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Title Page

Assignment No: 1

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Section: B

Pages : 18 (excluding title page)

Submitted to:

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(Question NO 1)

Given:

X = no of head obtaind by tipping fair coin 4 time.

Required:

a) S.S = ? and $P(x) = ?$

(b) mapping from S to S_X The range of X = ?

(c) $P_X(x)$ for Various Value of X.

Sol

(a) S.S = ? $P(x) = ?$

S.S = { HHHH, HHHT, HHTH, HTHH
THHH, HHTT, HTHT, THHT,
THTH, TTHH, HTTH, HTTT
THTT, TTHT, TTTH, TTTT }

Now $P(x) = ?$

As Coin is fair So

$P[HHHH] = P[HHHT] = \ldots\ldots P[TTTT] = 1/16$

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(b) mapping from S to S_x.

As

x = no of head in four flip of coin.

So

S	HHHH	HHHT	HHTH	HTHH	THHH	TTHH	THTH	THHT	HHTT
$X(S)=x$	4	3	3	3	3	2	2	2	2

S	HTHT	HTTH	HTTT	TTHT	THTT	TTTT	TTTH
$X(S)=x$	2	2	1	1	1	0	1

So

$$S_x = \{0, 1, 2, 3, 4\}$$

(c) $P_x(x) = ?$

$$P_x(0) = 1/16$$

$$P_x(1) = 4/16$$

$$P_x(2) = 6/16$$

$$P_x(3) = 4/16$$

$$P_x(4) = 1/16$$

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(Question No : 2)

Given

nine note of Rs. 10
one note of Rs. 50
X = total amount when two notes are selected.

Required

(a) Sample Space = ? $P(x) = ?$
(b) mapping $S \to to \to S_x$
(c) $P_X(x) = ?$

Sol

(a) $S.S = ?$ $P(x) = ?$

let $A_1, A_2, A_3, A_4 \cdots A_9$ represent notes of R.S. 10
& B represent note of Rs 50
Then

$S.S = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, B\}$.

or $S.S = \{A, B\}$ A = nine time Represent note of Rs 10
B = one note of Rs 50

Now

$P[A_1] = P[A_2] = \cdots P[A_9] = \frac{9}{10}$

And

$P[B] = \frac{1}{10}$

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(b) mapping $S \to$ to $\to S_x$.

X = total amount that two notes selected.

$$S_x = \{ A_1A_2, A_1A_3 \ldots\ldots A_1A_9$$
$$A_2A_1, A_2A_3 \ldots\ldots A_2A_9$$
$$\vdots \quad \vdots \quad \vdots$$
$$A_9A_1, A_9A_2 \ldots\ldots A_9A_8,$$
$$A_1B, BA_1, A_2B, BA_2 \ldots\ldots$$
$$A_9B, BA_9 \}$$

Now total no of outcome if both notes of Rs 10 = ?

we will use combination b/c without Replacement

So

$$\text{no of } (10,10) = {}^9C_2$$
$$= \frac{9!}{2!(9-2)!}$$
$$= \frac{9 \times \overset{4}{\cancel{8}} \times \cancel{7!}}{\cancel{2}! \times \cancel{7!}}$$

no of (10, 10) = 36

Now total no of outcome if ~~both~~ one notes of Rs. 10 and one of Rs. 50

one of (10, 50) = 18

we can write S. space of X h

$$S_x = \{(10, 10), (10, 50)\}$$

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As (10, 10) → Repeat 36 time
(10, 50) Repeat 18 time So

→ Part (c)

$$P_X(10,10) = \frac{36}{54}$$

$$\boxed{P_X(10,10) = 0.666}$$ Ans

$$P_X(10,50) = \frac{18}{54}$$

$$\boxed{P_X(10,50) = 0.333}$$ Ans

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Question No: 3

Given:

$$S_X = \{1, 2, 3, 4 \ldots\}$$

$$P_K = 0.6/K^2 \quad \text{for } K = 1, 2, 3, \ldots$$

Required:

a) $P[X > 4] = ?$

b) $P[6 \le X < 8] = ?$

Sol

$$S_X = \{1, 2, 3, 4 \ldots\}$$

$$P_1 = 0.6/(1)^2 = 0.6$$

$$P_2 = 0.6/4 = 0.15$$

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$P_3 = 0.6/9 = 0.067$

$P_4 = 0.6/16 = 0.0375$

$P_5 = 0.6/25 = 0.024$

$P_6 = 0.6/36 = 0.0167$

$P_7 = 0.6/49 = 0.0122$

$P_8 = 0.6/64 = 0.0093$

Part (a)

$$P[m>4] = ?$$

$$P[m>4] = 1 - \sum_{i=1}^{4} P_i$$

$$= 1 - [P_1 + P_2 + P_3 + P_4]$$

$$= 1 - [0.6 + 0.15 + 0.067 + 0.0375]$$

$$= 1 - 0.8545$$

$$\boxed{P[m>4] = 0.1455}$$

Part (b)

$$P[6 \leq m \leq 8] = ?$$

$$P[6 \leq m \leq 8] = P_6 + P_7 + P_8$$

$$= 0.0167 + 0.0122 + 0.0093$$

$$\boxed{P[6 \leq m \leq 8] = 0.0382}$$

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(Question No: 4)

Given:

To Show VAR[X] of binomial random variable X is npq.

