This linear combination can be written as the matrix product (verify)

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

which has the form of a linear system. Forming the corresponding augmented matrix and transforming it to reduced row echelon form, we obtain (verify)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

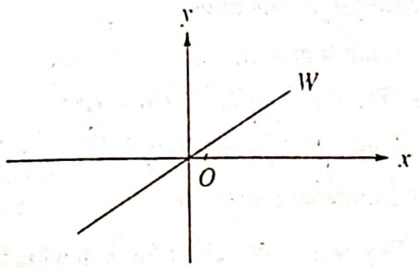
Hence the linear system is consistent with $c_1 = 0$, $c_2 = 1$, and $c_3 = 1$. Thus $\mathbf{u}$ is in span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. ■

Key Terms

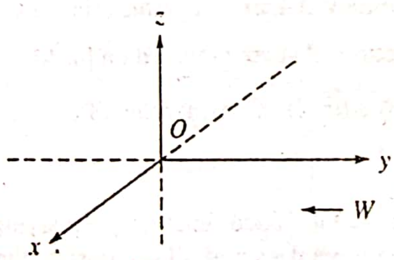
Subspace
Zero subspace
Closure property
Solution space
Linear combination

6.2 Exercises

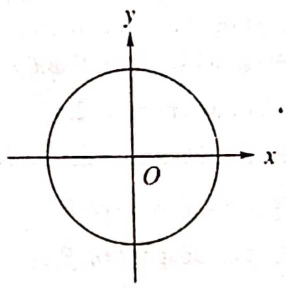
1. The set W consisting of all the points in R^2 of the form (x, x) is a straight line. Is W a subspace of R^2? Explain.



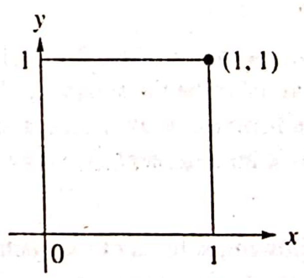
2. Let W be the set of all points in R^3 that lie in the xy-plane. Is W a subspace of R^3? Explain.



3. Consider the circle in the xy-plane centered at the origin whose equation is $x^2 + y^2 = 1$. Let W be the set of all vectors whose tail is at the origin and whose head is a point inside or on the circle. Is W a subspace of R^2? Explain.



4. Consider the unit square shown in the accompanying figure. Let W be the set of all vectors of the form $\begin{bmatrix} x \\ y \end{bmatrix}$, where $0 \leq x \leq 1$, $0 \leq y \leq 1$. That is, W is the set of all vectors whose tail is at the origin and whose head is a point inside or on the square. Is W a subspace of R^2? Explain.



5. Which of the following subsets of R^3 are subspaces of R^3? The set of all vectors of the form
   (a) $(a, b, 2)$
   (b) (a, b, c), where $c = a + b$
   (c) (a, b, c), where $c > 0$

6. Which of the following subsets of R^3 are subspaces of R^3? The set of all vectors of the form
   (a) (a, b, c), where $a = c = 0$
   (b) (a, b, c), where $a = -c$
   (c) (a, b, c), where $b = 2a + 1$

7. Which of the following subsets of R^4 are subspaces of R^4? The set of all vectors of the form
   (a) (a, b, c, d), where $a - b = 2$
   (b) (a, b, c, d), where $c = a + 2b$ and $d = a - 3b$
   (c) (a, b, c, d), where $a = 0$ and $b = -d$

8. Which of the following subsets of R^4 are subspaces of R^4? The set of all vectors of the form
   (a) (a, b, c, d), where $a = b = 0$
   (b) (a, b, c, d), where $a = 1, b = 0$, and $c + d = 1$
   (c) (a, b, c, d), where $a > 0$ and $b < 0$

9. Which of the following subsets of P_2 are subspaces? The set of all polynomials of the form
   (a) $a_2t^2 + a_1t + a_0$, where $a_0 = 0$
   (b) $a_2t^2 + a_1t + a_0$, where $a_0 = 2$
   (c) $a_2t^2 + a_1t + a_0$, where $a_2 + a_1 = a_0$

10. Which of the following subsets of P_2 are subspaces? The set of all polynomials of the form
   (a) $a_2t^2 + a_1t + a_0$, where $a_1 = 0$ and $a_0 = 0$
   (b) $a_2t^2 + a_1t + a_0$, where $a_1 = 2a_0$
   (c) $a_2t^2 + a_1t + a_0$, where $a_2 + a_1 + a_0 = 2$

11. (a) Show that P_2 is a subspace of P_3.
   (b) Show that P_n is a subspace of P_{n+1}.

