page (1)

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Final term paper

Linear Algebra:

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Reg No: 19PWCSE1795

Section: B

Date: 12/03/2021

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S M T W T F S Date:.....................

Q No ⇒ 1

Part (a)

Given.

$L: R^3 \to R^3$

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Is w in range L = ?

Sol

Here $w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ if w is in range L than there exist

$\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$ Such That $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w$

So

$$\begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Now we will find determinant

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$$-1\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 0$$

$$-1(1+1) - 2(1-2)$$
$$-2 + 2 = 0$$

Since $|\det| = 0$ But if we operate an augmented matrix (A/b)

$$(A/b) = \left[\begin{array}{ccc|c} -1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & -1 & 1 & -1 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} -1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & 3 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 + R_1 \\ R_3 + 2R_1 \end{array}$$

$$\underset{\sim}{R} \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - R_2$$

Since Rank $= 2$ of $(A) \neq$ Rank of $(A/b) = 3$.

So the system of equation is not Consistent. So we can say that no $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w, \quad \text{So} \quad w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

is not in Range L.

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Q No : (1)

Part (b)

Given.

$U = ai + 2j$

$V = ai + (-2j)$

$V = ai - 2j$

Sol

Since U & V are orthogonal
So

$U \cdot V = 0$

$(ai + 2j) \cdot (ai - 2j) = 0$

$a^2(i \cdot i) - 4(j \cdot j) = 0$

$a^2(1) - 4(1) = 0$

$a^2 - 4 = 0$

$a^2 = 4$

$a = \sqrt{4}$

$a = \pm 2$

So $a = (2, -2)$ Ans

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Q NO 2

Part (a)

Given:

$$u \oplus v = uv$$

&

$$c \otimes u^c = u^c$$

Sol

According to closure law

$u \in R^{\oplus}$, $v \in R^{\oplus} \Rightarrow u \oplus v = uv \in R^{\oplus}$

closed under $\oplus$.

$u \in R^{\oplus} \Rightarrow c \cdot v = u^c \in R^{\oplus}$

$c \in f$

$\Rightarrow$ closed under $\oplus$

$u \oplus v = uv = uv = v \oplus u$

$u \oplus (v \oplus w) = u(vw) = (uv)w$

$(v \oplus v) \oplus w$

$u \oplus 1 = u1 = u \Rightarrow 1$

$u \oplus \frac{1}{u} = |u| = 0$

hence $\frac{1}{u}$ is additive

$\Rightarrow a(bc)u = (u^{bc})^a = u^{abc}$

$(u^{ab})^c = c(ab)u$

$(a+b)u = u^{a+b} = u^a u^b = a \cdot u \oplus b \cdot v$

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$a(U+V) = aUV = (U \cdot V)^a = U^a . V^a = U^a + V^a$
$= aV$

$1 \cdot U = U' = U \quad \forall U \in R^{(+)}$

from above it is cleared that it is a vector space.

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Q No : 2
Part : b

Given:

$a_2 t^2 + a_1 t + a_0$ where $a_0 = 2$

Sol

let $W_2 = \{a_2 t^2 + a_1 t + a_0 \in P_2 \quad a_0 = 2\}$

we have that $P(t) = t^2 + 2 \in W_2$
but $2(p(t)) = 2t^2 + 4 \notin W_2$ thus
W_2 is not closed under
Scaler multiplication of vector
and therefore it is not a
Subspace of P_2.

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Q No : 3

Part — (a)

basis for a Vector Space:

let V be a Subspace of R^n for Same n. A Collection $B = \{V_1, V_2, \dots V_r\}$ of Vectors

from V is Said to be a basis for V if B is linearly Independent and Span V. if either one of These criterial is not Satisfied, then collection is not a basis for Vector V. if collection of vector spans V, then it contain Enough Vectors So that every Vector in V can be written as a linear Combination of those in the collection. if collection is linearly Independent, then it does not contain So many Vectors that Same become dependent on the others. Intuitively, then a basis has Just the right size: its's big enough to span the Space but not So big as to be dependent.

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Example:

the collection $\{i + i+j, 2j\}$ is not a basis for R^2. Although it spans R^2, it is not linearly independent. No collection of 3 or more vectors from R^2 can be independent.

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Part (b)

Given:

$$\{(3,2,2), (-1,2,1), (0,1,0)\}$$

Sol

the basis vectors are

$$\begin{bmatrix}3\\2\\2\end{bmatrix}, \begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$$

it will be for a basis of R^3 if

$$\det\begin{vmatrix}3 & -1 & 0\\2 & 2 & 1\\3 & 1 & 0\end{vmatrix} \neq 0$$

from coloumn 3

$$-1(3+2) \neq 0$$

$$-5 \neq 0$$

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Hence,

$$\begin{bmatrix}3\\2\\2\end{bmatrix}, \begin{bmatrix}-1\\2\\1\end{bmatrix} \text{ \& } \begin{bmatrix}0\\1\\0\end{bmatrix} \text{ form a}$$

basis for R^3.

