Q1: Think of ten 3-variable functions and design circuits for them using sum of minterms technique.

Q. Think of ten(10) 3-variable functions and design a circuit for it using sum of min term technique.

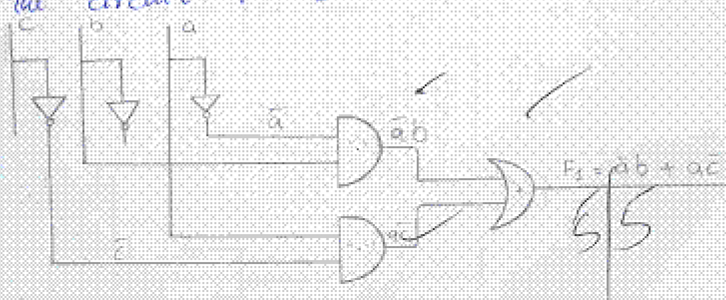
For three variables, there are $2^{2^{(3)}} = 256$ possible outputs. One of them is given below:

A	B	C	F_1	Min Term
0	0	0	0	$m_0 = \bar{a}\bar{b}\bar{c}$
0	0	1	0	$m_1 = \bar{a}\bar{b}c$
0	1	0	1	$m_2 = \bar{a}b\bar{c}$
0	1	1	1	$m_3 = \bar{a}bc$
1	0	0	1	$m_4 = \bar{a}\bar{b}\bar{c}$
1	0	1	0	$m_5 = a\bar{b}c$
1	1	0	1	$m_6 = ab\bar{c}$
1	1	1	0	$m_7 = abc$

From this truth table for F_1 we can write.

$$
\begin{aligned}
F_1(a,b,c) &= \Sigma_m(2,3,4,6) \\
&= \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} \rightarrow \text{Con. form} \\
&= \bar{a}b(\bar{c}+c) + a\bar{c}(\bar{b}+b) \\
&= \bar{a}b + a\bar{c}. \rightarrow \text{Standard form.}
\end{aligned}
$$

Now the circuit for F_1 can be drawn as,



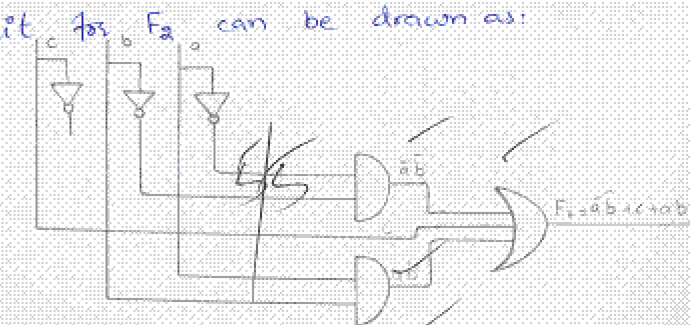
For any function F_2:

A	B	C	F_2	Min Term
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	1	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	1	abc

The function can be written as,

$$F_2(a,b,c) = \Sigma_m(0,1,3,5,6,7)$$
$$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}c + ab\bar{c} + abc$$
$$= \bar{a}\bar{b}(\bar{c}+c) + c(\bar{a}b + a\bar{b}) + ab(\bar{c}+c)$$
$$= \bar{a}\bar{b} + \bar{a}bc + a\bar{b}c + ab$$
$$= \bar{a}(\bar{b} + bc) + a(\bar{b}c + b)$$
$$= \bar{a}(\bar{b} + c) + a(\bar{b}c + b)$$
$$= \bar{a}\bar{b} + \bar{a}c + ac + ab$$
$$= \bar{a}\bar{b} + c(\bar{a}+a) + ab$$
$$= \bar{a}\bar{b} + c + ab \rightarrow \text{standard form}$$

The circuit for F_2 can be drawn as:



For F_3 the truth table is,

A	B	C	F_3	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	0	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	1	abc

