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Section: "A"

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Subject: Linear Algebra

Assignment: 1st

Department: CSE

Question #1:

$$2x - y = 5$$
$$4x - 2y = t$$

Solution:

a) Determine value of t, so that system has a solution.

$$2x - y = 5 \rightarrow (i)$$
$$4x - 2y = t \rightarrow (ii)$$

x ing eq (i) by '2' and subtract from (ii)

$$\cancel{4x} - \cancel{2y} = 10$$
$$\pm \cancel{4x} \mp \cancel{2y} = \pm t$$
$$0 = 10 - t \rightarrow (iii)$$

or $\boxed{t = 10}$

for $t = 10$ the system has a solution.

b) Determine value of t so system has no solution.

Put $t = 1$ in eq (iii)

$$0 = 10 - 1 \Rightarrow 0 \neq 9$$

for $t = 0$ the system has no solution.

c) How many different values of t can be selected in part (b)

for all values of t except 10 the system has no solution.

$t = 1, 2, 3 ---- 9, 11 .....$ $t \in R$ but $t \neq 10$

But $t \neq 10$

$$(-\infty, 10) \cup (10, \infty)$$

Question #2:

Discuss an oil refinery produces low suffer ----

Solution:

x_1 = Low suffer

x_2 = High Suffer

Low High Suffer

Blending Plant: $5x_1 + 4x_2 = 3h$

Refining Plant: $4x_1 + 2x_2 = 2h$

Convert hours into minutes.

$$5x_1 + 4x_2 = 3 \times 60 = 180$$

$$4x_1 + 2x_2 = 2 \times 60 = 120$$

$$\Rightarrow \quad 5x_1 + 4x_2 = 180 \quad \rightarrow (1)$$

$$4x_1 + 2x_2 = 120 \quad \rightarrow (2)$$

Multiplying equation (2) by '2'.

$$2(4x_1 + 2x_2) = 2 \times 120$$

$$8x_1 + 4x_2 = 240 \quad \rightarrow (3)$$

Subtract equation (1) from equation (3).

$$8x_1 + \cancel{4x_2} = 240$$

$$\underline{\pm 5x_1 \quad \pm 4x_2 \quad = \pm 180}$$

$$3x_1 = 60$$

$$x_1 = 60/3$$

$$\boxed{x_1 = 20}$$

Put $x_1 = 20$ in equation (1)

$$5(20) + 4x_2 = 180$$

$$100 + 4x_2 = 180$$

$$4x_2 = 180 - 100$$

$$4x_2 = 80$$

$$x_2 = 80/4$$

$$\boxed{x_2 = 20}$$

Each type of fuel should be manufactured of amount 20 tons.

$$x_1 = x_2 = 20 \text{ tons}$$

Solution is unique.

Question # 3:

a) A plastic manufacturer makes two types ----

Solution:

Regular Special plastic

Plant A : $2x + 2y = 8 \rightarrow (1)$

plant B : $5x + 3y = 15 \rightarrow (2)$

xing eq (1) by '5' and equation '2' with '2' and then subtract both equations.

eq ① ⇒ $\cancel{10x} + 10y = 40$

$\pm \cancel{10x} \pm 6y = \pm 30$

$$4y = 10$$

$$y = 10/4$$

$$y = 5/2 \quad \text{or} \quad y = 2.5$$

Put $y = 2.5$ in equation (1)

$$2x + 2(2.5) = 8$$

$$2x + 5 = 8$$

$$2x = 8 - 5$$

$$2x = 3$$

$$x = 3/2 \quad \text{or} \quad x = 1.5$$

So $x = 1.5$ tons

$y = 2.5$ tons

b) Let 0 represent "off" and 1 represent "ON".

$$A = \begin{bmatrix} ON & ON & OFF \\ OFF & ON & OFF \\ OFF & ON & ON \end{bmatrix}$$

Find ON/OFF matrix B so that A+B is a matrix with each entry ON.