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Q4

Part ⇒ (a)

Given:

$$A = \begin{bmatrix}1 & 0 & 0\\ -1 & 3 & 0\\ 3 & 2 & -2\end{bmatrix}$$

Sol

For characteristic polynomial ~~$\det(A-\lambda I)=0$~~

det of poly $(A - \lambda I) = 0$

$$|A| = \begin{vmatrix}1-\lambda & 0 & 0\\ -1 & 3-\lambda & 0\\ 3 & 1 & -2-\lambda\end{vmatrix}$$

expand first row

$$(1-\lambda)(3-\lambda)(-2-\lambda) - 0 = 0$$

$$(1-\lambda)(3-\lambda)(-2-\lambda) = 0$$

$$1-\lambda = 0$$

$$3-\lambda = 0$$

S M T W T F S Date:......................

$$-2-\lambda=0$$

$$\lambda=1 \quad , \lambda=3 \quad , \lambda=-2$$

$$\lambda=\{-2,1,3\}$$

(1) for $\lambda=1$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2V_1+2V_2=0$$
$$3V_1+V_2-3V_3=0$$

from solving it we get

$$V_1=\begin{bmatrix} 3/4 \\ 3/8 \\ 1 \end{bmatrix}$$

(2) for $\lambda=-2$

$$\begin{bmatrix} 3 & 0 & 0 \\ -1 & 5 & 0 \\ 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3V_1=0$$
$$-V_1+5V_2=0$$
$$3V_1+2V_2=0$$

by solving the equation we will get

$$V_2=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(3) for $\lambda=3$

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$$\begin{bmatrix} -2 & 0 & 0 \\ -1 & 0 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2V_1 = 0$

$-V_1 = 0$

$3V_1 + 2V_2 - 5V_3 = 0$

By solving we get

$$V_3 = \begin{bmatrix} 0 \\ 5/2 \\ 1 \end{bmatrix}$$

Result:

$\Rightarrow$ Eigen Value = 1 ~~Eigen vector~~

Eigen vector = $\begin{bmatrix} 3/4 \\ 3/8 \\ 1 \end{bmatrix}$

$\Rightarrow$ Eigen Value = -2

Eigen vector = $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow$ Eigen value = 3

Eigen vector = $\begin{bmatrix} 0 \\ 5/2 \\ 1 \end{bmatrix}$

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Q No ⇒ 4

Part (b)

Given:

2x2 nondiagonal matrix = ??
eigen values = 2, −3

Eigen vectors are,

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} \text{ \& } \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Sol

We have to find a 2x2 non-diagonal matrix $A \in M_2(R)$ Such that Eigen values are 2 & −3 and associated eigen vector resp are $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

from above Condition we get

$$A \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

Now $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are linearly Independent so it form a basis of R^2 - Now equating each component

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we get that

$(x, y) = (c_1 - c_2,\ c_1 + 2c_2)$ which implies that $x = c_1 - c_2$ and

$y = c_1 + 2c_2$

From this we get that

$c_1 = \frac{2x+y}{3}$ & $c_2 = \frac{y-x}{3}$

Hence

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{2x+y}{3} A\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{y-x}{3}$$

$A\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ since any matrix is a linear operator therefore we get

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{2x+y}{3}\begin{bmatrix} -3 \\ -3 \end{bmatrix} + \frac{y-x}{3}\begin{bmatrix} -2 \\ 4 \end{bmatrix}$$ which

Implies that

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -(2x+y) \\ -(2x+y) \end{bmatrix} + \frac{1}{3}\begin{bmatrix} -2y + 2x \\ 4y - 4z \end{bmatrix}$$

which implies that

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4z - 5y \\ -10z - 4z \end{bmatrix}$$

which implies that

$$A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4z - 5y \\ -10z + y \end{bmatrix}$$ which implies that

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$$A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4 & -5 \\ -10 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Now let us take

$A = \frac{1}{3}\begin{bmatrix} -4 & -5 \\ -10 & 1 \end{bmatrix}$ clearly A is a non-diagonal matrix

$$\frac{1}{3}\begin{bmatrix} -4 & -5 \\ -10 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$ and

$$\frac{1}{3}\begin{bmatrix} -4 & -5 \\ -10 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = -3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

thus we Satisfied ~~test~~ the the given Condition, Hence A is a Squared matrix.

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the END

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