function can be written as,

$$F_3(a,b,c) = \sum_m (0,1,2,6,7)$$

$$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + ab\bar{c} + abc$$

$$= \bar{a}\bar{b}(\bar{c}+c) + b\bar{c}(\bar{a}+a) + abc$$

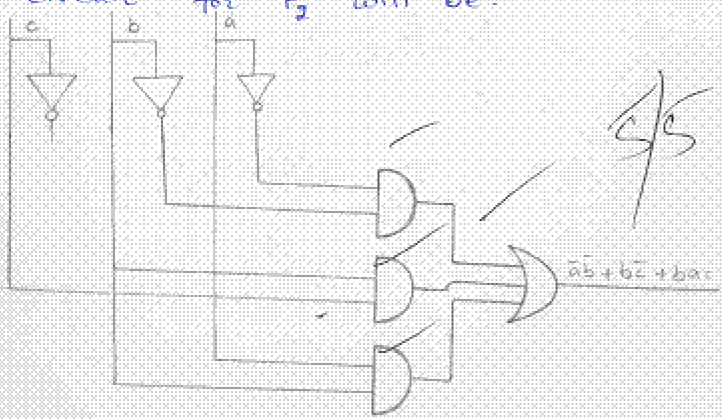
$$= \bar{a}\bar{b} + b\bar{c} + abc$$

$$= \bar{a}\bar{b} + b(\bar{c} + ac)$$

$$= \bar{a}\bar{b} + b(\bar{c} + a)$$

$$= \bar{a}\bar{b} + b\bar{c} + ba$$

The circuit for F_3 will be:



Now for F_4 ;

A	B	C	F_4	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	0	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	0	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	0	abc

The function can be written as,

$$F_4(a, b, c) = \Sigma_m(0, 2, 4, 6)$$
$$= \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + ab\bar{c}$$
$$= \bar{a}\bar{c}(\bar{b} + b) + a\bar{c}(\bar{b} + b)$$
$$= \bar{a}\bar{c} + a\bar{c}$$
$$= \bar{c}(\bar{a} + a)$$
$$= \bar{c}.$$

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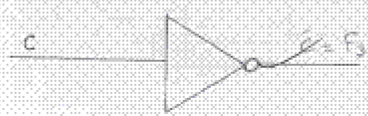
Circuit can be drawn as,



For any function F_s, let the truth table is,

A	B	C	F_s	Min Terms
0	0	0	1	$\bar a \bar b \bar c$
0	0	1	1	$\bar a\, b\, c$
0	1	0	0	$\bar a\, b \bar c$
0	1	1	1	$\bar a\, bc$
1	0	0	1	$a \bar b \bar c$
1	0	1	0	$a \bar b c$
1	1	0	1	$ab\bar c$
1	1	1	1	abc

The function can be written as,

$$F_s(a,b,c) = \Sigma_m(0,1,2,3,7)$$

$$\begin{aligned} F_s &= \bar a \bar b \bar c + \bar a \bar b c + \bar a b \bar c + a \bar b \bar c + a b \bar c + abc \\ &= \bar a \bar b(\bar c + c) + bc(\bar a + a) + ab(\bar c + c) \\ &= \bar a \bar b + bc + ab. \end{aligned}$$

Now looking at the standard form, the circuit can be drawn as,

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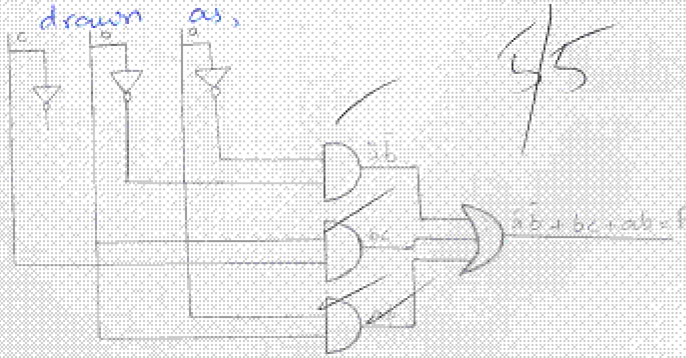


For any out put F_6, truth table.

A	B	C	F_6	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	1	abc

The function can be written as,

$$F_6(a,b,c) = \sum_m (0,1,2,3,7)$$

$$F_6 = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + abc.$$

$$= \bar{a}\bar{b}(\bar{c}+c) + \bar{a}b\bar{c} + bc(\bar{a}+a)$$

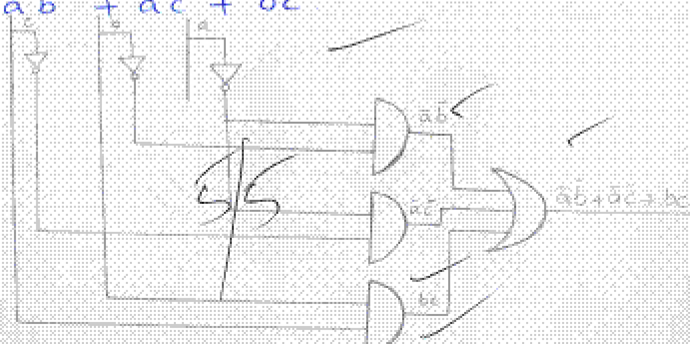
$$= \bar{a}\bar{b} + \bar{a}b\bar{c} + bc$$

$$= \bar{a}(\bar{b} + b\bar{c}) + bc$$

$$= \bar{a}(\bar{b}+\bar{c}) + bc$$

$$= \bar{a}\bar{b} + \bar{a}\bar{c} + bc$$

Circuit.



