

# Capturing the Spatio-Temporal Behavior of Real Traffic Data <sup>★</sup>

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## Abstract

Traffic data, like disk and memory accesses, typically exhibits burstiness, temporal locality, and spatial locality. However, except for qualitative speculations, it is not even known how to measure the spatio-temporal correlation, let alone how to reproduce it realistically. In this paper, we propose the 'entropy plots' to quantify the correlation and develop a new statistical model, the 'PQRS' model, to capture the burstiness and correlation of the real spatio-temporal traffic. Moreover, the model requires very few parameters and offers linear scalability. Experiments with multiple real data sets show that our model can mimic real traces very well.

*Key words:* traffic modeling, space-time correlation, spatio-temporal burstiness, locality, entropy

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## 1 Introduction

System design is typically evaluated through trace-driven simulations in which real traces are fed into the system [2,19]. Collecting traces involves system instrumentation or system simulation [17], which is expensive in terms of time and human resources. Modeling traffic data with a simple and accurate statistical model, on the other hand, has several advantages. First, we can run 'what if' scenarios, by generating as long or as short a trace as we want; or by varying the load, burstiness and other parameters of our statistical model. Second, we need much less storage space: a statistical model typically requires only a handful of parameters. Finally, we can do analytical performance studies: For example, if we know that our traffic is Poisson, we can estimate analytically queue length distributions at a server with a given service time distribution.

There has been significant effort on modeling the temporal and spatial locality of traffic data. However, none of them explicitly measures the spatio-temporal correlation or provides statistical models to incorporate the correlation. In this paper, we would like to answer the following questions:

- What is the spatial behavior of the traces? Are all the disk blocks equi-probable (i.e., random accesses in credit card applications), piece-wise uniform, or Gaussian-like?
- What is the spatio-temporal correlation? Should we worry about the issue? How close to reality is the (convenient) independence assumption?

Furthermore, we want to develop a statistical model that will naturally capture burstiness, uniformity, and correlation.

**Problem 1** *Given a two-dimensional trace,  $Y = \{(t, s)\}$ , (i.e.  $(t, s)$  defines a request of arrival time  $t$  on address  $s$ .), develop a mathematical model that can generate a synthetic trace,  $Y' = \{(t', s')\}$ , that has “similar” spatio-temporal behavior as  $Y$ .*

The goodness of the model can be evaluated by comparing the synthetic traces to the real traces in terms of both statistical measures (i.e. mean, burstiness, and correlation) and performance behavior (i.e. response time distributions for disk traces). The latter, in our opinion, is more important for practical reasons.

Compactness and efficiency of the model are two additional concerns. A naive model can

simply remember the given trace and reproduce it as a synthetic trace, but this hardly saves any space or effort, nor allows for generation of longer traces. The ideal model should (1) require few parameters, (2) exhibit burstiness over time and space, (3) preserve the spatio-temporal correlation, and (4) have linear scalability.

The paper is organized as follows. Section 2 reviews the related work. Section 3 studies the behavior of the real world traffic and Section 4 provides a measure for both the burstiness and the correlation. Section 5 introduces the PQRS model. Section 6 evaluates the model using real traffic data. Section 7 concludes the paper.

## 2 Related Work

Traffic modeling has recently attracted much attention, arguably thanks to the discovery of self-similarity in multiple forms of computer generated traffic [15,8,12,14], as well as thanks to the emergence of new applications like web caching.

Focusing only on temporal modeling, one approach is to employ some mathematical models to capture the self-similarity in the request arrival time exhibited in the real traffic data. Successful such models include the fractional Brownian motion [15], fractal ARIMA [11], the ON/OFF model used in the SURGE web trace generator [3], Multifractal Wavelets [16], and the b-model [20]. Related efforts have focused on memory and web proxy trace characterization ultimately aiming to estimate cache hit-ratios, for memory or web caches. Several groups identified Zipf-like distributions in document popularity and used the Independent Reference Model or segregated IRM to generate web proxy traces [1,4,5].

Contrary to the modeling of web server workloads, modeling I/O workloads can not afford to ignore the location of each request, because both the arrival time and the location are needed to determine its service time. Some of the few exceptions that pay attention to both time and location [19,13] use a mixture of sequential and random access patterns. Although a step to the correct direction, (a) these models require a large number of parameters and (b) they propose no way to measure the spatio-temporal locality, nor do they guarantee that their synthesized traces will exhibit similar locality properties.

The PQRS model is the first statistical model that captures not only the temporal and spatial behavior, but their correlation as well, and it does so in a surprisingly small number

of parameters. Next, we describe it in detail.

### 3 Observing Data: Naive Model

Understanding the real traffic data is crucial before we can start to build the model. This section studies the behavior of the real world traffic data.

#### 3.1 Burstiness

We use the time-space plot to get a basic feeling about the characteristics of real traffic data. The time-space plot  $C_{T,S}$  is similar to the memory reference map: The value of  $C_{T,S}(t, s)$  is the number of requests of arrival time  $t$  and address  $s$ . Figure 1 shows the time-space plot for the `cello` disk trace [17]. Further projection of the trace onto time and space gives the “marginal” one-dimensional traces  $C_T$  and  $C_S$ , in which  $C_T(t)$  ( $C_S(s)$ ) tells the number of requests of arrival time  $t$  (on disk block  $s$ ).

