Key Terms

Origin
Absolute value
Coordinate
Coordinate axis
x-axis
y-axis
Rectangular (or Cartesian) coordinate system
2-space
Direction of a directed line segment
Magnitude of a directed line segment
Vector
Components of a vector
Head of a vector
Tail of a vector
Equal vetors
Length (or magnitude) of a vector
Parallel vectors
Sum of vectors
Scalar multiple of vectors
Zero vector
Negative of a vector
Difference of vectors
Resultant force
Dot product
Perpendicular (or orthogonal) vectors
Unit vector

4.1 Exercises

1. Plot the following points in R^2.
(a) $(2, -1)$ (b) $(-1, 2)$ (c) $(3, 4)$
(d) $(-3, -2)$ (e) $(0, 2)$ (f) $(0, -3)$

2. Sketch a directed line segment in R^2 representing each of the following vectors.
(a) $\mathbf{u}_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ (b) $\mathbf{u}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
(c) $\mathbf{u}_3 = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$ (d) $\mathbf{u}_4 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$

3. Determine the head of the vector $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ whose tail is at $(3, 2)$. Make a sketch.

4. Determine the head of the vector $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ whose tail is at $(1, 2)$. Make a sketch.

5. Find $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, $2\mathbf{u}$, and $3\mathbf{u} - 2\mathbf{v}$ if
(a) $\mathbf{u} = (2, 3)$, $\mathbf{v} = (-2, 5)$
(b) $\mathbf{u} = (0, 3)$, $\mathbf{v} = (3, 2)$
(c) $\mathbf{u} = (2, 6)$, $\mathbf{v} = (3, 2)$

6. Repeat Exercise 5 for
(a) $\mathbf{u} = (-1, 3)$, $\mathbf{v} = (2, 4)$
(b) $\mathbf{u} = (-4, -3)$, $\mathbf{v} = (5, 2)$
(c) $\mathbf{u} = (3, 2)$, $\mathbf{v} = (-2, 0)$

7. Let $\mathbf{u} = (1, 2)$, $\mathbf{v} = (-3, 4)$, $\mathbf{w} = (w_1, 4)$, and $\mathbf{x} = (-2, x_2)$. Find w_1 and x_2 so that
(a) $\mathbf{w} = 2\mathbf{u}$ (b) $\frac{3}{2}\mathbf{x} = \mathbf{v}$ (c) $\mathbf{w} + \mathbf{x} = \mathbf{u}$

8. Let $\mathbf{u} = (-4, 3)$, $\mathbf{v} = (2, -5)$, and $\mathbf{w} = (w_1, w_2)$. Find w_1 and w_2 so that
(a) $\mathbf{w} = 2\mathbf{u} + 3\mathbf{v}$ (b) $\mathbf{u} + \mathbf{w} = 2\mathbf{u} - \mathbf{v}$
(c) $\mathbf{w} = \frac{5}{2}\mathbf{v}$

9. Find the length of the following vectors.
(a) $(1, 2)$ (b) $(-3, -4)$
(c) $(0, 2)$ (d) $(-4, 3)$

10. Find the length of the following vectors.
(a) $(-2, 3)$ (b) $(3, 0)$
(c) $(-4, -5)$ (d) $(3, 2)$

11. Find the distance between the following pairs of points.
(a) $(2, 3)$, $(3, 4)$ (b) $(0, 0)$, $(3, 4)$
(c) $(-3, 2)$, $(0, 1)$ (d) $(0, 3)$, $(2, 0)$

12. Find the distance between the following pairs of points.
(a) $(4, 2)$, $(1, 2)$ (b) $(-2, -3)$, $(0, 1)$
(c) $(2, 4)$, $(-1, 1)$ (d) $(2, 0)$, $(3, 2)$

13. Is it possible to write the vector $(-5, 6)$ as a linear combination (defined before Example 14 in Section 1.3) of the vectors $(1, 2)$ and $(3, 4)$?

14. If possible, find scalars c_1 and c_2, not both zero, so that

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

15. Find the area of the triangle with vertices $(3, 3)$, $(-1, -1)$, $(4, 1)$.

16. Find the area of the right triangle with vertices $(0, 0)$, $(0, 3)$, $(4, 0)$. Verify by using the formula $A = \frac{1}{2}$(base)(height).

17. Find the area of the parallelogram with vertices $(2, 3)$, $(5, 3)$, $(4, 5)$, $(7, 5)$.

18. Let Q be the quadrilateral with vertices $(-2, 3)$, $(1, 4)$, $(3, 0)$, and $(-1, -3)$. Find the area of Q.

19. Find a unit vector in the direction of $\mathbf{x}$.
(a) $\mathbf{x} = (3, 4)$ (b) $\mathbf{x} = (-2, -3)$ (c) $\mathbf{x} = (5, 0)$

20. Find a unit vector in the direction of $\mathbf{x}$.
(a) $\mathbf{x} = (2, 4)$ (b) $\mathbf{x} = (0, -2)$ (c) $\mathbf{x} = (-1, -3)$

21. Find the cosine of the angle between each pair of vectors $\mathbf{u}$ and $\mathbf{v}$.
(a) $\mathbf{u} = (1, 2)$, $\mathbf{v} = (2, -3)$
(b) $\mathbf{u} = (1, 0)$, $\mathbf{v} = (0, 1)$
(c) $\mathbf{u} = (-3, -4)$, $\mathbf{v} = (4, -3)$
(d) $\mathbf{u} = (2, 1)$, $\mathbf{v} = (-2, -1)$

22. Find the cosine of the angle between each pair of vectors **u** and **v**.
(a) $\mathbf{u} = (0, -1)$, $\mathbf{v} = (1, 0)$
(b) $\mathbf{u} = (2, 2)$, $\mathbf{v} = (4, -5)$
(c) $\mathbf{u} = (2, -1)$, $\mathbf{v} = (-3, -2)$
(d) $\mathbf{u} = (0, 2)$, $\mathbf{v} = (3, -3)$

23. Show that
(a) $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = 1$ (b) $\mathbf{i}\cdot\mathbf{j} = 0$

24. Which of the vectors $\mathbf{u}_1 = (1, 2)$, $\mathbf{u}_2 = (0, 1)$, $\mathbf{u}_3 = (-2, -4)$, $\mathbf{u}_4 = (-2, 1)$, $\mathbf{u}_5 = (2, 4)$, $\mathbf{u}_6 = (-6, 3)$ are
(a) orthogonal?
(b) in the same direction?
(c) in opposite directions?