Solution:

$$A = \begin{bmatrix} ON & ON & OFF \\ OFF & ON & OFF \\ OFF & ON & ON \end{bmatrix} \text{ or } A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and

$$A+B = \begin{bmatrix} ON & ON & ON \\ ON & ON & ON \\ ON & ON & ON \end{bmatrix} \text{ or } A+B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - A$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1-1 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

in ON/OFF form, The matrix B is

$$B = \begin{bmatrix} OFF & OFF & ON \\ ON & OFF & ON \\ ON & OFF & OFF \end{bmatrix}$$

Question # 4:

Let $S_1 = [18.95 \quad 14.75 \quad 8.98]$, $S_2 = [17.80 \quad 13.50 \quad 10.79]$ be a 3- vector ----

Solution:

$S_1 = [18.95 \quad 14.75 \quad 8.98]$

$S_2 = [17.80 \quad 13.50 \quad 10.79]$

i) Combined 2x3 Matrix:

$$\Psi = \begin{bmatrix} 18.95 & 14.75 & 8.98 \\ 17.80 & 13.50 & 10.79 \end{bmatrix}$$

is a matrix That contain combined information about The prices of items at Two Stores.

ii) **Items price reduce by 20%.**

As items price is reduce to 20% than actual price, Then a combained 2 x 3 matrix is,

$$\varphi = \begin{bmatrix} 18.95 \times 80\% & 14.75 \times 80\% & 8.98 \times 80\% \\ 17.80 \times 80\% & 13.50 \times 80\% & 10.79 \times 80\% \end{bmatrix}$$

$$\varphi = \begin{bmatrix} 18.95 \times 80/100 & 14.75 \times 80/100 & 8.98 \times 80/100 \\ 17.80 \times 80/100 & 13.50 \times 80/100 & 10.79 \times 80/100 \end{bmatrix}$$

$$\varphi = \begin{bmatrix} 15.16 & 11.8 & 7.184 \\ 14.24 & 10.8 & 8.632 \end{bmatrix}$$

Question # 5:

(a): The matrix transformation $f: R^2 \to R^2$, defined by $f(v) = Av$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$ ......

Solution:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$$

$$k = 1/2$$

R is a unit sqaure $(0,0)\ (0,1)\ (1,0)\ (1,1)$

$$v = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$f(v) = Av$$

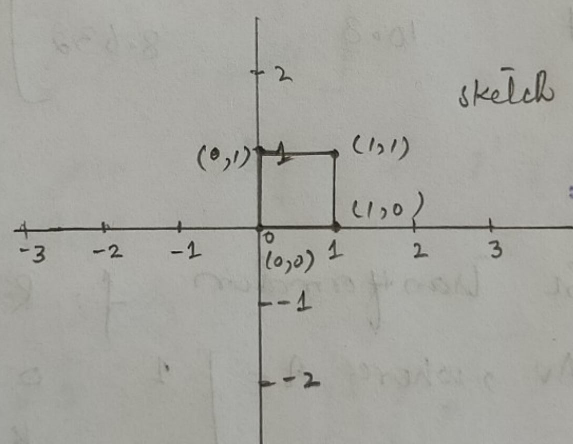
$$f(v) = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

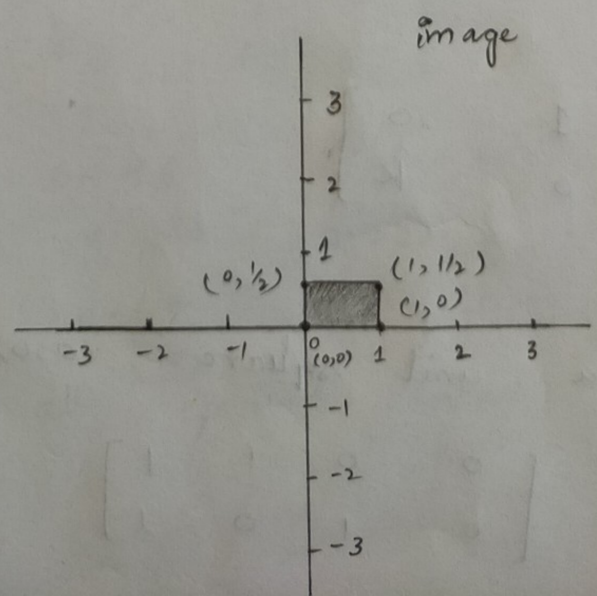
for $k = 1/2$

$$f(v) = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} 0+0 & 0+0 & 1+0 & 1+0 \\ 0+0 & 0+1/2 & 0+0 & 0+1/2 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$$





b) $$T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix}$$

Show That T is regular and find its steady state vector.

