Signals & Systems Laboratory

CSE- 301L

Lab # 10

OBJECTIVES OF THE LAB

This lab aims at the understanding of:

- Fourier Series Representation of Continuous Time Period Signals
- Convergence of Continuous Time Fourier Series

10.1 FOURIER SERIES REPRESENTATION OF CONTINUOUS TIME PERIOD SIGNALS

A signal expressed by the formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$$

is periodic with period T, as it is linear combination of *harmonically related complex exponentials* that are all periodic with T. Any well-behaving periodic function can be expressed as a linear combination of harmonically related complex exponentials. The representation of periodic signal in this way is known as *Fourier series* representation and the weight a_K's are referred to as Fourier series coefficients. Given a periodic signal x(t), it is possible to determine its Fourier series coefficients through the following integral.

$$a_k = \int_{-T}^{T} x(t) e^{-jkw_0 t} dt = \int_{-T}^{T} x(t) e^{-jk(2\pi/T)t} dt$$

This integral can be done over any time interval of length T, the period of the signal x(t).

10.1.1 Synthesis of a Simple Periodic Signal

Following example demonstrates that the linear combination of harmonically related complex exponentials leads to a periodic function. The signal used in example is:

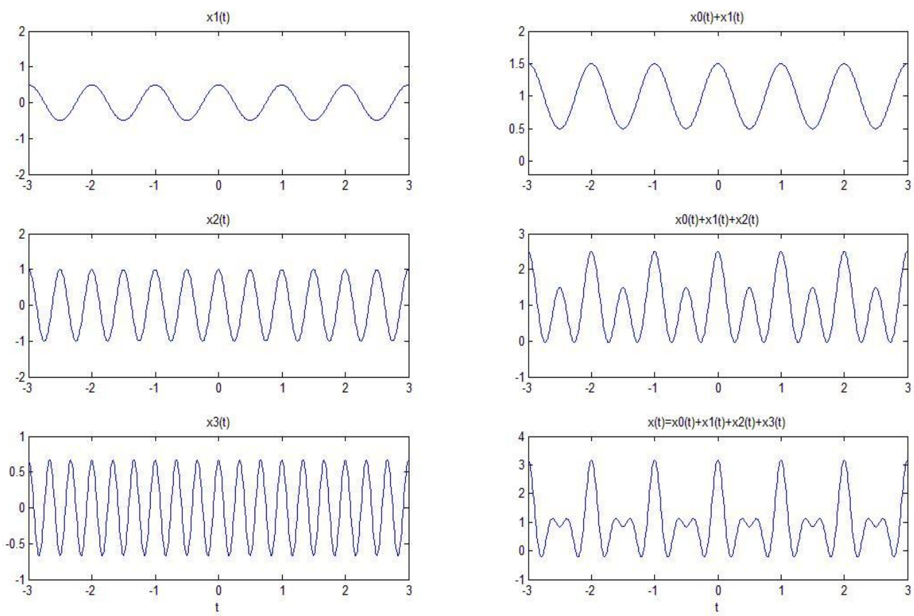
$$x(t) = \sum_{k=-3}^{3} a_k e^{jk\omega_0 t}, \quad where \;\; a_0 = 1, \;\; a_1 = a_{-1} = \frac{1}{4}, \;\; a_2 = a_{-2} = \frac{1}{2}, \;\; a_3 = a_{-3} = \frac{1}{3}$$

