Signals & Systems Laboratory

CSE- 301L

Lab # 11

OBJECTIVES OF THE LAB

This lab aims at the understanding of:

- Properties of CT Fourier Series
    - Linearity
    - Time Shifting
    - Time Scaling
    - Time Reversal

11.1 PROPERTIES OF CONTINUOUS TIME FOURIER SERIES

11.1.1 Linearity

Given two periodic signals x(t) and y(t) having same period, linearity property of FS representation can be expressed as

$$x(t) \overset{FS}{\leftrightarrow} a_k \text{ , } y(t) \overset{FS}{\leftrightarrow} b_k \Rightarrow x(t) + y(t) \overset{FS}{\leftrightarrow} a_k + b_k$$

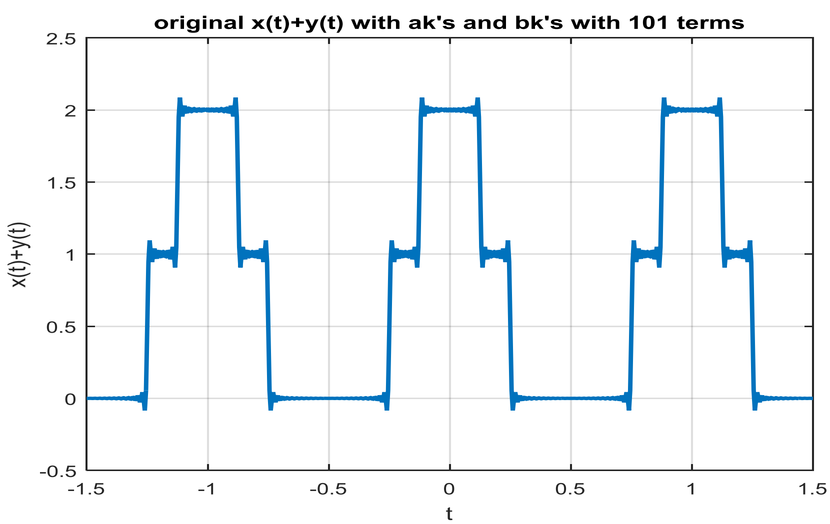
Where a_k and b_k are FS coefficients of x(t) and y(t) respectively. This property can be used in evaluating FS coefficients of a periodic signal that can be expressed as a linear combination of other periodic signals whose FS coefficients are known.

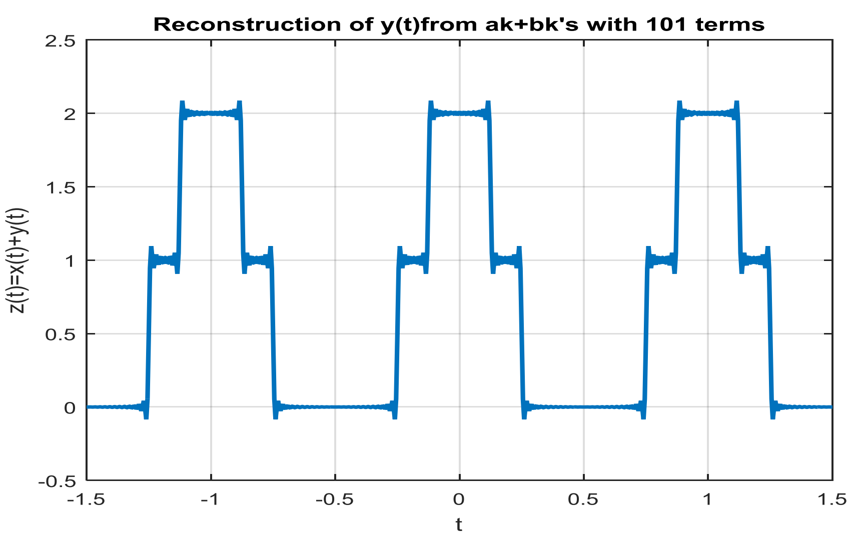
Example – Demonstration of Linearity Property of FS

```
clc
clear all
close all

% FS coefficients of periodic square waves
k = -50:50;
T1 = 0.25;
T=1;
ak = sin(k*2*pi*(T1/T))./(k*pi);

ak(51)=2*T1/T;  % Manual correction for a0 ?> ak(51)
t = -1.5:0.005:1.5;
xt = zeros(1,length(t));
for k = -50:50

xt = xt + ak(k+51)*exp(j*k*2*pi/T *t);
end

T1 = 0.125;
```

```
T=1;
k = -50:50;
bk = sin(k*2*pi*(T1/T))./(k*pi);

bk(51) = 2*T1/T;    % Manual correction for b0 ?> bk(51)

yt = zeros(1,length(t));
for k = -50:50

yt = yt + bk(k+51)*exp(j*k*2*pi/T *t);
end
sum=xt+yt;
%   Application of linearity property of FS
ck = ak+bk;

%   Reconstruction with M=50

w0 = 2*pi/T;

zt = zeros(1,length(t));
for k = -50:50

zt = zt + ck(k+51)*exp(j*k*w0*t);
end
figure(1);

plot(t,real(sum),'lineWidth',2);

xlabel('t');

ylabel('x(t)+y(t)');

title('original x(t)+y(t) with ak''s and bk''s with 101 terms');
```

```
grid;

figure(2);

plot(t,real(zt),'lineWidth',2);

xlabel('t');

ylabel('z(t)=x(t)+y(t)');

title('Reconstruction of y(t)from ak+bk''s with 101 terms');
grid;
```