Solution:

$$T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix}$$

i) Regularity of T:-

$$T^2 = T.T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 0 & 0.06 & 0.06 \\ 0.3 & 0.24 & 0.3 \\ 0.7 & 0.7 & 0.64 \end{bmatrix}$$

Since there is a zero entry in T^2 (Power of T)
So T is not regular.
Take another power of T.

$$T^3 = T.T^2 = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0.06 & 0.06 \\ 0.3 & 0.24 & 0.3 \\ 0.7 & 0.7 & 0.64 \end{bmatrix}$$

$$T^3 = \begin{bmatrix} 0.06 & 0.048 & 0.06 \\ 0.3 & 0.282 & 0.282 \\ 0.64 & 0.67 & 0.658 \end{bmatrix}$$

Now T^3 have all positive entries so T is regular.

Let $x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be a vector in steady state.

$$TX = X$$
$$(T - I)X = 0$$

$$\left(\begin{bmatrix} 0 & 0.2 & 0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) X = 0$$

$$\begin{bmatrix} -1 & 0.2 & 0 \\ 0 & -0.7 & 0.3 \\ 1 & 0.5 & -0.3 \end{bmatrix} X = 0$$

Augmented Matrix is

$$\sim \left[\begin{array}{ccc|c} -1 & 0.2 & 0 & 0 \\ 0 & -0.7 & 0.3 & 0 \\ 0 & 0.7 & -0.3 & 0 \end{array}\right] R_3 + R_1$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0.2 & 0 & 0 \\ 0 & -0.7 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim R_3 + R_2$$

From 2nd Row;

$$-0.7b + 0.3c = 0$$
$$0.3c = 0.7b$$
$$c = (0.7/0.3)b$$
$$c = 7/3\, b$$

From First Row;

$$-a + 0.2b + 0c = 0$$
$$-a + 0.2b = 0$$
$$0.2b = a$$

$$b = \frac{1}{0.2} a$$

$$\boxed{b = 5a}$$

∴ Sum of propability is 1

$$a + b + c = 1$$

$$a + 5a + \frac{7}{3} b = 1$$

$$6a + \frac{7}{3} b = 1$$

$$6a + \frac{7}{3}(5a) = 1$$

$$6a + \frac{35}{3} a = 1$$

$$\frac{18a + 35a}{3} = 1$$

$$\frac{53a}{3} = 1$$

$$\Rightarrow \boxed{a = 3/53}$$

$$\Rightarrow \quad b = 5(3/53)$$

$$\boxed{b = 15/53}$$

$$\Rightarrow \quad c = \frac{7}{3}\left(\frac{15}{53}\right)$$

$$\boxed{c = 35/53}$$

So The steady vector is

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3/53 \\ 15/53 \\ 35/53 \end{bmatrix}$$

Question # 6:

Find an LU - factorization ----.

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} ; b = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} ; b = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

~ Row operations on Matrix 'A' to obtain upper Triangular Matrix 'U'

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad \sim R_2 \quad R_2 - (0.5)R_1$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad \sim R_3 \quad R_3 - (-1R_1)$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \quad \sim R_4 \quad R_4 - 2R_1$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \sim R_3 \quad R_3 - (0.2)R_2$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0.4 & 6 \end{bmatrix} \sim R_4 \quad R_4 - (-0.4R_2)$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \sim R_4 \quad R_4 - 2R_3$$

Therefore the upper triangular matrix 'U' is

$$U = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

And the lower triangular matrix is,

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix}$$

As

$LZ = b$

$\because$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

$\Rightarrow \quad 1z_1 = -3 \qquad (i)$

$0.5z_1 + z_2 = -1.5 \qquad (ii)$

$-1z_1 + 0.2z_2 + z_3 = 5.6 \qquad (iii)$

$2z_1 - 0.4z_2 + 2z_3 + z_4 = 2.2 \qquad (iv)$

Solving these equations;

from eq (i), $\boxed{z_1 = -3}$

Put value of z_1 in eq (ii)

