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Assignment No: 2

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Section: B

Subject: Signal & Systems

Submitted to:

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Date: 1-08-2021

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(Question No: 1)

Fourier Series Representation:

(a) $x(t) = e^{j8t} + 2e^{j(12t+\pi/3)}$

Sol

$$x(t) = e^{j8t} + 2e^{j(12t+\pi/3)}$$

Now

$$e^{j\pi/3 + j12t} = e^{j12t} + e^{j\pi/3}$$

$$e^{j\pi/3} = \cos \pi/3 + j \sin \pi/3 = \frac{1}{2} + j\sqrt{3}/2 \quad — ①$$

$$e^{j\omega t} \Rightarrow \delta(\omega - \omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} e^{j\omega t}\Big]_{\omega=\omega_0}$$

$$e^{j\omega t} = 2\pi\delta(\omega - \omega_0)$$

putting eq ①

$$x(t) = e^{j8t} + 2\left(\tfrac{1}{2} e^{j12t} + j\sqrt{3}/2\, e^{j12t}\right)$$

$$x(\omega) = 2\pi\left[\delta(\omega - 8) + \delta(\omega-12) + j\sqrt{3} \times \delta(\omega-12)\right]$$

$$X(\omega) = 2\pi\left[\delta(\omega-8) + \delta(\omega-12)(1+j\sqrt{3})\right]$$

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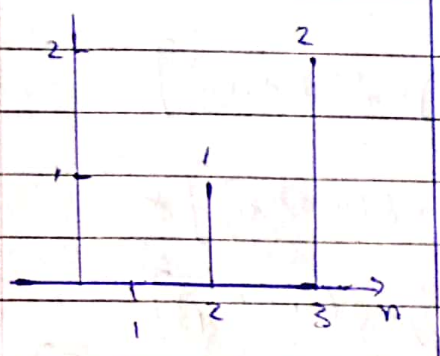
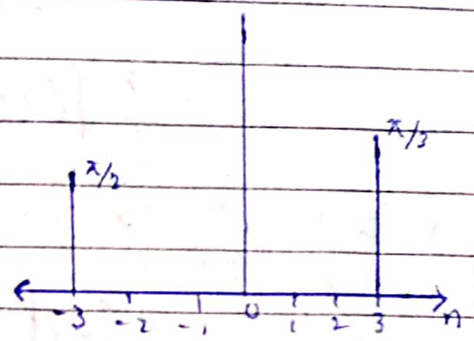
Spectrum:

Amplitude Spectrum



Phase Spectrum



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(b) $x(t) = \cos\left(\pi/4\, t - \pi/6\right) + \sin\left(3\pi/4 + \pi/4\right)$

Sol

Using formulae;

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

Now

$x(t) = \cos(\pi/4\, t - \pi/6) + \sin(3\pi/4\, t + \pi/4)$

$= (\cos \pi/4\, t \cdot \cos \pi/6 + \sin \pi/4\, t \sin \pi/6) +$
$(\sin 3\pi/4\, t \sin \pi/4 + \cos 3\pi/4 \cos \pi/4)$

$x(t) = \sqrt{3}/2 \cos \pi/4\, t + \frac{1}{2} \sin \pi/6\, t + \frac{1}{\sqrt{2}}\left[\sin 3\pi/4 + \cos 3\pi/4\, t\right]$

$$\cos w_0 t = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2}$$

$= \frac{1}{2}\left[\cancel{2\pi\,\delta(\omega - \omega_0)}\right.$

$$\cos w_0 t = \frac{1}{2}\left\{2\pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]\right\}$$

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$$\cos(\omega_0 t) = \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$\sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$= \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$X(\omega) = \frac{\sqrt{3}}{2}\left[\pi\left(\delta(\omega-\pi/4) + \delta(\omega+\pi/4)\right)\right] +$$

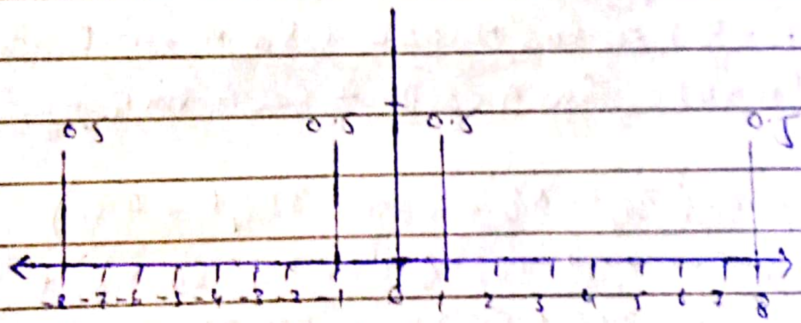
$$+ \frac{\pi}{2j}\left[\delta(\omega-\pi/4) - \delta(\omega+\pi/4)\right]$$

$$+ \frac{1}{\sqrt{2}}\left[\frac{\pi}{j}\left[\delta(\omega-3\pi/4) + \delta(\omega+3\pi/4)\right]\right]$$

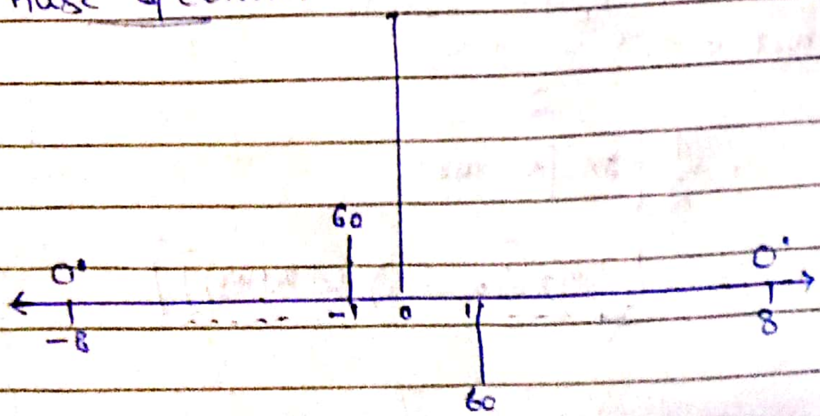
$$+ \frac{\pi}{\sqrt{2}}\left[\delta(\omega-3\pi/4) + \delta(\omega+3\pi/4)\right]$$

Spectrum:

Amplitude Spectrum.



Phase Spectrum.



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(C) $x(t) = \begin{cases} 1 & -1 \le t \le 0 \\ -1 & 0 \le t \le 1 \end{cases}$

Sol

$T = 2 \sec \Rightarrow \omega_0 = 2\pi / T$

$\boxed{\omega_0 = \pi}$

Now

$$C_0 = \frac{1}{T}\int_{-1}^{1} x(t)\,dt = \frac{1}{2}\left[\int_{-1}^{0} dt + \int_{0}^{1} -dt\right]$$

$$= \frac{1}{2}\left[t\Big|_{-1}^{0} - t\Big|_{0}^{1}\right]$$

$$= \frac{1}{2}\left[0 + 1 - 1\right]$$

$$\boxed{C_0 = 0}$$

$$C_n = \frac{1}{T}\int_{-1}^{1} x(t)\, e^{-jn\omega_0 t}\,dt \Rightarrow \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{2}\left[\int_{-1}^{0} e^{-jn\pi t}\,dt + \int_{0}^{1} -e^{-jn\pi t}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{e^{-jn\pi t}}{-jn\pi}\Big|_{-1}^{0} + \left[\frac{-e^{-jn\pi t}}{-jn\pi}\Big|_{0}^{1}\right]\right]$$

$$= \frac{1}{2}\left[\frac{1-e^{jn\pi}}{-jn\pi} - \left[\frac{-e^{0} + e^{-jn\pi}}{-jn\pi}\right]\right]$$

$$= \frac{1}{2}\left[\frac{e^{jn\pi} - 1 - 1 + e^{-jn\pi}}{jn\pi}\right]$$

$$= \frac{-2 + 2(-1)^n}{2jn\pi}$$

$$C_n = \frac{-1 + (-1)^n}{jn\pi}$$

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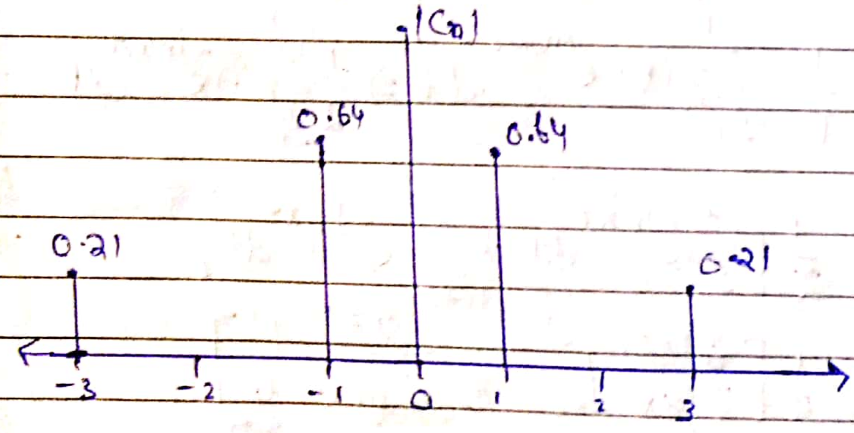
Now

n	C_n	$\lvert C_n \rvert$	$\angle C_n$
-3	$\frac{2}{3j\pi} = -\frac{2j}{3\pi}$	$2/3\pi$	-90°
-2	0	0	0°
-1	$\frac{2}{j\pi} = \frac{-j2}{\pi}$	$2/\pi$	-90°
0	0	0	0°
1	$-\frac{2}{j\pi} = 2j/\pi$	$2/\pi$	90°
2	0	0	0°
3	$-\frac{2}{3j\pi} = \frac{2j}{3\pi}$	$\frac{2}{3\pi}$	90

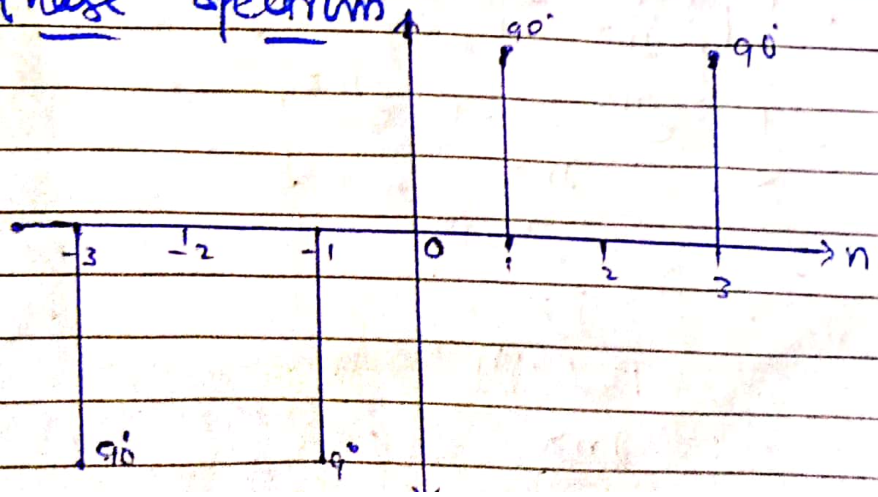
also $\frac{2}{\pi} = 0.64$

$\frac{2}{3\pi} = 0.21$

Magnitude Spectrum.



Phase Spectrum



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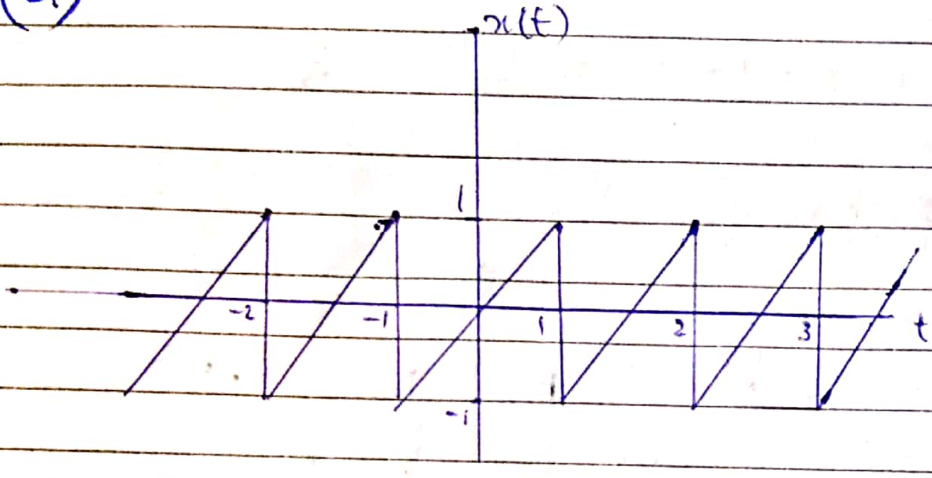
(Question No : 2)

Fourier Series Representation (a_{-3} to a_3):

(a)



$T = 0$

Since Signal is odd $a_n = 0$

$$a_0 = \frac{1}{2} \times 1 \times (-1) + \frac{1}{2} \times 1 \times 1 = 0$$

$$b_n = \frac{1}{T} \int x(t) \sin n\omega_0 t \, dt \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$= \frac{1}{2} \int_{-1}^{1} \sin n\pi t \, dt$$

$$= \frac{1}{2} \times 2 \int_{0}^{1} \sin n\pi t \, dt$$

$$= t \left(-\frac{\cos n\pi t}{n\pi} \right) \Big|_0^1 + \int_0^1 \frac{\cos n\pi t}{n\pi} dt$$

$$= -\cos n\pi + \frac{\sin n\pi t}{(n\pi)^2} \Big|_0^1$$

$$b_n = -\cos n\pi$$

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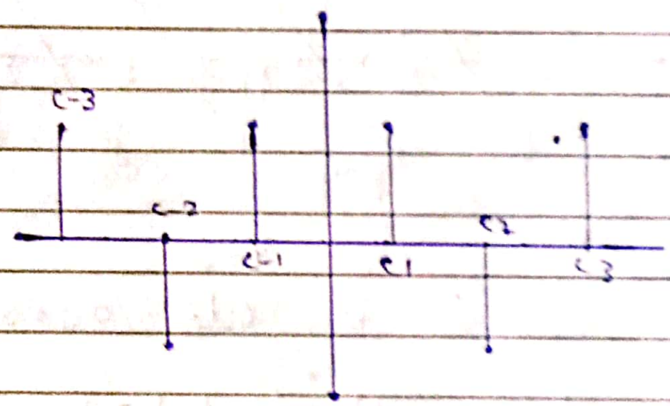
$C_0 = a_0 = 0, \quad C_n = \frac{a_n + jb_n}{2} = -\cos n\pi/2$

$C_{-1} = C_1 = -\frac{\cos 2\pi}{2} = -1/2$

$C_{-2} = C_2 = -\frac{\cos 2\pi}{2} = -1/2$

$C_{-3} = C_3 = -\frac{\cos 3\pi}{3} = 1/2$

Spectrum:



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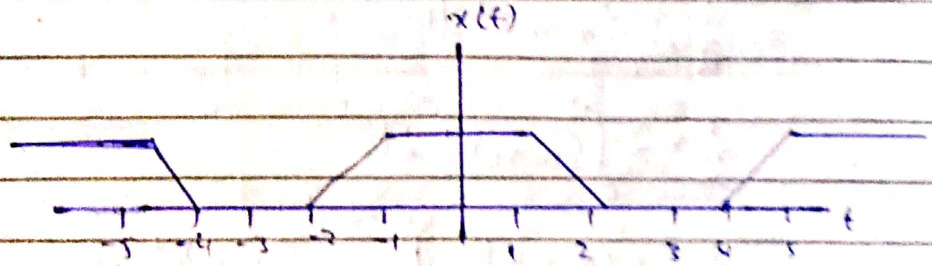
(b)



Sol

Now

$T = 8 \quad w_0 = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{8} \Rightarrow w_0 = \pi/4$

Signal is even ; $b_n = 0$

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$$a_0 = \frac{1}{T}\int x(t)\,dt$$

$$= \frac{1}{8}\int x(t)\,dt = \frac{1}{8}\times\frac{1}{2}\times(4+2))\times 1 = 3/8$$

$$a_n = \frac{1}{T}\int x(t)\,\cos n\omega_0 t\,dt$$

$$x(t) = t+2 \quad ; \quad -2 \le t \le -1$$
$$= 1 \quad ; \quad -1 \le t < 1$$
$$= 2-t \quad ; \quad 1 \le t \le 2$$

$$a_n = \frac{1}{8}\Big[\int_{-2}^{-1}(t+2)\cos n\omega_0 t\,dt + \int_{-1}^{1}\cos n\omega_0 t\,dt + \int_{1}^{2}(2-t)\cos n\omega_0 t\,dt\Big]$$

$$a_n = \frac{1}{8}\Big[\frac{t\sin \omega_0 t}{n\omega_0}\Big|_{-2}^{-1} - \int_{-2}^{-1}\frac{\sin n\omega_0 t}{n\omega_0}dt + \frac{2\sin n\omega_0 t}{n\omega_0}\Big|_{-2}^{-1}$$

$$+ \frac{\sin n\omega_0 t}{n\omega_0}\Big|_{-1}^{1} + \frac{2\sin n\omega_0 t}{n\omega_0}\Big|_{1}^{2} - \frac{t\sin n\omega_0 t}{n\omega_0}\Big|_{1}^{2}$$

$$+ \int_{-1}^{2}\frac{\sin n\omega_0 t\,dt}{n\omega_0}\,dt\Big]$$

$$a_n = \frac{1}{8}\Big[\frac{\cos n\omega_0 - \cos 2n\omega_0}{(n\omega_0)^2} - \frac{\cos 2n\omega_0 - \cos n\omega_0}{(n\omega_0)^2}\Big]$$

$$= \frac{1}{8}\Big[\frac{2(\cos n\omega_0 - \cos n\omega_0)}{(n\omega_0)^2}\Big]$$

$$= \frac{1}{4}\times\frac{-1}{(n\omega_0)^2}\Big[\cos\frac{n\pi}{4} - \cos\frac{n\pi}{4}\Big]$$

$$= \frac{1}{4}\times\frac{1}{(n\omega_0)^2}\times\cos n\pi/4$$

$$a_n = \frac{1}{4}\times\frac{16}{(n\pi)^2}\times\cos n\frac{\pi}{4} \qquad \omega_0 = \pi/4$$

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$$a_n = \frac{4}{(n\pi)^2} \cos n\pi/4$$

$$C_0 = a_0 = 3/8, \quad C_n = \frac{a_n + jb_n}{2}$$

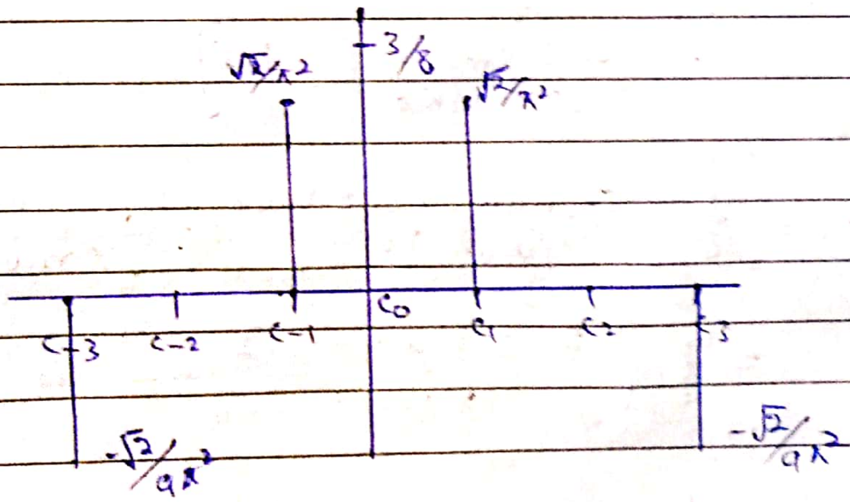
$$C_n = \frac{2}{(n\pi)^2} \cos\frac{n\pi}{4}$$

$$C_{-3} = C_3 = \frac{2}{(3\pi)^2} \cos 3\pi/4 = -\sqrt{2}/9\pi^2$$

$$C_{-2} = C_2 = \frac{2}{(2\pi)^2} \cos 2\pi/4 = 0$$

$$C_{-1} = C_1 = \frac{2}{(\pi)^2} \cos \pi/4 = \frac{\sqrt{2}}{\pi^2}$$

Spectrum:



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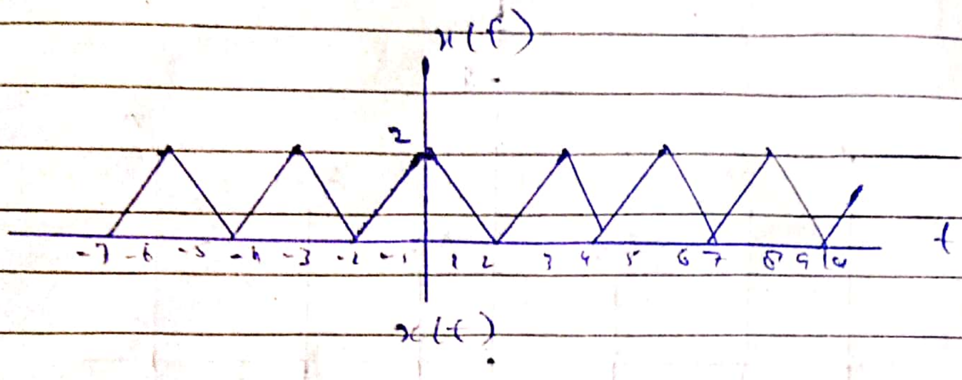
(C)



Now

$T = 3$; $\omega_0 = \frac{2\pi}{T} = 2\pi/3 \Rightarrow \boxed{\omega_0 = \frac{2\pi}{3}}$

$$x(t) = \begin{cases} 2+t ; & -2 < t < 0 \\ 2-t & 0 < t < 2 \end{cases}$$

$$a_0 = \frac{1}{T}\int x(t)\,dt = \frac{1}{3}\left[\int_{-2}^{0}(2+t)\,dt + \int_{0}^{1}(2-t)\,dt\right]$$

$a_0 = 1$

$$a_k = \frac{1}{3}\left[\int_{-2}^{0}(2+t)\,e^{-jk(2\pi/3)t}\,dt + \int_{0}^{1}(2-t)\,e^{-jk(2\pi/3)t}\,dt\right]$$

$$a_k = \frac{3j}{2\pi^2k^2}\left[e^{jk2\pi/3}\sin\left(\frac{k2\pi}{3}\right) + 2e^{j\pi/3 k}\sin\left(\frac{k\pi}{3}\right)\right]$$

$a_0 = C_0 = 1$

$C_{-3} = C_3 = \pi$

$C_{-2} = C_2 = 2\pi/3$

$C_1 = C_1 = \pi/3$

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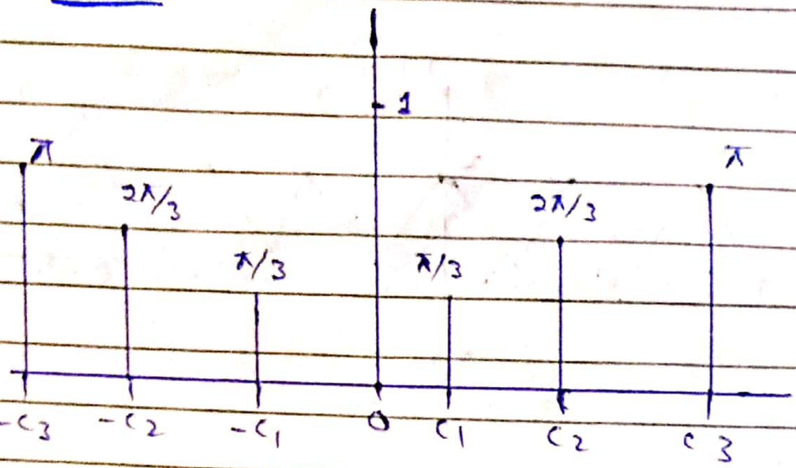
Spectrum.



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END of Q2:

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(Question No: 3)

Given:

$$y(t) = e^{-2t}u(t)$$

$$y(t) = ?$$

Sol

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \rightarrow \boxed{H(\omega)} \rightarrow y(t) = \sum_{n=-\infty}^{\infty} c_n H(n\omega_0) e^{jn\omega_0 t}$$

$$H(\omega) = \frac{1}{2+j\omega}$$

(i) $x(t) = e^{j8t} + 2e^{j\pi/3}e^{j12t}$

Sol

$$T_1 = \frac{2\pi}{8} = \frac{\pi}{4}$$

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$T_2 = \frac{2\pi}{12} = \pi/6$

$\frac{T_1}{T_2} = \frac{6}{4} = 3/2 = x$

$T = xT_1 = \frac{3}{2} \times \frac{\pi}{4} = \pi/2$

$\omega = \frac{2\pi}{T} = 4$

$C_2 = 1$

$C_3 = 2e^{j\pi/3} = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)$

$$y(t) = \sum_{n=-\infty}^{\infty} C_n H(n\omega_0) e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} C_n \frac{1}{2+jn\omega} e^{jn\omega t}$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} C_n \left(\frac{1}{2+4jn}\right) e^{j4nt}}$$

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(b)

$x(t) = \cos\left(\frac{\pi}{4}t - \frac{\pi}{6}\right) + \sin\left(\frac{3\pi t}{4} + \pi/4\right)$

$$x(t) = \frac{e^{j(\pi/4 t - \pi/6)} + e^{-j(\pi/4 t - \pi/6)}}{2}$$

$$\frac{e^{j(3\pi/4 t + \pi/4)} - e^{j(3\pi t/4 + \pi/4)}}{2}$$

$T_1 = \frac{2\pi}{\pi} \times 4 = 8$

$T_2 = \frac{2\pi}{3\pi} \times 4 = 8/3$

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$$\frac{T_1}{T_2} = \frac{8}{8} \times 3 = 3$$

$$T = 8$$

$$\omega = 2\pi/8 = \pi/4$$

$$x(t) = \underbrace{\frac{e^{-j\pi/4} e^{j\pi/4 t}}{2}}_{C_1} + \underbrace{\frac{e^{j\pi/4} e^{-j\pi/4 t}}{2}}_{C_{-1}} + \underbrace{\frac{e^{3\pi/4 t} - e^{j\pi/4}}{2j}}_{C_3} \; \underbrace{e^{-j(3\pi/4} - \frac{e^{-j(3\pi/4)}}{2j}}_{C_{-3}}$$

$$\boxed{v(t) = \sum_{n=-\infty}^{\infty} C_n \left(\frac{1}{2 + j\pi/4 n}\right) e^{j(\pi/4)nt}}$$

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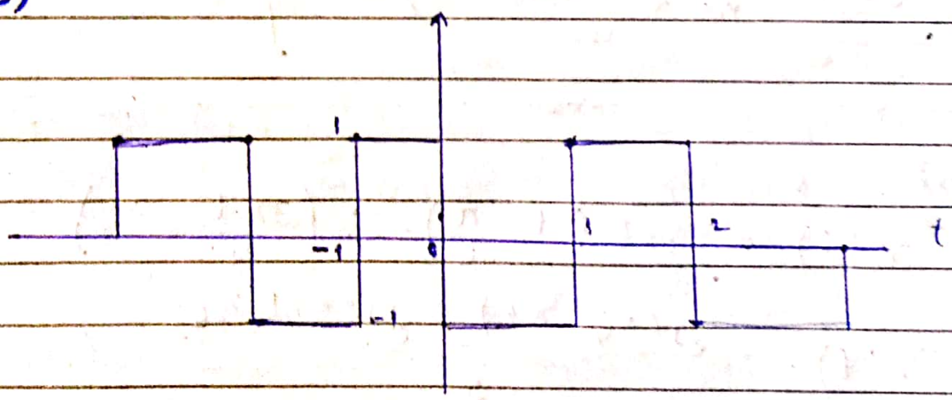
(C)



Period = 2

$$\omega_0 = 2\pi/2 = \pi$$

$\because$ Signal is odd;

$$C_n = -C_{-n}$$

$$C_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt$$

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$$C_o = \frac{1}{\alpha}\int_{1} dt + \int_{0} x\, dt$$

$$C_o = \frac{1}{\alpha}[1-1]$$

$$C_o = 0$$

$$C_n = \frac{1}{T}\left[\int_{-1}^{0} e^{jnw_ot} dt - \int_{0}^{1} e^{-jnw_ot} dt\right]$$

$$= \frac{1}{T}\left[\frac{e^{-jnw_ot}}{-jnw_ot}\right]_{-1}^{0} - \left(\frac{-1}{jnw_o}\right)\left[e^{-jnw_ot}\right]_0^1$$

$$C_n = \frac{1}{2}\left[\frac{1-e^{jnw_o}}{-jnw_o} + \frac{1}{jnw_o}\left[e^{-jnw_o} - 1\right]\right]$$

$$= \frac{1}{2jnw_o}\left[e^{jnw_o} - 1 - e^{-jnw_o} - 1\right]$$

$$C_n = \left[\frac{2\cos nw_o - 1}{2jnw_o}\right]$$

$$C_n = \frac{\cos nw_o - 1}{jnw_o}$$

$$\boxed{y(t) = \sum_{n=-\alpha}^{\alpha} C_n \frac{1}{2+jn\pi} e^{jn\pi t}}$$

x ———————— x

(a) it is mentioned that I have solved Fourier Series Co-efficients

$$T = 2$$

$$w = \frac{2\pi}{2} = \pi$$

$$\boxed{y(t) = \sum_{n=-\alpha}^{\alpha} C_n \frac{1}{2+jn\pi} e^{jn\pi t}}$$

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(b) $T = 6$

$W = 2\pi/T = \pi/3$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} C_n \frac{1}{2 + jn\pi/3} e^{j\pi/3 nt}}$$

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(c) $T = 3$

$w = \frac{2\pi}{3}$

$$\boxed{y(t) = \sum_{n=-\infty}^{n} C_n \frac{1}{2 + jn2\pi/3} e^{j\frac{2\pi}{3}nt}}$$

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END OF Assignment-2