We observe “bursty” behavior (i.e., non-uniformity) in both marginals.

- **Temporal burstiness.** The temporal burstiness is expected: various traffic traces, such as disk I/O traffic [12] and network traffic [15,8], have all been shown to be bursty.
- **Spatial burstiness.** Similarly,  $C_S$  is bursty, too, as noticed before [7]. Some blocks are more popular than others.

It is tempting to conjecture that the marginal distributions are Zipf. To settle the issue, we give Figure 2, which presents the “rank-frequency” plots for the temporal and spatial marginals. Specifically, Figure 2(a) gives the number of disk requests per time interval (10 seconds), versus the interval-ids sorted in decreasing popularity. Figure 2(b) gives the number of disk requests per 1000 disk blocks, versus the block-ids sorted in decreasing popularity. All axis are logarithmic. The plots are close to lines, leading to the conjecture that they follow generalized Zipf distributions, with slopes 0.40 and 0.79 respectively. However, closer inspection shows a flattening at the left top. This phenomenon had been observed in several data sets even by Zipf himself, who coined the term ‘top concavity’ [21], and who went into great lengths to try to explain it. On the contrary, models that use multifractals, like the

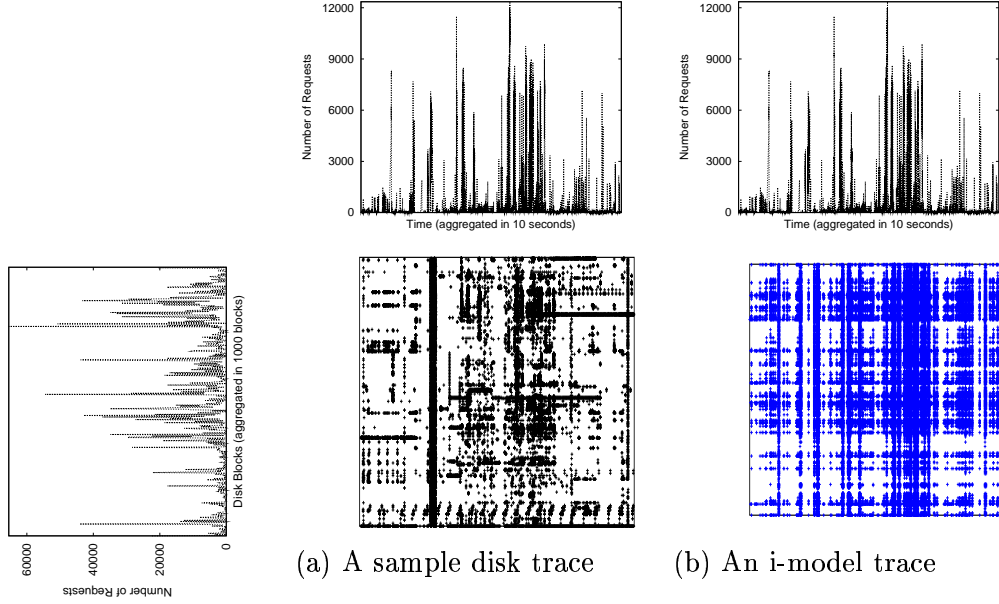


Fig. 1. Time-space plot of **cello** disk trace and an i-model trace. Both have the same marginals.

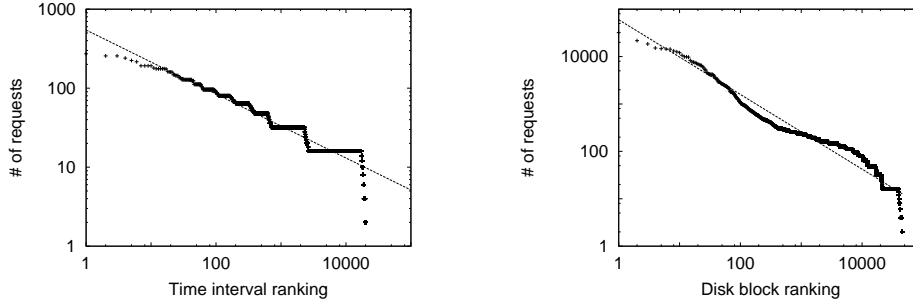


Fig. 2. Rank-frequency plots (in log-log scales) for time and location of the **cello** trace. Left: Rank of temporal interval (10 seconds) versus number of requests in that interval. Right: Rank of disk blocks (aggregated in 1000 disk blocks) versus number of requests on these disk blocks. The slopes of the fitting lines are 0.404 and 0.788 respectively.

upcoming PQRS model, naturally lead to curving rank-frequency distributions [9] and thus provide even better fits.

As mentioned earlier, there are many successful models to capture the temporal behavior of the traffic data. Similarly, they should be able to capture the spatial behavior since the temporal and spatial burstiness looks similar. That is, we can synthesize the two marginal traces using existing models. However, a combining algorithm is necessary to combine the marginals into two-dimensional traces. The straight-forward combining algorithm is the i-model.