$\Rightarrow \quad 0.5(-3) + z_2 = -1.5$

$-1.5 + z_2 = -1.5$

$z_2 = -1.5 + 1.5$

$\boxed{z_2 = 0}$

Put value of z_1, z_2 in eq (iii),

$\Rightarrow \quad -1(-3) + 0.2(0) + z_3 = 5.6$

$+3 + 0 + z_3 = 5.6$

$z_3 = 5.6 - 3$

$\boxed{z_3 = 2.6}$

Put all values in eq (iv)

$\Rightarrow \quad 2(-3) - 0.4(0) + 2(2.6) + z_4 = 2.2$

$-6 - 0 + 5.2 + z_4 = 2.2$

$-0.8 + z_4 = 2.2$

$$x_4 = 2.2 + 0.8$$

$$\boxed{x_4 = 3}$$

Therefore;

$$\boxed{\begin{aligned} x_1 &= -3 \\ x_2 &= 0 \\ x_3 &= 2.6 \\ x_4 &= 3 \end{aligned}}$$

Question # 7:

a) Discuss

$$\det\left(\begin{bmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}\right)$$

Solution:

Let $A = \begin{bmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$

det A is

$$|A| = \begin{vmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{vmatrix}$$

Row expansion Method;

expanding R_1.

$$|A| = (\lambda-1)\begin{vmatrix} \lambda-2 & 2 \\ 0 & \lambda-3 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 2 \\ 0 & \lambda-3 \end{vmatrix} - 2\begin{vmatrix} 0 & \lambda-2 \\ 0 & 0 \end{vmatrix}$$

det (A) or $|A| = (\lambda-1)\left[(\lambda-2)(\lambda-3) - 0\times 2\right]$

$+1\left[0\times(\lambda-3) - 0\times 2\right] - 2\left[0\times 0 - 0\times(\lambda-2)\right]$

$|A| = (\lambda-1)[(\lambda-2)(\lambda-3)] + 1(0) - 2(0)$

$|A| = (\lambda-1)(\lambda^2 - 3\lambda - 2\lambda + 6) + 0 - 0$

$|A| = (\lambda-1)(\lambda^2 - 5\lambda + 6)$

$|A| = \lambda^3 - 5\lambda^2 + 6\lambda - \lambda^2 + 5\lambda - 6$

$$\boxed{|A| = \lambda^3 - 6\lambda^2 + 11\lambda - 6}$$

b) Construct a linear system of equations to determine a quadratic polynomial ---- $P(x) = ax^2 + bx + c$ ---.

Solution :-

$P(x) = ax^2 + bx + c$

and $f(x) = xe^{x-1}$

Put $x = 1$ in $P(x)$

$P(1) = a(1)^2 + b(1) + c$

$P(1) = a + b + c$

Put $x = 1$ in $f(x)$

$f(1) = 1\,e^{1-1}$

$f(1) = 1$

But Given $P(1) = f(1)$

$\therefore \boxed{a+b+c=1} \longrightarrow (i)$

Differentiate $P(x)$ and $f(x)$ w.r.t 'x'

$p'(x) = 2ax + b$

$p'(1) = 2a(1) + b$

$p'(1) = 2a + b$

and $f'(x) = (1)e^{x-1} + x(e^{x-1}) \cdot d/dx(x-1)$

$f'(x) = e^{x-1} + xe^{x-1}(1)$

$f'(x) = e^{x-1} + xe^{x-1}$

Now $f'(1) = e^{1-1} + 1e^{1-1}$

$= e^0 + e^0$ $\qquad \therefore e^0 = 1$

$= 1 + 1$

$f'(1) = 2$

But Given $p'(1) = f'(1)$

$\therefore \boxed{2a + b = 2} \longrightarrow (ii)$

Now again differentiate $p'(x)$ and $f'(x)$

$p''(x) = 2a$

$p''(1) = 2a$

and

$f''(x) = e^{x-1} + 1e^{x-1} + xe^{x-1} d/dx(x-1)$

$= e^{x-1} + e^{x-1} + xe^{x-1}(1)$

$= e^{x-1}(1 + 1 + x)$

$f''(1) = e^{1-1}(2+1)$

$f''(1) = e^0(3)$ $\qquad \therefore e^0 = 1$

$f''(1) = 3.$

But Given $p''(1) = f''(1)$

$$\therefore \boxed{2a = 3} \rightarrow (iii)$$

From (iii) $\boxed{a = 3/2}$

Put value of 'a' in eq (ii)

