Circuits and System 1 - 2^{nd} Semester - Week 3 and Week 4

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Topics of this week

Till now, we finished chapter 2 including exercise problems. This week, we will be studying the following topics from chapter 3

- Combination of elements (series, parallel, delta and wye)
- Voltage divider and current divider circuits
- Kirchhoff current law
- Kirchhoff voltage law
- Applications of KCL and KVL in circuits
- Analyze circuits using MATLAB

Series Combination

Resistors is series add up

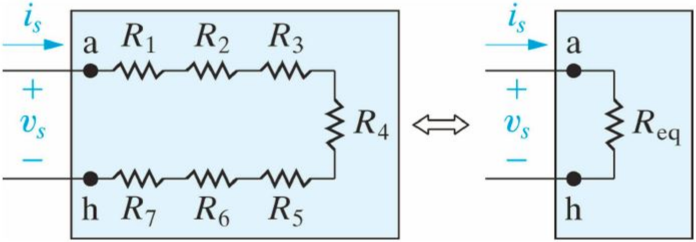

Figure: Resistors in Series Combination

If we have k resistors in series, the equivalent single resistors can be computed as follows:

$$R_{eq} = R_1 + R_2 + R_3 + ... + R_k = \sum_{i=1}^{k} R_i$$

Parallel Combination

In parallel combination, the formula for equivalent resistance is as follows:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_k} = \sum_{i=1}^{k} \frac{1}{R_i}$$

Conductance is opposite of resistance. The equivalent conductance in parallel combination can be computed as follows:

$$G_{eq} = G_1 + G_2 + G_3 + ... + G_k = \sum_{i=1}^{k} G_i$$

where

$$G_i = \frac{1}{R_i}$$

Parallel Combination - Special Cases

If $k = 2$, then we can write the following (for parallel combination):

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \quad \Longrightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

If all the resistors in parallel combination are of same values $(R_1 = R_2 = R_3 = R_k)$, then we can write the following:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_1} + \frac{1}{R_1} + \ldots + \frac{1}{R_1} = \frac{k}{R_1} \quad \Longrightarrow \quad R_{eq} = \frac{R_1}{k}$$

Voltage Divider Circuit

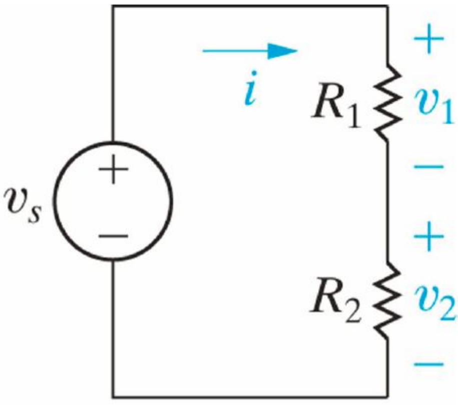

Figure: Voltage Divider Circuit

Voltage Divider Circuit

In series circuit, current is same i.e. $i = i_1 = i_2$, and we can write the following equation:

$$i = \frac{v_s}{R_1 + R_2}$$

If we want to compute the individual voltages across resistors, we can write the following equations:

$$v_1 = i \times R_1 = \frac{v_s}{R_1 + R_2} \times R_1 = v_s \frac{R_1}{R_1 + R_2}$$

$$v_2 = i \times R_2 = \frac{v_s}{R_1 + R_2} \times R_2 = v_s \frac{R_2}{R_1 + R_2}$$

Voltage Divider Circuit

Series combinations or connections of resistors are used to divide voltage among them. Generally, we obtain the following equation:

$$v_N = v_s \frac{R_N}{R_1 + R_2 + R_3 + ... + R_N}$$

$$v_N = v_s \frac{R_N}{R_{eq}}$$

Current Divider Circuit

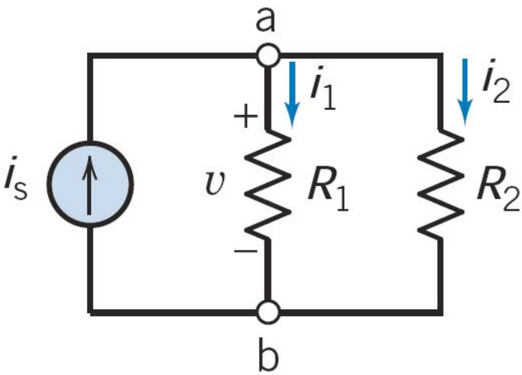

Figure: Current Divider Circuit

Current Divider Circuit

In parallel combination of resistors, voltage is the same and current is divided. We can write the following:

$$v = i_s R_{eq} = i_s \frac{R_1 R_2}{R_1 + R_2}$$

Now, for individual currents, we can write the following:

$$\begin{aligned} i_1 = \frac{v}{R_1} &= i_s \frac{R_1 R_2}{(R_1 + R_2) \times R_1} \\ &= i_s \frac{R_2}{R_1 + R_2} \end{aligned}$$