25. Find all constants a such that the vectors $(a, 4)$ and $(2, 5)$ are parallel.

26. Find all constants a such that the vectors $(a, 2)$ and $(a, -2)$ are orthogonal.

27. Write each of the following vectors in terms of **i** and **j**.
(a) $(1, 3)$ (b) $(-2, -3)$
(c) $(-2, 0)$ (d) $(0, 3)$

28. Write each of the following vectors as a 2×1 matrix.
(a) $3\mathbf{i} - 2\mathbf{j}$ (b) $2\mathbf{i}$ (c) $-2\mathbf{i} - 3\mathbf{j}$

29. A ship is being pushed by a tugboat with a force of 300 pounds along the negative y-axis while another tugboat is pushing along the negative x-axis with a force of 400 pounds. Find the magnitude and sketch the direction of the resultant force.

30. Suppose that an airplane is flying with an airspeed of 260 kilometers per hour while a wind is blowing to the west at 100 kilometers per hour. Indicate on a figure the approximate direction that the plane must follow to result in a flight directly south. What will be the resultant speed?

Theoretical Exercises

T.1. Show how we can associate a point in the plane with each ordered pair (x, y) of real numbers.

T.2. Show that $\mathbf{u} + \mathbf{0} = \mathbf{u}$.

T.3. Show that $\mathbf{u} + (-1)\mathbf{u} = \mathbf{0}$.

T.4. Show that if c is a scalar, then $\|c\mathbf{u}\| = |c|\|\mathbf{u}\|$.

T.5. Show that if $\mathbf{x}$ is a nonzero vector, then
$$\mathbf{u} = \frac{1}{\|\mathbf{x}\|}\mathbf{x}$$
is a unit vector in the direction of $\mathbf{x}$.

T.6. Show that
(a) $1\mathbf{u} = \mathbf{u}$
(b) $(rs)\mathbf{u} = r(s\mathbf{u})$, where r and s are scalars

T.7. Prove Theorem 4.1.

T.8. Show that if **w** is orthogonal to **u** and **v**, then **w** is orthogonal to $r\mathbf{u} + s\mathbf{v}$, where r and s are scalars.

T.9. Let θ be the angle between the nonzero vectors $\mathbf{u} = (x_1, y_1)$ and $\mathbf{v} = (x_2, y_2)$ in the plane. Show that if **u** and **v** are parallel, then $\cos\theta = \pm 1$.

MATLAB Exercises

The following exercises use the routine vec2demo, *which provides a graphical display of vectors in the plane. For a pair of vectors* $\mathbf{u} = (x_1, y_1)$ *and* $\mathbf{v} = (x_2, y_2)$, *routine* **vec2demo** *graphs* **u** *and* **v**, $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$, *and a scalar multiple. Once the vectors* **u** *and* **v** *are entered into* MATLAB, *type*

```
vec2demo(u, v)
```

For further information, use **help vec2demo**.

ML.1. Use the routine **vec2demo** with each of the following pairs of vectors. (Square brackets are used in MATLAB.)
(a) $\mathbf{u} = [2 \quad 0]$, $\mathbf{v} = [0 \quad 3]$
(b) $\mathbf{u} = [-3 \quad 1]$, $\mathbf{v} = [2 \quad 2]$
(c) $\mathbf{u} = [5 \quad 2]$, $\mathbf{v} = [-3 \quad 3]$

ML.2. Use the routine vec2demo with each of the following pairs of vectors. (Square brackets are used in MATLAB.)
(a) $\mathbf{u} = [2 \quad -2]$, $\mathbf{v} = [1 \quad 3]$
(b) $\mathbf{u} = [0 \quad 3]$, $\mathbf{v} = [-2 \quad 0]$
(c) $\mathbf{u} = [4 \quad -1]$, $\mathbf{v} = [-3 \quad 5]$

ML.3. Choose pairs of vectors **u** and **v** to use with vec2demo.

Q3 $\vec{PQ} = \begin{bmatrix}-2\\5\end{bmatrix}$ where $P=(3,2)$ $Q=?$ let $Q(a,b)$

then $\vec{PQ} = (a-3, b-2) = (-2,5)$

$a-3=-2 \Rightarrow \boxed{a=1}$ & $b-2=5 \Rightarrow \boxed{b=7}$

Q4 is similarly to Q3.

Q5 Find $u+v$, $u-v$, $2u$ & $3u-3v$ if.

(a) $u=(2,3)$, $v=(-2,5)$.

$u+v=(2-2,\ 3+5)=(0,8)$

$u-v=(2+2,\ 3-5)=(4,-2)$

$2u = 2[2,3] = [4,6]$.

$3u-2v = 3[2,3]-2[-2,5] = (6,9)-(-4,10)$

$= (6+4,\ 9-10)$

$= (10,-1)$.

(b) & (c) Part similarly to Part(a).

Q6 is similarly to Q5.

Q7 $u=(1,2)$, $v=(-3,4)$, $w=(w_1,4)$ & $x=(-2,x_2)$.

(a) $w=2u$

$(w_1,4)=2(1,2)$

$(w_1, 4) = (2, 4)$

$\Rightarrow w_1 = 2$

(b) $\frac{3}{2}x = v$

$\frac{3}{2}(-2, x_2) = (-3, 4)$

$(-3, \frac{3}{2}x_2) = (-3, 4)$

$\Rightarrow \frac{3}{2}x_2 = 4 \quad \Rightarrow \boxed{x_2 = 8/3}$

Q8 is similarly to Q7.

Q9 (a) $(1, 2)$

$\|u\| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$.

b, c, d is similarly to a

Q10 is similarly to Q9.

Q11 (a) $\|u\| = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{1+1} = \sqrt{2}$.

Q12 is similarly to Q11.

②

Ex # 4.1

Q13 $(-5,6) = c_1(1,2) + c_2(3,4)$ — ⊛

$= (c_1 + 2c_1) + (c_2 3 + c_2 4).$

$(-5,6) = (c_1 + 3c_2 , 2c_1 + 4c_2)$

$c_1 + 3c_2 = -5$ — ①

$2c_1 + 4c_2 = 6$

$\div 2$

$\Rightarrow c_1 + 2c_2 = 3$ — ②

① − ②

$\boxed{c_2 = -8}$

eq ① $\Rightarrow c_1 - 24 = -5 \Rightarrow \boxed{c_1 = 19}$

eq ⊛ $\Rightarrow (-5,6) = 19(1,2) - 8(3,4)$

Q14 $c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1\\2c_1\end{bmatrix} + \begin{bmatrix}3c_2\\4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1 + 3c_2\\2c_1 + 4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}.$

Augmented form

$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 4 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & -2 & 0 \end{array}\right] R_2 - 2R_1$$

$$-2c_2 = 0 \Rightarrow \boxed{c_2 = 0}$$

So it is impossible.

