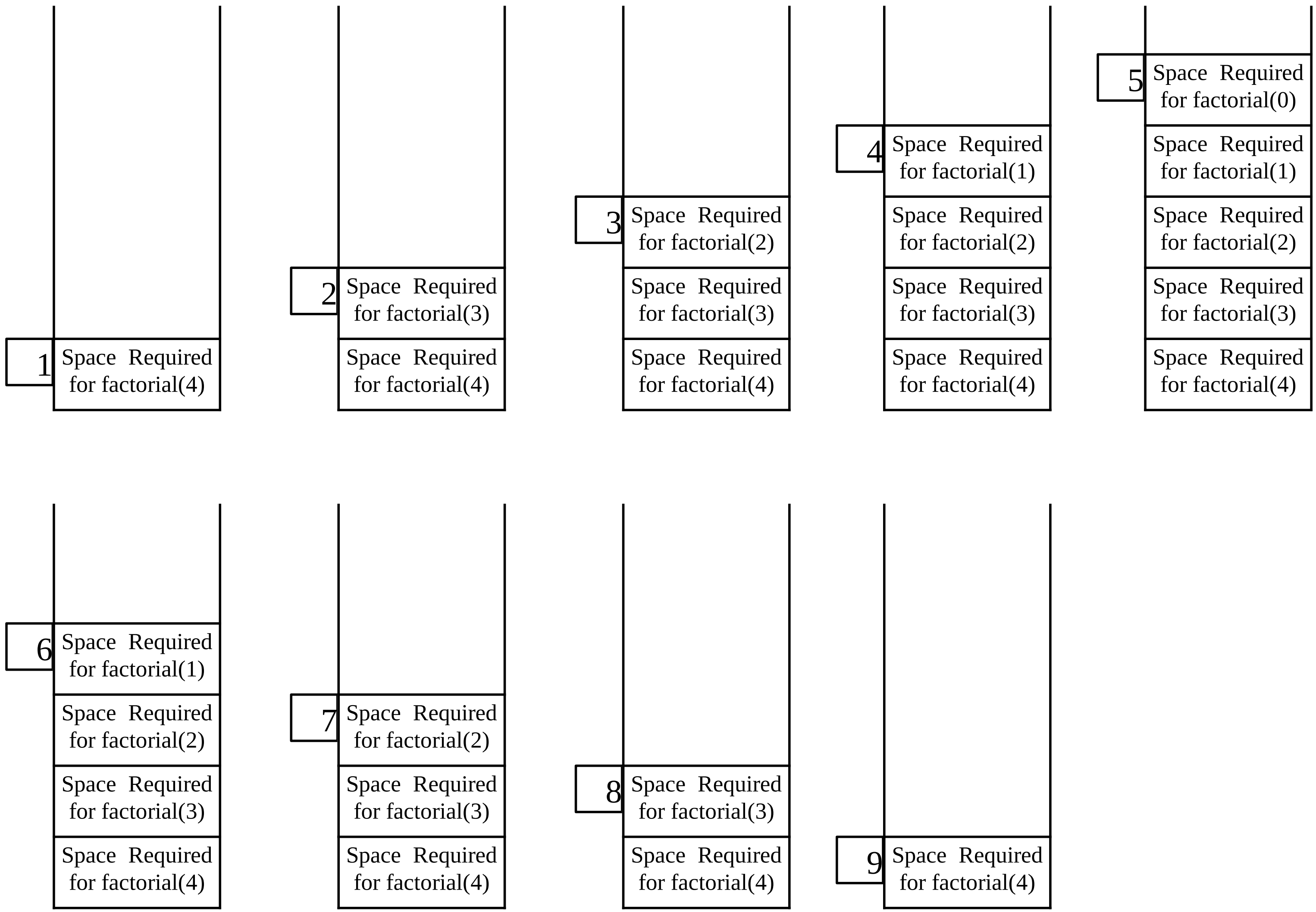
Trace Recursive factorial



Trace Recursive factorial



factorial(4) Stack Trace



Fibonacci Numbers

```
         indices:  0 1 2 3 4 5 6 7  8  9  10 11 ...
Fibonacci series:  0 1 1 2 3 5 8 13 21 34 55 89 ...
```

fib(0) = 0;

fib(1) = 1;