12. Show that P_n is a subspace of P.

13. Show that P is a subspace of the vector space defined in Example 5 of Section 6.1.

14. Let $\mathbf{u} = (1, 2, -3)$ and $\mathbf{v} = (-2, 3, 0)$ be two vectors in R^3 and let W be the subset of R^3 consisting of all vectors of the form $a\mathbf{u} + b\mathbf{v}$, where a and b are any real numbers. Give an argument to show that W is a subspace of R^3.

15. Let $\mathbf{u} = (2, 0, 3, -4)$ and $\mathbf{v} = (4, 2, -5, 1)$ be two vectors in R^4 and let W be the subset of R^4 consisting of all vectors of the form $a\mathbf{u} + b\mathbf{v}$, where a and b are any real numbers. Give an argument to show that W is a subspace of R^4.

16. Which of the following subsets of the vector space M_{23} defined in Example 4 of Section 6.1 are subspaces? The set of all matrices of the form
   (a) $\begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$, where $b = a + c$
   (b) $\begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$, where $c > 0$
   (c) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, where $a = -2c$ and $f = 2e + d$

17. Which of the following subsets of the vector space M_{23} defined in Example 4 of Section 6.1 are subspaces? The set of all matrices of the form
   (a) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, where $a = 2c + 1$
   (b) $\begin{bmatrix} 0 & 1 & a \\ b & c & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, where $a + c = 0$ and $b + d + f = 0$

18. Which of the following subsets of the vector space M_{nn} are subspaces?
   (a) The set of all $n \times n$ symmetric matrices
   (b) The set of all $n \times n$ nonsingular matrices
   (c) The set of all $n \times n$ diagonal matrices

19. Which of the following subsets of the vector space M_{nn} are subspaces?
   (a) The set of all $n \times n$ singular matrices
   (b) The set of all $n \times n$ upper triangular matrices
   (c) The set of all $n \times n$ matrices whose determinant is 1

20. (*Calculus Required*) Which of the following subsets are subspaces of the vector space $C(-\infty, \infty)$ defined in Example 8?
   (a) All nonnegative functions
   (b) All constant functions
   (c) All functions f such that $f(0) = 0$
   (d) All functions f such that $f(0) = 5$
   (e) All differentiable functions.

21. (*Calculus Required*) Which of the following subsets of the vector space $C(-\infty, \infty)$ defined in Example 8 are subspaces?
   (a) All integrable functions
   (b) All bounded functions
   (c) All functions that are integrable on $[a, b]$
   (d) All functions that are bounded on $[a, b]$

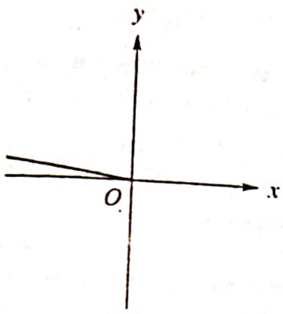
22. (*Calculus Required*) Consider the differential equation

$$y'' - y' + 2y = 0.$$

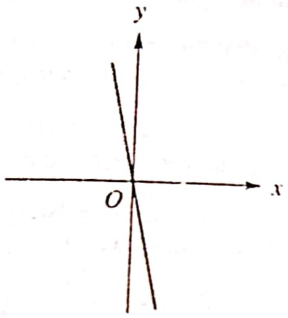
A solution is a real-valued function f satisfying the equation. Let V be the set of all solutions to the given differential equation; define $\oplus$ and $\odot$ as in Example 5 in Section 6.1. Show that V is a subspace of the vector space of all real-valued functions defined on $(-\infty, \infty)$. (See also Section 9.2.)

