Probability Methods in Engineering

Dr. Safdar Nawaz Khan Marwat
DCSE, UET Peshawar

Lecture 21

safdar@uetpeshawar.edu.pk

Important Continuous RVs

- Uniform Random Variable

- $S_X = [a, b]$
- $f_X(x) = 1/(b - a) \quad a \leq x \leq b$
- $E[X] = (a + b)/2$, $VAR[X] = (b - a)^2/12$

- Exponential Random Variable

- $S_X = [0, \infty)$
- $f_X(x) = \lambda\, e^{-\lambda t}$, $x \geq 0$, $\lambda > 0$
- $E[X] = 1/\lambda$, $VAR[X] = 1/\lambda^2$

Transform Methods

- Multiplication of huge numbers without calculators
  - Tedious and error-prone
- Instead, use log tables
  - x multiply by y can be computationally complex
  - Find $\text{antilog}(\log(x)+\log(y))$ instead

- Similarly, convolution of two functions (signals) is complex
  - Use "some" transform for convenience

- Two transform in our course
  - The characteristic function
  - The probability generating function

The Characteristic Function

- The characteristic function of a continuous RV is

$$\Phi_X(\omega) = E\left[e^{j\omega X}\right] = \int_{-\infty}^{\infty} f_X(x) e^{j\omega x} dx$$

- The inverse is taken as

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_X(\omega) e^{-j\omega x} d\omega$$

The Characteristic Function (cont.)

- The characteristic function of a discrete RV is

$$\Phi_X(\omega) = \sum_k p_X(x_k) e^{j\omega x_k}$$

- The characteristic function of an integer-valued RV is

$$\Phi_X(\omega) = \sum_{k=-\infty}^{\infty} p_X(k) e^{j\omega k}$$

- The inverse for integer-valued is taken as

$$p_X(k) = \frac{1}{2\pi} \int_0^{2\pi} \Phi_X(\omega) e^{-j\omega k} d\omega$$

Example

- Determine the characteristic function for an exponentially distributed random variable

Examples (cont.)

- Find the characteristic function for a geometric random variable