Proof:

Binomial (n, P) random variable as Sum of 'n' Independent Bernoulli (P) random Variable can be written as

$$X = X_1 + X_2 + X_3 + \cdots + X_n$$

then we can write

$$VAR(X) = VAR(X_1) + VAR(X_2) + \cdots + VAR(X_n).$$

if $X_i \sim$ Bernoulli (P), Then its variance is

$$VAR[X_i] = E[X_i^2] - E^2[X]$$

$$= [(1)^2 \times P + (0)^2 \times (1-P)] - P^2$$

$$= (P+0) - P^2$$

$$= P - P^2$$

$$VAR[X_i] = P(1-P)$$

from above discussion we can write it as

$$P + T + 0$$

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$$VAR[x] = P(1-P) + P(1-P) + \cdots + P(1-P)$$

$$VAR[x] = n \cdot P(1-P)$$

As $1-P = q$ so

$$\boxed{VAR[x] = npq}$$

* Hence Proved *

xx —— xx —— xx —— xx —— xx ——

(Question No : 5)

Given:

$E[x] = ?$

$E[x^2] = ?$

$VAR[X] = ?$

Sol

a) for equation (1):

Sample Space of X for equation (1) is.

$$S_x = \{0, 1, 2, 3, 4\}$$

We know that

$$E[x] = \sum_{\text{all } x} x \cdot P_x(x) \quad x = 0, 1, 2, 3, 4$$

So $E[x] = 0 \times 1/16 + 1 \times 4/16 + 2 \times 6/16 + 3 \times 4/16 + 4 \times 1/16$

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$E[x] = 0 + 0.25 + 0.75 + 0.75 + 0.25$

$\boxed{E[x] = 2}$ 1st moment

Now $E[x^2] = ?$

for $x = 0$ $x^2 = 0$

$x = 1$ $x^2 = 1$

$x = 2$ $x^2 = 4$

$x = 3$ $x^2 = 9$

$x = 4$ $x^2 = 16$

So

$S_x = \{0, 1, 4, 9, 16\}$

again

$E[x^2] = \sum_{all\, x} x \cdot P_x(x)$

→ Probabilities will be same as we finded in eq (1) so

$E[x^2] = 0 \times 1/16 + 1 \times 4/16 + 4 \times 6/16$
$+ 9 \times 4/16 + 16 \times 1/16$

$E[x^2] = 0 + 0.25 + 1.5 + 2.25 + 1$

$\boxed{E[x^2] = 5}$ 2nd moment.

Now $VAR[x] = ?$

As we know that

$VAR[x] = \sigma_x^2 = E[x^2] - E^2[x]$

Put values

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$VAR[x] = 5 - (2)^2$

$\boxed{VAR[x] = 1}$ variance of x

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(b) Now for equation (2).

Sol

As Sample for x in eq 2 is,

$S_x = \{(10,10), (10,50)\}$

or

$S_x = \{20, 60\}$

$P_x(20) = 0.666$

$P_x(60) = 0.333$

Now we know that

$E[x] = \sum_{all\,x} x \cdot P_x(x)$;

$E[x] = 20 \times 0.666 + 60 \times 0.333$

$E[x] = 13.32 + 19.98$

$\boxed{E[x] = 33.3}$ 1st moment

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Now for,

$x = 20 \qquad x^2 = 400$

$x = 60 \qquad x^2 = 3600 \qquad$ So

$S_x = \{400, 3600\}$

Now

$E[x^2] = \sum_{all\, x} x \cdot P_X(x)$

$E[x^2] = 400 \times 0.666 + 0.333 \times 3600$

$E[x^2] = 266.4 + 1198.8$

$\boxed{E[x^2] = 1465.2}$ 2nd moment.

Now $VAR[x] = ?$

$VAR[x] = E[x^2] - E^2[x]$

$= 1465.2 - (33.3)^2$

$= 1465.2 - 1108.89$

$\boxed{VAR[x] = 356.31}$

Variance of X.

—— XX —— XX —— XX —— XX

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(Question No : 6)

Given:

$S_c = \{1, 2, 3, 4\}$

$W = 3c^2$

Required:

→ $E[c] = ?$

→ $VAR[c] = ?$

→ $E[W] = ?$

→ $VAR[W] = ?$

→ $E[c^3]$

Sol

$S_c = \{1, 2, 3, 4\}$

As c is uniform Random variable

So

$P_c(1) = P_c(2) = P_c(3) = P_c(4) = 1/4$

Now $E[c] = ?$

As

$$E[c] = \sum_{\text{all } c} c \cdot P_c(c)$$

$$E[c] = 1 \times 1/4 + 2 \times \frac{1}{4} + 3 \times \frac{1}{4} + 4 \times \frac{1}{4}$$

$$E[c] = 0.25 + 0.5 + 0.75 + 1$$

$\boxed{E[c] = 2.5}$ mean of c.