For any F_7, the truth table,

A	B	C	F_7	Min Terms
0	0	0	0	$\bar{a}\,\bar{b}\,\bar{c}$
0	0	1	1	$\bar{a}\,\bar{b}\,c$
0	1	0	1	$\bar{a}\,b\,\bar{c}$
0	1	1	1	$\bar{a}\,b\,c$
1	0	0	0	$a\,\bar{b}\,\bar{c}$
1	0	1	1	$a\,\bar{b}\,c$
1	1	0	0	$a\,b\,\bar{c}$
1	1	1	0	$a\,b\,c$

function can be written as,

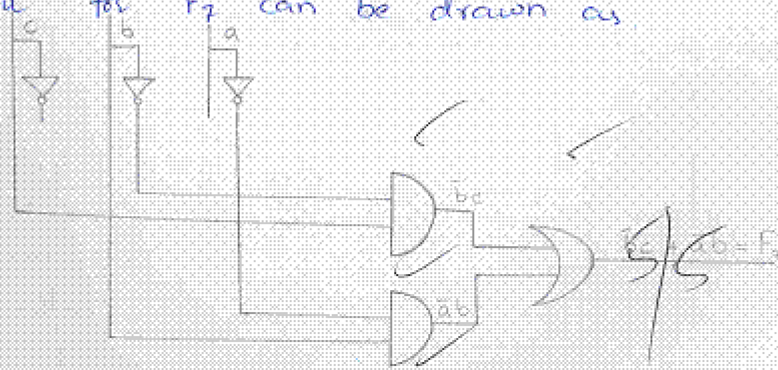
$$F_7(a, b, c) = \sum_m(\cancel{0}1, 2, 3, 5)$$

$$F_7 = \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c$$

$$= \bar{b}c(\bar{a} + a) + \bar{a}b(\bar{c} + c)$$

$$= \bar{b}c + \bar{a}b.$$

Circit for F_7 can be drawn as



For any function F_8,

A	B	C	F_8	Min Terms
0	0	0	0	$\bar a \bar b \bar c$
0	0	1	1	$\bar a \bar b c$
0	1	0	1	$\bar a b \bar c$
0	1	1	1	$\bar a b c$
1	0	0	0	$a \bar b \bar c$
1	0	1	1	$a \bar b c$
1	1	0	0	$a b \bar c$
1	1	1	0	$a b c$

function can be written as,

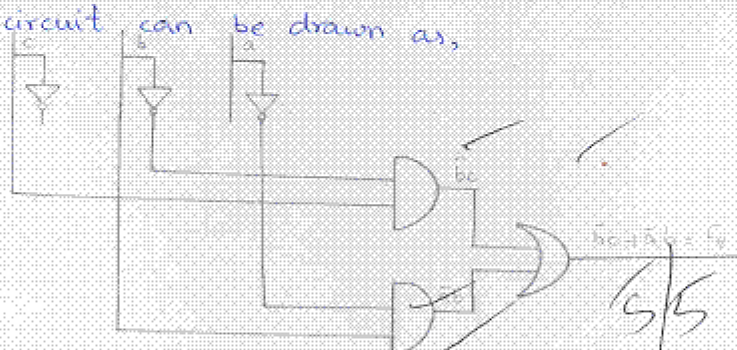
$$F_8(a, b, c) = \sum_m (1, 2, 3, 5)$$

$$F_8 = \bar a \bar b c + \bar a b \bar c + \bar a b c + a \bar b c$$

$$= \bar b c(\bar a + a) + \bar a b(\bar c + c)$$

$$= \bar b c + \bar a b.$$

The circuit can be drawn as,



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Let f_a be any output for which the truth table is,

A	B	C	F_a	Min Terms
0	0	0	0	$\bar{a}\bar{b}\bar{c}$
0	0	$\cancel{0}$1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	0	abc