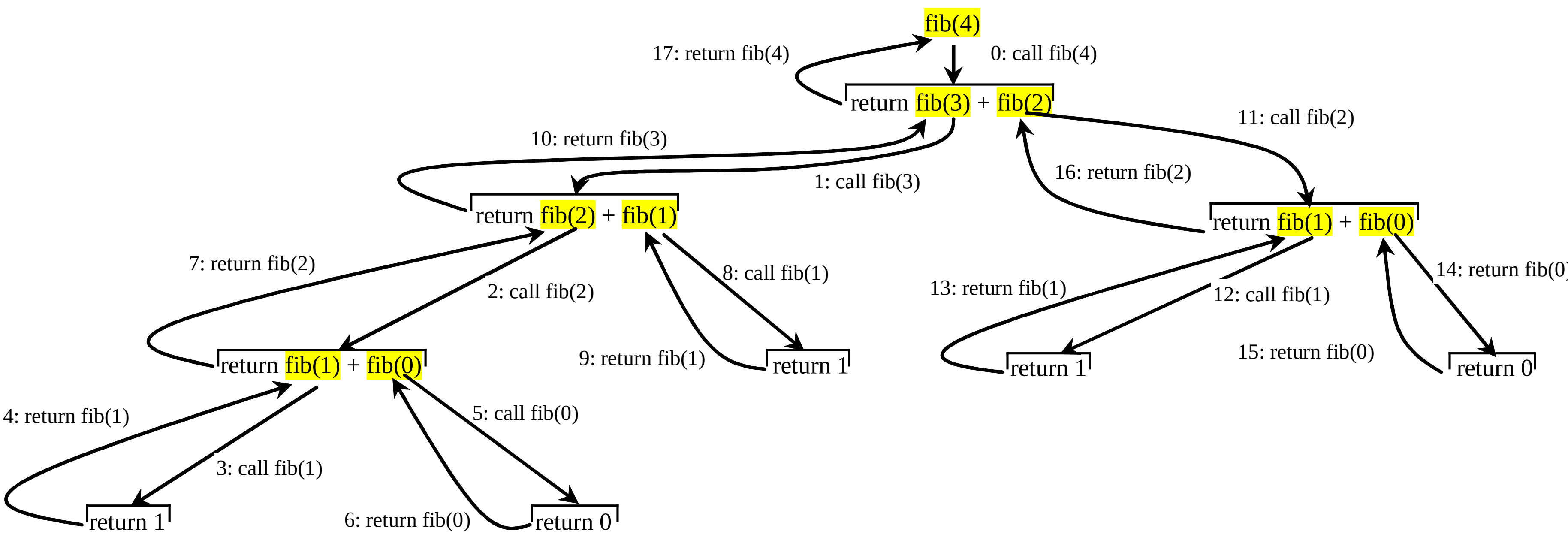
fib(index) = fib(index -1) + fib(index -2);   for integers index >=2

fib(3) = fib(2) + fib(1) = (fib(1) + fib(0)) + fib(1)

= (1 + 0) +fib(1) = 1 + fib(1) = 1 + 1 = 2

```
import java.util.Scanner;
public class ComputeFibonacci {
public static void main(String args[]) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter an index for the Fibonacci number: ");
    int index = input.nextInt();
    // Find and display the Fibonacci number
    System.out.println("Fibonacci(" + index + ") is " + fib(index));
  }
  /** The method for finding the Fibonacci number */
  public static long fib(long index) {
    if (index == 0) // Base case
      return 0;
    else if (index == 1) // Base case
      return 1;
    else  // Reduction and recursive calls
      return fib(index - 1) + fib(index - 2);
  }
}
```