Example – FS of CT Periodic Signal

```
clc
clear all
close all

t = -3:0.01:3;            % duration of signal
% dc component for k=0
x0 = 1;

% first harmonic components for k=-1 and k=1
x1 = (1/4)*exp(j*(-1)*2*pi*t)+(1/4)*exp(j*(1)*2*pi*t);
```

```
y1 = x0 + x1;                % sum of dc component and first harmonic

% second harmonic components for k=-2 and k=2
x2 = (1/2)*exp(j*(-2)*2*pi*t)+(1/2)*exp(j*(2)*2*pi*t);
y2 = y1 + x2;                % sum of all components until second harmonic

% third harmonic components for k=-3 and k=3
x3 = (1/3)*exp(j*(-3)*2*pi*t)+(1/3)*exp(j*(3)*2*pi*t);
x = x0 + x1 + x2 + x3;    % sum of all components until third harmonic

figure;
subplot(3,2,1);
plot(t,x1);
axis([-3 3 -2 2]);
title('x1(t)');

subplot(3,2,2);
plot(t,y1); axis([-3
3 -0.2 2]);
title('x0(t)+x1(t)');

subplot(3,2,3);
plot(t,x2);
axis([-3 3 -2 2]);
title('x2(t)');

subplot(3,2,4);
plot(t,y2);
axis([-3 3 -1 3]);
title('x0(t)+x1(t)+x2(t)');

subplot(3,2,5);
```

```
plot(t,x3);
xlabel('t'); axis([-3 3 -1 1]); title('x3(t)');

subplot(3,2,6);
plot(t,x);
xlabel('t'); axis([-3 3 -1 4]);
title('x(t)=x0(t)+x1(t)+x2(t)+x3(t)')
```



---------------------------TASK 1---------------------------

In above example, a_k's are chosen to be symmetric about the index k=0, i.e. $a_k = a_{-k}$. Select new a_k's on your own to alter this symmetry and form the new signal. What do you observe? Is x(t) a real signal when coefficients are not symmetric?

---------------------------TASK 2---------------------------

A discrete-time periodic signal x[n] is real valued and has a fundamental period of N = 5. The non-zero Fourier series coefficients for x[n] are:

$$a_0 = 1, \quad a_2 = a_{-2}^* = e^{j\frac{\pi}{4}}, \quad a_4 = a_{-4}^* = 2e^{j\frac{\pi}{3}}$$

Express x[n] as linear combination of given coefficients.

10.1.2 Synthesis of a Simple Periodic Signal

Once the Fourier series (FS) coefficient of a continuous time periodic signal is determined analytically using analysis equation, signal can be reconstructed using synthesis equation. Consider the periodic square wave signal defined as:

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < T/2 \end{cases}$$

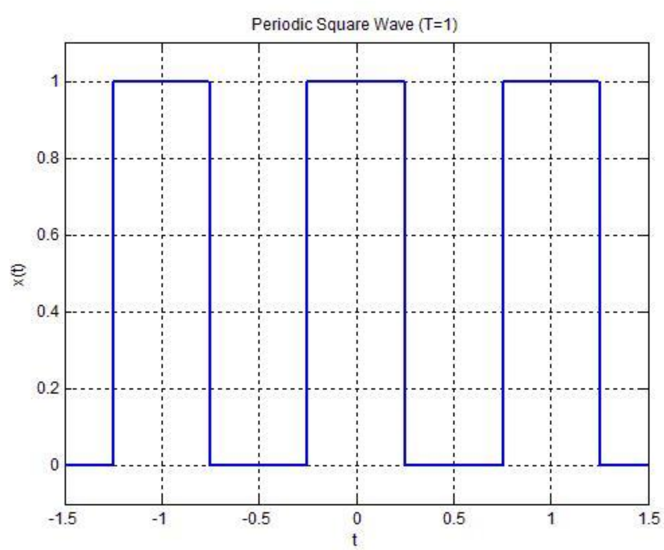
where T is the time period and T_1 is the duty cycle with FS coefficients

$$a_0 = \frac{2T_1}{T}\ ,\ a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} = \frac{\sin(k\,2\pi(T_1/T))}{k\pi} \quad for\ \ k \neq 0$$

In the following examples, first and ideal square wave is created and then square wave is approximated from its harmonics using synthesis equation by letting k in the partial sum go from –M to M instead of -∞ to + ∞ , where M is 10, 20, and 100. In all examples T is taken as 1 sec.