23. Determine which of the following subsets of R^2 are subspaces.

(a)

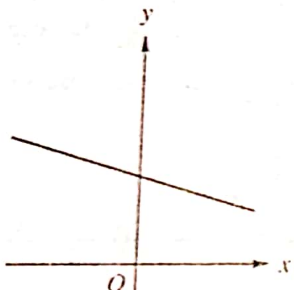


(b)

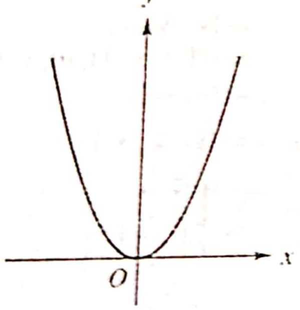


24. Determine which of the following subsets of R^2 are subspaces.

(a)



(b)



25. In each part, determine whether the given vector $\mathbf{v}$ belongs to span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = (1, 0, 0, 1), \qquad \mathbf{v}_2 = (1, -1, 0, 0).$$

and

$$\mathbf{v}_3 = (0, 1, 2, 1).$$

(a) $\mathbf{v} = (-1, 4, 2, 2)$ (b) $\mathbf{v} = (1, 2, 0, 1)$

(c) $\mathbf{v} = (-1, 1, 4, 3)$ (d) $\mathbf{v} = (0, 1, 1, 0)$

26. Which of the following vectors are linear combinations of

$$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}?$$

(a) $\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & -1 \\ 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

27. In each part, determine whether the given vector $p(t)$ belongs to span $\{p_1(t), p_2(t), p_3(t)\}$, where

$$p_1(t) = t^2 - t,$$
$$p_2(t) = t^2 - 2t + 1,$$
$$p_3(t) = -t^2 + 1.$$

(a) $p(t) = 3t^2 - 3t + 1$ (b) $p(t) = t^2 - t + 1$

(c) $p(t) = t + 1$ (d) $p(t) = 2t^2 - t - 1$

Exercises 28 through 33 use bit matrices.

28. Let $V = B^3$. Determine if

$$W = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

is a subspace of V.

29. Let $V = B^3$. Determine if

$$W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

is a subspace of V.

30. Let $V = B^4$. Determine if W, the set of all vectors in V with first entry zero, is a subspace of V.

31. Let $V = B^4$. Determine if W, the set of all vectors in V with second entry one, is a subspace of V.

32. Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Determine if $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ belongs to span S.

33. Let

$$S = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Determine if $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ belongs to span S.

Ex 6.2

Subspaces: let V be a vector space, then W will be subspace of V if

(a) $U \oplus V$ is in W

(b) $k \in R$, $k \odot U$ is in W.

Q: Which of the following subsets of R^3 are subspaces of R^3? the set of all vector of the form

(a) $(a, b, 2)$

let $\vec{U} = (a, b, 2)$ & $\vec{V} = (a', b', 2)$ are two vector in W then

$$\vec{U} \oplus \vec{V} = (a, b, 2) \oplus (a', b', 2)$$

$$= (a+a', b+b', 4) \notin W$$

Which is not in W Hence W is not a subspace.

(b) let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are two vector in W

then $a+b = c$, $a'+b' = c'$

(a) $\vec{U} \oplus \vec{V} = (a, b, c) \oplus (a', b', c')$

$$= (a+a', b+b', c+c') \in W$$

or

$\vec{U} \oplus \vec{V} = (a+a', b+b', a+b'+a'+b')$

$= (a+a', b+b', (a+a')+(b+b')) \in W.$

(b) $k \odot U = k \odot (a, b, c)$

$= k \odot (a, b, a+b)$

$= (k \odot a, k \odot b, k \odot (a+b)) \in W$

$= (k \odot a, k \odot b, k \odot c) \in W$

Hence W is the Subspace of R^3

Ex 6.2

Q6 (a) let $\vec{U} = (a,b,c)$ & $\vec{V} = (a',b',c')$ are the two vector in W then $a = c = 0$, $a' = c' = 0$.

(a) $U \oplus V = (a,b,c) \oplus (a',b',c')$

$= (a+a', b+b', c+c')$

$= (0, b+b', +0) \in W$

(b) $k \odot U = k \odot (a,b,c)$

$= (k \odot a, k \odot b, k \odot c)$

$= (k \odot 0, k \odot b, k \odot 0)$

$= (0, k \odot b, 0) \in W$

$= (0, kb, 0) \in W$

Hence W is the Sub space of R^3.