Fibonnaci Numbers



```
import java.util.Scanner;
public class ComputeFibonacciTabling {  // NO REPEATED COMPUTATION
public static void main(String args[]) {
    Scanner input = new Scanner(System.in);
    System.out.print("Enter an index for the Fibonacci number: ");
    int index = input.nextInt();
    f = new long[index+1];
    System.out.println("Fibonacci(" + index + ") is " + fib(index));
  }
  public static long[] f;
  public static long fib(long index) {
    if (index == 0)       return 0;
    if (index == 1) {      f[1]=1;      return 1;   }
    if(f[index]!=0)
      return f[index];
   else  // Reduction and recursive calls
      f[index] = fib(index - 1) + fib(index - 2);
   return f[index];
  }
}
```

Characteristics of Recursion

All recursive methods have the following characteristics:

- One or more base cases (the simplest case) are used to stop recursion.
- Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.

In general, to solve a problem using recursion, you break it into subproblems.

- If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively.
- This subproblem is almost the same as the original problem in nature with a smaller size.

Problem Solving Using Recursion

- Print a message for n times
  - break the problem into two subproblems:
    - print the message one time and
    - print the message for n-1 times
      - This new problem is the same as the original problem with a smaller size.
      - The base case for the problem is n==0.

```
public static void nPrintln(String message,int times) {
  if (times >= 1) {
    System.out.println(message);
    nPrintln(message, times - 1);
  } // The base case is times == 0
}
```

Think Recursively

- The palindrome problem (e.g., “eye”, “racecar”):

```
public static boolean isPalindrome(String s) {
  if (s.length() <= 1) // Base case
    return true;
  else if (s.charAt(0) != s.charAt(s.length() - 1))
    // Base case
    return false;
  else
    return isPalindrome(s.substring(1, s.length() - 1));
}
```

Recursive Helper Methods

- The preceding recursive isPalindrome method is not efficient, because it creates a new string for every recursive call.
- To avoid creating new strings, use a helper method:

```
public static boolean isPalindrome(String s) {
  return isPalindrome(s, 0, s.length() - 1);
}
public static boolean isPalindrome(String s, int low, int high) {
  if (high <= low) // Base case
    return true;
  else if (s.charAt(low) != s.charAt(high))
    // Base case
    return false;
  else
    return isPalindrome(s, low + 1, high - 1);
}
```

Recursive Selection Sort

1. Find the smallest number in the list and swap it with the first number.
2. Ignore the first number and sort the remaining smaller list recursively.

```java
public class SelectionSort {
  public static void sort(double[] list) {
    int low = 0, high = list.length - 1;
    while (low < high) {
      // Find the smallest number and its index in list(low .. high)
      int indexOfMin = low;
      double min = list[low];
      for (int i = low + 1; i <= high; i++)
        if (list[i] < min) {
          min = list[i];
          indexOfMin = i;
        }
      // Swap the smallest in list(low ... high) with list(low)
      list[indexOfMin] = list[low];
      list[low] = min;
      low = low + 1;
    }
  }
  public static void main(String[] args) {
    double[] list = { 2, 1, 3, 1, 2, 5, 2, -1, 0 };
    sort(list);
    for (int i = 0; i < list.length; i++)
      System.out.print(list[i] + " ");
  }
}
```

```java
public class RecursiveSelectionSort {
  public static void sort(double[] list) {
    sort(list, 0, list.length - 1); // Sort the entire list
  }
  public static void sort(double[] list, int low, int high) {
    if (low < high) {
      // Find the smallest number and its index in list(low .. high)
      int indexOfMin = low;
      double min = list[low];
      for (int i = low + 1; i <= high; i++) {
        if (list[i] < min) {
          min = list[i];
          indexOfMin = i;
        }}
      // Swap the smallest in list(low .. high) with list(low)
      list[indexOfMin] = list[low];
      list[low] = min;
      // Sort the remaining list(low+1 .. high)
      sort(list, low + 1, high);
    }}
  public static void main(String[] args) {
      double[] list = {2, 1, 3, 1, 2, 5, 2, -1, 0};
      sort(list);
      for (int i = 0; i < list.length; i++)
        System.out.print(list[i] + " ");
  }
}
```

