



# Probability Methods in Engineering

CSE-209

Dr. Safdar Nawaz Khan Marwat  
DCSE, UET Peshawar

Lecture 3



# Sample Space

➤ Same experimental procedure but different sample spaces

- ❑ Toss a coin three times and note the outcomes
- ❑ Toss a coin three times and note the number of heads

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$S = \{0, 1, 2, 3\}$$

➤ Same sample space but different representations

- ❑ Pick a number at random between zero and one
- ❑ Measure the time between page requests in a Web server

$$S = \{x : 0 \leq x \leq 1\}$$

$$S = \{x : x \geq 0\}$$

$$[0, 1]$$

$$[0, \infty)$$





# Sample Space (cont.)

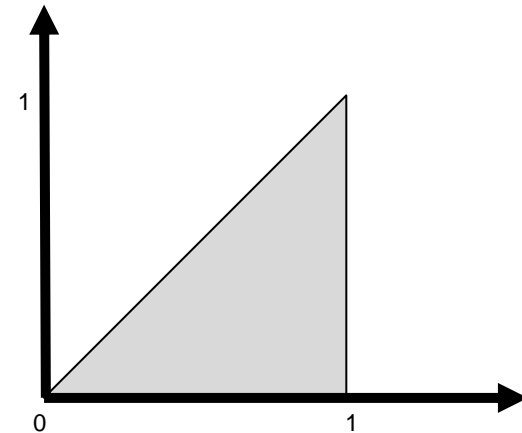
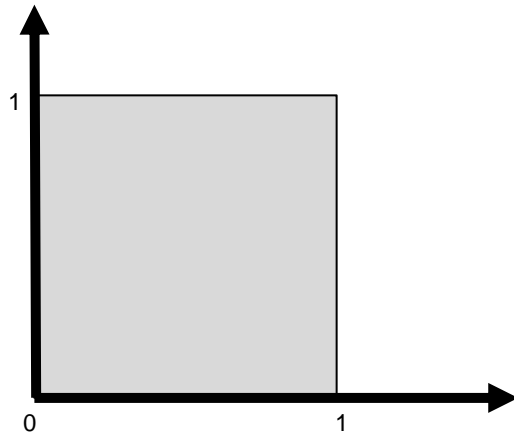
## ➤ Multidimensional sample spaces

- ❑ Pick two numbers at random between zero and one
- ❑ Pick a number  $x$  at random between zero and one, then pick a number  $y$  at random between zero and  $y$

$$S = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$$

$$S = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x\}$$

$$S = \{(x, y) : 0 \leq y \leq x \leq 1\}$$





# Set Theory

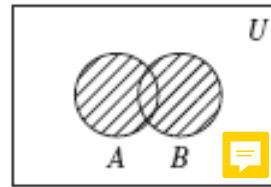
- Representation of events by sets
- Capital letters for names  $S, A, B, \dots$
- Small letters for elements  $a, b, x, y, \dots$
- Venn diagram illustrates sets and their interrelationship

Source: C. Görg, *Communication Networks II*, University of Bremen, Germany

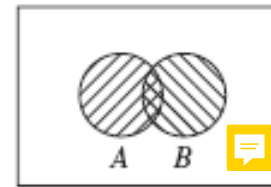




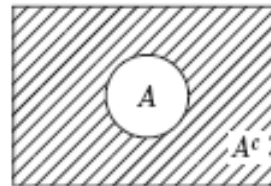
# Set Theory (cont.)



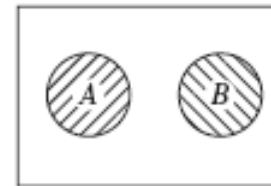
(a)  $A \cup B$



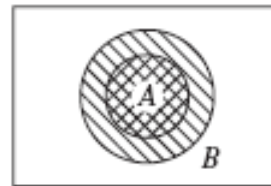
(b)  $A \cap B$



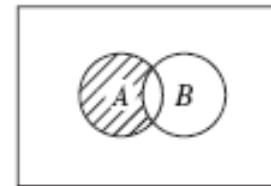
(c)  $A^c$



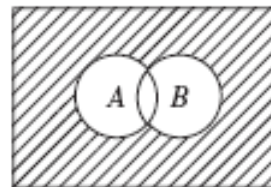
(d)  $A \cap B = \emptyset$



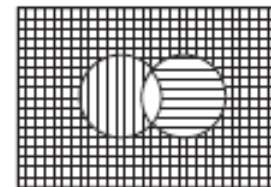
(e)  $A \subset B$



(f)  $A - B$



(g)  $(A \cup B)^c$



(h)  $A^c \cap B^c$



# Axioms of Probability

$$0 \leq P[A] \leq 1$$

$$P[S] = 1$$

➤ If  $A \cap B = \emptyset$

$$P[A \cup B] = P[A] + P[B]$$

➤ If  $A_i \cap A_j = \emptyset$  for all  $i \neq j$

$$P\left[\bigcup_{k=1}^{\infty} A_k\right] = \sum_{k=1}^{\infty} P[A_k]$$



# Corollaries

- Rules or propositions based on or derived from axiom(s)

$$P[A^c] = 1 - P[A]$$

$$P[\emptyset] = 0$$

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[B \cap C] - P[A \cap C] + P[A \cap B \cap C]$$



# Examples

## *Discrete countably finite sample space*

- An urn contains 10 identical balls numbered 0, 1, ... , 9. A random experiment involves selecting a ball from the urn and noting the number of the ball. Find the probability of the following events:
- ☐  $A = \text{"number of ball selected is odd,"}$
  - ☐  $B = \text{"number of ball selected is a non-zero multiple of 3,"}$
  - ☐  $C = \text{"number of ball selected is less than 5,"}$
  - ☐  $D = A \cup B$
  - ☐  $E = A \cup B \cup C$





# Examples (cont.)

## *Discrete countably infinite sample space*

- A fair coin is tossed repeatedly until the first heads shows up; the outcome of the experiment is the number of tosses required until the first heads occurs. Find a probability law for this experiment.



# Examples (cont.)

## *Continuous uncountably infinite sample space*

- Consider the random experiment “pick a number  $x$  at random between zero and one.” Let the probability that the outcome falls in a subinterval of  $S$  be proportional to the length of the subinterval.
  - ❑ What is the sample space of this random experiment?
  - ❑ What is the probability that the outcome falls in the interval  $[0, 0.5]$ ,  $[0.5, 1]$ ?
  - ❑ What is the probability that the outcome is 0.5?
  - ❑ What is the probability that the outcome falls in either  $[0, 0.2]$  or  $[0.8, 1]$ ?
  - ❑ What is the probability that the outcome falls in either  $[0.3, 0.6]$  or  $[0.5, 0.8]$ ?