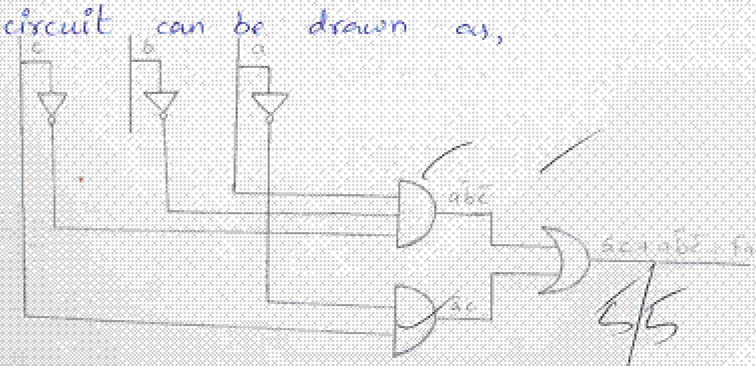
Function can be written as, $f_a(a,b,c) = \sum_m(1,3,4)$

$$f_a = \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}\bar{c}$$

$$= \bar{a}c(\bar{b}+b) + a\bar{b}\bar{c}$$

$$= \bar{a}c + a\bar{b}\bar{c}$$

The circuit can be drawn as,



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Let F_{10} be any output for which the function truth table is,

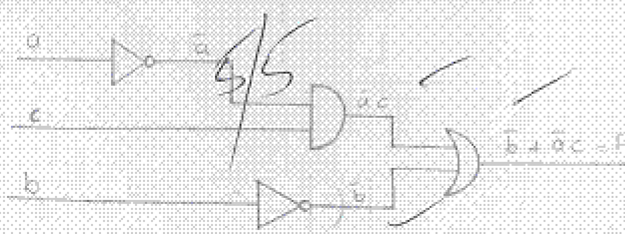
A	B	C	F_{10}	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	1	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	0	abc

The function can be written as,

$$F_{10}(a,b,c) = \Sigma_m(0,1,3,4,5)$$

$$\begin{aligned}
F_{10} &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}bc + a\bar{b}\bar{c} + a\bar{b}c \\
&= \bar{a}\bar{b}(\bar{c}+c) + \bar{a}bc + a\bar{b}(\bar{c}+c) \\
&= \bar{a}\bar{b} + \bar{a}bc + a\bar{b} = \bar{a}(\bar{b}+bc) + a\bar{b} \\
&= \bar{a}(\bar{b}+c) + a\bar{b} = \bar{a}\bar{b} + \bar{a}c + a\bar{b} \\
&= \bar{b}(\bar{a}+a) + \bar{a}c = \bar{b} + \bar{a}c
\end{aligned}$$

Circuit:-



Q1: Think of five 4-variable functions and design circuits for them using product of maxterms technique.

"BOOLEAN ALGEBRA and LOGIC GATES"

QNo1: Think of 5, 4-variables functions and design circuits for them using product of Sum (POS) technique.

Ans: As we have 4-variables, so by formula 2^n the total combinations are $2^4 = 16$.

Let F_1, F_2, F_3, F_4 and F_5 are the desired functions.

Now the truth table will be as:

Where A, B, C and D are the required inputs.

A	B	C	D	F_1	F_2	F_3	F_4	F_5	Max Terms
0	0	0	0	0	0	0	1	0	$A+B+C+D$
0	0	0	1	0	0	1	1	0	$A+B+C+\bar{D}$
0	0	1	0	1	0	1	1	1	$A+B+\bar{C}+D$
0	0	1	1	1	1	1	1	1	$A+B+\bar{C}+\bar{D}$
0	1	0	0	1	1	1	1	1	$A+\bar{B}+C+D$
0	1	0	1	0	0	1	1	1	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	1	0	0	1	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	1	0	0	1	$A+\bar{B}+\bar{C}+\bar{D}$
~~0~~1	0	0	0	1	1	0	1	1	$\bar{A}+B+C+D$
1	0	0	1	0	0	1	0	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	1	1	1	1	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	1	1	1	0	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	1	1	1	1	1	$\bar{A}+\bar{B}+C+D$
1	1	0	1	1	1	0	0	1	$\bar{A}+\bar{B}+C+\bar{D}$
1	1	1	0	1	1	0	0	1	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	1	1	1	1	1	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$

Fig: [illegible]

Now we take the product of these ma[illegible] terms whose output is zero as,

BOOLEAN EXPRESSIONS:

$$F_1 = (A+B+C+D)(A+B+C+\bar{D})(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+\bar{D})$$

$$F_2 = (A+B+C+D)(A+B+C+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})$$