(b) (a,b,c) where $a = -c$

let $\vec{U} = (a,b,c)$ & $\vec{V} = (a',b',c')$ are two vector in W then $a = -c$, $a' = -c$

(a) $U \oplus V = (a,b,c) \oplus (a',b',c')$

$= (a+a', b+b', c+c')$

$= (-c + c' + b+b', c+c')$

$\vec{U} \oplus \vec{V} = (-(c+c') + (b+b') + (c+c') \in W$

(b) $k \odot \vec{U} = k \odot (a, b, c)$

$= k \odot a, k \odot b, k \odot c$

$= k \odot (-c), k \odot b, k \odot c$

$= (-kc, kb, kc) \notin W$

if $k < 0$ the $kc > 0$.

(c) Let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ be two vector in W

then $b = 2a + 1$, $b' = 2a' + 1$

$\vec{U} \oplus \vec{V} = (a, b, c) \oplus (a', b', c')$

$= (a + a', b + b', c + c')$

$= (a + a', (2a + 1)(2a' + 1), c + c')$

$= (a + a', 2(a + a') + 2, c + c') \notin W$

So W is not a subspace of R^3.

Q7 (a) let $\vec{U}=(a,b,c,d)$ & $\vec{V}=(a',b',c',d')$ be two vector in $\mathbb{R}^4$ then $a-b=2$ & $a'-b'=2$

$a=2+b$ & $a'=2+b'$

(a) $\vec{U}\oplus\vec{V}=(a,b,c,d)\oplus(a',b',c',d')$

$=(a+a',\ b+b',\ c+c',\ d+d')$

$=(b+2+b'+2,\ b+b',\ c+c',\ d+d')$

$=(b+b'+4,\ b+b',\ c+c',\ d+d')\notin W$

So W is not a subspace of $\mathbb{R}^4$.

Part (b), (c) & Q8 is similarly to Q7

Q10 (a) $a_2t^2+a_1t+a_0$ where $a_1=0$ & $a_0=0$

let $\vec{U}=a_2t^2+a_1t+a_0$ & $\vec{V}=a_2't^2+a_1't+a_0'$

(a) $\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')\in W$

(b) $k\odot U=k\odot(a_2t^2+a_1t+a_0)$

$=(ka_2t^2+ka_1t+ka_0)\in W$ Subspace.

(b) $a_2t^2+a_1t+a_0$ where $a_1=2a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ & $\vec{V}=a_2't^2+a_1't+a_0'$

(a) $\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(a_2+a_2')t^2+2(a_0+a_0')t+(a_0+a_0')\in W$

(b) $k\odot U=k\odot(a_2t^2+a_1t+a_0)$

$=ka_2t^2+ka_1t+ka_0$

$=ka_2t^2+2ka_0t+ka_0\in W$ subspace.

(c) $a_2t^2+a_1t+a_0$, where $a_2+a_1+a_0=2.\Rightarrow a_2=2-a_1-a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ + $\vec{V}=a_2't^2+a_1't+a_0'$

$\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(2-a_1-a_0+2-a_1'-a_0')t^2+(a_1+a_1')t+(a_0+a_0')\notin W$

not a subspace.

Q9 is similarly to Q10

Q14 let $w_1 = a_1 U + b_1 V$
& $w_2 = a_2 U + b_2 V$.

(a) $w_1 \oplus w_2 = (a_1 U + b_1 V) \oplus (a_2 U + b_2 V)$

$= a_1 U + a_2 U + b_1 V + b_2 V$

$= (a_1 + a_2) U + (b_1 + b_2) V \in W$

(b) $k \odot w_1 = k \odot (a_1 U + b_1 V)$

$= (k a_1) U + (k b_1) V \in W$

So w is a Subspace of R^3.

Q15 is as same as Q14.

Q16 let $\vec{U} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$, $b = a + c$
$b_1 = a_1 + c_1$

(a) $\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$.

$= \begin{bmatrix} a + a_1 & b + b_1 & c + c_1 \\ d + d_1 & 0 & 0 \end{bmatrix}$.

$$U \oplus V = \begin{bmatrix} a+a_1 & (a+c)+(a_1+c_1) & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$$

(b) $$k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$$

$$= k \odot \begin{bmatrix} a & a+c & c \\ d & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} ka & k(a+b) & kc \\ kd & 0 & 0 \end{bmatrix} \in W$$

Subspace.