Example – Ideal Square Wave created by thresholding 1 Hz Cosine Wave

```
t = -1.5:0.005:1.5;          %duration of square wave
xcos = cos(2*pi*t);         %cosine wave of 1 Hz
xpsqw = xcos>0;             %thresholding cosine wave using relational operator
figure
plot(t,xpsqw,'lineWidth',2);
xlabel('t');
ylabel('x(t)');
title('Periodic Square Wave (T=1)');
axis([-1.5 1.5 -0.1 1.1]);
grid;
```



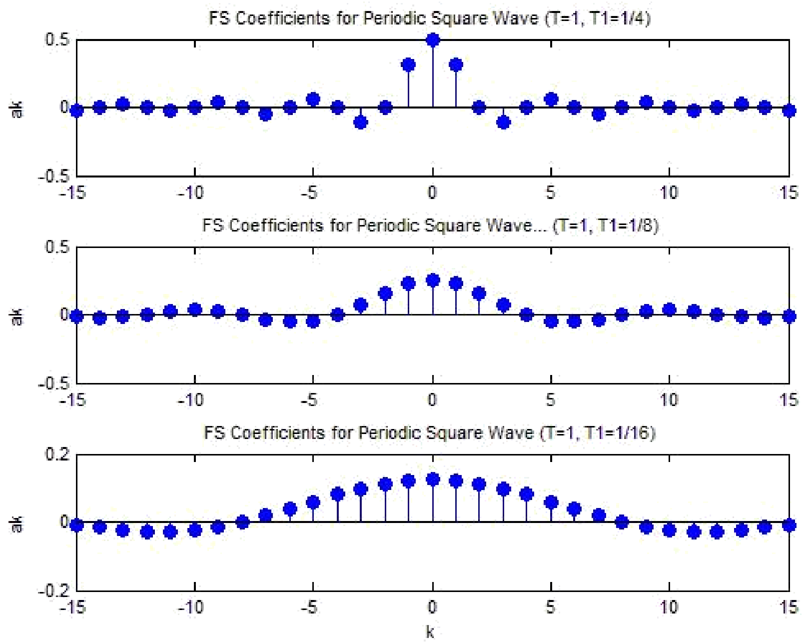
Example – FS coefficients for Square wave with period 1 sec & variable duty cycle

```
k = -15:15;                  %number of square wave coefficients
T = 1;                       %time period of square wave
T1 = 1/4;                    %duty cycle of square wave
ak1 = sin(k*2*pi*(T1/T))./(k*pi);          %square wave Fourier series coefficients

% Ignore the "divide by zero" warning that happens
% because k in the denominator hits 0. We will now do
% a manual correction for a0 -> ak1(16)

ak1(16) = 2*T1/T;

figure;
subplot(3,1,1);
stem(k,ak1,'filled');
ylabel('ak');
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/4)');

T1 = 1/8;
ak2 = sin(k*2*pi*(T1/T))./(k*pi);
ak2(16) = 2*T1/T;                % Manual correction for a0 -> ak2(16)

subplot(3,1,2);
stem(k,ak2,'filled');
ylabel('ak');
title('FS Coefficients for Periodic Square Wave... (T=1, T1=1/8)');

T1 = 1/16;
ak3 = sin(k*2*pi*(T1/T))./(k*pi);
ak3(16) = 2*T1/T;                % Manual correction for a0 -> ak3(16)
```

```
subplot(3,1,3);
stem(k,ak3,'filled');
xlabel('k');
ylabel('ak');
title('FS Coefficients for Periodic Square Wave (T=1, T1=1/16)');
```

---------------------------TASK 4---------------------------

Considering the FS coefficients plot given below, what do you observe happens to the envelope of the coefficients when T_1 is reduced from 1/4 to 1/16 with constant time period T?



---------------------------TASK 5---------------------------

Create the plots of square wave reconstructed using M = 10, 20, & 100 terms above, what do you observe about Gibb's phenomena?

---------------------------TASK 6---------------------------

Given the following FS coefficients:

$$a_k = \begin{cases} 1, & k \quad even \\ 2, & k \quad odd \end{cases}$$

Plot the coefficients & reconstructed signal. Take the terms for reconstructed signal to be M = 10, 20, & 50. What effect do you see when M is varied?

--------------------------TASK 7--------------------------

Given the following FS coefficients:

$$a_k = \begin{cases} jk, & |k| < 3 \\ 0, & otherwise \end{cases}$$

Plot the coefficients & reconstructed signal. Take 10 terms (M=10) for reconstructed signal.