Q15

$$= \frac{1}{2}\left| \det\left(\begin{bmatrix} 3 & 3 & 1 \\ -1 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix}\right)\right|$$

$$= \frac{1}{2}\left| 3\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left| 3(-1-1) + 1(3-1) + 4(3+1)\right|$$

$$= \frac{1}{2}\left| 3(-2) + 2 + 4(4)\right|$$

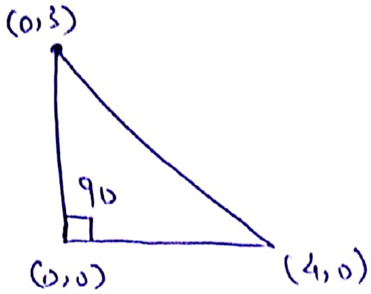
$$= \frac{1}{2}\left| -6 + 2 + 16\right|$$

$$= \frac{1}{2}\left| 18 - 6\right|$$

$$= \frac{1}{2}(12) = 6 \text{ A.}$$

Ex 4.1

Q16



$= \frac{1}{2}(\text{base})(\text{height})$

$= \frac{1}{2}(4)(3) = 6.$

or

$= \frac{1}{2}\left|\det\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 1 \\ 4 & 0 & 1 \end{bmatrix}\right)\right|$

$= \frac{1}{2}\left|4\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix}\right|$

$= \frac{1}{2}\left|(4(0-3)\right| = \frac{1}{2}|-12| = \frac{1}{2}(12) = 6$

Q17 $\text{Area} = \left|\det\left(\begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}\right)\right|$ (P-221).

$= \left|2\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix}\right|$

$= |2(3-5) - 5(3-5) + 4(3-3)|$

$= |-4+10| = |6|$

$= 6.$

Q19 (a) $X=(3,4)$

$= \frac{3,4}{\sqrt{3^2+4^2}} = \frac{3,4}{\sqrt{25}}, \frac{3,4}{5} = \left(\frac{3}{5}, 4/5\right)$

b, d c Part similarly to Part "a"

Q20 is similarly to Q19.

Q21 (a) $\cos\theta = \frac{U.V}{\|U\|\|V\|}$ — (1) $U=(1,2)$, $V=(2,-3)$

where $\|U\| = \sqrt{1^2+2^2} = \sqrt{5}$

& $\|V\| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$.

$U.V = (1,2)(2,-3) = (1)(2)+(2)(-3) = 2-6 = -4$

eq (1) $\Rightarrow$ $\cos\theta = \frac{-4}{\sqrt{5}.\sqrt{13}}$.

Q (b) (c) & (d) is similarly to Part (a).

Q22 is similarly to Q21.

Q23 is S.Y.S

Ex 4.1

Q24 (a) For orthogonal? $u \cdot v = 0$

$u_1 \cdot u_6 = (i+2j)\cdot(-6i+3j) = (-\cancel{6}+\cancel{6}) = 0$

So $u_1 \cdot u_4 = (i+2j)\cdot(-2i+j) = (-\cancel{2}+\cancel{2}) = 0$

$u_3 \cdot u_4 = (-2i-4j)\cdot(-2i+j) = (-\cancel{4}+\cancel{4}) = 0$

$u_3 \cdot u_6 = (-2i-4j)\cdot(-6i+3j) = (\cancel{12}-\cancel{12}) = 0$

$u_4 \cdot u_5 = (-2i+j)\cdot(2i+4j) = (-4+4) = 0$

$u_5 \cdot u_6 = (2i+4j)\cdot(-6i+3j) = (-12+12) = 0$

(b) u_1 & u_5 because $u_5 = 2u_1$

u_4 & u_6 because $u_6 = 3u_4$.

(c) u_1 & u_3 because $u_3 = -2u_1$

u_3 & u_5 because $u_3 = -u_5$

Q25 ~~Find~~ Slope m_1 = slope m_2 (vector are parallel if slopes are equal).

$m_1 = 4/a$, $m_2 = 5/2$

i.e $4/a = 5/2$

$5a = 8$

$\boxed{a = 8/5}$

Q26 vector are orthogonal if $\cos\theta = 0$.

$(a, +2).(a, -2) = 0$

$a^2 + (-4) = 0$

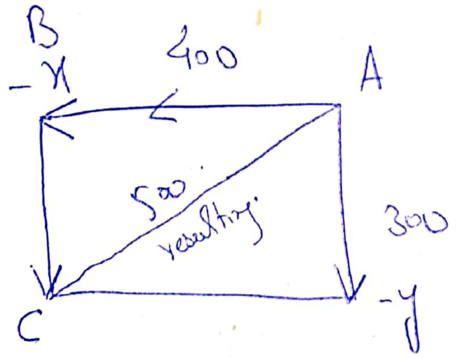
$a^2 = 4$

$\boxed{a = \pm 2}$

Q27 (a) $i + 3j$, (b) $-2i - 3j$ (c) $-2i$ (d) $3j$

Q28 (a) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}_{2\times 1}$, (b) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$.