11.1.2 Time Shifting

The time shifting property of FS states that

$$x(t) \overset{FS}{\leftrightarrow} a_k \;\Leftrightarrow\; x(t - t_0) \overset{FS}{\leftrightarrow} e^{-jk\omega_0 t} a_k \;\;, \omega_0 = 2\pi/T$$

Where x(t) is a periodic signal with FS coefficients a_k and x(t-t_0)is the time shifted version of it. This property can be used to evaluate FS coefficients of a periodic signal that can be expressed as time shifted version of another periodic signal whose FS coefficients are known.

Following example demonstrates the validity of time shifting property. Consider periodic square wave with period T = 1 and T_1 = 0.25, its FS coefficients a_k's are

$$a_0 = \frac{2T_1}{T} = 0.5,\; a_k = \frac{\sin(k\,2\pi(T_1/T))}{k\pi} = \frac{\sin(k\pi/2))}{k\pi} \;\; for \;\; k \neq 0$$

Let it be shifted by t_0 = 0.25, then FS coefficients b_k's for x(t-t_0) = x(t-0.25) can be found using time shifting property as

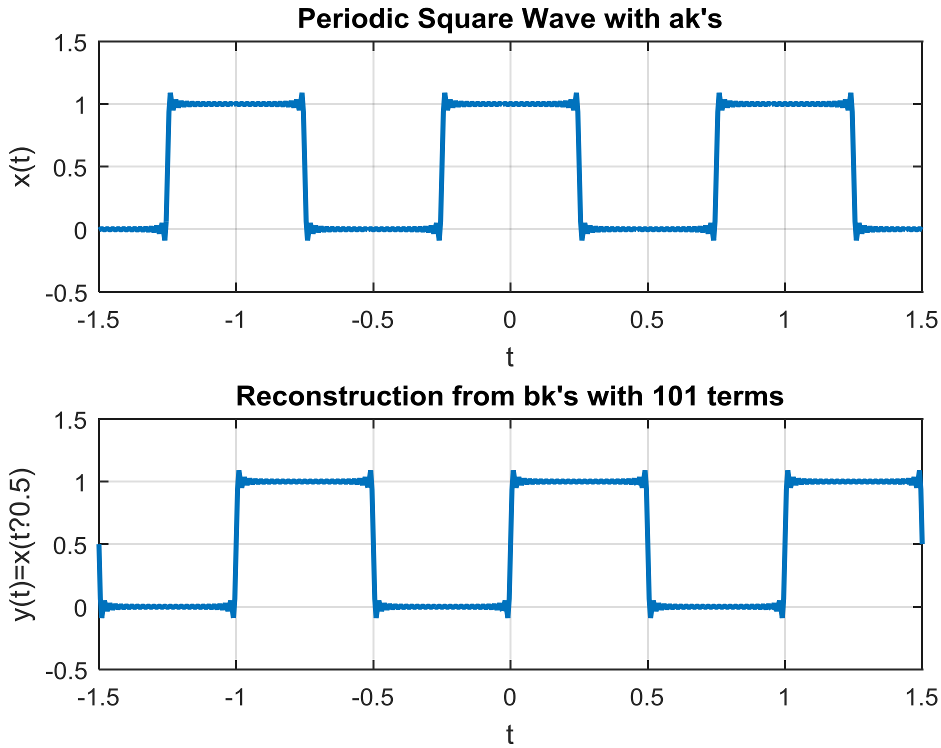
$$b_k = e^{-jk\omega_0 t_0} a_k \;\;, \quad t_0 = 0.25, \quad \omega_0 = 2\pi/T = 2\pi$$

