(Assignment No: 3)

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Section: B

Subject: PME

Submitted to:

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Date 09/08/2021

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(Question No. 1)

Given.

$$f_X(x) = \begin{cases} c(1-x^2) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Required:

a) c & plot pdf.

b) plot cdf of X

c) $P[X=0]$, $P[0<X<0.5]$ and $P[|X-0.5|<0.25]$

Sol:

(a) Since $F(x)$ is pdf we can write

$$\int_X F(x)\,dx = 1$$

$$\int_0^1 cx(1-x^2)\,dx = 1$$

$$c\left\{\int_0^1 x(1-x^2)\,dx\right\} = 1$$

$$c\left\{\int_0^1 (x-x^3)\,dx\right\} = 1$$

$$c\left\{\int_0^1 x\,dx - \int_0^1 x^3\,dx\right\} = 1$$

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$$c\left\{\left(\frac{x^2}{2}\right)\Big|_0^1 - \left(\frac{x^4}{4}\right)\Big|_0^1\right\} = 1$$

$$c\left\{\left(\frac{1}{2}-0\right) - \left(\frac{1}{4}-0\right)\right\} = 1$$

$$c\left\{\frac{1}{2} - \frac{1}{4}\right\} = 1$$

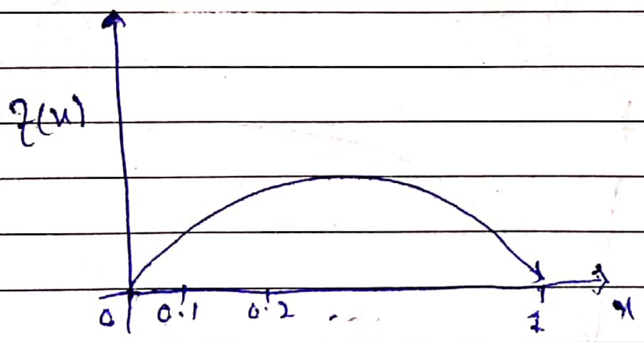
$$c\left\{2/8\right\} = 1$$

$$c\left(1/4\right) = 1$$

$$\boxed{c = 4}$$

$\therefore$ Pdf is,

$$f(x) = 4x(1-x^2) \qquad 0 \le x \le 1$$



Part (b)

Here cdf of X is

$$F_X(x) = P(X \le x)$$

$$= \int_0^x f(t)\, dt$$

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$$F_x(x) = \int_0^x 4t(1-t^2)\,dt$$

$$= 4\int_0^x (t - t^3)\,dt$$

$$= 4\left[\int_0^x t\,dt - \int_0^x t^3\,dt\right]$$

$$= 4\left\{\left(\frac{t^2}{2}\right)_0^x - \left(\frac{t^4}{4}\right)_0^x\right\}$$

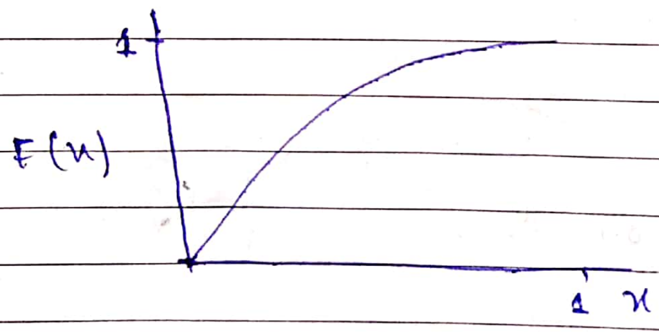
$$= 4\left\{\left(\frac{x^2}{2} - 0\right) - \left(\frac{x^4}{4} - 0\right)\right\}$$

$$= 4\left\{\frac{x^2}{2} - \frac{x^4}{4}\right\}$$

$$= \cancel{4}\left\{\frac{4x^2 - 2x^4}{\cancel{8}}\right\}$$

$$F_x(x) = \frac{4x^2 - 2x^4}{2}$$

Plot:



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Part (c)

$$P(0 < x < 0.5) = ?$$

$$P(0 < x < 0.5) = \int_0^{0.5} f(x)\,dx$$

P [illegible]

$$P(0<x<0.5) = \int_0^{0.5} 4x(1-x^2)\,dx$$

$$= 4\left[\int_0^{0.5} (x-x^3)\,dx\right]$$

$$= 4\left[\int_0^{0.5} x\,dx - \int_0^{0.5} x^3\,dx\right]$$

$$= 4\left[\left(\frac{x^2}{2}\right)_0^{0.5} - \left(\frac{x^4}{4}\right)_0^{0.5}\right]$$

$$= 4\left[\left(\frac{(0.5)^2}{2}\right) - \frac{(0.5)^4}{4}\right]$$

$$= \cancel{4}\left(\frac{0.25}{\cancel{2}}\right) - \cancel{4}\left(\frac{0.0625}{\cancel{4}}\right)$$

$$P[0<x<0.5] = 2(0.25) - 0.0625$$

$$\boxed{P[0<x<0.5] = 0.4375}$$

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(Question No - 2)

