Quizz NO 2

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Section: B

Subject: CV

Q1: Poles = ?

Given:

$$f(z) = \frac{z^2}{(z-1)^2(z+2)}$$

Sol

finding the poles first:

$z-1 = 0 \Rightarrow z = 1$ pole of order (2)

$z+2 = 0 \Rightarrow z = -2$ pole of order (1) (Simple poles)

finding Residue:-

we have formula for order "2"

$$\operatorname{Res} f(a) = \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{dz^{n-1}}(z-a)^n f(z)\right]_{z=a}$$

here $n = 2$, $a = 1$

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$$\Rightarrow \text{Res } f(1) = \frac{1}{(2-1)!}\left[\frac{d^{2-1}}{dz^{2-1}}(z-1)^2\left(\frac{z^2}{(z-1)^2(z+2)}\right)\right]_{z=1}$$

$$= \frac{1}{1!}\left[\frac{d}{dz}\left(\frac{z^2}{z+2}\right)\right]_{z=1}$$

$$= \frac{1}{1!}\left[\frac{(z+2)\cdot \frac{d}{dz}(z^2) - (z^2)\cdot \frac{d}{dz}(z+2)}{(z+2)^2}\right]$$

putting $z=1$

$$\Rightarrow \frac{1}{1!}\left[\frac{(z+2)(2z) - (z^2)}{(z+2)^2}\right]_{z=1}$$

$$= \frac{(1+2)\,(2(1) - (1)^2}{(1+2)^2}$$

$$= \frac{6-1}{9} \Rightarrow \boxed{\text{Res } f(1) = \frac{5}{9}}$$

find Residue of $f(-2)$

we have formula :-

$$\text{Res } f(a) = \lim_{z\to a}\left[(z-a)\,f(z)\right]$$

$$= \lim_{z\to -2}\left[\cancel{z+2}\,\frac{z^2}{(z-1)^2\cancel{(z+2)}}\right]$$

$$= \lim_{z\to -2}\left[\frac{z^2}{(z-1)^2}\right]$$

$$= \frac{(-2)^2}{(-2-1)^2} = \frac{4}{9}$$

$$\boxed{\text{Res } f(2) = \frac{4}{9}}$$

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Q3: find laurent's Expansion of the function.

$$f(z) = \frac{7z-2}{(z+1)(z+2)}$$

in region $1<|z+1|<3$

Sol

let $z+1 = U$

$z = U-1$

$$f(U) = \frac{7(U-1)-2}{(U-1+1)(U-1+2)} = \frac{7U-7-2}{U(U+1)}$$

$$f(U) = \frac{7U-9}{U(U+1)}$$ using partial fraction.

$$\frac{7U-9}{U(U+1)} = \frac{A}{U} + \frac{B}{U+1}$$

$$\frac{7U-9}{U(U+1)} = \frac{A(U+1)+B(U)}{U(U+1)}.$$

Xing by $U(U+1)$ on both side

$$\boxed{7U-9 = A(U+1) + B(U)} \quad \text{——} \ (1)$$

Putting $U=0$ in eq (1)

$7(0)-9 = A(0+1) + B(0)$

$\boxed{-9 = A}$

Putting $U=-1$ in eq (1)

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$$7(-1)-9 = A(-1+1)+B(-1)$$

$$\cancel{+}16 = \cancel{+}B$$

$$\boxed{B=16}$$

$$f(u) = \frac{-9}{u} + \frac{16}{u+1} \quad \text{in} \quad 1<u<3$$

$$= \frac{-9}{u} + \frac{16}{u\left(1+\frac{1}{u}\right)}$$

$$= \frac{-9}{u} + \frac{16}{u}\left(1+\frac{1}{u}\right)^{-1}$$

$$= -\frac{9}{u} + \frac{16}{u}\left[1-\frac{1}{u}+\frac{1}{u^2}-\frac{1}{u^3}+\cdots\right]$$

$$= \frac{-9}{u} + \frac{16}{u} - \frac{16}{u^2} + \frac{16}{u^3} - \frac{16}{u^4} + \cdots$$

$$f(u) = \frac{6}{u} - \frac{16}{u^2} + \frac{16}{u^3} - \frac{16}{u^4} + \cdots$$

here $u = z+1$

$$f(z) = \frac{6}{z+1} - \frac{16}{(z+1)^2} + \frac{16}{(z+1)^3} - \frac{16}{(z+1)^4} + \cdots$$

Hence this is valid for the region $1 < |z+1| < 3$.