Similarly, we can write the following:

$$i_2 = i_s \frac{R_1}{R_1 + R_2}$$

Current Divider Circuit

Remember, the previous formula of current division was only for $\mathbf{2}$ resistors. For more than 2, no formula exists.

For more than $\mathbf{2}$ resistors, we can apply the concept of conductance and obtain the following:

$$\boldsymbol{i_n = i_s \frac{G_n}{G_1 + G_2 + G_3 + ... + G_n}}$$

Important terms related to Kirchhoff Law

Node: A point where at least two circuit elements join

Loop: A path in which starting node and ending node are same

Another name for loop is mesh. Be careful: Every corner point is NOT necessarily a node

Important terms related to Kirchhoff Law

Identify all nodes, corner points and loops in this circuit

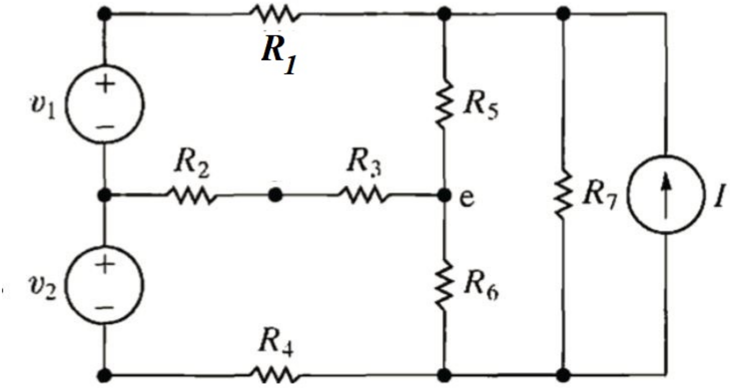

Figure: Example to demonstrate Kirchhoff Laws

Example related to Kirchhoff Law

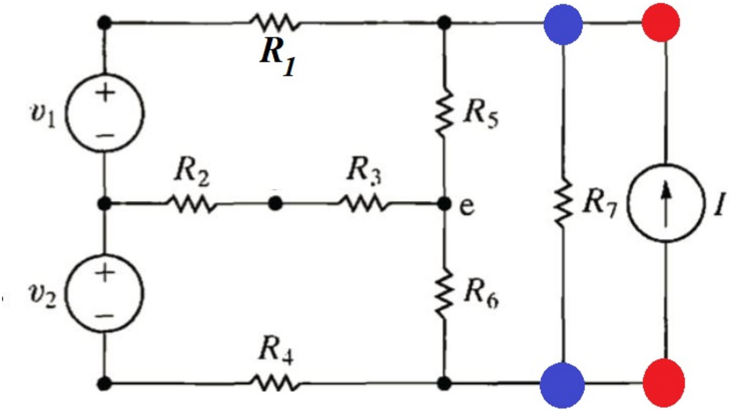

Figure: Example showing corner points - shown in red and blue color

Example related to Kirchhoff Law

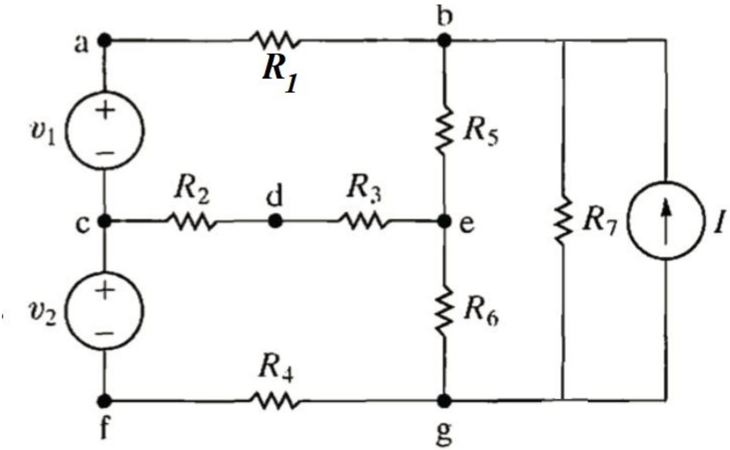

Figure: Example showing nodes and corner points

Example related to Kirchhoff Law

Nodes: a, b, c, d, e, f and g

Some of the possible loops:

$$\boldsymbol{v_1 - R_1 - R_5 - R_3 - R_2}$$
$$\boldsymbol{v_1 - R_1 - R_5 - R_6 - R_4 - v_2}$$
$$\boldsymbol{v_1 - R_1 - R_7 - R_4 - v_2}$$
$$\boldsymbol{v_1 - R_1 - R_7 - R_6 - R_3 - R_2}$$
$$\boldsymbol{v_1 - R_1 - I - R_4 - v_2}$$
$$\boldsymbol{v_1 - R_1 - I - R_6 - R_3 - R_2}$$
$$\boldsymbol{R_5 - R_7 - R_6}$$

Kirchhoff Law

KCL - Statement 1

Algebraic sum of currents in a node at a given time instant is equal to zero.

KCL - Statement 2

The sum of currents entering a node, at a given time instant, equals the sum of current leaving that node.

Kirchhoff Law

KVL - Statement 1

The algebraic sum of voltages in a loop at a given time instant is zero

KVL - Statement 2

In a loop at a given time instant, the sum of voltage rise is equal to sum of voltage drops.

Delta Combination of Resistors

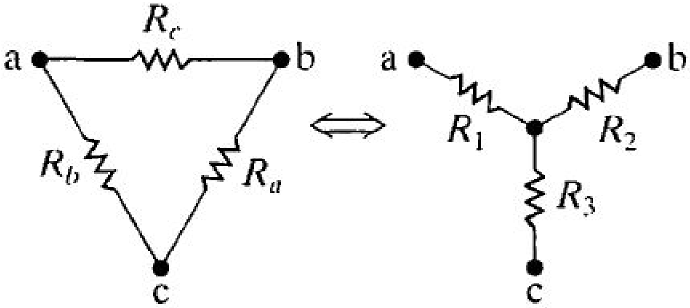

Figure: Delta to Wye Conversion

$$\boldsymbol{\nabla} = \triangle = \boldsymbol{\Pi}$$

$$\boldsymbol{Y} = \boldsymbol{T}$$

Delta Combination of Resistors

Equations for converting delta to wye interconnection:

$$\boldsymbol{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

$$\boldsymbol{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

$$\boldsymbol{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

Delta Combination of Resistors

Equations for converting wye to delta interconnection:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

You do NOT need to memorize these formulas. These formulas and equations are given in mid and final exams. You need to know the application of these formulas (where to apply these formulas)

General MATLAB Code

```
A=[2 3];

B=[1 2; 3 4 ];

inv(B)

B*A
```

MATLAB Code for Converting Delta to Wye

```
R1=(Rb * Rc)/(Ra + Rb + Rc)

R2=(Rc * Ra)/(Ra + Rb + Rc)

R3=(Ra * Rb)/(Ra + Rb + Rc)
```

P3.2.4 on Page 92

Problem 3.2.4 on page 92: Compute the power across each resistor as shown in circuit

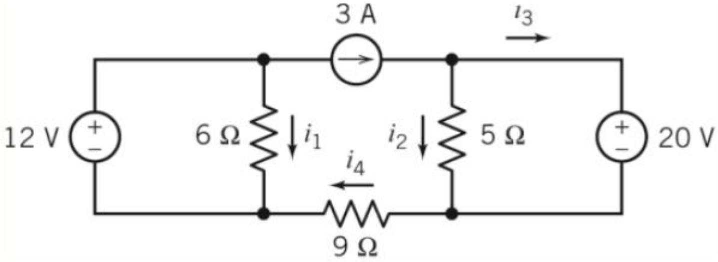

Figure: Problem 3.2.4 on Page 92

Obtain equation for i_1, i_2, i_3 and i_4.