Example – Demonstration of Time Shifting Property of FS

```
clc
clear all
close all

% FS coefficients of periodic square wave
k = -50:50;
T = 1;
T1 = 0.25;
ak = sin(k*2*pi*(T1/T))./(k*pi);
```

```
ak(51)=2*T1/T;        % Manual correction for a0 -> ak(51)
t = -1.5:0.005:1.5;
w0 = 2*pi/T;

xt = zeros(1,length(t));


%    Amount of time shift
t0 = 0.25;


%    FS coefficients of the time shifted signal w0 = 2*pi/T;
bk = ak.*exp(-j*k*w0*t0);

%construction of original square wave
for k = -50:50
xt = xt + ak(k+51)*exp(j*k*w0*t); end


%    Reconstruction from bk's with 101 terms (M=50) yt = zeros(1,length(t));
for k = -50:50
yt = yt + bk(k+51)*exp(j*k*w0*t);
end


figure(1);
subplot(2,1,1);
plot(t,xt,'lineWidth',2);
xlabel('t');
ylabel('x(t)');
title('Periodic Square Wave with ak’’s');
axis([-1.5 1.5 -0.2 1.2]);
grid;


subplot(2,1,2);
plot(t,real(yt),'lineWidth',2);
xlabel('t'); ylabel('y(t)=x(t-0.5)');
title('Reconstruction from bk''s with 101 terms'); axis([-1.5 1.5 -0.2 1.2]);
```

grid;



11.1.3 Time Reversal

The time reversal property of FS states tha

$$x(t) \overset{FS}{\leftrightarrow} a_k \Leftrightarrow x(-t) \overset{FS}{\leftrightarrow} a_{-k}$$

If FS representation for a periodic signal x(t) is known, then FS representation for the time reversed version of the signal x(-t) can be determined through this property.

--------------------------TASK 1--------------------------

Given the signal x(t) with ak’s

a) Plot the time reverse version of the signal x(-t) directly,
b) Plot FS coefficients a_{-k} of time reversed signal,
c) Plot the reconstructed time reversed signal using FS coefficients a_{-k}

Hint : use **bk = fliplr(ak);** for flipping the ak’s.

11.1.4 Time Scaling

The time scaling property of FS states that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{-\,jM\omega t} \;\Rightarrow\; x(\alpha t) = \sum_{k=-\infty}^{\infty} a_k\, e^{-\,jM\omega\alpha t}$$

That is, if x(t) is periodic with period T and fundamental frequency w=2π/T, then time scaled version of x(t), x(αt) where α being positive real number has αw as its fundamental frequency and the FS coefficients for x(αt) is same as those of x(t). Be careful about using the right i.e. scaled frequency (or period) in the reconstruction.