Q29



$U = 0i - 300j$ + $V = -400i + 0j$

$\vec{AC} = \vec{AB} + \vec{BC}$

$= \vec{V} + \vec{U}$

$= (-400i + 0j) + (0i - 300j) = -400i - 300j$

$|\vec{AC}| = \sqrt{(400)^2 + (300)^2} = 500$

Scanned with CamScanner

Key Terms

Components of a vector
Equal vectors
n-space
Scalars
Vector addition
Scalar multiplication of vectors
Zero vector
Negative of a vector
Difference of vectors
Coordinate system
Coordinate axes
x-, y-, and z-axes
Right-handed coordinate system
Left-handed coordinate system
x-, y-, and z coordinates
coordinates
Rectangular coordinate system
xy-, xz-, and yz-planes
Vector
Equal vectors
Length (or magnitude, or norm) of a vector
Distance between points (or vectors)
Standard inner product
Cauchy–Schwarz inequality
Angle between vectors
Triangle inequality
Unit vector
Parallelogram law

4.2 Exercises

1. Find $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $2\mathbf{u}$, and $3\mathbf{u}-2\mathbf{v}$ if
 (a) $\mathbf{u}=(1,2,-3)$, $\mathbf{v}=(0,1,-2)$
 (b) $\mathbf{u}=(4,-2,1,3)$, $\mathbf{v}=(-1,2,5,-4)$

2. Repeat Exercise 1 for
 (a) $\mathbf{u}=\begin{bmatrix}2\\0\\-4\end{bmatrix}$, $\mathbf{v}=\begin{bmatrix}3\\2\\1\end{bmatrix}$
 (b) $\mathbf{u}=\begin{bmatrix}-3\\5\\-3\\0\end{bmatrix}$, $\mathbf{v}=\begin{bmatrix}2\\1\\5\\-2\end{bmatrix}$

3. Let
$$\mathbf{u}=\begin{bmatrix}1\\-2\\3\end{bmatrix},\quad \mathbf{v}=\begin{bmatrix}-3\\-1\\3\end{bmatrix},$$
$$\mathbf{w}=\begin{bmatrix}a\\-1\\b\end{bmatrix},\quad \mathbf{x}=\begin{bmatrix}3\\c\\2\end{bmatrix}.$$
 Find a, b, and c so that
 (a) $\mathbf{w}=\frac{1}{2}\mathbf{u}$ (b) $\mathbf{w}+\mathbf{v}=\mathbf{u}$ (c) $\mathbf{w}+\mathbf{x}=\mathbf{v}$

4. Let $\mathbf{u}=(4,-1,-2,3)$, $\mathbf{v}=(3,-2,-4,1)$, $\mathbf{w}=(a,-3,-6,b)$, and $\mathbf{x}=(2,c,d,4)$. Find a, b, c, and d so that
 (a) $\mathbf{w}=3\mathbf{u}$ (b) $\mathbf{w}+\mathbf{x}=\mathbf{u}$ (c) $\mathbf{w}-\mathbf{u}=\mathbf{v}$

5. Let $\mathbf{u}=(4,5,-2,3)$, $\mathbf{v}=(3,-2,0,1)$, $\mathbf{w}=(-3,2,-5,3)$, $c=2$, and $d=3$. Verify properties (a) through (h) in Theorem 4.2.

6. Plot the following points in R^3.
 (a) $(3,-1,2)$ (b) $(1,0,2)$
 (c) $(0,0,-4)$ (d) $(1,0,0)$
 (e) $(0,-2,0)$

7. Sketch a directed line segment in R^3 representing each of the following vectors.
 (a) $\mathbf{u}_1=(2,-3,-1)$ (b) $\mathbf{u}_2=(0,1,4)$
 (c) $\mathbf{u}_3=(0,0,-1)$

8. For each of the following pairs of points in R^3, determine the vector that is associated with the directed line segment whose tail is the first point and whose head is the second point.
 (a) $(2,3,-1)$, $(0,0,2)$
 (b) $(1,1,0)$, $(0,1,1)$
 (c) $(-1,-2,-3)$, $(3,4,5)$
 (d) $(1,1,3)$, $(0,0,1)$

9. Determine the head of the vector $(3,4,-1)$ whose tail is $(1,-2,3)$.

10. Find the length of the following vectors.
 (a) $(1,2,-3)$ (b) $(2,3,-1,4)$
 (c) $(1,0,3)$ (d) $(0,0,3,4)$

Solution
$|(1,2,-3)|$
$\sqrt{(1)^2+(2)^2+(3)^2}$
$\sqrt{1+4+9}$
$=\sqrt{14}$

11. Find the length of the following vectors.
 (a) $(2,3,4)$ (b) $(0,-1,2,3)$
 (c) $(-1,-2,0)$ (d) $(1,2,-3,-4)$

12. Find the distance between the following pairs of points.
 (a) $(1,-1,2)$, $(3,0,2)$
 (b) $(4,2,-1,5)$, $(2,3,-1,4)$
 (c) $(0,0,2)$, $(-3,0,0)$
 (d) $(1,0,0,2)$, $(3,-1,5,2)$

13. Find the distance between the following pairs of points.
 (a) $(1,1,0)$, $(2,-3,1)$
 (b) $(4,2,-1,6)$, $(4,3,1,5)$
 (c) $(0,2,3)$, $(1,2,-4)$
 (d) $(3,4,0,1)$, $(2,2,1,-1)$

14. Is the vector $(2,-2,3)$ a linear combination of the vectors $(1,2,-3)$, $(-1,1,1)$, and $(-1,4,-1)$?

15. If possible, find scalars c_1, c_2, and c_3, not all zero, so that
$$c_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-2\end{bmatrix}+c_3\begin{bmatrix}3\\7\\ [illegible]\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

16. Find all constants a such that $\|(1, a, -3, 2)\| = 5$.

17. Find all constants a such that $\mathbf{u} \cdot \mathbf{v} = 0$, where $\mathbf{u} = (a, 2, 1, a)$ and $\mathbf{v} = (a, -1, -2, -3)$.

18. Verify Theorem 4.3 for $c = 3$ and $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (1, 2, -4)$, and $\mathbf{w} = (1, 0, 2)$.

19. Verify Theorem 4.4 for $\mathbf{u}$ and $\mathbf{v}$ as in Exercise 18.

20. Find the cosine of the angle between each pair of vectors $\mathbf{u}$ and $\mathbf{v}$.
(a) $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (-4, 4, 5)$
(b) $\mathbf{u} = (0, 2, 3, 1)$, $\mathbf{v} = (-3, 1, -2, 0)$
(c) $\mathbf{u} = (0, 0, 1)$, $\mathbf{v} = (2, 2, 0)$
(d) $\mathbf{u} = (2, 0, -1, 3)$, $\mathbf{v} = (-3, -5, 2, -1)$

21. Find the cosine of the angle between each pair of vectors $\mathbf{u}$ and $\mathbf{v}$.
(a) $\mathbf{u} = (2, 3, 1)$, $\mathbf{v} = (3, -2, 0)$
(b) $\mathbf{u} = (1, 2, -1, 3)$, $\mathbf{v} = (0, 0, -1, -2)$
(c) $\mathbf{u} = (2, 0, 1)$, $\mathbf{v} = (2, 2, -1)$
(d) $\mathbf{u} = (0, 4, 2, 3)$, $\mathbf{v} = (0, -1, 2, 0)$

22. Show that
(a) $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$
(b) $\mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{j} \cdot \mathbf{k} = 0$

23. Which of the vectors

$\mathbf{u}_1 = (4, 2, 6, -8)$, $\quad \mathbf{u}_2 = (-2, 3, -1, -1)$,
$\mathbf{u}_3 = (-2, -1, -3, 4)$, $\quad \mathbf{u}_4 = (1, 0, 0, 2)$,
$\mathbf{u}_5 = (1, 2, 3, -4)$, $\quad$ and $\mathbf{u}_6 = (0, -3, 1, 0)$

are
(a) orthogonal? (b) parallel?
(c) in the same direction?