$$\cancel{2}(3/\cancel{2}) + b = 2$$

$$3 + b = 2$$

$$b = 2 - 3$$

$$\boxed{b = -1}$$

Put values of 'a' and 'b' in eq (i)

$$3/2 - 1 + c = 1$$

$$3/2 + c = 1 + 1$$

$$c = 2 - 3/2$$

$$\boxed{c = 1/2}$$

Therefore the quadratic polynomial that satisfies the given equation is

$$\boxed{P(x) = 3/2\, x^2 - x + 1/2}$$

Question # 8:

A study has determined ———

$$\begin{array}{cc} & \text{Father's Occupation} \\ \begin{array}{c}\text{Son's} \\ \text{Occupation}\end{array} & \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \end{array}$$

Solution:

a) Probability that the grand child of a professional will also be a professional.

$$T = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$Tx = x'$$

$$x' = T \cdot x = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \cdot \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.8 \times 0.8 + 0.3 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.5 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.2 \times 0.1 + 0.6 \times 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.64 + 0.03 + 0.02 \\ 0.08 + 0.05 + 0.02 \\ 0.08 + 0.02 + 0.06 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.69 \\ 0.15 \\ 0.16 \end{bmatrix}$$

This is the probability that grandchild of a professional will also be a professional.

b) In the long run, what proportionals of the population will farmers?

Markov's Chain:-

$$Tx = x \quad , \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$(T - I)x = 0$$

$$\left(\begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) x = 0$$

$$\begin{bmatrix} -0.2 & 0.3 & 0.2 \\ 0.1 & -0.5 & 0.2 \\ 0.1 & 0.2 & -0.4 \end{bmatrix} x = 0$$

Homogenous System of linear equation.

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0.1 & 0.2 & -0.4 & 0 \end{array}\right] R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0 & 0.7 & -0.6 & 0 \end{array}\right] R_1 - R_2$$

$$\sim \begin{bmatrix} -0.3 & 0.8 & 0 \\ 0.1 & -0.5 & 0.2 \\ 0 & 0.7 & -0.6 \end{bmatrix}$$

$\Rightarrow$ 1st Row:

$$-0.3a + 0.8b = 0$$

$$\cancel{-}0.3a = \cancel{-}0.8b$$

$$0.3a = 0.8b$$

$$a = 0.8/0.3\, b$$

$$a = 8/3\, b$$

$\Rightarrow$ 3rd Row:

$$0.7b - 0.6c = 0$$

$$0.7b = 0.6c$$

$$b = 0.6/0.7\, c$$

$$b = 6/7\, c \Rightarrow \boxed{c = 7/6\, b}$$

Probability of vector is:

$$a + b + c = 1.$$

$$\frac{8}{3}b + b + \frac{7}{6}b = 1$$

$$\frac{29}{6}b = 1$$

$$\boxed{b = 6/29 = 0.207}$$

$$\Rightarrow \quad a = \frac{8}{3}\left(\frac{6}{29}\right)$$

$$\boxed{a = 16/29 = 0.551}$$

and

$$c = \frac{7}{6}\left(\frac{6}{29}\right)$$

$$\boxed{c = 7/29 = 0.241}$$

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16/29 \\ 6/29 \\ 7/29 \end{bmatrix} = \begin{bmatrix} 0.551 \\ 0.207 \\ 0.241 \end{bmatrix}$$

b) A ship is being ----.

Solution:

Tugboat along negative x-axis $= \bar{OA} = -400$

Tug boat along negative y-axis $= \bar{OB} = -300$

Using Pythogoras Theorem;

$$\bar{OC}^2 = \bar{OA}^2 + \bar{OB}^2$$

$$\bar{OC} = \sqrt{\bar{OA}^2 + \bar{OB}^2}$$

$$\bar{OC} = \sqrt{(-400)^2 + (-300)^2}$$

$$\bar{OC} = 500$$