### 3.2 I-Model

The i-model generates a two-dimensional trace by “multiplying” two marginal traces. For example, if 10% of the total requests arrive at time  $t$  and 5% of the total requests occur on address  $s$ ,  $10\% \times 5\%$  of the total requests have arrival time  $t$  and address  $s$ . Formally, the i-model specifies that given  $C_T$  and  $C_S$ ,

$$C_{T,S}(t, s) = C_T(t) \times C_S(s)/M, \quad t = 1, 2, \dots, \quad s = 1, 2, \dots, \quad (1)$$

where  $M$  is the total number of requests in the trace.

The i-model preserves the temporal and spatial burstiness because the marginals of  $C_{T,S}$  are exactly  $C_T$  and  $C_S$ . In addition, it requires no parameters.

Despite of these advantages, the i-model ignores a very important property of the traffic: *strong spatio-temporal* correlation. The strong correlation suggests that requests coming close in time tend to access nearby objects. Figure 1 (b) shows a two-dimensional i-model trace. Although the i-model trace has the same marginals as the real trace, we observe significant differences between the two traces. The difference is attribute to the existence of strong spatio-temporal correlation in the real trace. In fact, the independence assumption leads to grossly pessimistic results, as we will show in Section 6.

## 4 Proposed Method to Quantify Correlation

The i-model ignores the strong spatio-temporal correlation of the real traffic data. The correlation has a great impact on the performance behavior because requests to nearby objects take less time to finish. Therefore, it’s important to quantify the correlation and to incorporate it later in the model.

### 4.1 Definitions

Mutual information [18] measures the correlation between two events. We give a brief description of the related concepts. (Please refer to Table 1 for the symbols used in the paper.)

$P_{T,S}(t, s)$	Probability that a request on location $s$ arrives at time $t$ .
$P_T(t)$	Probability that a request arrives at time $t$ .
$P_S(s)$	Probability that a request is on location $s$
$H(E)$	Entropy of a random variable $E$
$H_T^{(n)}$	Temporal entropy at aggregation level $n$
$R_T$	Slope of the temporal entropy plot
$H_S^{(n)}$	Spatial entropy at aggregation level $n$
$R_S$	Slope of the spatial entropy plot
$H_{T,S}^{(n)}$	Joint entropy on time and space at aggregation level $n$
$R_{T,S}$	Slope of the joint entropy plot
$(p, q, r, s)$	Parameters to the PQRS model
$M$	Total number of requests in a trace

Table 1  
Symbols table.

Entropy measures the uniformity of a discrete probability function. **Entropy** on a random variable  $E$ , (e.g., disk block id of a random request), is defined as

$$H(E) = - \sum_{i=1}^N p_i \log_2 p_i, \quad (2)$$

where  $p_i$  is the probability that event  $E_i$  will happen (e.g., the  $i$ -th block will be hit) and  $N$  is the total number of possible outcomes (e.g., total number of disk blocks).  $H$  is close to 0 if the distribution is highly skewed and  $H$  reaches its maximum value of  $\log_2 N$  when the distribution is uniform. In another word,  $H$  measures the burstiness of the probability function.

The joint entropy on two random variables is defined similarly: for a given probability function  $P = \{p_{i,j}\}$  on two random variables  $\{E\}$  and  $\{F\}$ , (e.g., arrival time and disk block id of a random request), where  $p_{i,j}$  gives the probability that both event  $E_i$  and event  $F_j$  will happen, (e.g., a disk request at block id  $j$  arrives at  $i$ ), the **joint entropy** on  $E$  and  $F$  is

$$H(E, F) = - \sum_{i,j} p_{i,j} \log_2 p_{i,j}. \quad (3)$$

**Definition 2** The **mutual information**  $I(E; F)$  on two random variables  $E$  and  $F$  is

defined as

$$I(E; F) = H(E) + H(F) - H(E, F). \quad (4)$$

The mutual information  $I(E; F)$  indicates the degree of correlation between  $E$  and  $F$ . It becomes zero if  $E$  and  $F$  are independent.

## 4.2 Entropy Plots

We can apply the above definitions to traffic data to measure the burstiness and spatio-temporal correlation. The question is, then, at what the granularity. If we calculate the entropy on the finest resolution, the mutual information on time and space will be very close to zero because no correlation will be observed. Our answer is to calculate the entropy values at all “aggregation” levels.