24. Find c so that the vector $\mathbf{v} = (2, c, 3)$ is orthogonal to $\mathbf{w} = (1, -2, 1)$.

25. If possible, find a, b, and c not all zero so that $\mathbf{v} = (a, b, c)$ is orthogonal to both

$$\mathbf{w} = (1, 2, 1) \quad \text{and} \quad \mathbf{x} = (1, -1, 1).$$

26. Verify the triangle inequality for $\mathbf{u} = (1, 2, 3, -1)$ and $\mathbf{v} = (1, 0, -2, 3)$.

27. Find a unit vector in the direction of $\mathbf{x}$.
(a) $\mathbf{x} = (2, -1, 3)$ (b) $\mathbf{x} = (1, 2, 3, 4)$
(c) $\mathbf{x} = (0, 1, -1)$ (d) $\mathbf{x} = (0, -1, 2, -1)$

28. Find a unit vector in the direction of $\mathbf{x}$.
(a) $\mathbf{x} = (1, 2, -1)$ (b) $\mathbf{x} = (0, 0, 2, 0)$
(c) $\mathbf{x} = (-1, 0, -2)$ (d) $\mathbf{x} = (0, 0, 3, 4)$

29. Write each of the following vectors in R^3 in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.
(a) $(1, 2, -3)$ (b) $(2, 3, -1)$
(c) $(0, 1, 2)$ (d) $(0, 0, -2)$

30. Write each of the following vectors in R^3 as a 3×1 matrix.
(a) $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ (b) $\mathbf{i} + 2\mathbf{j}$
(c) $-3\mathbf{i}$ (d) $3\mathbf{i} - 2\mathbf{k}$

31. Verify that the triangle with vertices $P_1(2, 3, -4)$, $P_2(3, 1, 2)$, and $P_3(-3, 0, 4)$ is isosceles.

32. Verify that the triangle with vertices $P_1(2, 3, -4)$, $P_2(3, 1, 2)$, and $P_3(7, 0, 1)$ is a right triangle.

33. A large steel manufacturer, who has 2000 employees, lists each employee's salary as a component of a vector $\mathbf{u}$ in R^{2000}. If an 8% across-the-board salary increase has been approved, find an expression involving $\mathbf{u}$ giving all the new salaries.

34. The vector $\mathbf{u} = (20, 30, 80, 10)$ gives the number of receivers, CD players, speakers, and cassette recorders that are on hand in a stereo shop. The vector $\mathbf{v} = (200, 120, 80, 70)$ gives the price (in dollars) of each receiver, CD player, speaker, and cassette recorder, respectively. What does the dot product $\mathbf{u} \cdot \mathbf{v}$ tell the shop owner?

35. A brokerage firm records the high and low values of the price of IBM stock each day. The information for a given week is presented in two vectors, $\mathbf{t}$ and $\mathbf{b}$, in R^5, giving the high and low values, respectively. What expression gives the average daily values of the price of IBM stock for the entire 5-day week?

Exercises 36 through 39 involve bit matrices.

36. Let $\mathbf{u} = (1, 1, 0, 0)$. Find a vector $\mathbf{v}$ in B^4 so that $\mathbf{u} + \mathbf{v} = \mathbf{0}$. Is there more than one such vector $\mathbf{v}$? Explain.

37. Let $\mathbf{u} = (0, 1, 0, 1)$. Find a vector $\mathbf{v}$ in B^4 so that $\mathbf{u} + \mathbf{v} = \mathbf{0}$. Is there more than one such vector $\mathbf{v}$? Explain.

38. Let $\mathbf{u} = (1, 1, 0, 0)$. Find all vectors $\mathbf{v}$ in B^4 so that $\mathbf{u} \cdot \mathbf{v} = 0$.

39. Let $\mathbf{u} = (1, 0, 1)$. Find all vectors $\mathbf{v}$ in B^3 so that $\mathbf{u} \cdot \mathbf{v} = 0$.

Theoretical Exercises

T.1. Prove the rest of Theorem 4.2.

T.2. Show that $-\mathbf{u} = (-1)\mathbf{u}$.

T.3. Establish Equations (1) and (2) in R^3, for the length of a vector and the distance between two points, by using the Pythagorean theorem.

Ex 4.2

Q1 (a) $u = (1,2,-3)$, $v = (0,1,-2)$

$$u+v = \begin{bmatrix}1\\2\\-3\end{bmatrix} + \begin{bmatrix}0\\1\\-2\end{bmatrix} = \begin{bmatrix}1+0\\2+1\\-3-2\end{bmatrix} = \begin{bmatrix}1\\3\\-5\end{bmatrix}$$

$$u-v = \begin{bmatrix}1-0\\2-1\\-3+2\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}.$$

$$2u = 2\begin{bmatrix}1\\2\\-3\end{bmatrix} = \begin{bmatrix}2\\4\\-6\end{bmatrix}$$

$$3u-2v = 3\begin{bmatrix}1\\2\\-3\end{bmatrix} - 2\begin{bmatrix}0\\1\\-2\end{bmatrix} = \begin{bmatrix}3\\6\\-9\end{bmatrix} - \begin{bmatrix}0\\+2\\-4\end{bmatrix} = \begin{bmatrix}3-0\\6-2\\-9+4\end{bmatrix} = \begin{bmatrix}3\\4\\-5\end{bmatrix}.$$

Part b is same as 'a'

Q2 as same as Q1

Q3 $u = \begin{bmatrix}1\\-2\\3\end{bmatrix}$, $v = \begin{bmatrix}-3\\-1\\3\end{bmatrix}$, $w = \begin{bmatrix}a\\-1\\b\end{bmatrix}$, $x = \begin{bmatrix}3\\c\\2\end{bmatrix}$.