Given:

X is Continues uniform R.V in $[-b, b]$

Required:

$E[X] = ?$

$VAR[X] = ?$

Sol

let $X \sim U(-b, b)$

Its Pdf is given by

$$f(x) = \frac{1}{b-(-b)} = \frac{1}{2b} \quad -b \le x \le b$$

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The Characteristic function is

$$\phi_x(t) = E(e^{itx})$$

$$= \int_{-b}^{b} e^{itx} f(x)\,dx$$

$$= \int_{-b}^{b} \frac{e^{-itx}}{2b}\,dx$$

$$= \frac{1}{2b}\int_{-b}^{b} e^{itx}\,dx$$

$$= \frac{1}{2b}\left[\frac{e^{itx}}{it}\right]_{-b}^{b}$$

$$= \frac{1}{2}\left[\frac{e^{itb}}{it} - \frac{e^{-itb}}{it}\right]$$

$$\phi_x(t) = \frac{e^{ibt} - e^{-ibt}}{2ibt}$$

Consider $\phi_x(t) = \int_{-b}^{b} e^{itx} f(x)\,dx$

We know that exponential series is given by.

$$e^a = 1 + a + \frac{a^2}{2!} + \frac{a^3}{3!} + \cdots$$

$$\therefore \phi_x(t) = \int_{-b}^{b}\left[1 + itx + \frac{(itx)^2}{2!} + \cdots\right] f(x)\,dx.$$

here $E[x] = \int_x x f(x)\,dx = \int_{-b}^{b} \frac{x}{2b}\,dx$

$$= \frac{1}{2b}\left[\frac{x^2}{2}\right]_{-b}^{b} = \frac{1}{2b}\left[\frac{b^2}{2} - \frac{b^2}{2}\right]$$

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$E[x] = 0$.

$$E[x^2] = \int_{-b}^{b} x^2 f(x)\,dx = \frac{1}{2b}\left[\frac{x^3}{3}\right]_{-b}^{b}$$

$$= \frac{1}{2b}\left[\frac{b^3}{3} - \frac{-b^3}{3}\right]$$

$$\boxed{E[x^2] = b^3/3}$$ mean

$\therefore$

$$VAR[x] = E[x^2] - E^2[x] = ?$$

$$= \frac{b^3}{3} - 0$$

$$\boxed{VAR[x] = b^3/3}$$ Variance

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(Question NO => 3)

Given.

X is a geometric Random variable.

Required:

$E[x] = ?$

$VAR[x] = ?$

Sol

let $X \sim$ geometric (P)

the Pmf is given by

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$$P(x) = q^x p \qquad x = 0, 1, 2, \dots$$

its characteristic function is given

$$\phi_x(t) = E[e^{itx}]$$

$$= \sum_{x=0}^{\infty} e^{itx} q^x \cdot p$$

$$= p \sum_{x=0}^{\infty} (e^{it} q)^x$$

$$\phi_x(t) = p(1 + e^{it} q + e^{i2t} q^2 + \dots)$$

using geometric series

$$1 + r + r^2 + \dots = \frac{1}{1-r} \qquad \text{So}$$

$$\phi_x(t) = \frac{p}{1 - qe^{it}}$$

its mean is

$$E[x] = \sum x\, p(x)$$

$$E[x] = \sum_{x=0}^{\infty} x q^x p$$

$$= p \sum_{x=0}^{\infty} x q^x$$

$$= p[q + 2q^2 + 3q^3 + \dots]$$

$$= pq(1 + 2q + 3q^2 + \dots)$$

$$E[x] = \frac{pq}{(1-q)^2} = \frac{\cancel{p}q}{p^{\cancel{2}}}$$

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$$\boxed{E[x] = q/p}$$

Now

$$\boxed{VAR[x] = q/p^2}$$

Ans

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(Question No = 4)

Given:

X is an geometric Random variable

Required:

$E[x] = ?$

$VAR[x] = ?$

Sol

let

$X \sim \text{geometric}(p)$

its pmf is $P(x) = q^x p \quad x = 0, 1, 2, \ldots$

its Probability generating function is given by

$$P_x(s) = E[s^x]$$

$$= \sum_{x=0}^{\infty} s^x q^x p$$

$$= p \sum_{x=0}^{\infty} (sq)^x$$

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$P_x(s) = P[1 + qs + (qs)^2 + \cdots]$

