University of engineering & technology Peshawar

Complex Veriable
Quiz no#01

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Section: B

Reg No: 19PWCSE1795

Semester: 3rd

"On my honor, as a student of University of Engineering and Technology Peshawar, I have neither given nor received unauthorized assistance on this academic work"

Student signature: ____________________

Submitted to:
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Quizz No 1 Complex Variable.

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Q 1.

Given

$$f(z) = z - i$$

$$f(z) = x + yi - i$$

Using CRE

$U = x$ $\quad\quad$ $V = y - 1$

$U_x = \dfrac{\partial x}{\partial x}$ $\quad\quad$ $V_x = \dfrac{\partial}{\partial x}(y-1)$

$U_x = 1$ $\quad\quad$ $V_x = 0$

$U_y = \dfrac{\partial}{\partial y} x = 0$ $\quad\quad$ $V_y = \dfrac{\partial}{\partial y}(y-1)$

$U_y = 0$ $\quad\quad$ $V_y = 1$

Now if we notice that

$U_x = V_y$ & $U_y = -V_x$

i.e

P.T.O

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$1 = 1$ $\qquad 0 = 0$

it is Analytic

Now put $x = 0$ & $y = 1$

in $f(z)$

$$f(z) = x + (y-1)i$$

$$= 0 + (1-1)i$$

$$f(z) = 0$$

it is not analytic if

$y = 1$ & $x \leq 0$.

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Q 2: Given:

$$f(z) = e^{-z^4}$$

Sol

$$f(z) = e^{-z^4}$$

$$f(0) = e^{-0} = 1$$

but given $f(0) = 0 \neq 1$

So function is no Continuous

Obviously not analytic

but $\frac{\partial f}{\partial \bar{z}} = \frac{\partial}{\partial \bar{z}} e^{-z^4} = 0$

hence Cauchy - Riemann eq

Satisfied.

Q3: Given

Sol $z = \sin \cosh v + i \cos u \sinh v$

Ceases to be analytic at

$w = u + iv$

$$\frac{\partial z}{\partial w} = \frac{\partial z}{\partial u} = \frac{\partial}{\partial u}(\sin u \cdot \cosh v + i \cos u \sinh v)$$

$$= \cosh v \frac{\partial \sin u}{\partial u} + i \sinh v \frac{\partial \cos u}{\partial u}$$

$$= \cosh v \cos u + i \sinh v (-\sin u)$$

$$\frac{\partial z}{\partial w} = \cosh v \cos u - i \sin u \cdot \sinh v$$

Squaring both side

$$\left(\frac{\partial z}{\partial w}\right)^2 = (\cos u \cdot \cosh v - i \sin u \cdot \sinh v)^2$$

$$= \cos^2 u \cosh^2 v + i \sin^2 u \cdot \sinh^2 v$$
$$- 2i(\cos u \cdot \cosh v)(\sin u \cdot \sinh v)$$

$$= (1 - \sin^2 u) \cosh^2 v + (-1)(1 - \cos^2 u)$$
$$\cdot \sinh^2 v - 2(\sin u \cosh v)(i \cos u \sinh v)$$

$$= \cosh^2 v - (\cosh^2 v)(\sin^2 u) - \sinh^2 v$$
$$+ \cos^2 u \sinh^2 v - 2(\sin u \cosh v)(i \cos u \sinh v)$$

P T O

$$= (\cosh^2 v - \sinh^2 v) - [\sin^2 u \cosh^2 v - \cos^2 u \sinh^2 v$$

$$+ 2(\sin u \cosh v)(i \cos u \sinh v)$$

$$= 1 - [(\sin u \cosh v)^2 + i^2 (\cos u \sinh v)^2$$

$$+ 2(\sin u \cosh v)(i \cos u \sinh v)]$$

$$= 1 - [(\sin u \cosh v)^2 + (i \cos u \sinh v)^2$$

$$+ 2(\sin u \cosh v)(i \cos u \sinh v)]$$

$$= 1 - [\sin u \cosh v + i \cos u \sinh v]^2$$

$$\left(\frac{\partial z}{\partial w}\right)^2 = 1 - z^2$$

$$\frac{\partial z}{\partial w} = \sqrt{1 - z^2}$$

$$\frac{\partial w}{\partial z} = \frac{1}{\sqrt{1 - z^2}}$$

$\Rightarrow$ for w be analytic then $\frac{dw}{dz}$ is define/finite therefor w ceases to be analytic

i get to know here that

$$1 - z^2 = 0$$

xx ——— xx ——— xx

Q) 4: Given

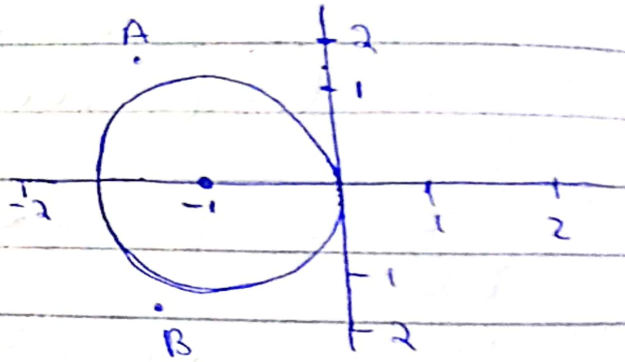
$$\int_C \frac{z+4}{z^2+2z+5}\,dz$$

C is circle $|z+1|=1$

Sol

finding the root of z^2+2z+5

we get:-



→ finding roots of z^2+2z+5

we get:-

$z=(-1+2i)$, $z=(-1,-2)$

↳ $|z+1|=1$

⇒ Center $(-1,0)$

⇒ radius $=1$

Point $A=(-1,2)$

Point $B=(-1,-2)$

x —— x x ——— x x ——— x x ———