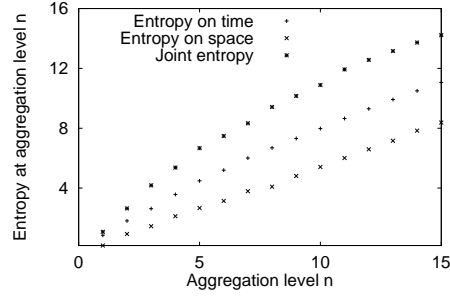
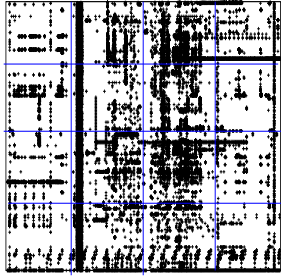
To find the entropy values at aggregation level  $n$ , the time-space plot of the trace is divided into  $2^n \times 2^n$  grids. For example, Figure 3 (a) shows the  $4 \times 4$  grids at aggregation level 2.  $P_{T,S}^{(n)}$  describes the probability that a request falls into each grid; that is, the request has arrival time  $t$ , ( $t_1 \leq t < t_2$ ), and address  $s$ , ( $s_1 \leq s < s_2$ ).  $P_T^{(n)}$  and  $P_S^{(n)}$  are the projections of  $P_{T,S}^{(n)}$  on time and space. All the  $P$ s can be easily derived from the given trace.

**Definition 3** *The **entropy plot** for a given trace is defined by plotting the entropy values against the aggregation level  $n$ , where the entropy on time, space and the joint entropy at aggregation level  $n$  are*

$$\begin{cases} H_T^{(n)} = H(P_T^{(n)}); \\ H_S^{(n)} = H(P_S^{(n)}); \\ H_{T,S}^{(n)} = H(P_{T,S}^{(n)}). \end{cases} \quad (5)$$

The entropy plot provides an insight on how the burstiness and correlation change across different resolution levels. The points form a line if the burstiness and correlation are stable at all granularities. Interestingly, real traffic has stable burstiness and correlation as the linear entropy plot of the sample disk trace suggests (Figure 3 (b)). This further confirms the self-similarity of the disk I/O workloads.





(a) Aggregation level 2 ( $2^2 \times 2^2$  grids) (b) Entropy plot for the sample disk trace.

Fig. 3. Entropy plot for the `cello` disk trace.

**Lemma 4** *For a trace of stable temporal and spatial burstiness and spatio-temporal correlation, all the entropy plots are linear:*

$$\begin{cases} H_T^{(n)} = nR_T; \\ H_S^{(n)} = nR_S; \\ H_{T,S}^{(n)} = nR_{T,S}. \end{cases} \quad (6)$$

$R_T$  and  $R_S$  measure the temporal and spatial burstiness respectively. The intuition behind  $R_T$  is the rate of information contained in one more bit of time-stamp. For example, when all the requests come in a burst, all the time-stamps will be the same and all the bits are useless, which leads to  $R_T = 0$ . When the requests are uniformly distributed along time, all the bits in the time-stamps are useful and  $R_T$ , in this case, is 1. Similarly,  $R_S$  gives the rate of information in the location bit.

$R_I$ , defined as  $R_T + R_S - R_{T,S}$ , tells the mutual information per bit (e.g., how much information the time-stamp bit tells about the location of the request). When  $R_I$  equals to 0, the time-stamps and the addresses of the requests is independent.

The real traffic data shows strong spatio-temporal correlation. The estimated value of  $R_T$ ,  $R_S$ , and  $R_{T,S}$  (Figure 3 (b)) are 0.722, 0.573, and 0.881, leading to  $R_I \approx 0.414$ . The large value of  $R_I$  indicates strong spatio-temporal correlation in the real traffic data.  $R_I$  for the i-model trace, on the other hand, is 0.001, suggesting independence between time and space. (Hence the name i-model.)

## 5 Proposed Model: PQRS Model

This section presents a new statistical model, called the “PQRS” model, to capture the stable burstiness and correlation of the real traffic data.

### 5.1 Generation: PQRS Model

The PQRS model generates a two-dimensional trace using four parameters, namely,  $p, q, r, s$ , where  $p + q + r + s = 1$ . The recursive construction is the reverse process of the aggregation in entropy plot calculation (Figure 5 (a)). At the starting point, the probability that a request falls into the whole time-space plot is 1. In step 1, the time-space plot is divided into  $2 \times 2$  grids and the probability that a request falls in each grid is  $p, q, r, s$  respectively. In step 2, each grid is further divided into 4 smaller grids and the requests in the grid are distributed to the 4 smaller grids with the same probabilities,  $p, q, r, s$ . The process goes on recursively until the required resolution on time and space is achieved (e.g., the size of the grids is smaller than a microsecond or a disk block).

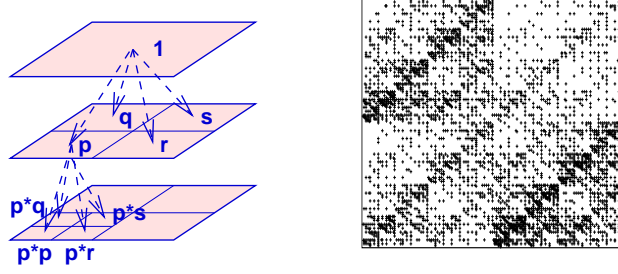
```
initialize the stack;
push the whole trace onto the stack;
while (stack is not empty) do
  pop a square from the stack;
  if required resolution is met, then
    output the requests of the square;
  else
    divide the square into  $2 \times 2$  grids;
    distribute the requests to the grids;
    push the four grids onto the stack;
```

**Fig. 4:** PQRS trace generation

The above algorithm assumes the same order of  $p, q, r, s$  in all the levels. We can generate a random PQRS trace by imposing a different order at each step. Figure 5 (b) gives a sample trace generated by the PQRS model with  $p, q, r, s$  of 0.2, 0.3, 0.4, 0.1. More requests cluster at bottom right corner since  $r$  has the greatest value among the four parameters.

