Circuits and Systems - 1 - Week 10

Source Transformation

A voltage source along with a resistor in series can be replaced by a current source with parallel resistor.

Similarly, a current source along with a resistor in parallel can be replaced by a voltage source with series resistor.

Let us take an example of voltage source having voltage $\boldsymbol{v_s}$ and a series resistor connected with it, denoted by $\boldsymbol{R_s}$. The load is denoted by $\boldsymbol{R_L}$.

Source Transformation - Voltage Source to Current Source

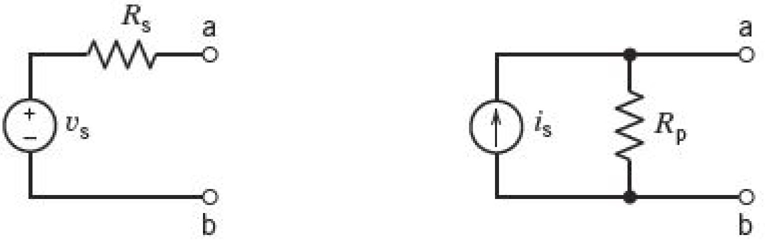

Figure: A voltage source and current source

We can convert this voltage source $\boldsymbol{v_s}$ along with $\boldsymbol{R_s}$ to a current source as shown in next slide.

Source Transformation - Voltage Source to Current Source

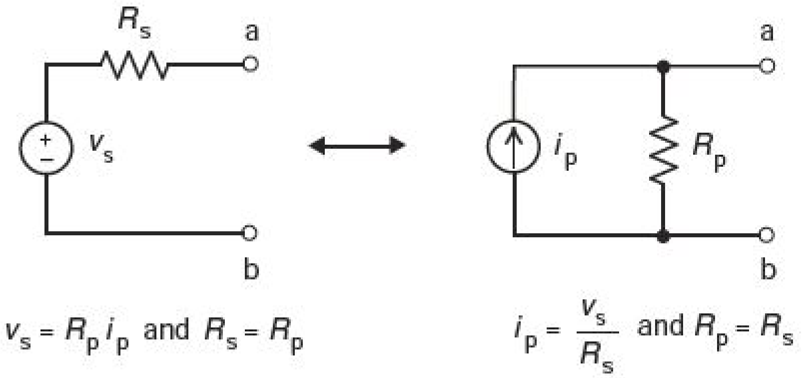

Figure: Transforming voltage source to current source

Source Transformation - Voltage Source Example

If between point $\boldsymbol{a}$ and $\boldsymbol{b}$ is the load resistance of $\mathbf{2k\Omega}$, then compute the value of $\boldsymbol{i_p}$ and $\boldsymbol{R}$ such that the following holds:

- The current across $\mathbf{2k\Omega}$ in circuit $\mathbf{1}$ and circuit $\mathbf{2}$ is the same
- The voltage across $\mathbf{2k\Omega}$ in circuit $\mathbf{1}$ and circuit $\mathbf{2}$ is the same

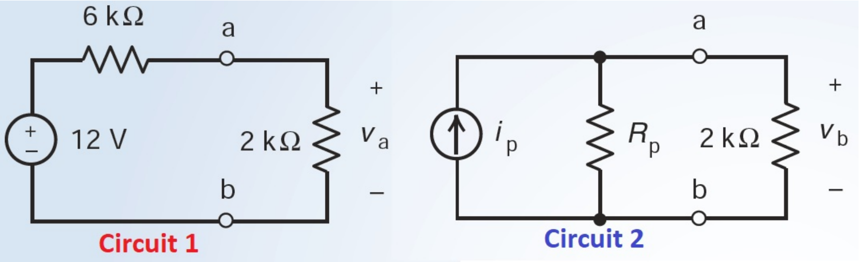

Figure: Example for source transformation

Source Transformation - Voltage Source Example

I will apply the theory just we studied now.

$$\boldsymbol{i_p = \frac{12}{6k} = 0.002A = 2mA}$$

$$\boldsymbol{R_p = 6k\Omega}$$

Can you compute $\boldsymbol{v_a}$ and $\boldsymbol{v_b}$? Similarly, compute the current across load resistor $\boldsymbol{2k\Omega}$.

Source Transformation - Voltage to Current Source

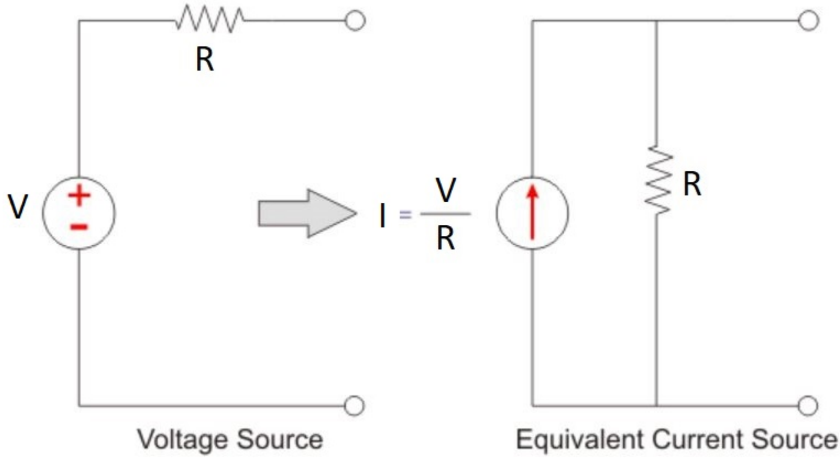

Figure: Voltage to current source transformation

Source Transformation - Current to Voltage Source

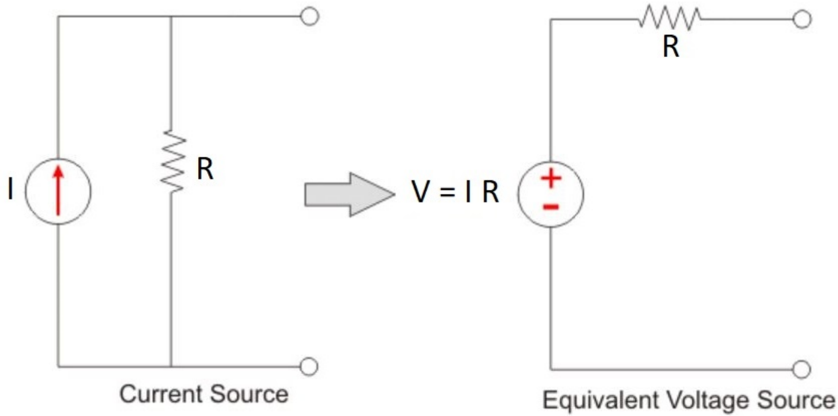

Figure: Current to voltage source transformation

Source Transformation - Look into polarity signs

Convert the voltage source into current source and simplify the circuit

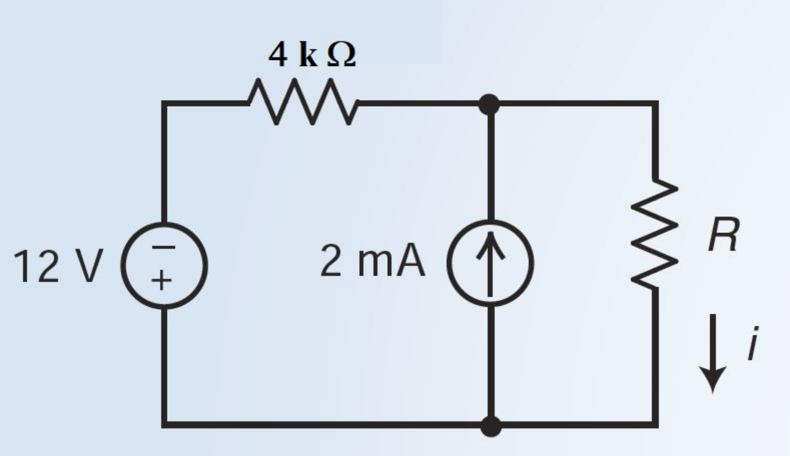

Figure: Convert voltage source into current source

Source Transformation - Look into polarity signs

Now, we have to be careful with the polarity signs. The equivalent circuit is as follows:

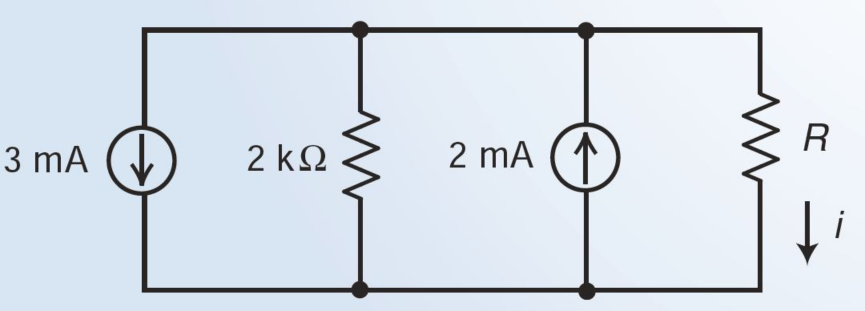

Figure: Equivalent circuit after conversion

We can further simplify the current sources by applying rules for parallel combination of current sources.

Source Transformation

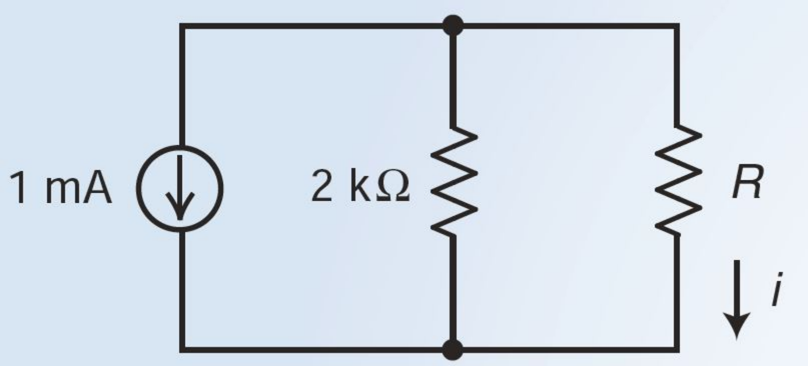

Figure: Equivalent circuit after conversion

Thevenin Theorem Motivation

Sometimes we want to vary the load across a circuit and study its behavior. So, applying nodal or mesh analysis consume a lot of time (if the circuit load changes).

Perhaps, it is possible that one component in a circuit varies or can be changed (e.g. the value of a resistor can be changed).

This leads us to separate the variable or load element from the rest of circuit

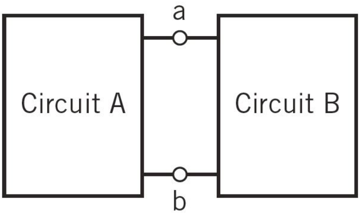

Figure: Division of a circuit into 2 sub-circuits

Thevenin Theorem Terminologies

In order to separate the load (or variable element) from the rest of circuit, we need to introduce a few new terminologies:

- Open-circuit voltage denoted by $\boldsymbol{v_{oc}}$
- Short-circuit current denoted by $\boldsymbol{i_{sc}}$
- Thevenin resistance denoted by $\boldsymbol{R_{th}}$ or $\boldsymbol{R_t}$

We will study these terminologies in the example on next slide.