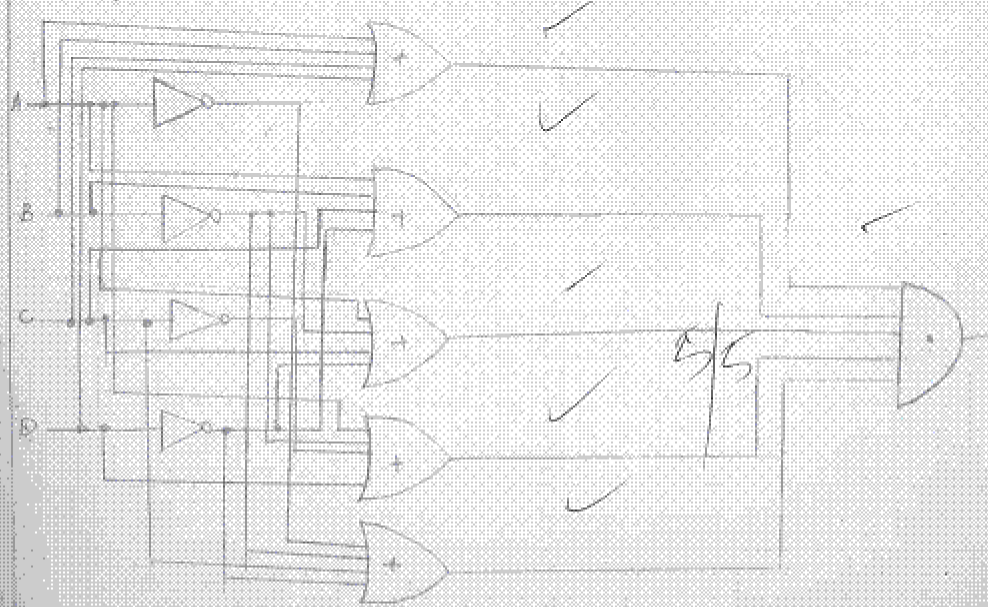
$$F_3 = (A+B+C+D)(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)$$

$$F_4 = (A+\bar{B}+\bar{C}+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

$$F_5 = (A+B+C+D)(A+B+C+\bar{D})(\bar{A}+B+\bar{C}+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})$$

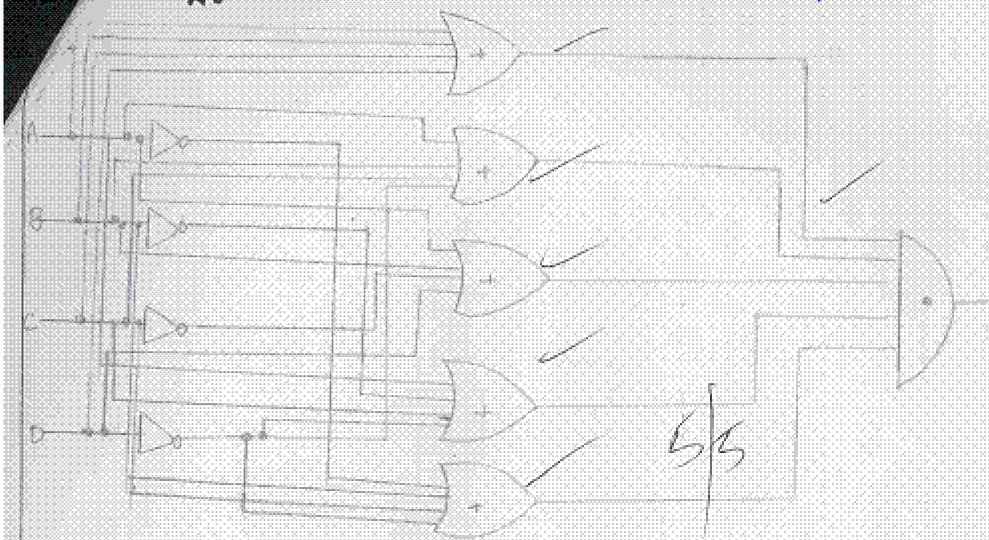
Circuits:

F_1:

