Assignment no#01

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Complex veriable

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"On my honor, as student of University of Engineering and Technology, I have neither given nor received unauthorized assistance on this academic work."

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Answer To Q.7 part (i)

(i) $1-i$

Sol

As we know

$z = r(\cos\theta + i\sin\theta)$

$r\cos\theta = 1$ — (1)

$r^2\cos^2\theta = 1$ — (2)

$r\sin\theta = -1$ — (3)

$r^2\sin\theta = 1$ — (4)

$r^2\cos^2\theta = 1$

$r^2\sin^2\theta = 1$

$r^2(\cos^2\theta + \sin^2) = 2$

$r^2 = 2 \Rightarrow r = \sqrt{2}$

So

$\sqrt{2}\cos\theta = 1$

$\cos\theta = 1/\sqrt{2}$

$\theta = \cos^{-1} 1/\sqrt{2}$

$\boxed{\theta = 45}$

Answer To Q-1 part (ii)

$3 \pm 4i$

Sol: $3+i4$

As we know

$Z = r(\cos\theta + \sin\theta)$

From Given Question $r\cos\theta = 3$ —(1)

$r^2\cos^2\theta = 9$ —(2)

$r\sin\theta = 4$

$r^2\sin^2\theta = 16$ —(3)

$r^2\cos^2\theta = 9$ Adding (2) and (3)

$+\ r^2\sin^2\theta = 16$

$r^2(\cos^2\theta + \sin^2\theta) = 25$

$r^2 = 25 \Rightarrow \boxed{r = 5}$ put in (1)

$5\cos\theta = 3 \Rightarrow \cos\theta = 3/5$

$\theta = \cos^{-1} 3/5$

$\boxed{\theta = 53.1°}$

Now For 3-4

$r\cos\theta = 3 \Rightarrow r^2\cos^2\theta = 9$ —(4)

eq (B) $\rightarrow r\sin\theta = -4 \Rightarrow r^2\sin^2\theta = 16$ —(5)

Adding (4) and (5)

$r^2(\cos^2\theta + \sin^2\theta) = 25$

$r^2 = 25$

$r = 5$ put in B

$5\sin\theta = -4$

$\sin\theta = -4/5$

$\theta = \sin^{-1} -4/5$

$\boxed{\theta = -53.1°}$

(3)

Answer To Q 1 part (iii)

$-5+5i$

Solution: As we know

$z = r(\cos\theta + \sin\theta)$

from given Question

$r\cos\theta = -5$ — (1)

$r^2\cos^2\theta = 25$ — (2)

$r\sin\theta = 5$

$r^2\sin^2\theta = 25$ — (3)

Adding (2) and (3)

$$\begin{array}{rl} & r^2\cos^2\theta = 25 \\ + & r^2\sin^2\theta = 25 \\ \hline & r^2(\sin^2\theta + \cos^2\theta) = 50 \end{array}$$

$r^2 = 50 \Rightarrow r = \sqrt{50}$ put in (1)

$\sqrt{50}\cos\theta = -5$

$\Rightarrow \cos\theta = -5/\sqrt{50}$

$\theta = \cos^{-1}(-5/\sqrt{50})$

$\boxed{\theta = -45^\circ}$

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(4)

Question # 2
part
(i) $\sqrt[3]{1+i}$

Sol Here $k = 0, 1, 2$

Let $w = \sqrt[3]{1+i}$

$k = 2^{1/6} e^{i\pi/12}$ if $k=0$

$k = 2^{1/6} e^{i(\pi/12 + 2\pi/3)}$

$e = \sqrt{1+1}$

$e = \sqrt{2}$

$k = 2^{1/6} e^{i(\frac{\pi + 8\pi}{12})}$

$= e^{1/6} e^{i(3\pi/4)}$

$r = \sqrt[n]{e}$

If $k=1$

$= e^{1/6} e^{i(\pi/12 + 4\pi/3)}$

$r = \sqrt[n]{\sqrt{2}}$

$= 2^{1/6} e^{i(17\pi/12)}$

$r = 2^{1/6}$

If $k=2$

$w = n^{1/n}\sqrt{e}\, e^{i(\theta/n + \frac{2k\pi}{n})}$ $k = 2^{1/6} e^{i3\pi/4}$

$\theta = \pi/4$ $= 2^{1/6}(\cos \frac{3}{4}\pi + i\sin\frac{3\pi}{4})$

$w^{1/n} = 2^{1/6} e^{i(\pi/12 + \frac{2k\pi}{7})}$

$2^{\pi/6} e^{i\pi/12} = e^{1/6}(\cos \pi/12 + i \sin \pi/12)$

So these are the required roots.