Thevenin Theorem Example

Transform the following circuit into Thevenin equivalent circuit

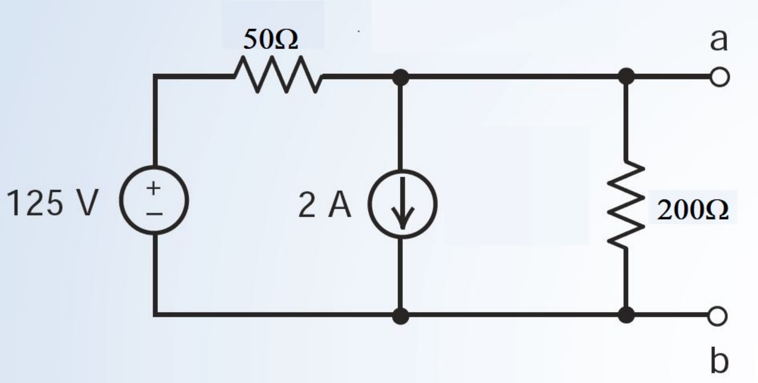

Figure: Transform this circuit into Thevenin equivalent circuit

Can you simplify the above circuit using source transformation theory?

Thevenin Theorem Example - using source transformation

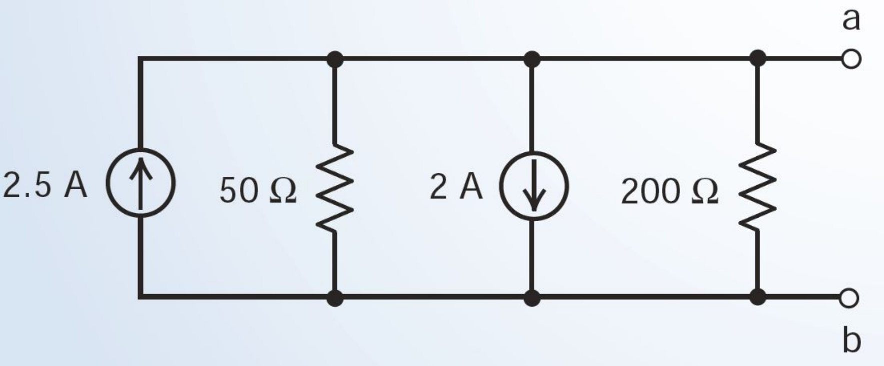

Figure: Transforming the circuit using source transformation theory

Thevenin Theorem Example - using source transformation

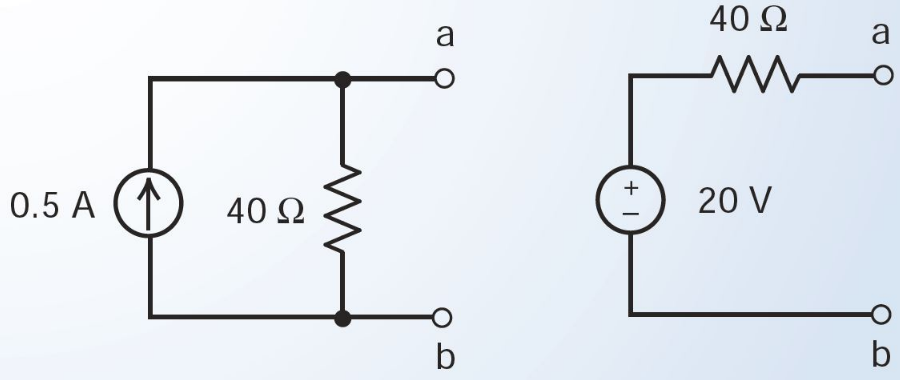

Figure: Transforming the circuit using source transformation theory

Thevenin Theorem Example

Let us first compute v_{oc}, the open circuit voltage. The voltage across terminals a and b or 200Ω resistor is v_{oc}.

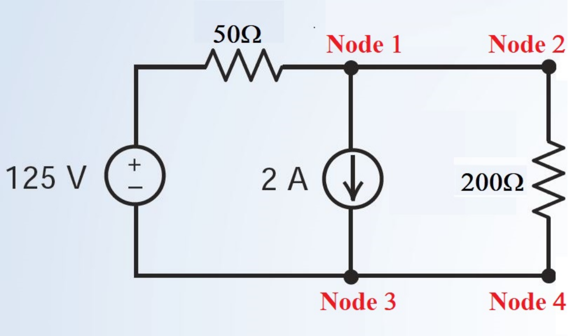

Figure: Applying nodal analysis to compute v_{oc}

Actually, node 3 and 4 are the same nodes. Similarly, node 1 and 2 are the same nodes.