Recursive Binary Search

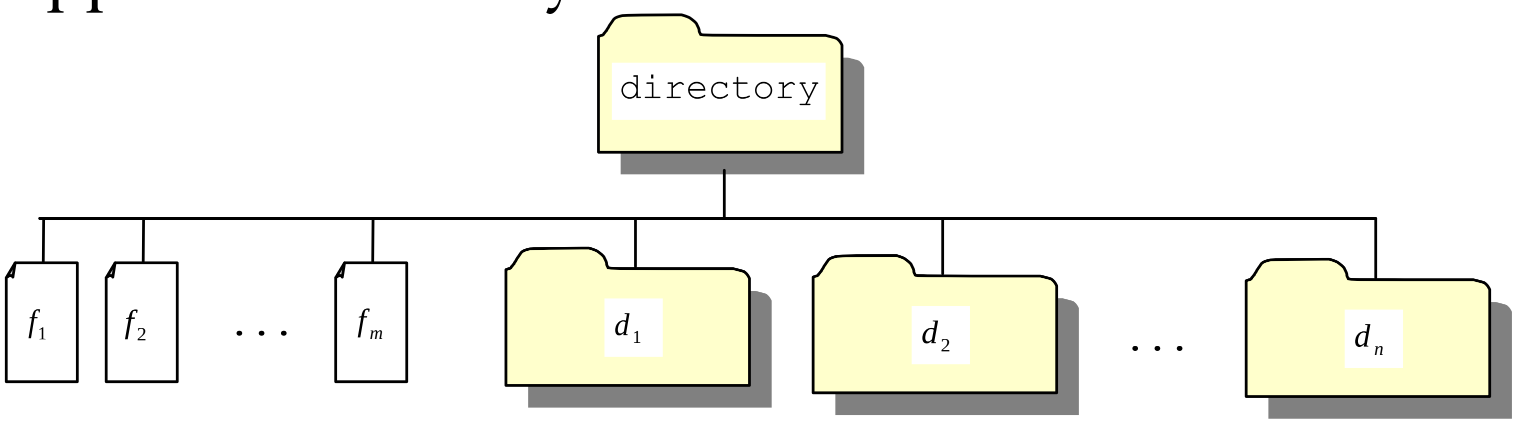
- Case 1: If the key is less than the middle element, recursively search the key in the first half of the array.
- Case 2: If the key is equal to the middle element, the search ends with a match (Base case).
- Case 3: If the key is greater than the middle element, recursively search the key in the second half of the array.

```java
public class BinarySearch {
  public static int binarySearch(int[] list, int key) {
    int low = 0;
    int high = list.length - 1;
    while(low <= high){
      int mid = (low + high) / 2;
      if (key < list[mid])
        high = mid - 1;
      else if (key == list[mid])
        return mid;
      else
        low = mid + 1;
    }
    // The list has been exhausted without a match
    return -low - 1;
  }
  public static void main(String[] args) {
    int[] list = { 1,2,3,4,5,6,10 };
    System.out.print(binarySearch(list,6));
  }
}
```

```
public class RecursiveBinarySearch {
  public static int recursiveBinarySearch(int[] list, int key) {
    int low = 0;
    int high = list.length - 1;
    return recursiveBinarySearch(list, key, low, high);
  }
  public static int recursiveBinarySearch(int[] list, int key,
      int low, int high) {
    if (low > high)  // The list has been exhausted without a match
      return -low - 1;
    int mid = (low + high) / 2;
    if (key < list[mid])
      return recursiveBinarySearch(list, key, low, mid - 1);
    else if (key == list[mid])
      return mid;
    else
      return recursiveBinarySearch(list, key, mid + 1, high);
  }
}
```