Find a, b & c so that.

(a) $w = \frac{1}{2}u \Rightarrow \begin{bmatrix}a\\-1\\b\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\-2\\3\end{bmatrix} = \begin{bmatrix}1/2\\-1\\2/3\end{bmatrix}$

$\Rightarrow a = 1/2$
$b = 2/3$.

(b) $W+V=U$

$$\Rightarrow \begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} a-3 \\ -2 \\ b+3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$a-3=1 \Rightarrow a=1+3=4 \Rightarrow \boxed{a=4}$

$b+3=3 \Rightarrow \boxed{b=0}$

(c) $W+X=V$

$$\begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+3 \\ c-1 \\ b+2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} \quad \Rightarrow a+3=-3 = -3-3 = -6 \Rightarrow \boxed{a=-6}$$

$c-1=-1 \Rightarrow \boxed{c=0}$ $\quad b+2=3 \Rightarrow \boxed{b=1}$

Q 4 as same as Q 3.

Ex 4.2

Q5 $U=(4,5,-2,3)$ & $V=(3,-2,0,1)$, $c=2$, $d=3$

(a) $U+V=V+U$

$U+V=(4+3,5-2,-2+0,3+1)=(7,3,-2,4)$

$V+U=(3+4,-2+5,0-2,1+3)=(7,3,-2,4)$.

(b) $U+(V+W)=(U+V)+W$.

L.H.S $U+(V+W)$

$(4,5,-2,3)+[(3,-2,0,1)+(-3,2,-5,3)]$.

$(4,5,-2,3)+(0,0,-5,4)$

$(4,5,-7,7)$.

R.H.S $(U+V)+W$

$[(4,5,-2,3)+(3,-2,0,1)]+(-3,2,-5,3)$.

$(7,3,-2,4)+(-3,2,-5,3)$

$(4,5,-7,7)$.

Hence $U+(V+W)=(U+V)+W$

(c) $U+0=0+U$.

L.H.S $=(4,5,-2,3)+(0,0,0,0)=(4,5,-2,3)$

R.H.S $(0,0,0,0)+(4,5,-2,3)=(4,5,-2,3)$.

(d) $u + (-u) = 0$

$(4,5,-2,3) + (-(4,5,-2,3))$

$(4,5,-2,3) + (-4,-5,+2,-3)$

$(4-4, 5-5, -2+2, 3-3)$

$(0,0,0,0)$.

(e) $c(u+v) = cu + cv$

$L.H.S = 2[(4,5,-2,3) + (3,-2,0,1)]$

$= 2(7,3,-2,4)$

$= (14,6,-4,8)$.

$R.H.S \; 2(4,5,-2,3) + 2(3,-2,0,1)$

$= (8,10,-4,6) + (6,-4,0,2)$

$= (14,6,-4,8)$.

(f) $(c+d)u = cu + du$

$L.H.S \; (2+3)(4,5,-2,3) = 5(4,5,-2,3) = (20,25,-10,15)$

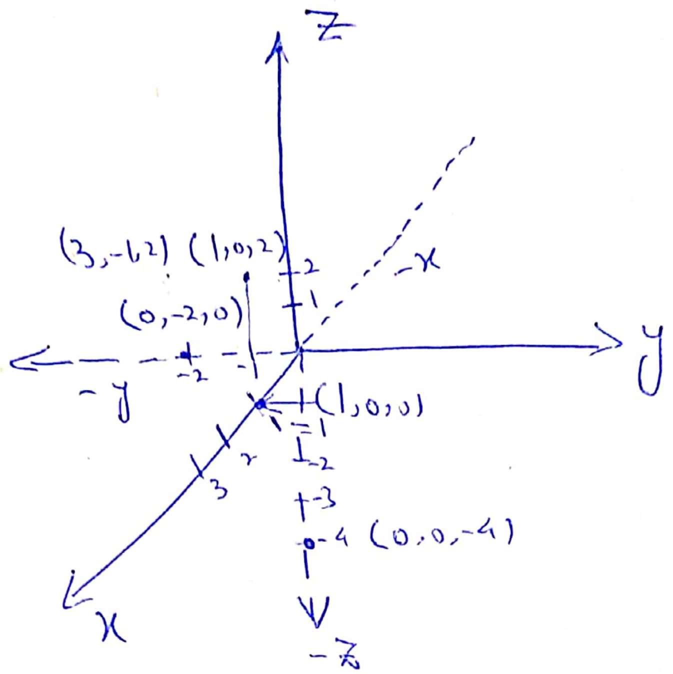
$R.H.S \; 2(4,5,-2,3) + 3(4,5,-2,3)$

$(8,10,-4,6) + (12,15,-6,9)$

$= (20,25,-10,15)$

Ex 4.2

Q6



Q7 Same as Q6

Q8 (a) P(1st Point), Q(Second Point).

$$\vec{PQ} = \vec{Q} - \vec{P} = (0,0,2) - (2,3-1)$$
$$= (-2,-3,3)$$

$$\vec{PQ} = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix}.$$

Part b, c, d Same as Part a.

Q9 head Q=? $P=(1,-2,3)$ $\vec{PQ}$(vector) $=(3,4,-1)$.

$$\vec{PQ} = \vec{Q} - \vec{P}$$

$$\Rightarrow Q = \vec{PQ} + \vec{P} = (3,4,-1)+(1,-2,3)$$

$$\vec{Q} = (3+1,\ 4-2,\ -1+3)$$

$$\vec{Q} = (4,2,2).$$

Q10 (a) $||u|| = \sqrt{(1)^2+(2)^2+(-3)^2} = \sqrt{1+4+9} = \sqrt{14} = \sqrt{14}$.

Part b, c, d same as part a.

Q11 same Q10.

Q12 (a) $(1,-1,2),(3,0,2)$.

$$||u-v|| = \sqrt{(1-3)^2+(-1+0)^2+(2-2)^2}.$$

$$= \sqrt{4+1+0}$$

$$= \sqrt{5}.$$

Part b, c, d same as part a.

Q13 same as Q12.