P3.2.4 on page 92 - Solution

P3.2.7 on page 93

Problem 3.2.7 on page 93: Compute the values of $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ in this circuit

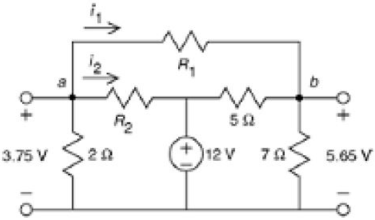

Figure: Problem 3.2.7 on Page 93

Solution: Apply KCL at node $\boldsymbol{a}$ and node $\boldsymbol{b}$. (Remember: if $\boldsymbol{i_1}$ is leaving node $\boldsymbol{a}$, so it must enter node $\boldsymbol{b}$). For sake of easiness, let us assume all currents are leaving node $\boldsymbol{a}$.

P3.2.7 on page 93 - Solution

P3.2.7 on page 93 - Solution

MATLAB Code for P3.2.7 on Page 93

MATLAB has a function named `solve` which is used to solve simultaneous equations

```
ans=solve('3.75/2 + (3.75-5.65)/R1 + (3.75-12)/R2=0',

          '(3.75-5.65)/R1+(12-5.65)/5=5.65/7')

ans.R1

ans.R2
```

P3.2.11 on page 93

Problem 3.2.11 on page 93: Compute the value of current labeled as i_m in the circuit shown below

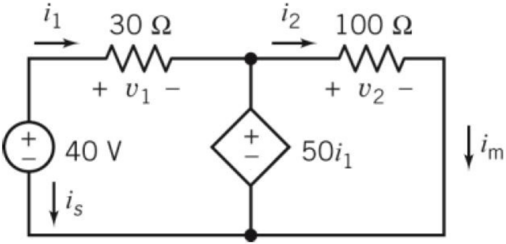

Figure: Problem 3.2.11 on Page 93

Write KVL equations for left loop and right loop.

P3.2.11 on page 93 - Solution

P3.2.14 on page 94

Determine i in the circuit shown below:

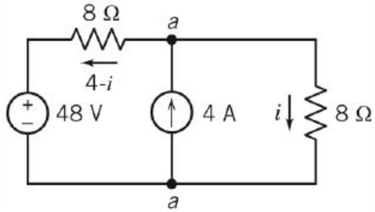

Figure: Problem 3.2.14 on Page 94

Can you write KVL equation for the outer loop?

P3.2.14 on page 94 - Solution

Inclass problem

Determine the value of voltage measured by the voltmeter in the circuit shown below:

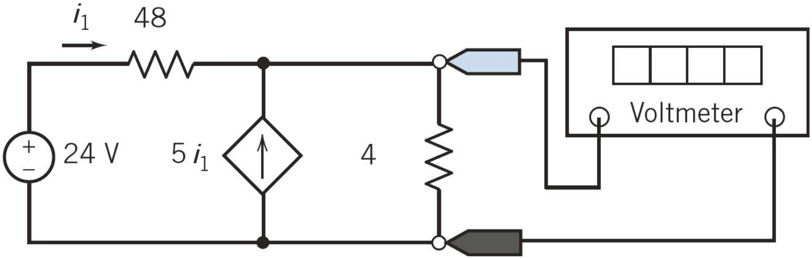

Figure: Problem for practice in class

Inclass problem solution

MATLAB Code for In class problem

```
ans=solve('6 * (24-v)/48 = v/4')
```

P3.2.20 on Page 94

Compute the value of G and R_1 in the circuit shown below if $v_2 = 4$ and the power supplied by $20V$ is $2W$.

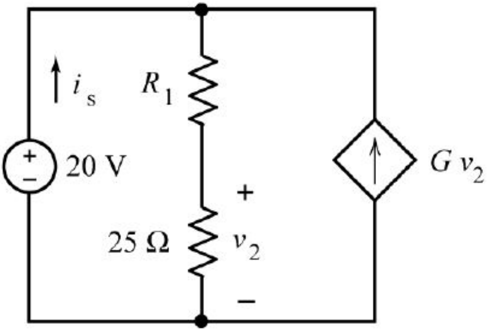

Figure: Problem 3.2.20 on Page 94

P3.2.20 on page 94 - Solution

P3.2.22 on Page 95

Compute the value of $\boldsymbol{R_a}$, $\boldsymbol{R_b}$ and $\boldsymbol{R_c}$ in the circuit shown below

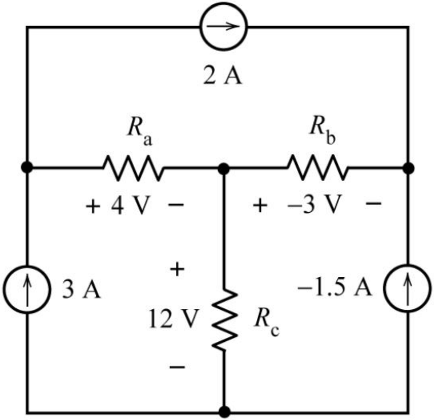

Figure: Problem 3.2.22 on Page 95

P3.2.22 on Page 95 - Solution

P3.3.1 on Page 95

Compute the values of $\boldsymbol{v_1}$, $\boldsymbol{v_2}$, $\boldsymbol{v_3}$ and $\boldsymbol{v_4}$ in the circuit shown below (using voltage division rule).