Question 2 part (ii)

$\sqrt[4]{-7+24i}$

Sol

Let $w = -7 + 2i$

$$w^{1/n} = \sqrt[n]{e}\; e^{i\left(\theta/n + \frac{2k\pi}{n}\right)} \quad —①$$

$$e_1 = \sqrt{(-7)^2 + (24)^2} = \sqrt{625}$$

$e = 25$ and $n = 4$

Now put e θ and n in eq ①

$$w^{1/4} = \sqrt[4]{25}\; e^{i\left(\frac{4\pi}{40} + \frac{2\pi k}{4}\right)}$$

$$= \sqrt[4]{25}\; e^{i(4\pi/40)} \qquad k = 0$$

$$= \sqrt[4]{25}\; e^{i(4\pi/40 + 2\pi/4)}$$

$$= \sqrt[4]{25}\; e^{2\left(\frac{4\pi}{40} + \frac{4\pi}{4}\right)}$$

for $k=0$ $\quad \sqrt[4]{25}\, e^{2(4\pi/40)}$

$$= \sqrt[4]{25}\left(\cos \pi/10 + i \sin \frac{\pi}{2}\right)$$

$k = 2 \Rightarrow \sqrt[4]{25}\, e^{i\, 11\pi/10} = \sqrt[4]{25}\left(\cos\frac{11\pi}{10} + i \sin\frac{11\pi}{10}\right)$

$k = 3 \quad \sqrt[4]{25}\; e^{i\frac{8\pi}{2}} = \sqrt[4]{25}\left(\cos\frac{8\pi}{5} + i \sin\frac{8\pi}{5}\right)$

These are roots.

Question 2 part (iii)

$\sqrt[4]{-4}$

Sol let $w = -4 + 0i$

$w^{1/n} = \sqrt[n]{e}\; e^{i\left(\theta/n + \frac{2k\pi}{n}\right)}$ — ①

As we know $\theta = \tan^{-1}(y/x)$

$\theta = \tan^{-1}\left(\frac{0}{-4}\right)$

$\boxed{\theta = 0^\circ}$

$w^{1/4} = \sqrt[4]{4}\; e^{i\left(\frac{2k\pi}{4}\right)}$

$k=0;$ $\sqrt[4]{4}\, e^{i(0)} = \sqrt[4]{4}\,(\cos\theta + i\sin\theta)$

$k=1;$ $\sqrt[4]{4}\, e^{i(\pi/2)} = \sqrt[4]{4}\,\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$k=2;$ $\sqrt[4]{4}\; e^{i(\pi)} = \sqrt[4]{4}\,(\cos\pi + i\sin\pi)$

$k=3;$ $\sqrt[4]{4}\; e^{i(3\pi/2)} = \sqrt[4]{4}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

These are the required roots

(7)

Question. 3, part (i)

Sol $z^2-(5+i)z+8+i=0$

Here $a=1$, $b=-5-i$, $C=8+i$

Now By quadratic formula

$$z=\frac{-(-5-i)\pm\sqrt{(-5-i)^2-4(1)(8+i)}}{2(1)}$$

$$=\frac{5+i\pm\sqrt{25-1-2(-5)(i)-12-4i}}{2}$$

$$=\frac{5+i\pm\sqrt{12-14i}}{2}$$

$$=\frac{5+i}{2}\pm\sqrt{\left(\frac{1}{2}\right)^2(12-14i)}$$

$$=\frac{1}{2}\left(5+i\pm\sqrt{12-14i}\right)$$

$$=\frac{1}{2}\left(\sqrt{(5+i)^2}\pm\sqrt{12-14i}\right)=\frac{1}{2}\left(\sqrt{25-1+10i}\pm\sqrt{12-14i}\right)$$

$$=\frac{1}{2}\left(\sqrt{24+10i}\pm\sqrt{12-14i}\right)$$

Question 3 part(ii)

Sol: $z^2 - (7+i)z + 24 + 7i = 0$

Here $a = 1$ $b = 7+i$ $c = 24+7i$

Now By quadratic formula

$$z = \frac{(7+i) \pm \sqrt{(7+i)^2 - 4(1)(24+7i)}}{2}$$

$$= \frac{7+i \pm \sqrt{48 + 14i - 96 - 28i}}{2}$$

$$= \frac{(7+i) \pm \sqrt{-48 - 14i}}{2}$$

$$= \cancel{(7+i)} \pm \left[\sqrt{\tfrac{1}{2}(50-48)} - i\sqrt{\tfrac{1}{2}(50+48)}\right]$$

$$\pm\sqrt{-48-14i} = \pm(1-7i)$$

$$z = \frac{(7+i) \pm (1-7i)}{2}$$

$$z = \frac{(7+i) - (1-7i)}{2}, \quad z = \frac{(7+i) - (1-7i)}{2}$$

$$z = \frac{8-6i}{2}, \quad z = \frac{6-6i}{2}$$

$$z = 4-3i, \quad z = 3-3i$$

$$\boxed{z = 4-3i,\ 3-3i}$$

Q4: part (i)