Directory Size

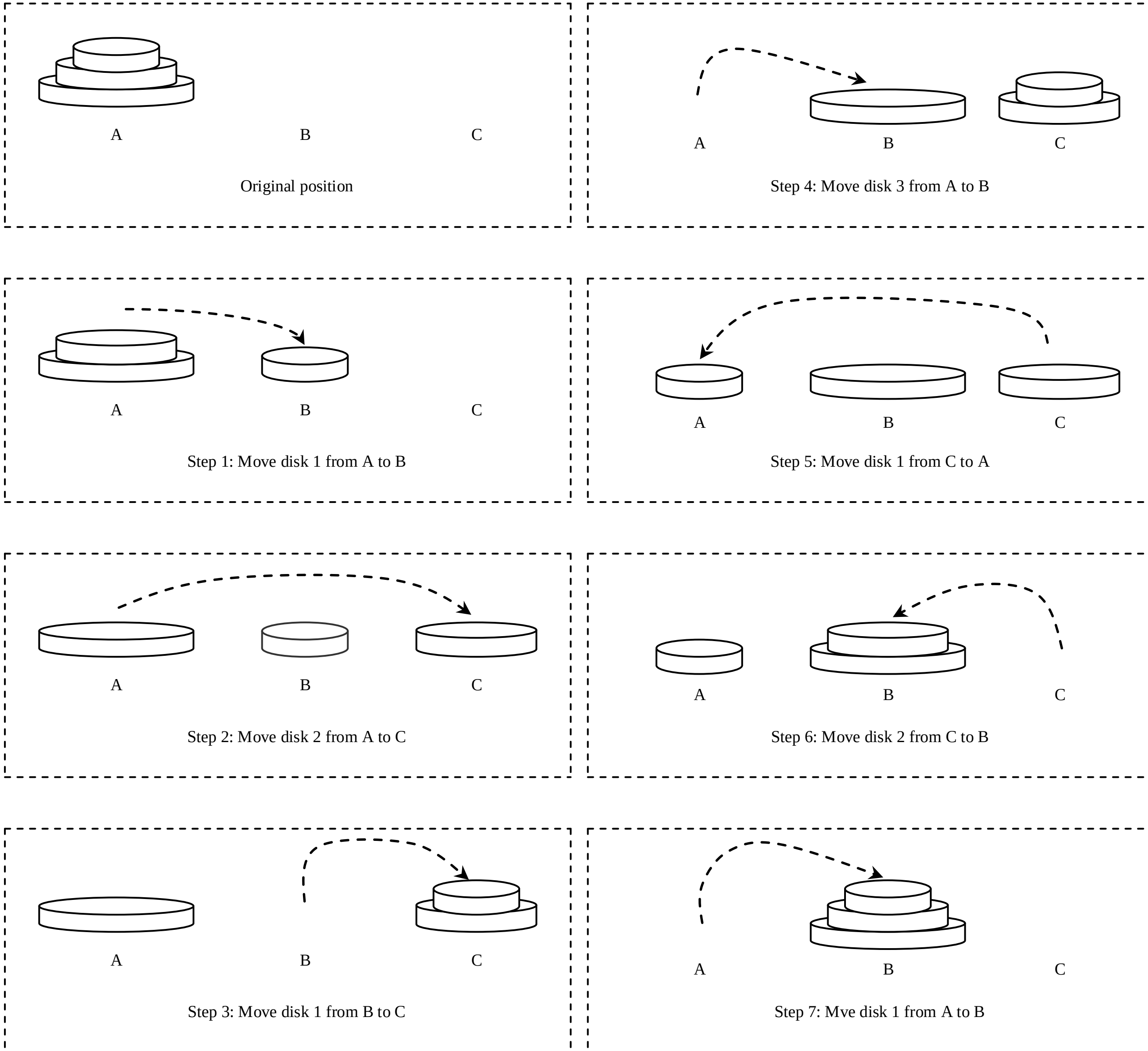
- Some problems are impossible to solve without recursion.
- Example: find the size of a directory.
  - The size of a directory is the sum of the sizes of all files in the directory.
  - A directory may contain subdirectories.
  - Suppose a directory contains files and subdirectories