Now $VAR[c] = ?$

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As we know that

$$VAR[C] = \sum_{all\ c} (c - E[c])^2 \cdot P_c(c)$$

$$VAR[C] = (1-2.5)^2 \times \frac{1}{4} + (2-2.5)^2 \times 1/4 + (3-2.5)^2 \times 1/4 + (4-2.5)^2 \times 1/4$$

$$= (-1.5)^2 \times \frac{1}{4} + (-0.5)^2 \times 1/4 + (0.5)^2 \times 1/4 + (1.5)^2 \times 1/4$$

$$VAR[C] = 2.25 \times 1/4 + 0.25 \times 1/4 + 0.25 \times 1/4 + 2.25 \times 1/4$$

$$= 0.562 + 0.0625 + 0.0625 + 0.562$$

$$\boxed{VAR[C] = 1.249}$$ variance of C

Now $E[W] = ?$

Sol

for W)

$c = 1$ $W = 3(1)^2 = 3$

$c = 2$ $W = 3(2)^2 = 12$

$c = 3$ $W = 3(3)^2 = 27$

$c = 4$ $W = 3(4)^2 = 48$

So

$$S_W = \{3, 12, 27, 48\}$$

→ Probability is same as in case of C.

Now

$$E[w] = \sum_{all\ w} w \cdot P_W(w)$$

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$$E[W] = 3 \times 1/4 + 12 \times 1/4 + 27 \times 1/4 + 48 \times 1/4$$

$$E[W] = 0.75 + 3 + 6.75 + 12$$

$$\boxed{E[W] = 22.5}$$ mean of W

Now $VAR[W] = ?$

Sol

As

$$VAR[W] = E[W - E[W]]^2$$

(or)

$$VAR[W] = \sum_{\text{all } w} (w - E[W])^2 P_W(w)$$

$$VAR[W] = (3-22.5)^2 \times 1/4 + (12-22.5)^2 \times 1/4 + (27-22.5)^2 \times 1/4 + (48-22.5)^2 \times 1/4$$

$$VAR[W] = (-19.5)^2 \times 1/4 + (-10.5)^2 \times 1/4 + (4.5)^2 \times 1/4 + (25.5)^2 \times 1/4$$

$$= 380.25 \times 1/4 + 110.25 \times 1/4 + (20.25) \times 1/4 + 650.25 \times 1/4$$

$$= 95.06 + 27.56 + 5.063 + 162.56$$

$$\boxed{VAR[W] = 290.25}$$ variance of W

Now $E[c^3] = ?$

for

$c = 1 \quad c^3 = 1$

$c = 2 \quad c^3 = 8$

$c = 3 \quad c^3 = 27$

$c = 4 \quad c^3 = 64$

So
$S_c = \{1, 8, 27, 64\}$

Now

$$E[c^3] = \sum_{all\ c} c \cdot P_c(c)$$

$$E[c^3] = 1 \times 1/4 + 8 \times 1/4 + 27 \times 1/4 + 64 \times 1/4$$

$$E[c^3] = 0.25 + 2 + 6.75 + 16$$

$\boxed{E[c^3] = 25}$ 3rd moment of c.

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(Question NO # 07)

Given:

N = no of Customers

$\alpha = \lambda t$

λ = arrival rate = Customers/second.

λ = 6 customers/hour

Required:

P = ?

a) more than 12 Customers in 2 hours.

b) less than or equall to 12 Customer's in 2 hours.

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Sol

$$x = \lambda t$$

$$\lambda = 6 \text{ Customers}/\text{hour} = 6/60 \times 60 s$$

$$\lambda = 6/3600 s$$

$$\lambda = \frac{1}{600} s$$

a) more than 18 Customers in 18 hours.

$$P[N > 18] = 1 - P[N \leq 18]$$

$$= 1 - \sum_{k=0}^{18} \left(\frac{(\alpha)^k}{k!} \cdot e^{-\alpha} \right)$$

here $t = 2$ hours $= 2 \times 3600 = 7200 s$

$$\alpha = \lambda t = \frac{1}{\cancel{600}} \times \cancel{7200}^{12}$$

$$\alpha = 12$$

$$P[N > 18] = 1 - \sum_{k=0}^{18} \left(\frac{(12)^k}{k!} \cdot e^{-12} \right)$$

$$= 1 - e^{-12} \left[\frac{(12)^0}{0!} + \frac{(12)^1}{1!} + \frac{(12)^2}{2!} + \cdots \frac{(12)^{18}}{18!} \right]$$

by Solving it we get.

$$\boxed{P[N > 18] = 0.03742}$$

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(b) less than or equall to 12 Customer in 2 hours.

Sol

again

$t = 2\text{ hours} = 7200\,s$

$\alpha = \left(\frac{1}{600\,s}\right)\left(\cancel{7200\,s}^{12}\right)$

$\alpha = 12$

$$\boxed{P[N \leq 12] = 0.5760}$$

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END OF Assignment - 1