Sol $f = z^2 + 2z + 2$ at $1-i$

put the given Value instead of z

$f = (1-i)^2 + 2(1-i) + 2$

$f = 1 - 1 - 2i + 2 - 2i + 2$

$f = 4 - 4i$

So

$\boxed{Rel = 4}$ $\boxed{Im = -4}$

part (ii) $f = \frac{1}{1-z}$ at $7+2i$

put the given value instead of z

$f = \frac{1}{1-7-2i} = \frac{1}{-6-2i}$

$f = \frac{1}{-6-2i} \times \frac{-6+2i}{-6+2i} \Rightarrow \frac{-6+2i}{36+4}$

$f = \frac{-6+2i}{40} = \frac{-6}{40} + \frac{2i}{40}$

$f = \frac{-3}{20} + \frac{i}{20}$

So $\boxed{Rel = -3/20}$ & $\boxed{Im = \frac{1}{20}}$

Q5 part (i)

Sol $f(z) = \dfrac{\operatorname{Im} z}{|z|}$

let $z = a+bi$

$\operatorname{Im} z = b$ and $|z| = \sqrt{a^2+b^2}$

$$\lim_{z\to 0} f(z) = \lim_{z\to 0} \frac{b}{\sqrt{a^2+b^2}}$$

$a \to 0$

$b \to 0$

$$\text{Limit}_{\substack{a\to 0\\ b\to 0}} \frac{b}{\sqrt{a^2+b^2}}$$

$\lim_{b\to 0} = 1$

$b - 0$

$a \to 0$

$$\text{Limit}_{\substack{b\to 0\\ a\to 0}} \frac{b}{\sqrt{a^2+b^2}}$$

$\text{limit}_{a\to 0} = 0$

$$\text{limit}_{\substack{a\to 0\\ b\to 0}} f(z) \neq \text{limit}_{\substack{b\to 0\\ a\to 0}} f(z)$$

Hence the limit does'nt exist

So the $f(z)$ is not Continuous.

Q:5 part (ii)

Sol $f(z) = \frac{ReZ}{(1+|z|)}$

let $z = a+bi$

$Re(z) = a$ $Im(z) = b$

and $|z| = \sqrt{a^2+b^2}$

Then

$f(z) = \frac{a}{1+\sqrt{a^2+y^2}}$ Take limit of Both side

$a \to 0$	$b \to 0$
$b \to 0$	$a \to 0$
$\lim_{\substack{a \to 0 \\ b \to 0}} \frac{a}{1+\sqrt{a^2+b^2}}$	$\lim_{\substack{b \to 0 \\ a \to 0}} \frac{a}{1+\sqrt{a^2+b^2}}$
$\lim_{b \to 0} \frac{0}{1+\sqrt{b^2}}$	$\lim_{a \to 0} \frac{a}{1+\sqrt{a^2}}$
$\lim_{b \to 0} 0$	$\lim_{a \to 0} \frac{0}{1+0} = 0$

$$\lim_{\substack{a \to 0 \\ b \to 0}} f(z) = \lim_{\substack{b \to 0 \\ a \to 0}} f(z)$$

Hence all the Conditions are statisfied by the $f(z)$. So $f(x)$ is Continuous.

Q6 part(i)

Sol: $(z-1)/(z+i)$ at i

$$f(z) = \frac{z-i}{z+i}$$

$$f'(z) = \frac{d}{dz}\left(\frac{z-i}{z+i}\right)$$

$$f'(z) = \frac{(z+i) - (z-i)(1)}{(z+i)^2}$$

$$f'(z) = \frac{z+i-z+i}{(z+i)^2}$$

$$f'(z) = \frac{2i}{(z+i)^2} = \frac{2i}{(i+i)^2} = \frac{2i}{(2i)^2}$$

$$f'(z) = \frac{2i}{-4}$$

$$f'(i) = -i/2$$

x———x———x———x

part (ii)