### 5.2 Parameter Fitting

The recursive construction algorithm guarantees that PQRS traces have stable burstiness and spatio-temporal correlation because all the steps use the same parameters to distribute the requests.



(a) Generation algorithm (b) A Sample PQRS trace

Fig. 5. Recursive trace generation process for the PQRS model.

**Lemma 5** *Traces generated by the PQRS model have stable burstiness and correlation as they have linear entropy plots.*

$$\begin{cases} H_T^{(n)} = nH_T^{(1)} = nR_T; \\ H_S^{(n)} = nH_S^{(1)} = nR_S; \\ H_{T,S}^{(n)} = nH_{T,S}^{(1)} = nR_{T,S}; \end{cases} \quad (7)$$

**Lemma 6** *For a PQRS trace generated with parameter  $p, q, r, s$ ,  $p+q+r+s = 1$ , the entropy rates are*

$$\begin{cases} R_T = -(p+q) \log_2(p+q) - (r+s) \log_2(r+s); \\ R_S = -(p+r) \log_2(p+r) - (q+s) \log_2(q+s); \\ R_{T,S} = -p \log_2 p - q \log_2 q - r \log_2 r - s \log_2 s. \end{cases} \quad (8)$$

All the proofs are omitted from the paper for brevity. Equation 8 suggests that the difference between  $(p+q)$  and  $(r+s)$  determines the temporal burstiness of the synthetic traces and the difference between  $(p+r)$  and  $(q+s)$  determines the spatial burstiness. Varying the value of  $p$  changes the degree of the spatio-temporal correlation when temporal and spatial burstiness is fixed.

The parameter fitting algorithm for the PQRS model is simple. For a given trace, we estimate the value for  $p, q, r, s$  by plugging the slopes of the entropy plots in Equation 8.

The following two lemmas give some additional features of the PQRS model.

**Lemma 7** *The Poisson model is a special case of the PQRS model where  $p = q = r = s = 0.25$ .*

**Lemma 8** *The  $i$ -model is a special case the PQRS model where  $\frac{p}{q} = \frac{r}{s}$ .*

### 5.3 Complexity

The computational complexity of the algorithm is an important property of the model. Our analysis shows that both the trace generation and the parameter fitting algorithms for the PQRS model offer linear scalability to the number of requests in traces.

**Lemma 9** *The computational complexity in generating a PQRS trace is  $O(M \times N)$ , where  $M$  is the total number of requests and  $N$  is the resolution level.*

We outline the proof here. We upper-bound the trace generation through a naive implementation of the algorithm. The recursive generation conceptually forms a quad tree of height  $N$ . (See Figure 5 (a).) The  $4^n$  grids in step  $n$  form the  $4^n$  nodes at level  $n$  in the quad tree. We decide the arrival time  $t$  and address  $s$  of each request by walking down the quad tree from the root. At each level of the tree, we choose one of the four subnodes with probability  $p, q, r, s$  until we reach a leaf node. Enumerating all the requests gives the final trace. Therefore, the complexity of the trace generation is  $O(M \times N)$ . In reality,  $N$  is usually logarithm to the length of the trace in time (or space).

**Lemma 10** *The computation complexity for the parameter fitting algorithm of the PQRS model is  $O(M \times N)$ .*

We sketch the proof here. The number of non-zero grids in each aggregation level is at most  $M$ ; therefore, it takes  $O(M)$  computations to generate a point in the entropy plot. Given that there are  $N$  points in the entropy plot, the total computational complexity is  $O(M \times N)$ .

In summary, the strength of the PQRS model lies in its power as well as in its simplicity. The model generates traces with stable burstiness and correlation as the real traffic data exhibits. In addition, the model offers linear scalability.

## 6 Experiments

We evaluate the PQRS model using two types of traffic data: disk and memory reference traces. The experiments examine the validity of the PQRS model and compare the performance behavior of the PQRS model traces to the real ones.

### 6.1 Experiment Setup

Two types of traffic data are in use. Table 2 summarizes the data sets.

**Cello disk traces.** The disk traces were collected on a UNIX file server in HP on June 12, 1992 [17]. The server has 8 disks attached to it. Total of six traces are in use: **Disk-a** for the aggregation of all the disk requests, **Disk-r** for all the read requests, **Disk-w** for all the write requests, and **Disk-0**, **Disk-2**, **Disk-7** for individual disk 0, 2, 7. All the traces are one day long. The other five disks are not studied because of the small volume of disk requests. The arrival time is accurate up to microseconds. The disk block number ranges from 0 to more than 5,000,000.

**TPC-C memory reference traces.** The TPC-C memory traces were collected on a realistic processor simulator running TPC-C workloads on Shore [6]. There are total of six traces: five for five types of transactions and one for a mixture of different types of transactions. Only references to the heap area are studied here.