(h) $\vec{U} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$ & $c > 0$

(a) $$\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} a_1, & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$$

(b) $$k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & 0 & 0 \end{bmatrix}$$

If $k < 0$ $ka < 0$ so $\notin W$ not subspace.

5

Ex 6.2

(c) Let $\vec{U}=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ & $\vec{V}=\begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$ & $a=-2c$.
$f=2e+d$.

(a) $\vec{U}\oplus\vec{V}=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\oplus\begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$

$$=\begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & f+f_1 \end{bmatrix}$$

$$=\begin{bmatrix} -2(c+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & (2e+d)+(2e_1+d_1) \end{bmatrix}$$

$$=\begin{bmatrix} -2(c_1+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & 2(e+e_1)+(d+d_1) \end{bmatrix} \in W$$

(b) $k\odot U=k\odot\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$=\begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$$

$$=\begin{bmatrix} -2ck & kb & kc \\ kd & ke & k(2e+d) \end{bmatrix} \in W$$

which is subspace.

Q17 is similarly Q16.

Definition: If $S = \{V_1, V_2, \dots V_k\}$ is a set of vector in a vector space V. then the set of all vectors in V that are linear combinations of the vectors in S is denoted by span S or span $\{V_1, V_2 \dots V_k\}$.

Q25 $V_1 = (1,0,0,1)$ $V_2 = (1,-1,0,0)$, $V_3 = (0,1,2,1)$

(a) $V = (-1,4,2,2)$.

let $c_1V_1 + c_2V_2 + c_3V_3 = V$ then $V_1, V_2, V_3 \in V$.

$c_1(1,0,0,1) + c_2(1,-1,0,0) + c_3(0,1,2,1) = (-1,4,2,2)$

$(c_1, 0, 0, c_1) + (c_2, -c_2, 0, 0) + (0, c_3, 2c_3, c_3) = (-1,4,2,2)$

$c_1 + c_2 = -1$ — (i)

$-c_2 + c_3 = 4$ — (ii)

$2c_3 = 2$ — (iii)

$c_1 + c_3 = 2$ — (iv)

eq (iii) $\Rightarrow 2c_3 = 2 \Rightarrow \boxed{c_3 = 1}$

eq (ii) $\Rightarrow -c_2 + 1 = 4 \Rightarrow -c_2 = 4 - 1 = 3 \Rightarrow \boxed{c_2 = -3}$

eq (iv) $\Rightarrow c_1 + 1 = 2 \Rightarrow c_1 = 2 - 1 = 1 \Rightarrow \boxed{c_1 = 1}$

eq (i) $\Rightarrow c_1 + (-3) = -1 \Rightarrow c_1 = -1 + 3 = 2 \Rightarrow \boxed{c_1 = 2}$

So solution is not possible so V is not span of (V_1, V_2, V_3).

Ex 6.2

Part (b), (c), (d) is same as Part (a).

Q26 $A_1=\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}, A_2=\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}, A_3=\begin{bmatrix}2 & 2\\ -1 & 1\end{bmatrix}.$

(a) $\begin{bmatrix}5 & 1\\ -1 & 9\end{bmatrix}.$

Sol. $\begin{bmatrix}5 & 1\\ -1 & 9\end{bmatrix}= c_1\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}+c_2\begin{bmatrix}1 & 1\\ 0 & 2\end{bmatrix}+c_3\begin{bmatrix}2 & 2\\ -1 & 1\end{bmatrix}.$

$$=\begin{bmatrix}c_1 & -c_1\\ 0 & 3c_1\end{bmatrix}+\begin{bmatrix}c_2 & c_2\\ 0 & 2c_2\end{bmatrix}+\begin{bmatrix}2c_3 & 2c_3\\ -c_3 & c_3\end{bmatrix}.$$

$$\begin{bmatrix}5 & 1\\ -1 & 9\end{bmatrix}=\begin{bmatrix}c_1+c_2+2c_3 & -c_1+c_2+2c_3\\ -c_3 & 3c_1+2c_2+c_3\end{bmatrix}$$

$c_1+c_2+2c_3=5$ —— (1)

$-c_1+c_2+2c_3=1$ —— (2)

$-c_3=-1$ —— (3)