$P_x(s) = \frac{P}{1-qs}$ using geometric series

Now differentiate w.r.t. s we get

$P_x'(s) = \frac{d}{ds}\left(\frac{P}{1-qs}\right)$

$= P\left[\frac{-1}{(1-qs)^2}\right](-q)$

$= \frac{pq}{(1-qs)^2}$

at $s=1$ $P_x'(s) = \frac{pq}{(1-q)^2}$

$= \frac{\cancel{p}q}{p^{\cancel{2}}}$

$\boxed{P_x' = q/p}$ → mean

$\therefore \boxed{E[X] = P_x'(s) = q/p}$

Now taking 2nd order derivative

$\frac{d^2}{ds^2} P_x(s) = \frac{d^2}{ds^2}\left(\frac{pq}{(1-qs)^2}\right)$

$= pq\left(\frac{-2}{(1-qs)^3}\right)(-q)$

$= \frac{2pq^2}{(1-qs)^3}$

at $S=1$ $P_x''(s) = \frac{2pq^2}{(1-q)^3}$

$= \frac{2\cancel{p}q^2}{p^{\cancel{3}}}$

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$$P_x''(s) = 2q^2/p^2$$

Using pgf variance is given by

$$V(X) = P_x''(s) + P_x'(s) - [P_x'(s)]^2$$

$$= \frac{2q^2}{p^2} + \frac{q}{p} - q^2/p^2$$

$$V(X) = \frac{q^2}{p^2} + q/p$$

$$\boxed{V[X] = q/p^2}$$ → variance of X.

Ans

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(Question NO ⇒ 5)

Given:

Urn contain 16 balls
4 ball lebeled 1
4 labeled 2
2 " 3
2 " 4
Remaining labled 5, 6, 7, 8.
→ one ball is drawn

Required:

entropy of X = ?

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Sol

The Pmf of x is

$$P(x) = P[X = x] = \frac{\text{no of ball labeled "x"}}{\text{Total no of balls}}$$

$$= \begin{cases} 4/16 & x = 1 \\ 4/16 & x = 2 \\ 2/16 & x = 3 \\ 2/16 & x = 4 \\ 1/16 & x = 5, 6, 7, 8 \\ 0 & \text{otherwise} \end{cases}$$

Entropy in x is given by

$$E_x = -\sum_x P(x) \log P(x)$$

here base of log is 2 b/c entropy is measured in bits

$$E_x = -\sum_{x=1}^{8} P(x) \log(P(x))$$

$$= -\frac{4}{16} \log \frac{4}{16} - \frac{4}{16} \log \frac{4}{16} + 2 \times \frac{2}{16} \log \frac{2}{16}$$
$$- 4 \times \frac{1}{16} \log \frac{1}{16}$$

$$= -\frac{1}{2} \log \frac{4}{16} - \frac{1}{4} \log \frac{2}{16} - \frac{1}{4} \log \frac{1}{16}$$

$$= \log\left(\sqrt{\frac{16}{4}}\right) + \log\left(\frac{16}{2}\right)^{1/4} + \log(16)^{1/4}$$

$$= \log\left(\frac{4}{2} \times \frac{2}{2^{1/4}} \times 2\right)$$

$$E_x = \frac{1}{4} \log \frac{8^4}{2} = \frac{1}{4} \log 2048$$

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$E_x = 2.75$ (it measured in bits]

$E_x = \frac{1}{4} \log 2048$

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(Question No – 6)

Given:

X = outcomes of toss of fair ~~dice~~ die.

Required:

→ (a) $E_x = ?$

→ (b) If x – even & Reduction in $E_x = ?$

Sol

(a) The Pmf of X is,

$$P(x) = P[X = x] = \begin{cases} 1/6 & x = 1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

As die is fair

$$\text{Entropy} = -\sum_{x=1}^{6} P(x) \log P(x)$$

$$= -\sum_{x=1}^{6} \frac{1}{6} \log 1/6$$

$$E_x = -\log 1/6 = \log 6 \approx 2.58$$

(it's measured in bit)

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$$\boxed{E_X = \log 6}$$ Ans

(b)

The Conditional Pmf of X given X is even is,

$$P_1(x) = P[X=x \mid X \text{ is even}] = \frac{P[X=x \;\; x \text{ is even}]}{P[X \text{ is even}]}$$

$$P_1(x) = \begin{cases} \frac{1/6}{3\times 1/6} & x = 2,4,6 \\ 0 & \text{otherwise} \end{cases}$$

$$P_1(x) = \begin{cases} 1/3 & x = 2,4,6 \\ 0 & \text{therwise} \end{cases}$$

∴ Now Entropy (Conditional) $= -\sum_{x=2,4,6} P_1(x) \log P_1(x)$

$$E_X = -3\times \tfrac{1}{3} \log \tfrac{1}{3} = \log 3 \; (\approx 1.58 \text{ in bits})$$

Reduction in Entropy = old entropy − New reduced entropy

$$= \log 6 - \log 3 = \log 2 \; (\approx 1 \text{ in bit})$$

$$\boxed{\text{Reduction in Entropy is } \log 2}$$ Ans

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The END of 3rd & last Assignment

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