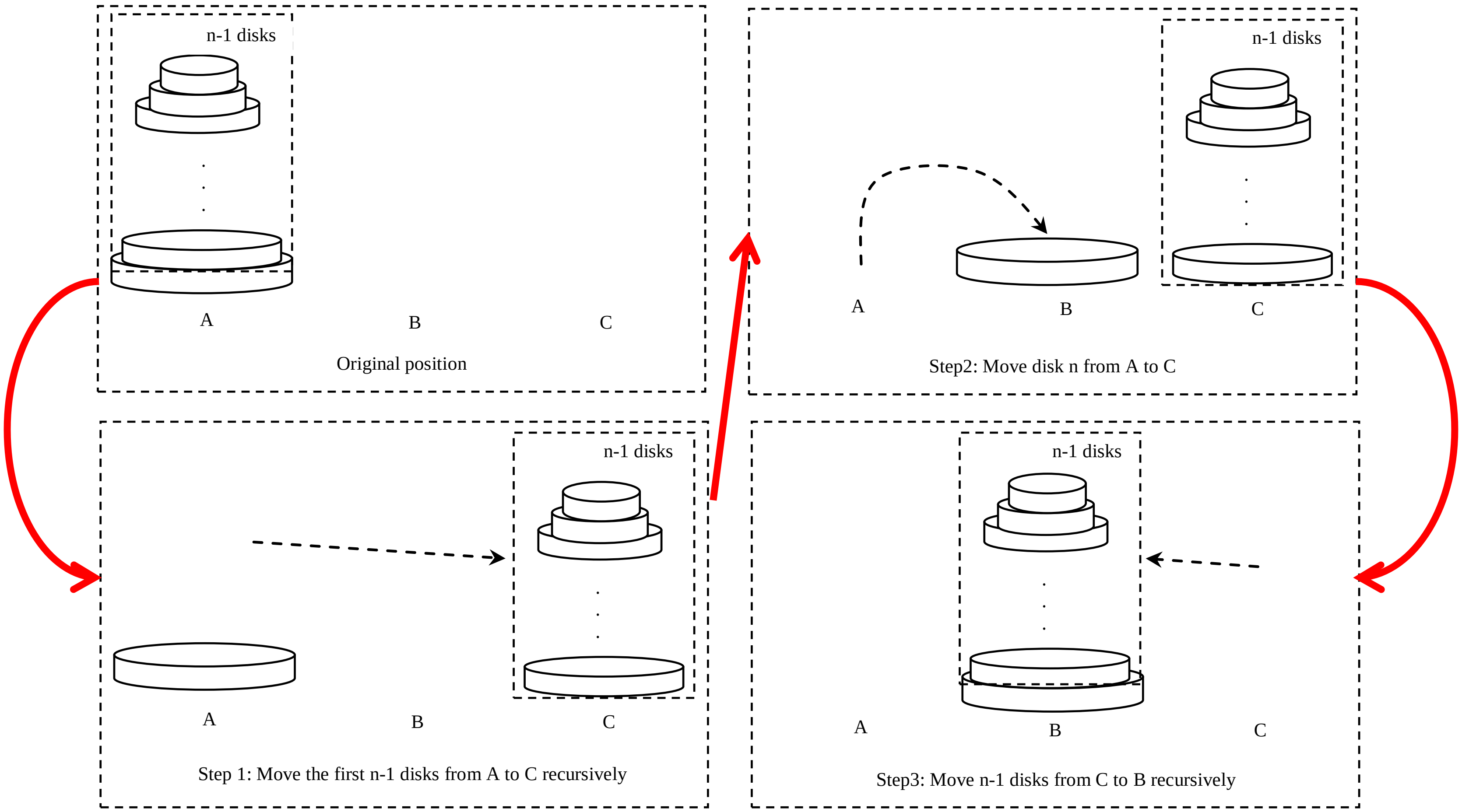
```java
import java.io.File;
import java.util.Scanner;
public class DirectorySize {
  public static void main(String[] args) {
    System.out.print("Enter a directory or a file: ");
    Scanner input = new Scanner(System.in);
    String directory = input.nextLine();
    System.out.println(getSize(new File(directory)) + " bytes");
  }
  public static long getSize(File file) {
    long size = 0; // Store the total size of all files
    if (file.isDirectory()) {
      File[] files = file.listFiles(); // All files and subdirectories
      for (int i = 0; i < files.length; i++) {
        size += getSize(files[i]); // Recursive call
      }
    }else { // Base case
      size += file.length();
    }
    return size;
}}
```

Towers of Hanoi

- There are *n* disks labeled 1, 2, 3, . . ., *n*, and three towers labeled A, B, and C.
- No disk can be on top of a smaller disk at any time.
- All the disks are initially placed on tower A.
- Only one disk can be moved at a time, and it must be the top disk on the tower.


