

Derivation of Linear Index Distribution

Initial Setup

For a sequence of length n , we want weights proportional to the position:

$$\text{weights} = \{1, 2, 3, \dots, n\}$$

Total Weight

Total weight is the sum of arithmetic sequence:

$$\text{total_weight} = \frac{n(n+1)}{2}$$

Random Value Generation

Generate random value r in $[0, \text{total_weight})$:

$$r \in [0, \frac{n(n+1)}{2})$$

Finding the Index

To find index i , we need to solve:

$$\frac{i(i+1)}{2} = r$$

Multiply both sides by 2:

$$i^2 + i = 2r$$

Rearrange to standard quadratic form:

$$i^2 + i - 2r = 0$$

Using quadratic formula:

$$i = \frac{-1 \pm \sqrt{1 + 8r}}{2}$$

Since we need positive i , we take:

$$i = \frac{-1 + \sqrt{1 + 8r}}{2}$$