Thevenin Theorem Example

Applying KCL to node 1, we obtain the following:

$$\frac{125 - v_{oc}}{50} = 2 + \frac{v_{oc}}{200}$$

Simplifying this equation gives us $v_{oc} = 20V$. Next, we need to compute another term which is called as short circuit current i_{sc}

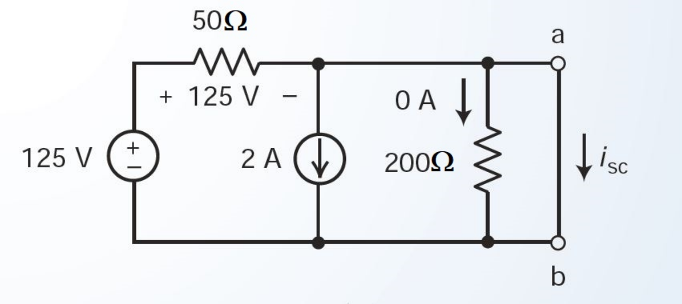

Figure: Computing i_{sc}

Thevenin Theorem Example

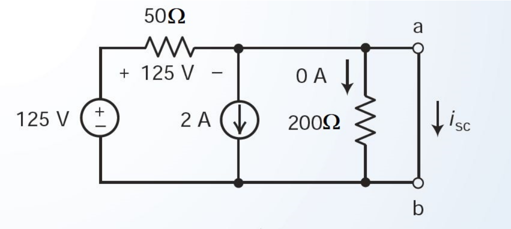

Figure: Computing $\boldsymbol{i_{sc}}$

As the $\mathbf{200\Omega}$ resistor has short circuit resistance, so no current flows through $\mathbf{200\Omega}$ resistor and the voltage across $\mathbf{200\Omega}$ resistor is $\boldsymbol{0V}$. Hence, the voltage across $\mathbf{50\Omega}$ resistor would be $\boldsymbol{125V}$ (can be verified using KVL also).

The current across $\mathbf{50\Omega}$ resistor would be $\boldsymbol{\frac{125}{50} = 2.5A}$.

Thevenin Theorem Example

To compute i_{sc}, we apply KCL to node 1 again and obtain the following:

$$\mathbf{2.5}A = \mathbf{2}A + \mathbf{0} + i_{sc}$$

From the above equation, we obtain $i_{sc} = \mathbf{0.5}A$. Next, we need to compute the third term which is called Thevenin resistance. To do so, we need to short-circuit the voltage sources and open-circuit the current sources as shown below:

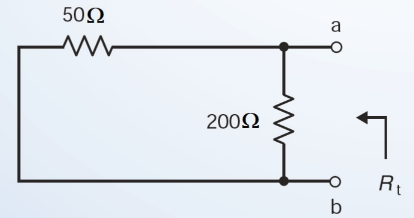

Figure: Computing R_t

$$R_t = \mathbf{200} \parallel \mathbf{50} = \mathbf{40}\Omega$$

Thevenin Theorem Example

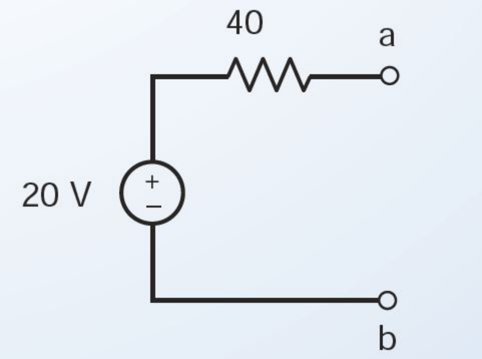

Figure: Thevenin Equivalent Circuit

The value of $\boldsymbol{i_{sc}}$ indicate the Thevenin equivalent circuit is correct.

Thevenin Theorem Practice Problem

Practice Problem 5.4-1 on page 201 of textbook. The answers of the practice problem is already given in the book.

Next week, we will start with Norton Theorem, then study Maximum Power Transfer theorem and solve problems of chapter 5.