**Evaluation tools.** The ultimate goal of traffic modeling is to facilitate system designs. Therefore, we focus on the performance behavior of the traces. We use the response time and queue length distributions for disk traces and the cache miss ratio for memory reference traces as our performance metrics.

**Methodology.** We want to answer the following questions. First, does real traffic have stable burstiness and correlation over aggregation? Second, how does the PQRS model perform? The synthetic traces should have the same performance behavior as the real ones if the synthetic traces accurately capture the characteristics of the real traffic data.

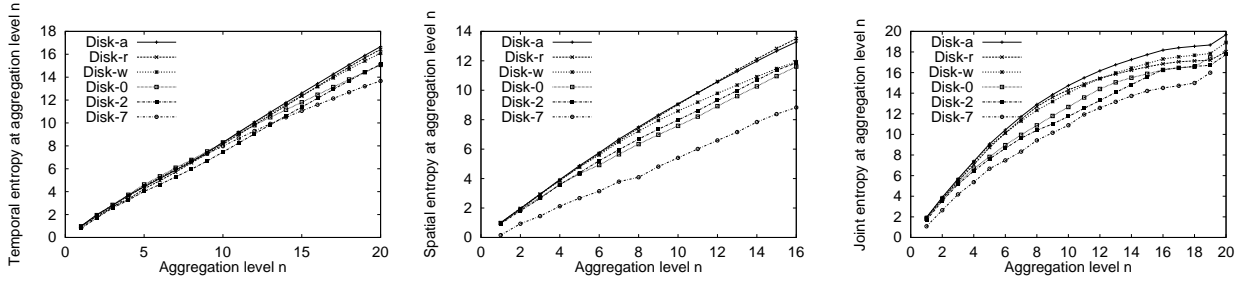
Trace	Total disk requests	$\hat{R}_T$	$\hat{R}_S$	$\hat{R}_{T,S}$	$I_{T,S}$	$(\hat{p}, \hat{q}, \hat{r}, \hat{s})$
Disk-a	4,575,798	0.641	0.819	1.058	0.402	(0.862,0.001,0.257,0.741)
Disk-r	1,822,781	0.847	0.833	0.984	0.696	(0.016,0.258,0.720,0.006)
Disk-w	3,300,628	0.641	0.728	0.992	0.377	(0.150,0.013,0.053,0.784)
Disk-0	1,101,416	0.814	0.690	0.941	0.563	(0.043,0.184,0.772,0.001)
Disk-2	1,396,649	0.790	0.723	0.904	0.609	(0.200,0.027,0.001,0.772)
Disk-7	371,320	0.722	0.573	0.881	0.414	(0.056,0.135,0.808,0.001)

(a) Cello disk trace summary

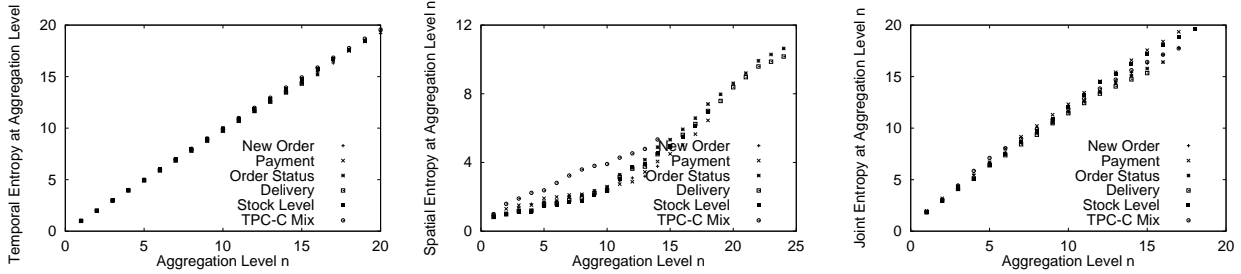
Trace	Length	Total requests	$\hat{R}_T$	$\hat{R}_S$	$\hat{R}_{T,S}$	$I_{T,S}$	$(\hat{p}, \hat{q}, \hat{r}, \hat{s})$
New Order	14,990,636	4,000,000	0.962	0.200	0.996	0.166	(0.030,0.255,0.001,0.714)
Payment	17,242,172	4,573,044	0.963	0.281	1.042	0.202	(0.239,0.047,0.713,0.001)
Order Status	1,355,168	268,943	0.950	0.456	0.989	0.417	(0.095,0.185,0.001,0.722)
Delivery	525,100	129,388	0.957	0.439	0.987	0.409	(0.090,0.192,0.001,0.717)
Stock Level	14,453,440	3,613,360	0.974	0.349	1.052	0.271	(0.231,0.064,0.704,0.001)
Mix	12,268,876	4,000,000	0.983	0.309	0.990	0.302	(0.248,0.054,0.697,0.001)

(b) TPC-C memory reference trace summary (Trace length in CPU cycles)

Table 2  
Trace Summary



(a) Entropy plots on time, space and the joint entropy plot (from left to right) for cello traces.



(b) Entropy plots on time, space and the joint entropy plot (from left to right) for TPC-C traces.

Fig. 6. Entropy plots for the two data sets.

## 6.2 Model Checking

The PQRS model is designed for the traffic data with stable burstiness and spatio-temporal correlation. Therefore, the traffic data should have linear entropy plots for the PQRS model to work. In addition, we rely on linear entropy plots for estimation of parameter  $p, q, r, s$ .

Figure 6 shows the entropy plots for the disk and memory traces. We have made the following observations.

- The entropy plots are reasonably linear, suggesting stable burstiness and correlation in the traces. This ensures that the traces are well within the capability of the PQRS model.
- Strong spatio-temporal correlation exists.  $R_I$  ranges from 0.313 to 0.696 for disk traces and from 0.166 to 0.417 for memory traces, indicating strong correlation.
- The PQRS model is able to model uniform traces as well.  $R_T$  for the memory traces is close to 1, suggesting a uniform distribution of the memory accesses on time. This is because the program is consistently accessing data during its course of execution.

In summary, real traffic data has stable burstiness and correlation over aggregation and is within the capability of the PQRS model. Strong correlation exists, invalidating the independence assumption of the i-model.

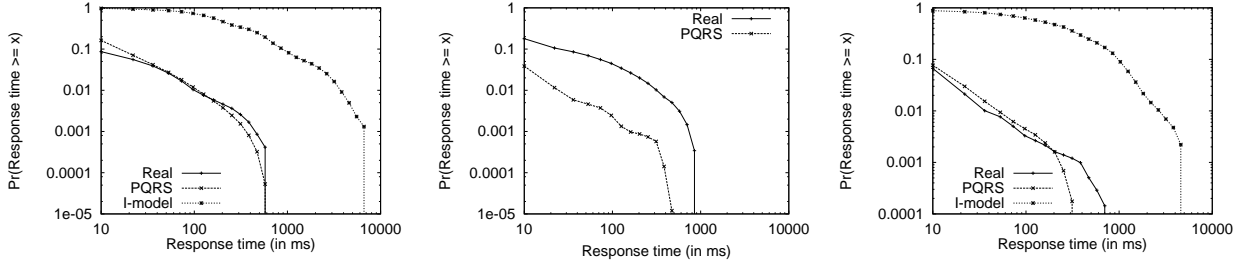
Table 2 lists the estimated  $p, q, r, s$  value for the data sets. The following sections compare the performance behavior of the real traces and the PQRS traces generated using these values.

### 6.3 Disk Trace Evaluation

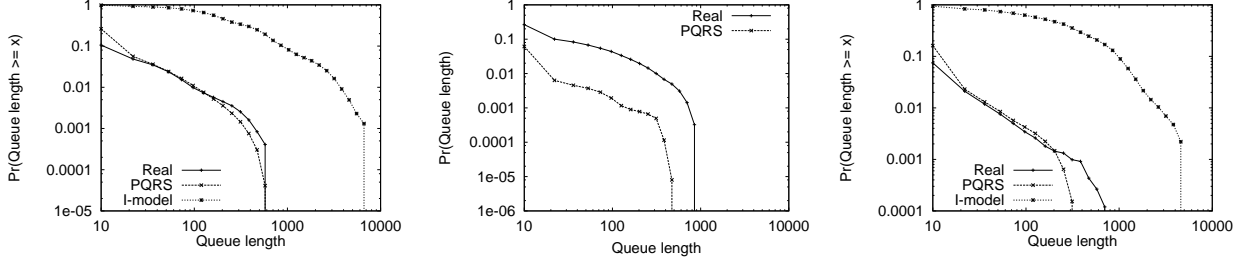
Figure 7 compares the response time and queue length distributions of the real disk traces and the PQRS traces on a realistic disk simulator [10]. Both distributions are in negative cumulative form (NCDF) and in log-scale. That is, point (10, 0.01) in the response time distribution plot tells that 1% of the disk requests have response time greater than 10 milliseconds. Traces with strong spatio-temporal correlation should have short tails in these distributions as requests close in address can be served faster.

The comparison shows that the PQRS traces, by accurately capturing the burstiness and the correlation, simulates the performance behavior of the real traces very well.

- Strong spatio-temporal correlation plays an important role in performance behavior. The i-model traces produce extremely large response time because of the independence assumption although they have exactly the same burstiness along time and space as the real traces. The i-model results for **Disk-2** are missing because the queue becomes long enough to saturate the system.



(a) Response time distribution in NCDF (From left to right: Disk-0, Disk-2, and Disk-7)



(b) Queue length distribution in NCDF (From left to right: Disk-0, Disk-2, and Disk-7)

Fig. 7. Disk Trace Performance Evaluation. (I-model traces for disk-2 are missing due to queue saturation).

- The PQRS model works amazingly well in simulating the real traffic by accurately capturing both the burstiness as well as the strong correlation at all aggregation levels.

The above comparison has shown that the PQRS model mimic the real disk I/O traffic very well in performance behavior.