Ex 4.2

Q4

$$a\begin{bmatrix}1\\2\\-3\end{bmatrix}+b\begin{bmatrix}-1\\1\\1\end{bmatrix}+c\begin{bmatrix}-1\\4\\-1\end{bmatrix}=\begin{bmatrix}2\\-2\\3\end{bmatrix}.$$

$$\begin{bmatrix}a\\2a\\-3a\end{bmatrix}+\begin{bmatrix}-b\\b\\b\end{bmatrix}+\begin{bmatrix}-c\\4c\\-c\end{bmatrix}=\begin{bmatrix}2\\-2\\3\end{bmatrix}.$$

$$\begin{bmatrix}a-b-c\\2a+b+4c\\-3a+b-c\end{bmatrix}=\begin{bmatrix}2\\-2\\3\end{bmatrix}.$$

$a-b-c=2$ —— (1)

$2a+b+4c=-2$ —— (2)

$-3a+b-c=3$ —— (3)

add (−2) time first to (2)

$-2\cancel{a}+2b+2c=-4$

$\cancel{2a}+b+4c=-2$

$3b+6c=-6$

$b+2c=-2$ —— $(2)'$

add (3) times (1) to (3).

$3\cancel{a}-3b-3c=6$

$-\cancel{3}a+b-c=3$

$-2b-4c=9$

$2b-4c=-9$ —— $(3)'$

add (−2) times $(2)'$ to $(3)'$

$$-2\cancel{b}-4\cancel{c}=4$$
$$\cancel{2b}\ \cancel{+4c}=-9$$

$$0=-5$$

which is impossible.

Q15 $c_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-2\end{bmatrix}+c_3\begin{bmatrix}3\\7\\-4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$.

$$\begin{bmatrix}c_1\\2c_1\\-c_1\end{bmatrix}+\begin{bmatrix}c_2\\3c_2\\-2c_2\end{bmatrix}+\begin{bmatrix}3c_3\\7c_3\\-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$\begin{bmatrix}c_1+c_2+3c_3\\2c_1+3c_2+7c_3\\-c_1-2c_2-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$c_1+c_2+3c_3=0 \quad \text{—— } 1$$
$$2c_1+3c_2+7c_3=0 \quad \text{—— } 2$$
$$-c_1-2c_2-4c_3=0 \quad \text{—— } 3$$

add (-2) times 1 to 2.

$$-2\cancel{c_1}-2c_2-6c_3=0$$
$$\cancel{2c_1}+3c_2+7c_3=0$$

$$c_2+c_3=0$$

$$\boxed{c_2=-c_3}$$

Ex 4.2

add (+2) times ① to ③:

$$2c_1 + 2\cancel{c_2} + 6c_3 = 0$$
$$-c_1 - 2\cancel{c_2} - 4c_3 = 0$$

$$c_1 + 2c_3 = 0$$

$$\boxed{c_1 = -2c_3}$$

let $c_3 = r$

then $c_2 = -r$ & $c_1 = -2r$

For Possible Solution Put $r = 1$ (unit value)

$c_3 = 1$, $c_2 = -1$, $c_3 = -2(1) = -2$

then $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$.

Q16 $\|(1, a, -3, 2)\| = 5$

$$\sqrt{(1)^2 + (a)^2 + (-3)^2 + (2)^2} = 5$$

$$\sqrt{14 + a^2} = 5$$

Sq: B.S

$$14 + a^2 = 25$$
$$a^2 = 25 - 14 = 11 \Rightarrow \boxed{a = \pm\sqrt{11}}$$

Q17 $U=(a,2,1,a)$ & $V=(a,-1,-2,-3)$.

& $U\cdot V=0$.

$(a,2,1,a)\cdot(a,-1,-2,-3)=0$

$(a^2-2,-2-3a)=0$

$a^2-3a-4=0$

$$a=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}$$

$$a=\frac{3\pm\sqrt{9+16}}{2}$$

$$a=\frac{3\pm\sqrt{25}}{2}=\frac{3\pm5}{2}.\ \frac{8}{2},-2/2$$

$$\boxed{a=4,-1}$$

Q18 $c=3$, $U=(1,2,3)$, $V=(1,2,-4)$, $W=(1,0,2)$.

(a) $U\cdot U\geq 0$.

$(1,2,3)\cdot(1,2,3)\geq 0$.

$1+4+9\geq 0$

$14\geq 0$.

(b) $u \cdot v = -7 = v \cdot u$.

(c) $(u+v) \cdot w = u \cdot w + v \cdot w$.

L.H.S $[(1,2,3)+(1,2,-4)] \cdot (1,0,2)$

$= (2,4,-1) \cdot (1,0,2)$

$= \cancel{(2+1, 4)}$

$= (\cancel{2}-\cancel{2}) = 0$.

R.H.S $(1,2,3) \cdot (1,0,2) + (1,2,-4)(1,0,2)$

$(1+6)+(1-8)$

$(\cancel{2}-\cancel{2}) = 0$.

Q9 $|u \cdot v| \leq \|u\| \|v\|$

$|u \cdot v| = |(1,2,3) \cdot (1,2,-4)|$

$= |1^2+2^2+3(-4)| = |1+4-12| = |-7| = 7$.

$\|u\| = \sqrt{u \cdot u} = \sqrt{(1,2,3) \cdot (1,2,3)} = \sqrt{1^2+4+9} = \sqrt{14}$

$\|v\| = \sqrt{v \cdot v} = \sqrt{(1,2,-4)(1,2,-4)} = \sqrt{1+4+16} = \sqrt{21}$

$\|u\| \|v\| = \sqrt{14}\sqrt{21} = \sqrt{14 \times 21} = 7\sqrt{6}$.

So $7 \leq 7\sqrt{6}$ verified.

Q20 (a) $U=(1,2,3)$, $V=(-4,4,5)$.

$$\cos\theta = \frac{U\cdot V}{\|U\|\|V\|} \quad \text{— ①}$$

$$U\cdot V = (1,2,3)\cdot(-4,4,5) = (-4+8+15) = 19.$$

$$\|U\| = \sqrt{1+2^2+3^2} = \sqrt{14}$$

$$\|V\| = \sqrt{(-4)^2+(4)^2+5^2} = \sqrt{57}.$$

$$\text{eq ①} \Rightarrow \cos\theta = \frac{19}{\sqrt{14}\sqrt{57}} = 0.673.$$

(b) $U=(0,2,3,1)$ $V=(-3,1,-2,0)$

$$\cos\theta = \frac{U\cdot V}{\|U\|\cdot\|V\|} \quad \text{— ①.}$$

$$U\cdot V = (0,2,3,1)\cdot(-3,1,-2,0) = (0+2-6+0) = -4.$$

$$\|U\| = \sqrt{4+9+1} = \sqrt{14} \quad \|V\| = \sqrt{9+4+1} = \sqrt{14}.$$

$$\text{eq ①} \Rightarrow \cos\theta = \frac{-4}{\sqrt{14}\sqrt{14}} = \frac{-4}{14} = \frac{-2}{7}$$

$$\boxed{\cos\theta = -2/7}$$

c & d part similarly.