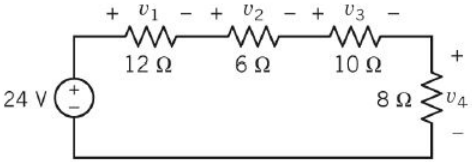

Figure: Problem 3.3.1 on Page 95

P3.3.1 on page 95 - Solution

P3.3.3 on page 95

Solve the following two parts:
Part a - If $\boldsymbol{R_2 = 50\Omega}$, then compute $\boldsymbol{R_1}$.
Part b - If $\boldsymbol{R_1 = 50\Omega}$, then compute $\boldsymbol{R_1}$.

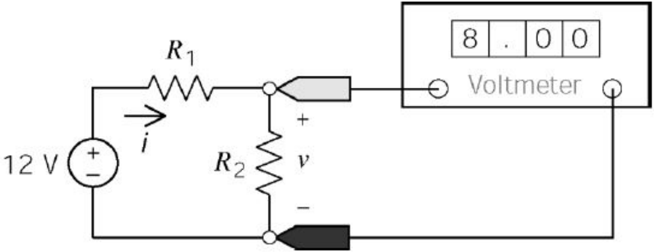

Figure: Problem 3.3.3 on page 95

P3.3.3 on page 95 - Solution

P3.3.6 on Page 96

Using voltage-division rule, compute the value of $\boldsymbol{v_b}$ when $\boldsymbol{v_a = 24V}$ and $\boldsymbol{R = 240\Omega}$

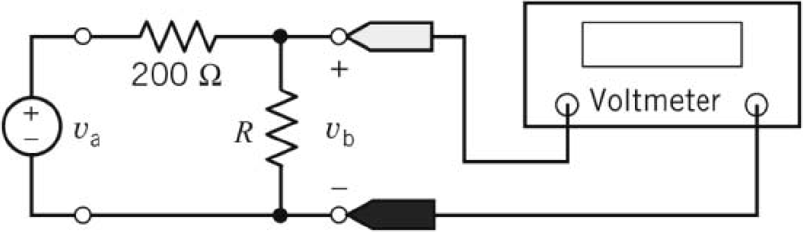

Figure: Problem 3.3.6 on Page 96

P3.3.6 on Page 96 - Solution

P3.3.7 on Page 96

Refer to the circuit in the book, compute the value of voltage v in the circuit

Solution: You can add or subtract voltage source in series based on their polarities
- same argument can be applied to obtain equivalent resistor

P3.3.7 on Page 96 - Solution

P3.3.8 on Page 96

Determine the power supplied by the dependant source in the following circuit.

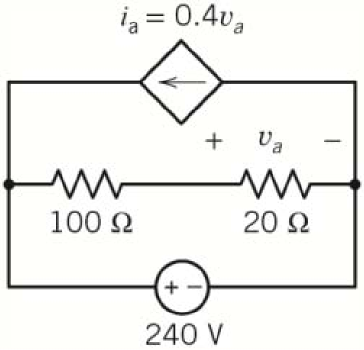

Figure: Problem 3.3.8 on Page 96

P3.3.8 on Page 96 - Solution

Using voltage division rule, we obtain the following:

$$\boldsymbol{v_a = \frac{20}{20+100} \times 240 = 40V}$$

Then

$$\boldsymbol{i_a = 0.4 \times 40 = 16A}$$

The power supplied by dependant source can be computed as follows:

$$\boldsymbol{p = 240 \times i_a = 3840W}$$

P3.3.9 on Page 96

Using voltage division rule, obtain the expression for v_m if $a = \frac{\theta}{360}$

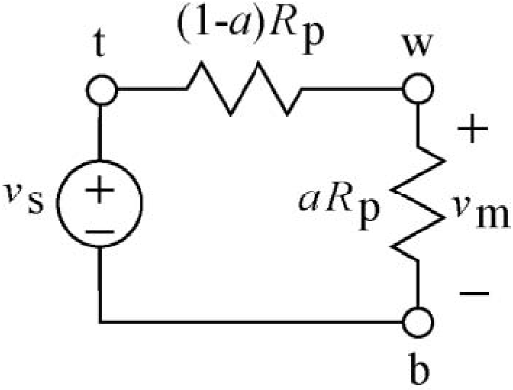

Figure: Problem 3.3.9 on Page 96

P3.3.9 on Page 96 - Solution

Using voltage division rule, we obtain the following:

$$\begin{aligned} v_m &= \frac{aR_p}{(1-a)R_p + aR_p} \times v_s \\ &= \frac{aR_p}{R_p - aR_p + aR_p} \times v_s \\ &= \frac{aR_p}{R_p} \times v_s \\ &= a \times v_s \end{aligned}$$

Substituting the value of a which is $a = \frac{\theta}{360}$, we obtain the following:

$$v_m = \frac{\theta}{360} v_s$$

P3.3.13 on Page 97

Consider the voltage divider circuit shown below. The resistor $\boldsymbol{R}$ represents a temperature sensor and is expressed as $\boldsymbol{R = 50 + \frac{1}{2}T}$. Determine $\boldsymbol{v_m}$ corresponding to $\boldsymbol{25^oC}$.

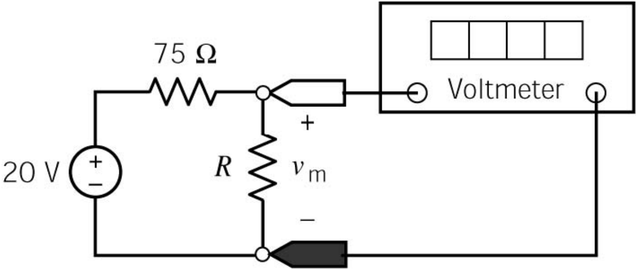

Figure: Problem 3.3.13 on Page 97

P3.3.13 on Page 97 - Solution

Using voltage division rule, we obtain the following:

$$\boldsymbol{v_m = \frac{R}{75 + R} \times 20}$$
$$\boldsymbol{75v_m + Rv_m = 20R}$$
$$\boldsymbol{75v_m = R(20 - v_m)}$$
$$\boldsymbol{R = \frac{75v_m}{(20 - v_m)}}$$

If we plug-in the temperature and resistance equation, then we obtain the following:

$$\boldsymbol{T = 2(R - 50)}$$
$$\boldsymbol{T = 2\left(\frac{75v_m}{20 - v_m} - 50\right)}$$

At $\boldsymbol{25^oC}$, we obtain $\boldsymbol{v_m = 9.1V}$.

P3.4.4 on Page 98

Determine i in the circuit using current division rule

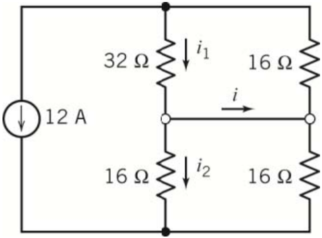

Figure: Problem 3.4.4 on Page 98

P3.4.4 on Page 98 - Solution

Using current division rule, we obtain the following:

$$\boldsymbol{i_1 = \frac{16}{32+16} \times -12 = -4A}$$
$$\boldsymbol{i_2 = \frac{16}{16+16} \times -12 = -6A}$$

Finally, we obtain $\boldsymbol{i = i_1 - i_2 = 2A}$.

DP3.4 on page 109

A Christmas tree light set is required that will operate from a 6V battery on a tree in a city park. The battery can provide 9A for 4 hours period of operation each night. Design a circuit having parallel set of bulbs and determine the number of bulbs where each bulb can be treated as a resistor of $\mathbf{12\Omega}$.

DP3.4

We have $\boldsymbol{6V}$ battery which can supply $\boldsymbol{9A}$. The resistance of bulb is $\boldsymbol{12\Omega}$ and we need to determine how much current each bulb consumes.

$$\boldsymbol{i_{bulb} = \frac{V}{R} = \frac{6}{12} = 0.5A}$$

If each bulb requires $\boldsymbol{0.5A}$ and you have $\boldsymbol{9A}$ available, so the total number of bulbs is $\boldsymbol{18}$.