$(z-4i)^8$ at $5+4i$

Sol $f(z) = (z-4i)^8$

$$f'(z) = 8(z-4i)^7$$

$$f'(z) = 8(5+4i-4i)^7 = 8(5)^7$$

$$\boxed{f'(z) = 625000}$$

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Q 7 part (i)

Sol $x/x^2+y^2 = U_x$

$$U_x = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2}$$

$$U_x = \frac{y^2 - x^2}{(x^2+y^2)^2}$$

$$U_{xx} = \frac{2x^5 - 6xy^2 - 4x^3y^2}{(x^2+y^2)^4} \quad — ②$$

$$U_y = \frac{(-1)(x\,dy)}{(x^2+y^2)^2}$$

$$U_y = -2xy/(x^2+y^2)^2$$

$$U_{yy} = \frac{(x^4+y^4+2x^2y^2)(-2x)+2xy(2)(2y)(x^2+y^2)}{(x^2+y^2)^4}$$

$$U_{yy} = -\frac{2x^2-6xy^2-4x^3y^2}{(x^2+y^2)^2} \quad — ①$$

adding eq ① and ②

$U_{xx} + U_{yy} = 0$ The function is normal

Now CRE's in

$$U_x = U_y$$

$$U_y = U_x = \frac{-2xy}{(x^2+y^2)^2} \quad — a$$

$$U_y = \frac{y^2-x^2}{(x^2+y^2)^2} \quad — ②$$

$U_x = -2xy/(x^2+y^2)^2$

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Q.7 part (ii)

Sol $u = \sin x \cosh y$

$u_x = \cos x \cosh y$

$u_{xx} = -\sin x \cosh y$ — (1)

$u_y = \sin x \sinh y$

$u_{yy} = +\sin x \cosh y$ — (2)

adding (1) and (2)

$u_{xx} + u_{yy} = 0$

$u_x = v_y = \cos x \cosh y$ — (3)

$-v_y = u_x = -\sin x \sinh y$

$v_y = \cos x \cosh y$

Integrating Both the Sides.

$V = \cos x \sinh y + k(x)$ (i) partial differ. with r to x

$V_x = -\sin x \sinh y + k'(x)$ — (ii)

comparing (i) and (ii)

$-\sin x \sinh y = -\sin x \sinh y + k'(x)$

$k(x) = 0$ Integrating w.r.t x

$k(x)' = c'$

$V = \cos x \sinh y + c_1$

$f(z) = V + k = \sin x \cosh y + \cos x \sinh x$

$f(z) = \sin z + c$

Q.8 part (i)

Sol $z+5i$

$z = 2+3\pi i$

$e^z = e^{2+3\pi i} = e^2(\cos 3\pi + i\sin 3\pi)$

$e^2(0.98 + i\,0.16) = e^2 0.98 + i\,0.16$

$e^z = 7.24 + i\,1.18$

$u = 7.28 \qquad v = i\,18$

$|e^z| = \sqrt{[e^2(\cos 3\pi)]^2 + [e^2(\sin 3\pi)]^2}$

$= \sqrt{e^4(\cos^2 3\pi + \sin^2 3\pi)}$

$= \sqrt{e^4\,(1)}$

$= e^2$

$\boxed{|e| = 7.38}$

part (ii) $-\pi i/2$

$z = 0 - \frac{\pi i}{2}$

$e^0 = 1$

$e^{-i\pi/2} = (\cos \pi/2 - i\sin\frac{\pi}{2}$

$e^z = 0.99 - i\,0.027$

$u = 0.99 \qquad v = 0.027$

$|e^z| = \sqrt{\cos^2\frac{\pi}{2} + \sin^2\frac{\pi}{2}}$

$|e^z| = \sqrt{1}$

$\boxed{|e^z| = 1}$

Q10 part (i)

Sol $\cos(1+i)$

$\cos(z) = \cos x \cosh y - i \sin x \sinh y$

$x = 1 \quad y = i$ put values

$\cos(1+i) = \cos(1)\cosh(1) - \sin(1)\sinh i$

$= (0.54)(1.54) - i(0.84)(1.17$

$\cos(1+i) = 0.8316 - i0.98$

x ———— x ———— x

part (ii)

$\cos(\frac{1}{2}\pi - \pi i)$

$x = \frac{1}{2}\pi, \quad y = -\pi$

$\cos z = \cos(\frac{1}{2}\pi)\cosh(-\pi) - i\sin(\frac{\pi}{2})\sinh(-\pi)$

$= 0 - i(1)(-11.54)$

$= 0 + i\,11.54$

$\boxed{\cos(\frac{\pi}{2} - \pi i) = 0 + 11.54i}$