Scanned with CamScanner

Ex 4.2

Q21 is similaly to Q20.

Q22 S.Y.S

Q23 is similaly Q24 (Ex 4.1).

Q24 For orthogonal $V \cdot W = 0$.

$(2, c, 3) \cdot (1, -2, 1) = 0$

$2 - 2c + 3 = 0$

$-2c = -5$

$\boxed{c = 5/2}$

Q25 $V = (a, b, c)$, $W = (1, 2, 1)$ $X = (1, -1, 1)$

$V \cdot W = (a, b, c) \cdot (1, 2, 1) = (a + 2b + c) = 0$ — 1

$V \cdot X = (a, b, c) \cdot (1, -1, 1) = (a - b + c) = 0$ — 2

1 - 2

$\cancel{a} + 2b + \cancel{c} = 0$

$\cancel{a} - b + \cancel{c} = 0$

$3b = 0 \Rightarrow \boxed{b = 0}$

eq ① $\Rightarrow a+c=0 \Rightarrow a=-c$

eq ② $\Rightarrow a+c=0 \Rightarrow a=-c$.

Let $c=r$ then $a=-r$, $b=0$

For possible solution put $r=1$

$c=1, a=-1, b=0$

$\boxed{a=-1, b=0, c=1}$

Q2 triangle inequality $\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

$u=(1,2,3,-1)$ & $v=(1,0,-2,3)$

$u+v=(2,2,1,2)$

$\|u+v\| = \sqrt{(2)^2+(2)^2+(1)^2+(2)^2} = \sqrt{4+4+1+4} = \sqrt{13}$.

$\|u+v\|^2 = (\sqrt{13})^2 = 13$ — ①

$\|u\| = \sqrt{1+4+9+1} = \sqrt{15}$

$\|v\| = \sqrt{(1)^2+(0)^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$

Now $(\|u\|+\|v\|)^2 = (\sqrt{15}+\sqrt{14})^2 = 57.98=58$ — ②

From ① & ② we

$\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

Ex 4.2

Q27 $X = (2, -1, 3)$

unit vector in the direction of X

then $U = \dfrac{X}{\|X\|}$ — (1)

$\|X\| = \sqrt{4+1+9} = \sqrt{14}$

eq (1) $\Rightarrow U = \dfrac{(2,-1,3)}{\sqrt{14}} = \left(\dfrac{2}{\sqrt{14}}, \dfrac{-1}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right).$

Part b, c, d are same as part (a).

Q28 is same as Q27.

Q29 (a) $(1, 2, -3) = i + 2j - 3k$.

Part b, c, d are same as part (a).

Q30 (a) $2i + 3j - 4k$

$2i = 2I_1 = 2\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\0\\0\end{bmatrix}$

$3j = 3I_2 = 3\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\3\\0\end{bmatrix}$

$-4k = -4I_3 = -4\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\-4\end{bmatrix}.$

3X1 matrix $\begin{bmatrix}2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\3\\0\end{bmatrix} + \begin{bmatrix}0\\0\\-4\end{bmatrix} = \begin{bmatrix}2\\3\\-4\end{bmatrix}_{3\times 1}.$ (Part b, c, d same as part a)

Q31 For isosale triangle two sides are equal.

$||P_1-P_2|| = \sqrt{(2-3)^2+(3-1)^2+(4-2)^2} = \sqrt{41}$

$||P_1-P_3|| = \sqrt{(2+3)^2+(3)^2+(-4-4)^2} = \sqrt{100} = 10$

$||P_2-P_3|| = \sqrt{(3+3)^2+(1)^2+(2-4)^2} = \sqrt{41}.$

From above we have.

$||P_1-P_2|| = ||P_2-P_3||$ Varified.

———— X ————

Q32 For right triangle angle b/w two sides are 90° &

$\cos\theta = 0$

$\vec{u} = \overrightarrow{P_1P_2} = (3-2, 1-3, 2+4) = (1,-2,6)$

$\vec{v} = \overrightarrow{P_1P_3} = (7-2, 0-3, 1+4) = (5,-3,5)$

$\vec{w} = \overrightarrow{P_2P_3} = (7-3, 0-1, 1-2) = (4,-1,-1)$

$\vec{u}\cdot\vec{v} = (5+6+30) = 41$,

$\vec{v}\cdot\vec{w} = (5+6+30) = 41$.

& $\vec{u}\cdot\vec{w} = (4+2-6) = 0.$

Ex 4.2

Q33

$$U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{2000} \end{bmatrix}$$

8 % increase

$$U = \begin{bmatrix} u_1 + 0.08u_1 \\ u_2 + 0.08u_2 \\ \vdots \\ u_{2000} + 0.08u_{2000} \end{bmatrix} = \begin{bmatrix} (1+0.08)u_1 \\ (1+0.08)u_2 \\ \vdots \\ (1+0.08)u_{2000} \end{bmatrix}$$

$$U = \begin{bmatrix} 1.08u_1 \\ 1.08u_2 \\ \vdots \\ 1.08u_{2000} \end{bmatrix} = 1.08U.$$

—————— X ——————

Q34. U.V will show Price of total CD players, Speakers and cassette recorders.

—————— X ——————

Q35 $\frac{1}{2}(t+b)$ (average).

Q36 $U = (1,1,0,0)$ $U+V=0$ let $V = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

Now $U+V=0$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a+1 \\ b+1 \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a=1,\ b=1,\ c=0,\ d=0$

$V = (1,1,0,0)$.

Q37 Same as Q36.

Q39 $U = (1,0,1)$ & let $V = (a,b,c)$.

$U \cdot V = 0$

$(1,0,1).(a,b,c) = 0$

$a+0+c = 0$

$a+c = 0$. —①

For bit matrix $b=0$, $b=1$.

if ① $\Rightarrow b=0$. then $a=c=0$

if $b=1$ then $a=c=1$.

So $V = (0,0,0).,\ (0,1,0),\ (1,0,1),\ (1,1,1)$.

Q38 Same as Q39.

Scanned with CamScanner