#### 6.4 Memory Trace Evaluation

Memory trace evaluation involves comparing the cache miss rates of the real traces to the PQRS traces. The miss rate is an important performance metric in computer architecture research and it reflects the temporal and spatial locality. Memory references on nearby locations have a better chance to be cache hits if they are close to each other in arrival time. Therefore, strong spatio-temporal correlation leads to low cache miss rates.

Figure 8 compares the cache miss rates for three sets of traces: R for the real traces, I for the i-model traces of the same marginals as the real ones, and P for the PQRS traces with parameters listed in Table 2. Six groups of bars show the cache miss rates on six different



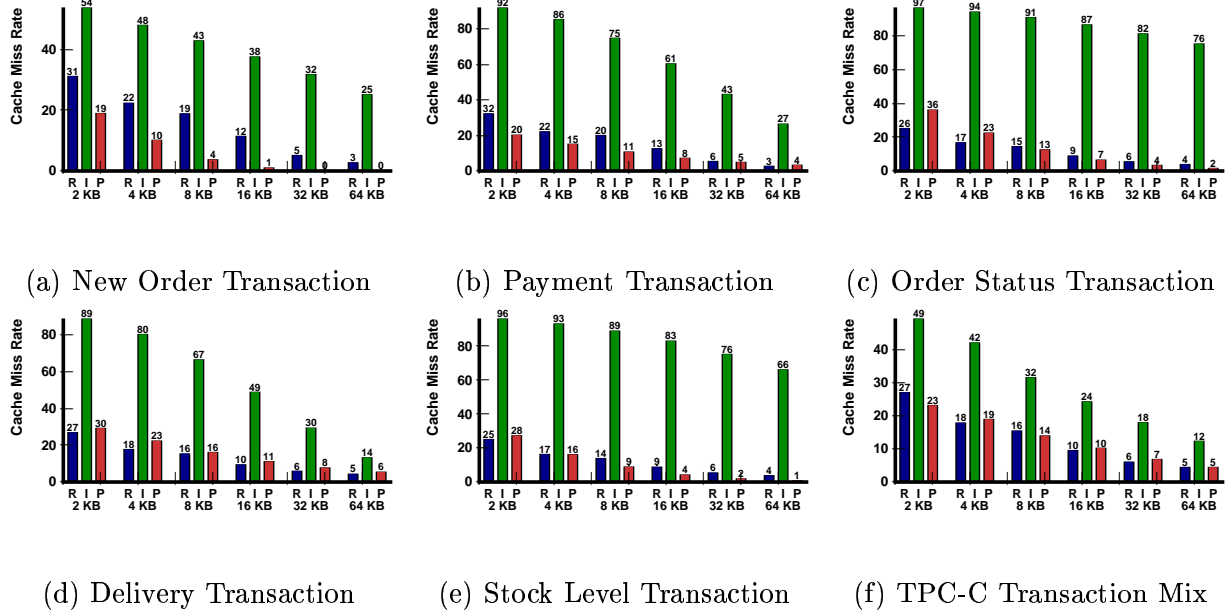


Fig. 8. Comparison of the performance behavior of the memory reference traces. Three types of traces are evaluated against the a realistic cache simulator. Trace “R” stands for “Real”; “I” for the i-model, and “P” for the PQRS model.

cache sizes in each graph.

We observe that the traces with high degree of spatio-temporal correlation, such as the R and P traces, suffer low cache miss rates as we have expected. The relative error of the PQRS traces is within 30%. On the other hand, the I traces, assuming independence between time and space, experience extremely high miss rates and have relative error as high as 1800%.

## 6.5 Summary

Both disk traces and memory references traces have shown reasonably stable burstiness and spatio-temporal correlation over aggregation as suggested by the linear entropy plots. Strong temporal correlation exists in both types of traffic data and it has a significant impact on the performance behavior of the traces. Therefore, traffic modeling should take the correlation into consideration.

The PQRS model, carefully designed for such traffic data, is able to replicate the behavior of real traces very well. The i-model, on the other hand, fails to do so by ignoring the correlation.

## 7 Conclusions

Modeling disk traffic is a hard problem [10], especially when we need to capture both the temporal as well as spatial correlations. In this paper, we propose the entropy plot to measure the spatio-temporal correlation and we discover that the burstiness and correlation remain stable for many scales for real traffic data, which is another evidence of self-similarity. We develop a simple statistical model, the PQRS model, to capture the characteristics of such traffic: it can be bursty or uniform in time, bursty or uniform in space, and it can give zero to 100% correlation between space and time.

Smaller contributions include

- We are the first to quantify the popular, but vague intuition that memory and disk accesses exhibit locality, not only in space or time, but in space-time as well.
- We give fast, scalable algorithms to run our model: both the parameter fitting and trace generation algorithms require linear time on the number of requests.
- Experiments on multiple real data sets show that the simple PQRS model can mimic them very well, leading to good performance predictions (cache-hit-ratios, queue length distributions). In contrast, the independence model (i-model), fails miserably.

One promising research direction could focus on applying the PQRS model to other spatio-temporal settings (e.g., earthquakes over space and time). Another direction could be the analytical derivation of performance measures of interest (like the cache-hit ratio, or disk queue length distributions), given the  $p, q, r, s$  values of a trace.

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