

# **Signals & Systems Laboratory**

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**CSE- 301L**

**Lab # 05**

## OBJECTIVES OF THE LAB

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In this lab, we will cover the following topics:

- *Gain familiarity with Complex Numbers and plot them*
  - *Complex exponential signals*
  - *Real exponential signals*
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## 5.1 COMPLEX NUMBERS

A complex number  $z$  is an ordered pair  $(x, y)$  of real numbers. Complex numbers can be represented in rectangular form as  $z = x + iy$ , which is the vector in two-dimensional plane. The horizontal coordinate  $x$  is called the *real part* of  $z$  and can be represented as  $x = \text{Re}\{z\}$ , while the vertical coordinate  $y$  is called the *imaginary part* of  $z$  and represented as  $y = \text{Imag}\{z\}$ . That is:

$$\begin{aligned} z &= (x, y) \\ &= x + iy \\ &= \text{Re}\{z\} + i \text{Imag}\{z\} \end{aligned}$$

Another way to represent a complex number is in polar form. In polar form, the vector is defined by its length ( $r$ ) or magnitude ( $|z|$ ) and its direction ( $\theta$ ). A rectangular form can be converted into polar form using formulas:

$$\begin{aligned} |z| &= r = (x^2 + y^2)^{1/2} \\ \theta &= \arctan(y/x) \\ z &= r e^{j\theta} \end{aligned}$$

where  $e^{j\theta} = \cos \theta + i \sin \theta$ , and known as the Euler's formula.

## 5.2 BUILT-IN MATRIX FUNCTIONS

### Function Description

<b>real</b>	returns the real part $x$ of $z$
<b>imag</b>	returns the imaginary part $y$ of $z$
<b>abs</b>	returns the length $r$ of $z$
<b>angle</b>	returns the direction $\theta$ of $z$
<b>conj</b>	returns the complex conjugate $\bar{z}$ of $z$

Here are some examples:

### Example

To define the complex number, for instance,  $z = (3, 4)$  in matlab write in matlab

```
editor >> z = 3 + 4i

z =

    3.0000 + 4.0000i
```

### Example

To find the real and imaginary parts of the complex number,

```
write >> x = real(z)

x =

    3

>> y = imag(z)

y =

    4
```

### Example

To find the length and direction of z,

```
write >> r = abs(z)

r =

    5

>> θ = angle(z)

θ =

    0.9273
```

### Example

To find the conjugate of z, write

```
>> zx = conj(z)

zx =

    3.0000 - 4.0000i
```

## -----TASK 01-----

Write matlab function **zprint**, which takes a complex number and returns its real part, imaginary part, magnitude, phase in radians, and phase in degrees.

A sample run of program is:

```
>> zprint(z)

Z = X + jY Magnitude Phase Ph(deg)
    3    4    5    0.927 53.13
```

## -----TASK 02-----

Compute the conjugate  $\bar{z}$  (i.e. `z_conj` [give variable name]) and the inverse  $1/z$  (i.e. `z_inv` [give variable name]) for any complex number  $z$ . Display the results numerically with `zprint`.

## -----TASK 03-----

Take two complex number and compute  $z_1 + z_2$  and display the results numerically using `zprint`.

## -----TASK 04-----

Take two complex numbers and compute  $z_1 z_2$  and  $z_1/z_2$ . Use `zprint` to display the results numerically.

### 5.3 COMPLEX EXPONENTIAL SIGNALS

The complex exponential signal is defined as

$$x'(t) = A e^{(j \omega_0 t + \phi)}$$

which is a complex-valued function of  $t$ , where the magnitude of  $x'(t)$  is

$$|x'(t)| = A \quad \Rightarrow \quad \text{magnitude or length of } x'(t)$$

$$\arg x'(t) = (\omega_0 t) \quad \Rightarrow \quad \text{angle or direction of } x'(t)$$