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Q4: find the Fourier Series of the function.

$$f(x) = \begin{cases} -1 & \text{for} \quad -\pi < x < -\pi/2 \\ 0 & \text{for} \quad -\pi/2 < x < \pi/2 \\ 1 & \text{for} \quad \pi/2 < x < \pi \end{cases}$$

Sol

$$f(x) = \begin{cases} -1 & \text{for} \quad -\pi < x < -\pi/2 \\ 0 & \text{for} \quad -\pi/2 < x < \pi/2 \\ 1 & \text{for} \quad \pi/2 < x < \pi \end{cases}$$

Now

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx] \quad (1)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(0)\, dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(0)\, dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{-\pi/2} 0\, dx + \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} -1\, dx + \frac{1}{\pi} \int_{\pi/2}^{\pi} 1\, dx$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} 0\, dx + \frac{1}{\pi} \int_{0}^{\pi/2} -1 \cos nx\, dx + \frac{1}{\pi} \int_{\pi/2}^{\pi} (1) \cos nx\, dx.$$

$$\boxed{a_n = \frac{-2}{n\pi} \sin \frac{n\pi}{2} + \frac{1}{n\pi} \sin n\pi}$$

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Now

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{\pi}\int_{0}^{\pi/2}(-1)\sin nx\,dx + \frac{1}{\pi}\int_{\pi/2}^{\pi} 1\sin nx\,dx$$

$$\boxed{b_n = \frac{2}{n\pi}\cos\frac{n\pi}{2} - \frac{1}{n\pi}\cos n\pi - \frac{1}{n\pi}}$$

eq ① become.

$$f(x) = \frac{0}{2} + \sum_{n=1}^{\infty}\left[\left(\frac{-2}{n\pi}\sin\frac{n\pi}{2} + \frac{1}{n\pi}\sin n\pi\right]\cos nx + \left(\frac{2}{n\pi}\cos\frac{n\pi}{2} - \frac{1}{n\pi}\cos n\pi - \frac{1}{n\pi}\right]\sin nx\right]$$

$$= \frac{1}{\pi}\left[(-2\cos x) + (-2\sin 2x) + \left(\frac{2}{3}\cos 3x\right) + \cdots\cdots\right]$$

$$\boxed{f(x) = \frac{1}{\pi}\left[-2\cos x - 2\sin 2x + \frac{2}{3}\cos 3x + \cdots\right]}$$

Ans

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Q2: Evaluate $\int_c \frac{z}{z^2+1} dz$ where

i) c is $|z+\frac{1}{2}|=2$

ii) c is $|z+i|=1$

Sol

(i) c is $|z+\frac{1}{2}|=2$ —— eq(i)

$$\int_c \frac{z}{z^2+1} dz \quad \text{—— (1)}$$

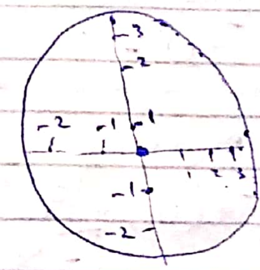
for Singular point

$$z^2+1=0$$
$$\sqrt{z^2}=\sqrt{-1}$$
$$z = \pm\sqrt{-1}$$
$$z = \pm 1$$

Point $(z) = (\pm 1, 0)$ or $(+1, 0)$ $(-1, 0)$

radius of circle is $r=2$

Center of Circle is



$$z_0 = \frac{1}{z_1}$$
$$z_0^2 = 1$$
$$z = \pm 1 \ (+1, -1)$$

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So the point lies inside the given path so applying Cauchy integral theorem.

$$\int_c \frac{z}{z^2+1}\,dz$$

$$\oint_c \frac{2z}{2(z^2+1)}\,dz \Rightarrow \frac{1}{2}\int_0^{2\pi} \frac{2z}{(z^2+1)}$$

So derivative of denominator is equal to numerator.

$$\left|\ln|z^2+1|\right|_0^{2\pi} = \ln|2\pi+1|$$

$$= \ln(2\pi+1)$$

$$= \ln(7.28) = (3.48) \text{ Ans}$$

(ii) C: is $|z+i| = 1$

Here

radius $= 1$

Singular point is

$$z^2+1 = 0$$

$$z^2 = -1$$

$$z = \pm\sqrt{-1}$$

$$z = \pm 1 \ (+1,0) \ (-1,0)$$

$$z_0 = 1 \ (0,1)$$

So the pole singular point lies outside the given path. So the Cauchy integral theorem already equal to zero

$$\boxed{\oint_0^{2\pi} \frac{z}{z^2+1}\,dz = 0}$$

The END

Ashfaq Ahmad