Mathematically it can be expressed as:

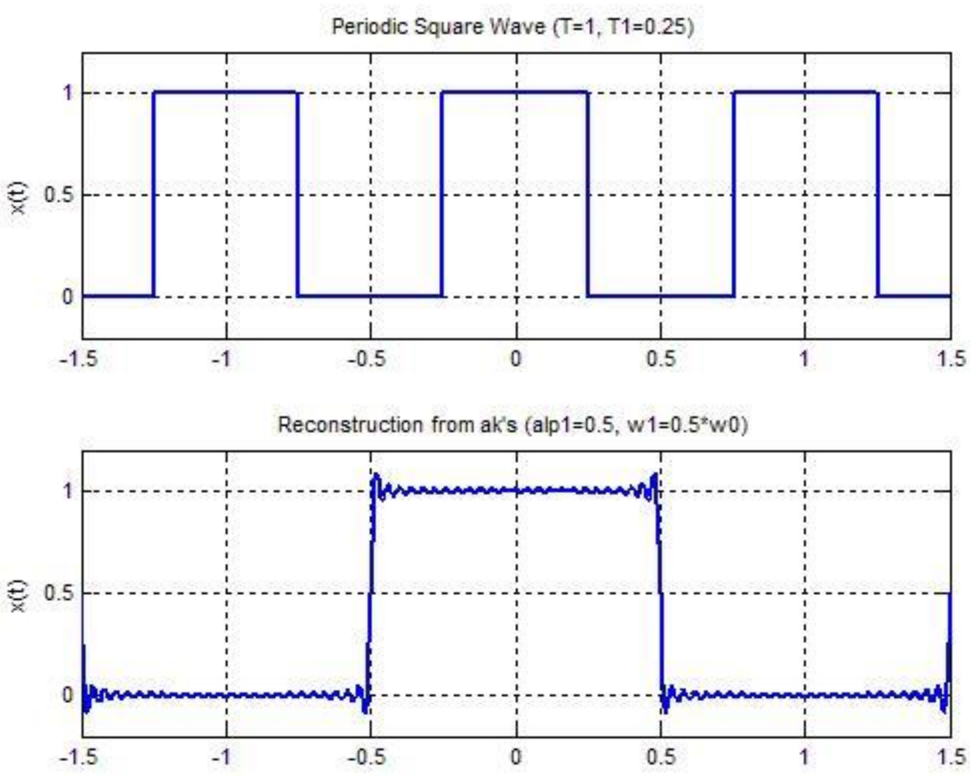
$$x(t) \overset{FS}{\leftrightarrow} a_k \qquad \Leftrightarrow \qquad x(\alpha t) \overset{FS}{\leftrightarrow} a_k$$

$$x(t) = x(t+T), \omega = \frac{2\pi}{T} \qquad\qquad x(\alpha t) = x(\alpha(t+\frac{T}{\alpha})), \omega = \alpha\frac{2\pi}{T}$$

Example – Demonstration of Time Scaling Property of FS having α = 0.5

```
clc
clear all
close all

%	Generation of periodic square wave t = -1.5:0.005:1.5;

xcos = cos(2*pi*t);
xt = xcos>0;

%	FS coefficients of periodic square wave k = -50:50;

T = 1;
T1 = 0.25;
ak = sin(k*2*pi*(T1/T))./(k*pi);
ak(51) = 2*T1/T;	% Manual correction for a0 -> ak(51)

%	Time scaling parameters
alp1 = 0.5;

%	w's for the time scaled signals w0 = 2*pi/T;
w1 = alp1*w0;
```

```
%       Reconstruction from ak's with 101 terms (M=50)

xat1 = zeros(1,length(t));

for k = -50:50
xat1 = xat1 +ak(k+51)*exp(j*k*w1*t);

end
figure(1);

subplot(2,1,1);
plot(t,xt,'lineWidth',2);
ylabel('x(t)');

title('Periodic Square Wave (T=1, T1=0.25)');
axis([-1.5 1.5 -0.2 1.2]);
grid;

subplot(2,1,2);
plot(t,real(xat1),'lineWidth',2);
ylabel('x(t)');
title('Reconstruction from ak''s (alp1=0.5, w1=0.5*w0)');
 axis([-1.5 1.5 -0.2 1.2]);
 grid;
```



---------------------------TASK 3---------------------------

Given the periodic square wave x(t) with T = 1 & T_1 = 0.25, rewrite the above code for time scaling when value of alpha is 2 i.e. x(αt) = x(2t).

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