Using Euler's formula, it can be expressed in rectangular or Cartesian form,

$$\text{i.e. } x'(t) = A e^{(j \omega_0 t + \phi)} = A \cos(\omega_0 t + \phi) + j A \sin(\omega_0 t + \phi)$$

where

$A$  = amplitude,

$\phi$ =phase shift

$\omega_0$  = frequency in rad/sec

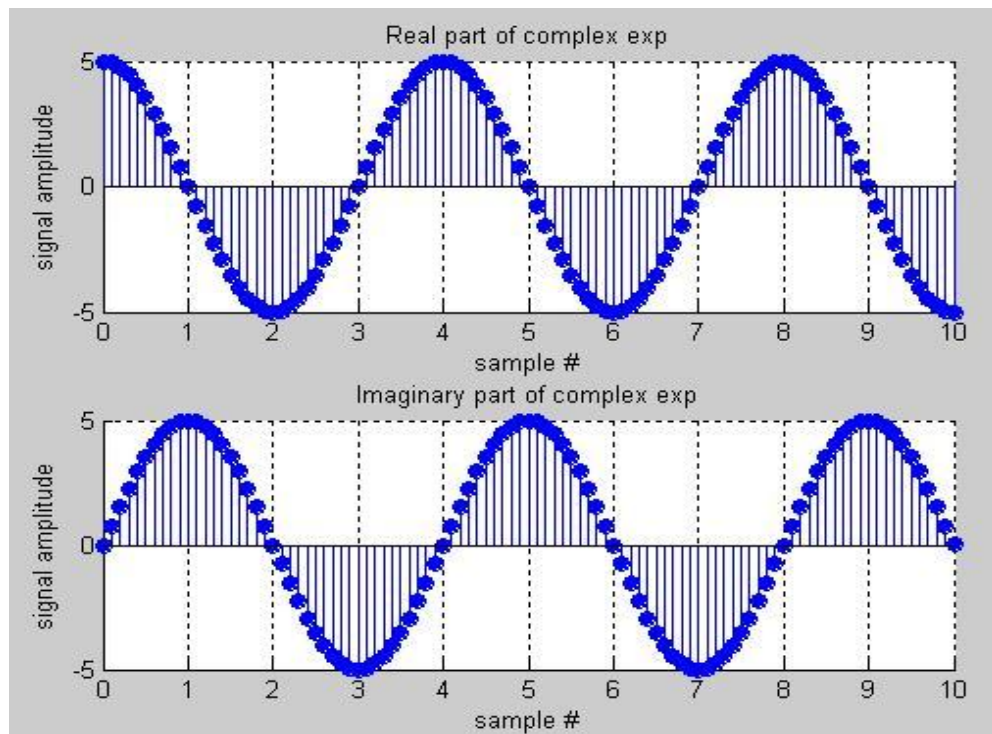
#### Example

```
clear, close all, clc
n=0:1/10:10;
k=5;
a=pi/2;
x=k * exp(a*n*i);
% plot the real part
subplot(2,1,1)
stem(n, real(x), 'filled')
```

```

title('Real part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid
% plot the imaginary
part subplot(2,1,2)
stem(n, imag(x), 'filled')
title('Imaginary part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid

```



## -----TASK 05-----

Determine the complex conjugate of the exponential signal given in above example and plot its real and imaginary portions.

### -----TASK 06-----

Generate the complex valued signal and plot its magnitude, phase, the real part, and the imaginary part in separate subplots.

$$y(n) = \exp^{(-0.2 + j0.5n)}, -10 \leq n \leq 10$$

### -----TASK 07-----

- Generate a real-exponential  $x = a * \exp^{(n)}$  for  $a=0.7$  and  $n$  ranging from 0-10. Find the discrete time as well as the continuous time version of this signal. Plot the two signals on same graph (holding both the graphs).
- Repeat the same program with value of  $a=1.3$ .

### -----TASK 08-----

Multiply the two discrete signals  $x_1 = 5 \exp^{(i*n*\pi/4)}$  and  $x_2 = a * \exp^{(n)}$  (use point-by-point multiplication of the two signals). Plot the real as well as the exponential parts for  $0 < a < 1$  and  $a > 1$ .

### -----TASK 09-----

Plot the discrete signal  $x = a^{|n|}$  for  $n$  ranging from -10 to 10. Draw two subplots for  $0 < a < 1$  and  $a > 1$ .

### -----TASK 10-----

- Generate the signal  $x(t) = A e^{(j\omega t + \pi)}$  for  $A = 3$ ,  $\pi = -0.4$ , and  $\omega = 2\pi(1250)$ . Take a range for  $t$  that will cover 2 or 3 periods.
- Plot the real part versus  $t$  and the imaginary part versus  $t$ . Use subplot(2,1,i) to put both plots in the same window.
- Verify that the real and imaginary parts are sinusoids and that they have the correct frequency, phase, and amplitude.