(1)

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Subject:- Signals & System
Assignment:- 02
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X———X

Solution 01:-

a) $x(t) = e^{j8t} + 2e^{(12t+\pi/3)}$

Now

$$e^{j12+j\pi/3} = e^{j12t} + e^{j\pi/3}$$

$$e^{j\pi/3} = \cos\pi/3 + j\sin\pi/3 \Rightarrow \frac{1}{2} + j\frac{\sqrt{3}}{2} \quad —(1)$$

$$e^{j\omega t} \Rightarrow \delta(\omega-\omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi} e^{j\omega t}\Big]_{\omega=\omega_0}$$

$$\therefore e^{j\omega t} = 2\pi\delta(\omega-\omega_0)$$

Putting eq (1)

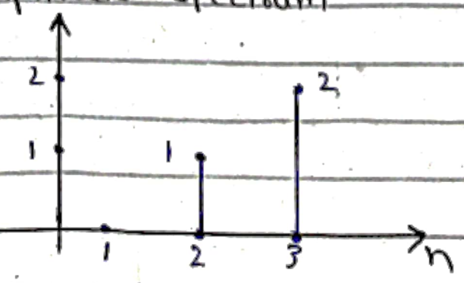
$$x(t) = e^{j8t} + 2\left(\frac{1}{2}e^{j12t} + j\frac{\sqrt{3}}{2}e^{j12t}\right)$$

$$X(\omega) = 2\pi\left[\delta(\omega-8) + \delta(\omega-12) + j\sqrt{3}\times\delta(\omega-12)\right]$$

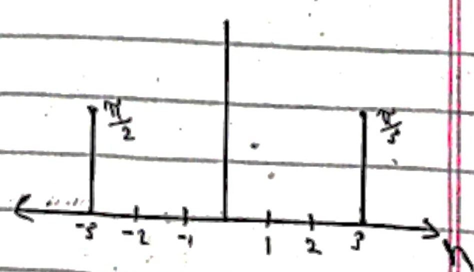
$$X(\omega) = 2\pi\left[\delta(\omega-8) + \delta(\omega-12)(1+j\sqrt{3})\right]$$

Spectrum:-

Amplitude spectrum



Phase spectrum



②

(b) $x(t) = \cos\left(\frac{\pi}{4}t - \frac{\pi}{6}\right) + \sin\left(\frac{3\pi}{4}t + \frac{\pi}{4}\right)$

Using Formulas;

$\cos(A-B) = \cos A\cdot\cos B + \sin A \sin B$

$\sin(A+B) = \sin A\cdot\sin B + \cos A\cdot\cos B$

Now,

$x(t) = \cos\left(\frac{\pi}{4}t - \frac{\pi}{6}\right) + \sin\left(\frac{3\pi}{4}t + \frac{\pi}{4}\right)$

$= \left(\cos\frac{\pi}{4}t\cdot\cos\frac{\pi}{6} + \sin\frac{\pi}{4}t\cdot\sin\frac{\pi}{6}\right) + \left(\sin\frac{3\pi}{4}t\cdot\sin\frac{\pi}{4} + \cos\frac{3\pi}{4}t\cdot\cos\frac{\pi}{4}\right)$

$x(t) = \frac{\sqrt{3}}{2}\cos\frac{\pi}{4}t + \frac{1}{2}\sin\frac{\pi}{4}t + \frac{1}{\sqrt{2}}\left[\sin\frac{3\pi}{4}t + \cos\frac{3\pi}{4}t\right]$

$\cos w_0 t = \frac{e^{jw_0t} + e^{-jw_0t}}{2}$

$= \frac{1}{2}\left[2\pi\left[\delta(w-w_0) + \delta(w+w_0)\right]\right]$

$= \pi\left[\delta(w-w_0) + \delta(w+w_0)\right]$

$\sin w_0 t = \frac{e^{jw_0t} - e^{-jw_0t}}{2j}$

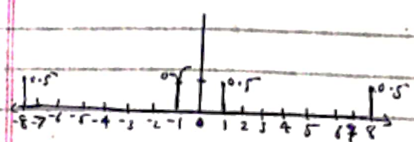
$= \frac{\pi}{j}\left[\delta(w-w_0) - \delta(w+w_0)\right]$

$X(w) = \frac{\sqrt{3}}{2}\left[\pi\left(\delta\left(w-\frac{\pi}{4}\right) + \delta\left(w+\frac{\pi}{4}\right)\right)\right] + \frac{\pi}{2j}\left[\delta\left(w-\frac{\pi}{4}\right) - \delta\left(w+\frac{\pi}{4}\right)\right] + \frac{1}{\sqrt{2}}\left[\frac{\pi}{j}\left(\delta\left(w-\frac{3\pi}{4}\right) + \delta\left(w+\ldots\right)\right)\right]$

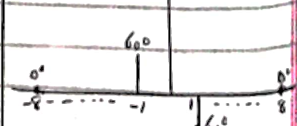
$+ \frac{\pi}{\sqrt{2}}\left[\delta\left(w-\frac{3\pi}{4}\right) + \delta\left(w+\frac{3\pi}{4}\right)\right]$

Spectrum:–

Amplitude spectrum



Phase spectrum



③

$x(t) = \begin{cases} 1 & -1 \le t \le 0 \\ -1 & 0 \le t \le 1 \end{cases}$

$\because T = 2\,sec \Rightarrow w_0 = \frac{2\pi}{T} = \frac{2\pi}{2} \rightarrow \boxed{w_0 = \pi}$

Now

$C_0 = \frac{1}{T}\int_{-1}^{1} x(t)\,d(t) = \frac{1}{2}\left[\int_{-1}^{0} dt + \int_{0}^{1} -dt\right]$

$= \frac{1}{2}\left[t\Big|_{-1}^{0} - t\Big|_{0}^{1}\right] = \frac{1}{2}\left[0+1-1\right] = \boxed{0}$

$\boxed{C_0 = 0}$

$C_n = \frac{1}{T}\int_{-1}^{1} x(t)e^{-jnw_0t}dt \Rightarrow \frac{1}{2}\int_{-1}^{1} x(t)e^{-jnw_0t}dt$

$= \frac{1}{2}\left[\int_{-1}^{0} e^{-jn\pi t}dt + \int_{0}^{1} -e^{-jn\pi t}dt\right]$

$= \frac{1}{2}\left[\frac{e^{-jn\pi t}}{-jn\pi}\Big|_{-1}^{0} + \left[\frac{-e^{-jn\pi t}}{-jn\pi}\right]\Big|_{0}^{1}\right]$

$= \frac{1}{2}\left[\frac{1-e^{jn\pi}}{-jn\pi} - \left[\frac{-e^{0} + e^{-jn\pi}}{-jn\pi}\right]\right]$

$= \frac{1}{2}\left[\frac{e^{jn\pi} - 1 - 1 + e^{-jn\pi}}{jn\pi}\right] \quad (\because e^{jn\pi} = e^{-jn\pi})$

$= \frac{1}{2}\left[\frac{-2 + 2e^{jn\pi}}{jn\pi}\right] = \frac{-2+2(-1)^n}{2j\pi n}$

$= \frac{-1 + (-1)^n}{j\pi n}$

④

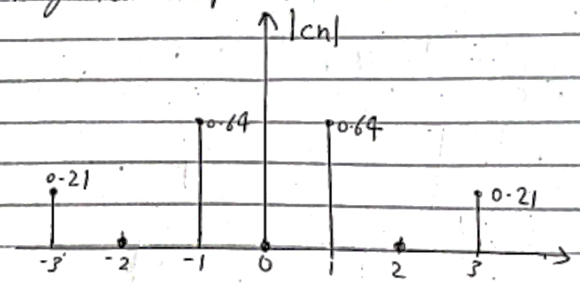
Now,

n	C_n	$\lvert C_n \rvert$	$\angle C_n$
-3	$2/3j\pi = -\frac{2j}{3\pi}$	$2/3\pi$	-90°
-2	0	0	0
-1	$2/j\pi = -\frac{j2}{\pi}$	$\frac{2}{\pi}$	-90°
0	0	0	0
1	$-2/j\pi = 2j/\pi$	$2/\pi$	90°
2	0	0	0
3	$-2/3j\pi = \frac{2j}{3\pi}$	$2/3\pi$	90°

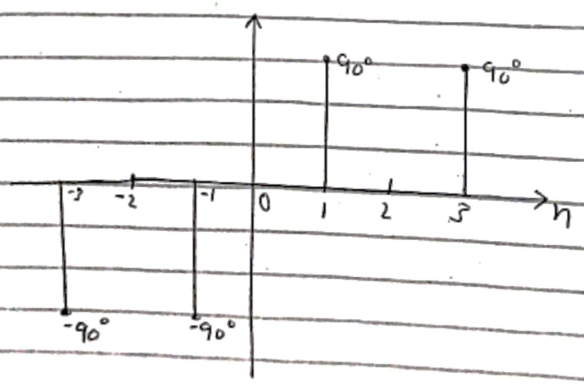
also ; $\frac{2}{\pi} = 0.64$

$\frac{2}{3\pi} = 0.21$

Magnitude spectrum:-

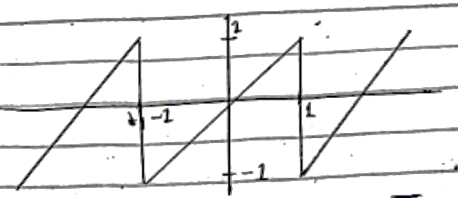


Phase spectrum



⑤

Solution 2:-



T=0

Since signal is Odd ; $a_n = 0$

$$a_0 = \frac{1}{2} \times 1 \times (-1) + \frac{1}{2} \times 1 \times 1 = 0$$

$$b_n = \frac{1}{T}\int x(t)\sin n\omega_0 t\,dt \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

$$= \frac{1}{2}\int_{-1}^{1} \sin n\pi t\,dt$$

$$= \frac{1}{2}\times 2\int_{0}^{1} \sin n\pi t\,dt$$

$$= t\left(\frac{-\cos n\pi t}{n\pi}\right)\Big|_0^1 + \int_0^1 \frac{\cos n\pi t}{n\pi}\,dt$$

$$= -\cos n\pi + \frac{\sin n\pi t}{(n\pi)^2}\Big|_0^1$$

$$= -\cos n\pi$$

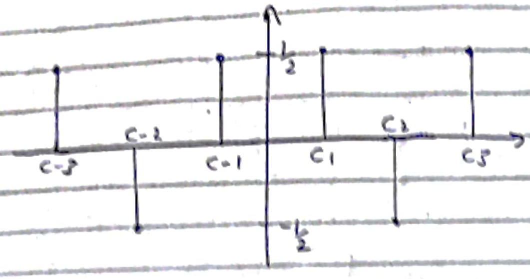
$$C_0 = a_0 = 0, \quad C_n = \frac{a_n + jb_n}{2} = \frac{-\cos n\pi}{2}$$

$$C_{-1} = C_1 = \frac{-\cos\pi}{2} = \frac{1}{2}$$

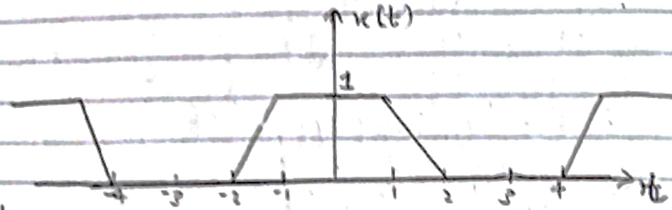
$$C_{-2} = C_2 = \frac{-\cos 2\pi}{2} = -\frac{1}{2}$$

$$C_{-3} = C_3 = \frac{-\cos 3\pi}{2} = \frac{1}{2}$$

Spectrum:-



b)



Now,

$T=8$, $\omega_0 = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{8} \Rightarrow \omega_0 = \frac{\pi}{4}$

Signal is Even; $b_n = 0$

$a_0 = \frac{1}{T}\int x(t)\,dt$

$= \frac{1}{8}\int x(t)\,dt = \frac{1}{8} \times \frac{1}{2} \times (4+2) \times 1 = \boxed{\frac{3}{8}}$

$a_n = \frac{1}{T}\int x(t)\cos n\omega_0 t\,dt$

$x(t) = t+2 \quad ; \; -2 \le t \le -1$
$\quad = 1 \quad ; \; -1 \le t < 1$
$\quad = 2-t \quad ; \; 1 \le t < 2$

$a_n = \frac{1}{8}\Big[\int_{-2}^{-1}(t+2)\cos n\omega_0 t\,dt + \int_{-1}^{1}\cos n\omega_0 t\,dt + \int_{1}^{2}(2-t)\cos$

$= \frac{1}{8}\Big[\frac{t\sin n\omega_0 t}{n\omega_0}\Big|_{-2}^{-1} - \int_{-2}^{-1}\frac{\sin n\omega_0 t}{n\omega_0}\,dt + \frac{2\sin n\omega_0 t}{n\omega_0}\Big|_{-2}^{-1}\Big)$

$+ \frac{\sin n\omega_0 t}{n\omega_0}\Big|_{-1}^{1} + \frac{2\sin n\omega_0 t}{n\omega_0}\Big|_{1}^{2} - \frac{t\sin n\omega_0 t}{n\omega_0}\Big|_{1}^{2} + \int_{1}^{2}\frac{\sin n\omega_0 t\,dt}{n\omega_0}\Big]$

$= \frac{1}{8}\Big[\frac{\cos n\omega_0 - \cos 2n\omega_0}{(n\omega_0)^2} - \frac{\cos 2n\omega_0 - \cos n\omega_0}{(n\omega_0)^2}\Big]$

$= \frac{1}{8}\Big[\frac{2(\cos n\omega_0 - \cos 2n\omega_0)}{(n\omega_0)^2}\Big]$

$= \frac{1}{4} \times \frac{-1}{(n\omega_0)^2}\Big[\cos\frac{n\pi}{4} - \cos\frac{n\pi}{2}\Big]$

$= \frac{1}{4} \times \frac{1}{(n\omega_0)^2} \times \cos\frac{n\pi}{4}$

$= \frac{1}{4} \times \frac{16}{(n\pi)^2} \times \cos\frac{n\pi}{4} \qquad (\because \omega_0 = \frac{\pi}{4})$

$= \frac{4}{(n\pi)^2}\cos\frac{n\pi}{4}$

$C_0 = a_0 = \frac{3}{8}$, $C_n = \frac{a_n + jb_n}{2}$

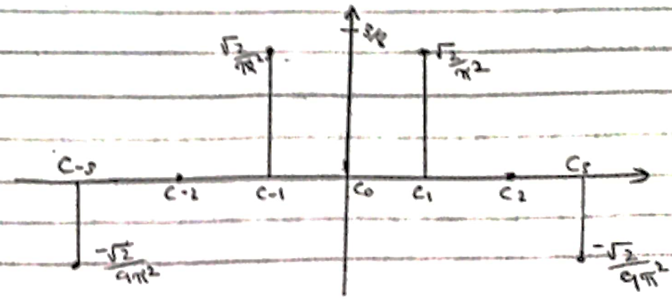
$C_n = \frac{2}{(n\pi)^2}\cos\frac{n\pi}{4}$

$C_{-3} = C_3 = \frac{2}{(3\pi)^2}\cos\frac{3\pi}{4} = \frac{-\sqrt{2}}{9\pi^2}$

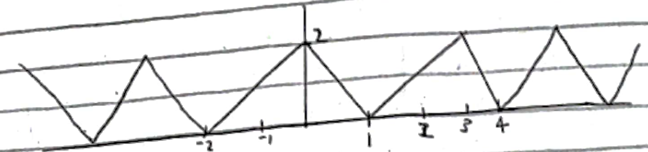
$C_{-2} = C_2 = \frac{2}{(2\pi)^2}\cos\frac{2\pi}{4} = 0$

$C_{-1} = C_1 = \frac{2}{(\pi)^2}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{\pi^2}$

Spectrum:-



c)



Now,

$\because T = 3$; $w_0 = \frac{2\pi}{T} = \frac{2\pi}{3} \Rightarrow \boxed{w_0 = \frac{2\pi}{3}}$

$$x(t) = \begin{cases} 2+t & ; -2 < t < 0 \\ 2-t & ; 0 < t < 1 \end{cases}$$

$$a_0 = \frac{1}{T}\int x(t)\,dt = \frac{1}{3}\left[\int_{-2}^{0}(2+t)dt + \int_{0}^{1}(2-t)dt\right]$$

$$a_0 = 1$$

$$a_k = \frac{1}{3}\left[\int_{-2}^{0}(2+t)e^{-jk\left(\frac{2\pi}{3}\right)t}dt + \int_{0}^{1}(2-t)e^{-jk\left(\frac{2\pi}{3}\right)t}dt\right]$$

$$= \frac{3j}{2\pi^2 k^2}\left[e^{jk\frac{2\pi}{3}}\sin\left(\frac{k2\pi}{3}\right) + 2e^{j\frac{\pi}{3}k}\sin\left(\frac{k\pi}{3}\right)\right]$$

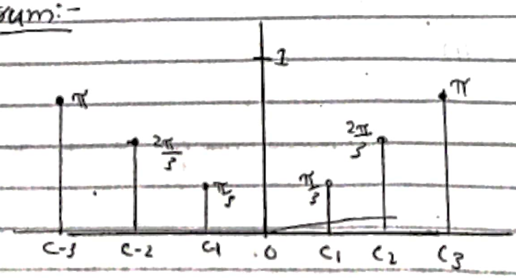
$a_0 = c_0 = 1$

$c_3 = c_{-3} = \pi$

$c_2 = c_{-2} = \frac{2\pi}{3}$

$c_1 = c_{-1} = \frac{\pi}{3}$

Spectrum:-



Solution 03:-

$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jw_0nt}$ $\boxed{H(w)}$ $y(t) = \sum_{n=-\infty}^{\infty} c_n H(nw_0) e^{jw_0nt}$

$$H(w) = \frac{1}{2+jw}$$

$$x(t) = e^{j8t} + 2e^{j\pi/3}e^{j12t}$$

$$T_1 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$T_2 = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$\frac{T_1}{T_2} = \frac{6}{4} = \frac{3}{2} = x$$

$$T = xT_1 = \frac{3}{2} \times \frac{\pi}{4} = \frac{\pi}{2}$$

$$w = \frac{2\pi}{T} = 4$$

$C_2 = 1$

$C_3 = 2e^{j\pi/3} = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)$

$$y(t) = \sum_{n=-\infty}^{\infty} c_n H(nw_0)e^{jw_0nt}$$

$$= \sum_{n=-\infty}^{\infty} c_n \frac{1}{2+jnw} e^{jnwt}$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} c_n\left(\frac{1}{2+4jn}\right)e^{j4nt}}$$

b) $x(t) = \cos\left(\frac{\pi}{4}t - \frac{\pi}{6}\right) + \sin\left(\frac{3\pi t}{4} + \frac{\pi}{4}\right)$

$$x(t) = \frac{e^{j(\pi/4\,t - \pi/6)} + e^{-j(\pi/4\,t - \pi/6)}}{2} + \frac{e^{j(\frac{3\pi}{4}t + \frac{\pi}{4})} - e^{-j(\frac{3\pi}{4}t + \frac{\pi}{4})}}{2}$$

$$T_1 = \frac{2\pi}{\pi} \times 4 = 8$$

$$T_2 = \frac{2\pi}{3\pi} \times 4 = \frac{8}{3}$$

$$\frac{T_1}{T_2} = \frac{8}{8} \times 3 = 3$$

$$T = 8$$

$$w = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$x(t) = \underbrace{\frac{e^{-j\pi/8} e^{j\frac{\pi}{4}t}}{2}}_{c_1} + \underbrace{\frac{e^{-j\pi/8 - j\pi/4 t}}{2}}_{c_{-1}} + \underbrace{\frac{e^{j\frac{3}{4}\pi t} e^{j\pi/4}}{2j}}_{c_3} - \underbrace{\frac{e^{-j(3\pi\ldots}}{2j}}_{c_{-3}}$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} C_n \left(\frac{1}{2+j\pi/4\, n}\right) e^{j(\pi/4)nt}}$$

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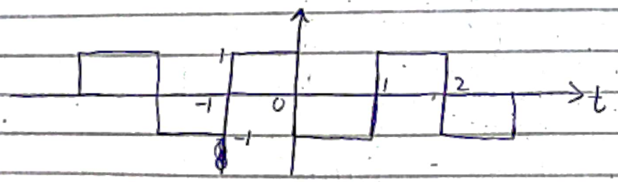
c)



Period $= 2$

$$w_0 = \frac{2\pi}{2} = \pi$$

$\because$ Signal is odd;

$$C_n = -C_{-n}$$

$$C_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\,dt$$

$$= \frac{1}{\alpha} \int_{-1}^{0} dt + \int_{0}^{1} x\,dt$$

$$= \frac{1}{\alpha} [1 - 1]$$

$$C_0 = 0$$

$$C_n = \frac{1}{T} \left[\int_{-1}^{0} e^{-jn\omega_0 t}\,dt - \int_{0}^{1} e^{-jn\omega_0 t}\,dt \right]$$

$$= \frac{1}{T} \left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0 t} \right]_{-1}^{0} - \left(\frac{-1}{jn\omega_0}\right) \left[e^{-jn\omega_0 t} \right]_{0}^{1}$$

$$= \frac{1}{2} \left[\frac{1 - e^{jn\omega_0}}{-jn\omega_0} + \frac{1}{jn\omega_0} \left[e^{-jn\omega_0} - 1 \right] \right]$$

$$= \frac{1}{2jn\omega_0} \left[e^{jn\omega_0} - 1 - e^{-jn\omega_0} - 1 \right]$$

$$\therefore = \left[\frac{2\cos n\omega_0 - 1}{2jn\omega_0} \right]$$

$$C_n = \frac{\cos n\omega_0 - 1}{jn\omega_0}$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} C_n \frac{1}{2+jn\pi} e^{jn\pi t}}$$

(a) It is mentioned that I have solved Fourier Series co-efficients

$$T = 2$$

$$w = \frac{2\pi}{2} = \pi$$

$$\boxed{y(t) = \sum_{n=-\infty}^{\infty} C_n \frac{1}{2+jn\pi} e^{jn\pi t}}$$

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b) $T = 6$

$\omega = \frac{2\pi}{T} = \pi/3$

$$\therefore \boxed{y(t) = \sum_{n=-\alpha}^{\alpha} C_n \frac{1}{2+jn\pi/3} e^{j\pi/3 nt}}$$

X ——— X ——— X ——— X

c) $T = 3$

$\omega = \frac{2\pi}{3}$

$$\boxed{\therefore y(t) = \sum_{n=-\alpha}^{n} C_n \frac{1}{2+j n \frac{2\pi}{3}} e^{j\frac{2\pi}{3}nt}}$$

X ——— X

THE END