A
B
C
Original position
A
B
C
Step 1: Move disk 1 from A to B
A
B
C
Step 2: Move disk 2 from A to C
A
B
C
Step 3: Move disk 1 from B to C
A
B
C
Step 4: Move disk 3 from A to B
A
B
C
Step 5: Move disk 1 from C to A
A
B
C
Step 6: Move disk 2 from C to B
A
B
C
Step 7: Mve disk 1 from A to B

The Towers of Hanoi problem can be decomposed into three subproblems:



Solution to Towers of Hanoi

- Move the first n - 1 disks from A to C with the assistance of tower B.
- Move disk n from A to B.
- Move n - 1 disks from C to B with the assistance of tower A.

```java
import java.util.Scanner;
public class TowersOfHanoi {
  public static void main(String[] args) {
    Scanner input = new Scanner(System.in);
    System.out.print("Enter number of disks: ");
    int n = input.nextInt(); System.out.println("The moves are:");
    moveDisks(n, 'A', 'B', 'C');
  }
  public static void moveDisks(int n, char fromTower, char toTower,
       char auxTower) {
    if (n == 1) // Stopping condition
      System.out.println("Move disk " + n + " from " +
        fromTower + " to " + toTower);
    else {
      moveDisks(n - 1, fromTower, auxTower, toTower);
      System.out.println("Move disk " + n + " from " +
        fromTower + " to " + toTower);
      moveDisks(n - 1, auxTower, toTower, fromTower);
    }
  }
}
```

Greatest Common Divisor (GCD)

$\gcd(2, 3) = 1$

$\gcd(2, 10) = 2$

$\gcd(25, 35) = 5$

$\gcd(205, 5) = 5$

gcd(m, n):

- Approach 1: Brute-force, start from min(n, m) down to 1, to check if a number is common divisor for both m and n, if so, it is the greatest common divisor.
- Approach 2: `Euclid's` algorithm
- Approach 3: Recursive method

Approach 1: GCD

```
public static int gcd(int m,int n){
   int min = n;
  if(m < n) min = m;
  for(int i=min; i>1; i--)
     if(m%i==0 && n%i==0)
        return i;
  return 1;
}
```

Approach 2: Euclid's algorithm

```
// Get absolute value of m and n;
t1 = Math.abs(m); t2 = Math.abs(n);
// r is the remainder of t1 divided by t2
r = t1 % t2;
while (r != 0) {
  t1 = t2;
  t2 = r;
  r = t1 % t2;
}
// When r is 0, t2 is the greatest
// common divisor between t1 and t2
return t2;
```

Approach 3: Recursive Method

```
gcd(m, n) = n                if m % n = 0
gcd(m, n) = gcd(n, m % n);  otherwise

public static int gcd(int m,int n){
    if (m % n == 0) return n;
    else return gcd(n, m % n);
}
```

From Iteration to Recursion

```
public static void m(int n){
    for(int i=1; i<=n; i++){
        for(int j=1; j<=n; j++){
            System.out.print(i+j);
        }
        System.out.println(i);
    }
}
public static void main(String[] args) {
    m(10);
}
```

```
public static void mr(int n){
        mr(1,n);
}
public static void mr(int i, int n){
        if(i<=n){
                mr(1, i, n);
                System.out.println(i);
                mr(i+1,n);
        }
}
public static void mr(int j, int i, int n){
        if(j<=n){
                System.out.print(i+j);
                mr(j+1,i,n);
        }
}
```