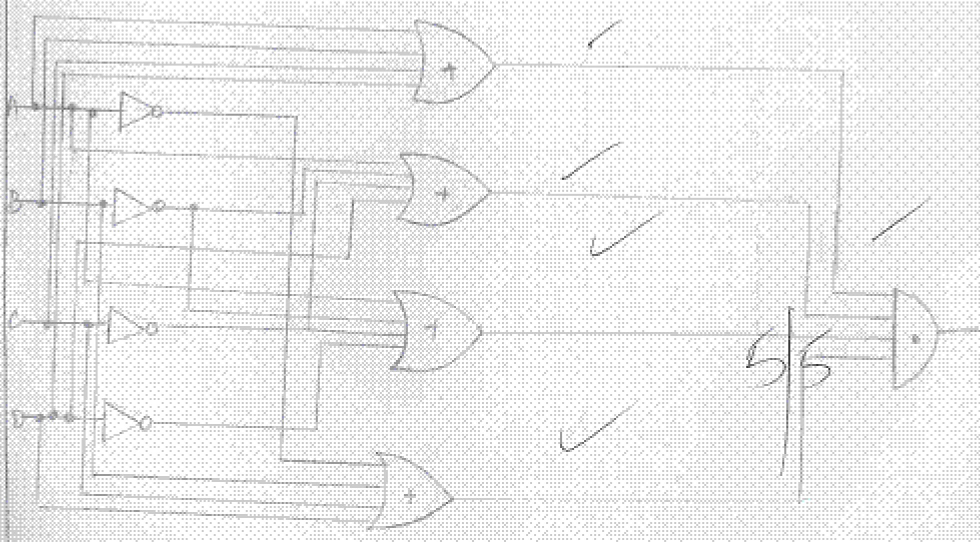


Pg :

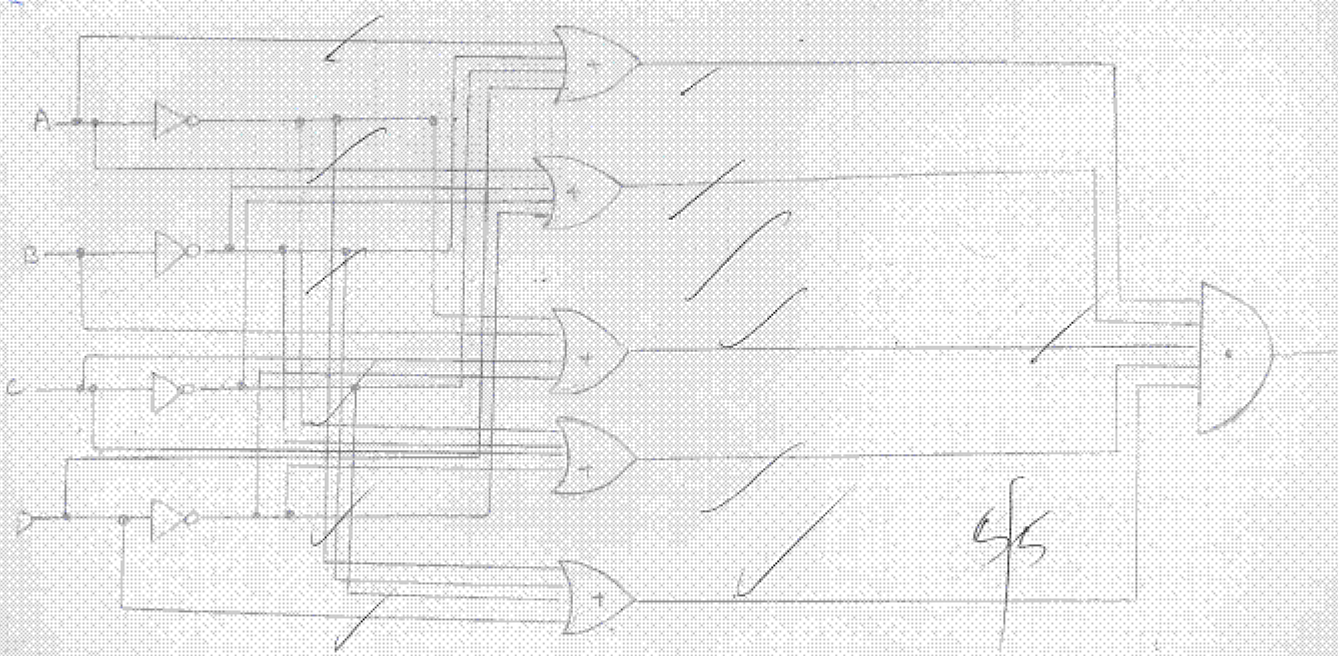
F21:



F8:



F4:



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F5: