Circuits and Systems 1 - Week 6 - Solution of 2 Equations

Solution of Eg1 Nodal - Slide 1

At node 1:

$$\frac{10 - v_1}{1} = \frac{v_1 - v_2}{2} + \frac{v_1}{5} \tag{1}$$

At node 2:

$$\frac{v_1 - v_2}{2} + 2 = \frac{v_2}{10} \tag{2}$$

Let us convert solve these 2 equations as follows:

$$10 - v_1 = \frac{v_1}{2} - \frac{v_2}{2} + \frac{v_1}{5}$$
$$-v_1 - \frac{v_1}{2} - \frac{v_1}{5} + \frac{v_2}{2} = -10 \tag{3}$$

$$\frac{v_1}{2} - \frac{v_2}{2} + 2 = \frac{v_2}{10}$$
$$\frac{v_1}{2} - \frac{v_2}{2} - \frac{v_2}{10} = -2 \tag{4}$$

Solution of Eg1 Nodal - Slide 2

$$(-1-\frac{1}{2}-\frac{1}{5})v_1+(\frac{1}{2})v_2=-10 \quad (5)$$

$$(-1.7)v_1+(-0.5)v_2=-10 \quad (6)$$

$$(\frac{1}{2})v_1+(-\frac{1}{2}-\frac{1}{10})v_2=-2 \quad (7)$$

$$(0.5)v_1+(-0.6)v_2=-2 \quad (8)$$

Let us convert these 2 equations into matrices form as shown in next slide:

$$(-1.7)v_1+(0.5)v_2=-10 \quad (9)$$

$$(0.5)v_1+(-0.6)v_2=-2 \quad (10)$$

Solution of Eg1 Nodal - Slide 3

$$\begin{bmatrix} \mathbf{-1.7} & \mathbf{0.5} \\ \mathbf{0.5} & \mathbf{-0.6} \end{bmatrix} \begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \begin{bmatrix} \mathbf{-10} \\ \mathbf{-2} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \left(\begin{bmatrix} \mathbf{-1.7} & \mathbf{0.5} \\ \mathbf{0.5} & \mathbf{-0.6} \end{bmatrix} \right)^{\mathbf{-1}} \begin{bmatrix} \mathbf{-10} \\ \mathbf{-2} \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \begin{bmatrix} \mathbf{-0.7792} & \mathbf{-0.6494} \\ \mathbf{-0.6494} & \mathbf{-2.2078} \end{bmatrix} \begin{bmatrix} \mathbf{-10} \\ \mathbf{-2} \end{bmatrix} \tag{13}$$

Final Solution of Example 1

The final values are as follows:

$$\begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \end{bmatrix} = \begin{bmatrix} \mathbf{9.0909} \\ \mathbf{10.9091} \end{bmatrix} \tag{14}$$

Nodal Analysis Example 2

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$
$$\frac{v_1 - v_2}{5} = \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2}$$
$$i_\phi = \frac{v_1 - v_2}{5}$$

If we substitute i_ϕ, we obtain the following 2 equations

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$
$$\frac{v_1 - v_2}{5} = \frac{v_2}{10} + \frac{v_2}{2} - \frac{8}{2}\frac{v_1 - v_2}{5} \quad (15)$$

Nodal Analysis Example 2 - Slide 2

$$\begin{aligned} \frac{v_1}{2} - \frac{20}{2} + \frac{v_1}{20} + \frac{v_1}{5} - \frac{v_2}{5} &= 0 \\ \frac{v_1}{5} - \frac{v_2}{5} = \frac{v_2}{10} + \frac{v_2}{2} - \frac{8v_1}{10} + \frac{8v_2}{10} & \end{aligned} \tag{16}$$

Let us combine common terms to obtain more simple solution as follows:

$$\begin{aligned} \left(\frac{1}{2} + \frac{1}{20} + \frac{1}{5}\right) v_1 + \frac{-1}{5} v_2 &= 10 \\ \left(\frac{1}{5} + \frac{8}{10}\right) v_1 + \left(\frac{-1}{5} - \frac{1}{10} - \frac{1}{2} - \frac{8}{10}\right) v_2 &= 0 \end{aligned} \tag{17}$$

Nodal Analysis Example 2 - Slide 2

$$\begin{aligned} (\mathbf{0.75})\, v_1 + (-\mathbf{0.2}) v_2 &= \mathbf{10} \\ v_1 + (-\mathbf{1.6}) v_2 &= \mathbf{0} \end{aligned} \tag{18}$$

Now let us substitute $v_1 = \mathbf{1.6} v_2$ in the above equation, we obtain the following:

$$\begin{aligned} \mathbf{0.75}(\mathbf{1.6} v_2) + (-\mathbf{0.2}) v_2 &= \mathbf{10} \\ \mathbf{1.2} v_2 - \mathbf{0.2} v_2 &= \mathbf{10} \\ v_2 = \mathbf{10} \Longrightarrow v_1 &= \mathbf{1.6}(\mathbf{10}) = \mathbf{16} \end{aligned} \tag{19}$$