Complex Numbers:

Definition:- let (a,b) be a complex number denoted by z & $Z = (a,b)$ or $a+ib$, where "a" is called the real Part of "z" is denoted by $Re(Z)$. Similarly "b" is called the imaginary Part of "Z" & is written as $Im(Z)$.

Imaginary unit:- The number $(0,1)$ is called the imaginary unit & is denoted by "i" We have
$i^2 = -1$ or $i = \sqrt{-1}$

Note:- Any complex number whose real Part is zero is called Pure imaginary e.g $(0,3)$, $-4i$ etc.

Binary operation:-

Def:- "A binary operation is a function which converts an order Pair into a Single number"

Addition of complex Numbers:

let $Z_1 = (a_1, b_1)$ & $Z_2 = (a_2, b_2)$, addition of two complex numbers are defined by

$$Z_1+Z_2=(a_1,b_1)+(a_2,b_2)$$
$$=(a_1+ib_1)+(a_2+ib_2)$$
$$=a_1+a_2+i(b_1+b_2)$$
$$=(a_1+a_2,\ b_1+b_2)$$

Note:- commutative ~~Property~~ law w.r.t addition also hold for complex No: ie

$$Z_1+Z_2=Z_2+Z_1$$

Multiplication of complex No:

Let us consider two complex numbers $Z_1=(x_1,y_1)$ & $Z_2=(x_2,y_2)$ Now their multiplication is defined as:

$$Z_1\cdot Z_2=(x_1,y_1)\cdot(x_2,y_2)$$
$$=(x_1+iy_1)\cdot(x_2+iy_2)$$
$$=x_1(x_2+iy_2)+iy_1(x_2+iy_2)$$
$$=x_1x_2+ix_1y_2+iy_1x_2+i^2y_1y_2$$
$$=x_1x_2+i(x_1y_2+y_1x_2)-y_1y_2$$
$$=(x_1x_2-y_1y_2)+i(x_1y_2+y_1x_2)$$

$$\boxed{Z_1.Z_2=(x_1x_2-y_1y_2,\ x_1y_2+x_2y_1)}$$

Note:- commutative law w.r.t multiplication also hold for complex No: i.e

$$Z_1 \cdot Z_2 = Z_2 \cdot Z_1.$$

Division of complex Numbers :-

let $Z_1 = (x_1 + iy_1)$ & $Z_2 = (x_2 + iy_2)$ be the two complex number. Then their quotient is defined as:

$$Z = \frac{Z_1}{Z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}$$

$$Z = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2}$$

$$= \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{(x_2)^2 - (iy_2)^2}$$

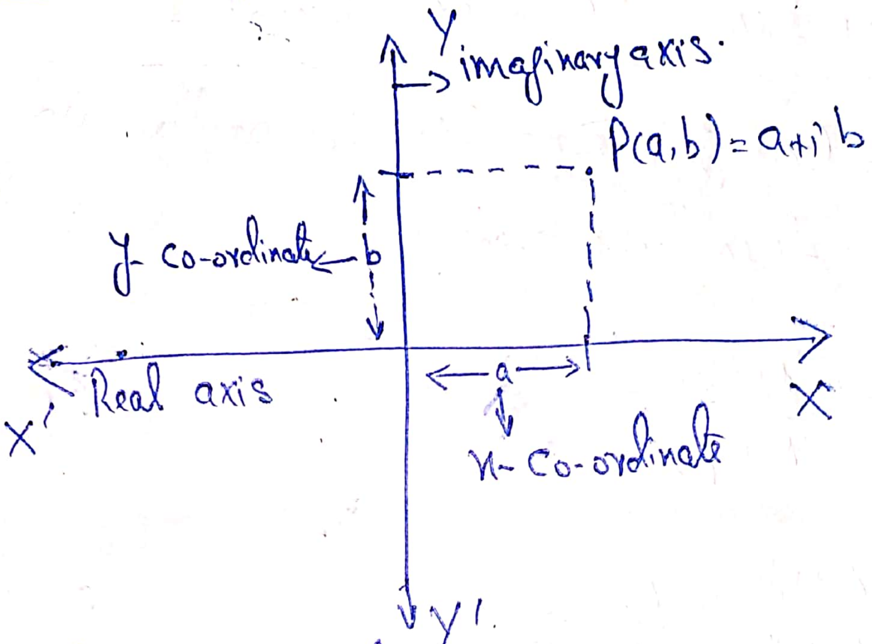
$$= \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\frac{(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

$Re\left(\frac{z_1}{z_2}\right) = \frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}$ & $Im(R) = \frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$.

— X —

Complex plane :-



Complex conjugate:-

Let $Z = a+ib$ be a complex number, then a number of the form $(a-ib)$ is called the complex conjugate of $(a+ib)$. The complex conjugate of a complex number is denoted by "$\bar{Z}$"

e.g. (i) $Z_1 = 3+4i \Rightarrow \bar{Z}_1 = 3-4i$

(ii) $Z_2 = -3-4i \Rightarrow \bar{Z}_2 = -3+4i$

(iii) $Z_3 = 3-4i \Rightarrow \bar{Z}_3 = 3+4i$

Properties of complex conjugates:

Let z_1 & z_2 be two complex numbers. Then it satisfied the following Properties.

① $\overline{z_1+z_2} = \bar{z}_1+\bar{z}_2$

② $\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

③ $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

④ $\overline{z_1-z_2} = \bar{z}_1-\bar{z}_2$.

~o~o~o~o

Exercise # 12.1

Let $z_1 = 4-5i$ and $z_2 = 2+3i$. Find (in form $x+iy$).

Q2 $z_1 z_2$

Sol: $z_1 z_2 = (4-5i)(2+3i)$

$= \boxed{23+2i}$

Q3 $(z_1+z_2)^2$

Sol: $z_1+z_2 = (4-5i)+(2+3i) = 6-2i$

Sq: Both sides.

$(z_1+z_2)^2 = (6-2i)^2 = \boxed{32-24i}$