$3c_1+2c_2+c_3=9$ —— (4)

eq (3) $\Rightarrow -c_3=-1 \Rightarrow \boxed{c_3=1}$

Put $c_3=1$ eq (1) & eq (2)

eq (1) $\Rightarrow c_1+c_2+2=5 \Rightarrow c_1+c_2=3$ —— (5)

eq (2) $\Rightarrow -c_1+c_2+2=1 \Rightarrow -c_1+c_2=-1$ —— (6)

Put $c_3=1$ in eq (4)

$3c_1+2c_2+1=9$

$3c_1+2c_2=8$ ——— (7)

(5) + (6)

$$\cancel{c_1}+c_2=3$$
$$-\cancel{c_1}+c_2=-1$$
$$2c_2=2$$
$$\boxed{c_2=1}$$

Xing eq (6) by (3) & add with (7)

$$-\cancel{3c_1}+3c_2=-3$$
$$\cancel{3c_1}+2c_2=8$$
$$5c_2=5$$
$$\boxed{c_2=1}$$

eq (7) => $3c_1+2(1)=8$

$3c_1=8-2=6$

$$\boxed{c_1=2}$$

So $\begin{bmatrix} 5 & 1 \\ -9 & \end{bmatrix}$ belongs to space if $c_1=2$, $c_2=1$, $c_3=1$.

Part (b), (c) & (d) same as part (a).

Ex 6.2

Q28 $W = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. bit matrices

let $W_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $W_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $W_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

$W_1 + W_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = W_3$

$W_1 + W_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = W_2$

$W_2 + W_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = W_1$

$W_1 + W_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \notin W$ So not a subspace.

Q29 $W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

let $W_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $W_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $W_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $W_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$W_1 + W_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = W_3$

$W_1 + W_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = W_2$

$W_1 + W_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = W_1$

$W_1 + W_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = W_4$

$W_2 + W_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = W_4$

$W_3 + W_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = W_4$

$W_4 + W_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = W_4$

$W_2 + W_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = W_1$

$W_2 + W_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = W_2$

$W_3 + W_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = W_3$

it's subspace.

Q30 let $V = B^4$. Determine If W the set of all vector in V with first entry zero is a subspace of V.

$$W = \left\{ \begin{array}{cccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\ \begin{bmatrix}0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \end{array} \right\}$$

$$w_1 + w_2 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = w_2$$

$$w_1 + w_3 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix} = w_3$$

$$w_1 + w_4 = \begin{bmatrix}0\\1\\1\\1\end{bmatrix} = w_4$$

$$w_1 + w_5 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = w_5$$

$w_1 + w_6 = w_6$

$w_1 + w_7 = w_7$

$w_1 + w_8 = w_8$

$w_1 + w_1 = w_1$

$$w_2 + w_2 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = w_1$$

Similarly other also hold.

Ex 6.2

Q3 let $V = B^4$. Determine if W, the set of all vector in V with second entry one is a subspace of V

$$W = \left\{ \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\\1\end{bmatrix} \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \begin{bmatrix}1\\1\\1\\0\end{bmatrix} \begin{bmatrix}1\\1\\0\\1\end{bmatrix} \begin{bmatrix}1\\1\\0\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\\0\end{bmatrix} \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \right\}$$

$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \quad w_7 \quad w_8$

Not vector space as $w_1 + w_8 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \notin W$

Q32 $u \in \text{Si}$ f

$$u = c_1 v_1 + c_2 v_2 + c_3 v_3$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1\\0\\0\end{bmatrix} + \begin{bmatrix}c_2\\c_2\\0\end{bmatrix} + \begin{bmatrix}c_3\\c_3\\c_3\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1+c_2+c_3\\c_2+c_3\\c_3\end{bmatrix}.$$

$c_1 + c_2 + c_3 = 1$ —— (1)

$c_2 + c_3 = 0$ —— (2)

$c_3 = 1$ —— (3)

$\boxed{c_3 = 1}$

bit max

(2) $\Rightarrow c_2 + 1 = 0$

$\boxed{c_2 = -1}$ or $\boxed{c_2 = 1}$

(1) $\Rightarrow c_1 + 1 + 1 = 1$

$c_1 = 1 - 2 = -1 = 1$ (for b.m)

$\boxed{c_1 = 1}$

It belong to when $c_1 = c_2 = c_3 = 1$.

Q33 is Same as Q 32.

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Electrical Engg: