University of engineering & technology Peshawar

Linear Algebra

Assignment no#01

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Submitted by: Ashfaq Ahmad

Section: B

Reg No: 19PWCSE1795

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"On my honor, as a student of University of Engineering and Technology Peshawar, I have neither given nor received unauthorized assistance on this academic work"

Student signature: ____________________

Submitted to:
Eng: Jamal Nasir

Department Of Computer System Engineering

Q1: Sol

Given

$$2x - y = 5$$
$$4x - 2y = t$$

(i) the Augmented matrix of system is

$$\left[\begin{array}{cc|c} 2 & -1 & 5 \\ 4 & -2 & t \end{array}\right] \quad R_2 - 2R_1$$

$$\left[\begin{array}{cc|c} 2 & -1 & 5 \\ 0 & 0 & t-10 \end{array}\right] \quad R_1/2$$

$$\left[\begin{array}{cc|c} 1 & -1/2 & 5/2 \\ 0 & 0 & t-10 \end{array}\right]$$

Since Rank of Augmented matrix $[A|B]$ is not equal to Rank of Coffrient matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$

So system has no unique Solution. i.e $\underset{R\text{-}A}{1} \neq \underset{R\text{-}[A|B]}{2} = 2$ → no. of variable

(ii) No Solution:

if $t - 10 \neq 0$

$t \neq 10$

then the $\rho[A] \neq \rho[A|B]$

So the System has no Solution

~~(iii) if $t-10=0$~~

~~$\Rightarrow t=10$~~

~~Then $\rho[A] = \rho[A|B] < n$~~

~~So the system has infinite many~~
~~solution.~~

iii) different value of t at (ii)

(a)

* Set of all real no rather
then 10.

or ——— x xx ——— xx ——— x x ———

Q2: Sol

let x reprent lower Sulphur.
& y " higher Sulphur

then L.S H.S hours

B.P $5x + 4y = 3$

R.P $4x + 2y = 2$

in Second

$$5x + 4y = 3 \times 60 = 180 \quad \text{—(i)}$$

$$4x + 2y = 2 \times 6 = 120 \quad \text{—(ii)}$$

xing (ii) by 2 then subtract from (i)

$$5x + 4y = 180$$
$$\underline{8x + 4y = 240}$$
$$-3x = -60$$
$$\boxed{x = 20}$$

Put x in (i)

$$5(20) + 4y = 180$$
$$100 + 4y = 180$$
$$\frac{4y}{4} = \frac{80}{4}$$
$$\boxed{y = 20}$$

S. Set $= \{20, 20\}$ Ans

Unique Solution

—— xx —— xx —— xx —— xx —— xx

P T O

Q3 (a)

Given:

let x denote Regular plastic ton,

& y " Special plastic ton.

then

	R.P		S.P		
P.A	$2x$	+	$2y$	$= 8$	— (1)
P.B	$5x$	+	$3y$	$= 15$	— (2)

Xing (i) by 5 & (ii) by (2) then

Subtract (1) from (2)

$$\begin{array}{r} \cancel{10x} + 6y = 30 \\ -\cancel{10x} \mp 10y = \mp 40 \\ \hline -4y = -10 \end{array}$$

$$\boxed{y = 2.5 \text{ tons}}$$

put y in (1)

~~2(2.5)~~

$$2x + 2(2.5) = 8$$

$$2x + 5 = 8$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{3}{2}$$

$$\boxed{x = 1.5} \text{ tons}$$

S. Set $= \{1.5, 2.5\}$

(b) Given:

$0 \rightarrow$ OFF

$1 \rightarrow$ ON

$$A = \begin{bmatrix} ON & ON & OFF \\ OF & ON & OFF \\ OFF & ON & ON \end{bmatrix}$$

matrix A in binary form

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

According to given condition.

$$A + B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - A$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1-1 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ Ans

Q4: Given

$S_1 = [18.95 \quad 14.75 \quad 8.98]$

$S_2 = [17.80 \quad 13.50 \quad 10.79]$

(i)

In matrix form

$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} \text{Item-1} & \text{Item-2} & \text{Item-3} \\ \end{array}$$

$$A = \begin{bmatrix} 18.95 & 14.75 & 8.98 \\ 17.80 & 13.50 & 10.79 \end{bmatrix} \begin{array}{l} \rightarrow \text{Store A} \\ \rightarrow \text{Store B} \end{array}$$

(ii) price of each item reduced 20% So

$$A = \begin{bmatrix} 18.95 \times 80\% & 14.75 \times 80\% & 8.98 \times 80\% \\ 17.80 \times 80\% & 13.50 \times 50\% & 10.79 \times 80\% \end{bmatrix}$$

$$A = \begin{bmatrix} 15.16 & 11.8 & 7.18 \\ 14.24 & 10.8 & 8.63 \end{bmatrix} \text{Ans}$$

—— xx —— xx —— xx —— xx

Q5: (a) Given:

$f : R^2 \longrightarrow R^2 \qquad f(V) = AV$

where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix}$$

$K = 1/2$

Sketch & image of R = ???

Sol

$K = 1/2$

And

$$V = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

As

$f(V) = AV$

$$f(V) = AV$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad K = 1/2 = 0.5$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$f(V) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0.5 & 0 & 0.5 \end{bmatrix} \text{ image}$$

P P T P O

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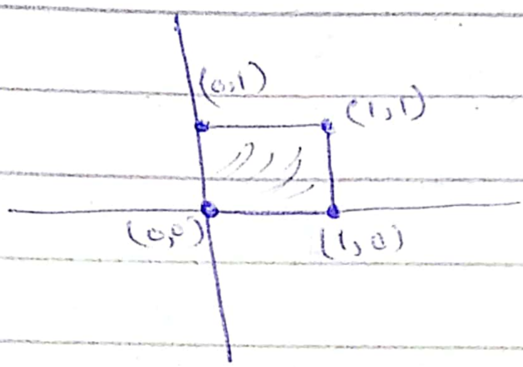
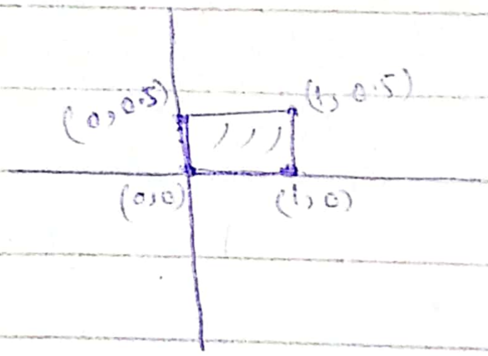
graph:



sketch

Image



x x x —— x x x

—xx————————xx—

Q5: (b)

$$T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 0 & 0.5 & 0.7 \end{bmatrix}$$

Sol

we have need to find Some values of T

$$\begin{bmatrix} 0.06 & 0+0.018+0.03 & 0+0.018+0.042 \\ 0.3 & 0.06+0.72+0.15 & 0+0.072+0.21 \\ 0.64 & 0.14+0.21+0.32 & 0+0.21+0.448 \end{bmatrix}$$

$$\begin{bmatrix} 0.06 & 0.048 & 0.06 \\ 0.3 & 0.282 & 0.282 \\ 0.64 & 0.67 & 0.658 \end{bmatrix}$$

Since all the entries of T^5 are possible thus we set T is regular.

$$TU = U$$

$$(I - T) = 0$$

In this case we have to Solve the homogenous System.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -0.2 & 0.0 \\ 0 & 0.7 & -0.3 \\ -1 & -0.5 & 0.3 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = 0$$

$\cancel{M_1 - 0.2 M_2 = 0}$

$\cancel{0.7 M_2 = 0.3 M_3} =$

$M_1 = 0.2 U_2 = 0$

$0.1 m_2 - 0.3 U_3 = 0$

$1.M_1 - 0.5 M_2 + 0.3 M_3 = 0$

therefore we get

$M_1 = 0.2 U_2$

$= \frac{2}{10} U_2$

thus

$M_1 = \frac{1}{5} U_2$

and

$0.3 U_2 = 0.7 U_2$

$M_1 = \frac{0.7}{0.3} U_3 = \frac{7}{3} M_3$

Since $m_1+m_2+m_3=1$ So

$$\frac{1}{5}m_2+m_3+7/3=1$$

$$\frac{3m_2+15m_2+35m_2}{15}=1$$

on Substituting the value of m_2 in $m_1=1/5\,m_2$ we get.

$$m_1=1/5\,m_2$$

$$m_1=\frac{1}{\cancel{5}}\times\frac{\cancel{15}}{35}$$

$$m_1=3/35$$

on Substituting value of m_2 in $m_1=7/3$

$$m_1=7/\cancel{3}\left(\cancel{15}/\cancel{53}\right)$$

$$m_1=\frac{35}{53}$$

Hence $U=\begin{bmatrix}m_1\\m_2\\m_3\end{bmatrix}\begin{bmatrix}3/53\\15/53\\35/53\end{bmatrix}$

$$U=\begin{bmatrix}0.057\\0.283\\0.660\end{bmatrix}$$ Ans

Q6: LU-Factorization

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \qquad b = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

Sol

for upper matrix

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad 2R_2 + R_3$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ 0 & -2.2 & 0.8 & 10 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad 2R_3 + R_4$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ 0 & 0 & 0.8 & 10 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \quad R_4 - 2R_1$$

(.8

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ 0 & 0 & 0.8 & 10 \\ 0 & 0 & 0.45 & -1.56 \end{bmatrix} \quad R_4 + 0.2R_2$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ 0 & 0 & 0.8 & 10 \\ 0 & 0 & 0 & -7.2 \end{bmatrix} \quad 1.78R_4 - R_3$$

$\cancel{R_1}$

Now for L-Triangular matrix

$$\begin{bmatrix} 2 & 1 & 0 & 4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0.25 & -1 \\ 2 & 1 & 0 & 4 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad R_1 \longleftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & -1 \\ 2 & 1 & 0 & 4 \\ -2 & -1.1 & 1 & 6.2 \\ 4 & 2.2 & 0.83 & -2.58 \end{bmatrix} \quad C_3/0.25$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 1 & 4 & 4 \\ -2 & -1.1 & 7.2 & 6.2 \\ 4 & 2.2 & -1.75 & -2.58 \end{bmatrix} \quad R_3 + C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 1 & 0 & 4 \\ -2 & -1.1 & 11.6 & 6.2 \\ 4 & 2.2 & -10.55 & -2.58 \end{bmatrix} \quad C_3 - 4C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 6 \\ -2 & -1.1 & 11.6 & 4 \\ +4 & 2.2 & -10.55 & 2.58 \end{bmatrix} \quad C_1 + C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -2 & -1.1 & 11.6 & 10.6 \\ 4 & 2.2 & -10.55 & -10.63 \end{bmatrix} \quad C_4 - 6C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -2 & -1.1 & 11.6 & 0 \\ 4 & 2.2 & -10.55 & -1.13 \end{bmatrix} \quad 1.1C_4 - C_3$$

Now

$LZ = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -2 & -1.1 & 11.6 & 0 \\ 4 & 2.2 & -10.55 & -1.13 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

$\boxed{z_1 = -3}$

$2z_1 + z_2 = -1.5$

$2(-3) + z_2 = -1.5$

$-6 + z_2 = -1.5$

$\boxed{z_2 = 4.5}$

$z_3 = ??$

$-2z_1 - 1.1z_2 + 11.6z_3 = 5.6$

$-2(-3) - 1.1(4.5) + 11.6z_3 = 5.6$

$+6 - 4.95 + 11.6z_3 = 5.1$

$11.6z_3 = 5.1 - 1.05$

$11.6z_3 = -4.55$

$$\boxed{z_3 = 0.392}$$

$4z_1 + 2.2z_2 \mp 10.55z_3 - 1.13z_4 = 2.2$

$4(-3) + 2.2(4.5) - 10.55(0.392) - 1.13z_4 = 2.2$

$-12 + 9.9 - 4.1356 - 1.13z_4 = 2.2$

$-1.13z_4 = 2.2 - (-6.24)$

$-1.13z_4 = +8.44$

$$\boxed{z_4 = -7.46}$$

Now $Ux = Z$

$$\begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -1.1 & 0.75 & 4.2 \\ 0 & 0 & 0.8 & 10 \\ 0 & 0 & 0 & -7.2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ 4.5 \\ 0.392 \\ -7.46 \end{bmatrix}$$

$-7.2x_4 = -7.46$

$$\boxed{x_4 = 1.036}$$

$$0.8x_3 + 10x_4 = 0.392$$
$$0.8x_3 + 10(1.036) = 0.392$$
$$0.8x_3 + 10.36 = 0.392$$
$$0.8x_3 = -9.96$$
$$\boxed{x_3 = -12.46}$$

$$-1.1x_2 + 0.75x_3 + 4.2x_4 = 4.5$$
$$-1.1x_2 + 0.75(-12.46) + 4.2(1.036) = 4.$$
$$-1.1x_2 + (-9.34) + 4.35 = 4.5$$
$$-1.1x_2 = 9.49$$
$$\boxed{x_2 = -8.63}$$

$$2x_1 + x_2 + 0 - 4(x_4) = -3$$
$$\cancel{2(-8.63)}$$
$$2x_1 - 8.63 - 4(1.036) = -3$$
$$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{9.77}{2}$$
$$\boxed{x_1 = 4.88}$$

Thus

$$X = \begin{bmatrix} 4.88 \\ -8.63 \\ -12.46 \\ 1.036 \end{bmatrix} \quad \underline{\text{Ans}}$$

Q7: (a)

$$\det\left(\begin{bmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}\right)$$

Sol

Expand by 3rd row.

$$\Rightarrow \begin{vmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{vmatrix} =$$

$$0 \begin{vmatrix} -1 & 2 \\ \lambda-2 & 2 \end{vmatrix} - 0 \begin{vmatrix} \lambda-1 & -2 \\ 0 & 2 \end{vmatrix} +$$

$$+ (\lambda-3) \begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{vmatrix}$$

$$\begin{vmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{vmatrix} = 0 - 0 + \lambda-3 \begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{vmatrix}$$

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$$\begin{vmatrix} \lambda-1 & -1 & 2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{vmatrix} = (\lambda-3)(\lambda-1)(\lambda-2)$$

Ans

Q7. (b)

$\Rightarrow p(x) = ax^2 + bx + c$

$\Rightarrow f(x) = xe^{x-1}$

$\Rightarrow p(1) = a+b+c \quad \Rightarrow f(1) = 1$

$p'(x) = 2ax + b \quad \Rightarrow f'(x) = e^{x-1} + e^{x-1}$

$p'(1) = 2a+b \quad \Rightarrow f'(1) = 1+1 = 2$

$p''(x) = 2a$

$f''(x) = e^{x-1} + e^{x-1} + e^{x-1}$

$p''(1) = 2a$

$f''(1) = 1+1+1 = 3$

from the problem

$f(1) = p(1) = a+b+c = 1$ — (1)

$f'(1) = p'(1) = 2a+b = 2$ — (2)

$f''(1) = p''(1) = 2a = 3$ — (3)

from eq (3)

$\boxed{a = 3/2}$

put in (2)

$2(3/2) + b = 2$

$\Rightarrow b = 2-3$

$\boxed{b = -1}$

Put (a) (b) in (1)

$$\frac{3}{2} + (-1) + c = 1$$

$$\boxed{c = \frac{1}{2}}$$

So

$$\boxed{P(x) = \frac{3}{2}x^2 - x + \frac{1}{2}}$$ Ans

— xx — xx — xx —

Q 8: Sol

(i) As we know that

$$T = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}, \quad x = \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$X' = TX$$

$$= \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.64 + 0.3 + 0.02 \\ 0.08 + 0.05 + 0.02 \\ 0.08 + 0.02 + 0.06 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.69 \\ 0.15 \\ 0.16 \end{bmatrix}$$

ii) To find proportion of population of farmers we use homogenous system.

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{pmatrix}\right]\begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.2 & -0.3 & -0.2 \\ -0.1 & 0.5 & -0.2 \\ -0.1 & -0.2 & -0.4 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0$$

$$0.2A - 0.3B - 0.2C = 0$$
$$-0.1A + 0.5B - 0.2C = 0$$
$$-0.1A - 0.2B - 0.4C = 0$$

$$\Rightarrow \quad 0.3A - 0.8B = 0$$
$$A = \frac{8}{3}B \quad \text{—— (1)}$$

Now

$$0.6C = 0.7B$$
$$\frac{7B}{6} = C \quad \text{—— (2)}$$

Now

$$\frac{8}{3}B + B + \frac{7}{6}B = 1$$

(22)

$$B\frac{(16+6+7)}{6} = 1$$

$$\frac{29}{6} B = 1$$

$$\Rightarrow \boxed{B = \frac{6}{29}}$$

Put (B) in (1)

$$A = \frac{\cancel{6}^{2}}{29} \times \frac{8}{\cancel{3}}$$

$$\boxed{A = \frac{16}{29}}$$

Put (B) in (C)

$$C = \frac{7}{\cancel{6}} \times \frac{\cancel{6}}{29}$$

$$\boxed{C = 7/29}$$

So

$$U = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 16/29 \\ 6/29 \\ 7/29 \end{bmatrix} = \begin{bmatrix} 0.552 \\ 0.207 \\ 0.241 \end{bmatrix} \text{ Ans}$$

---x---

Q8 (b)

Sol

As we know that

$$\bar{OA} = -400$$

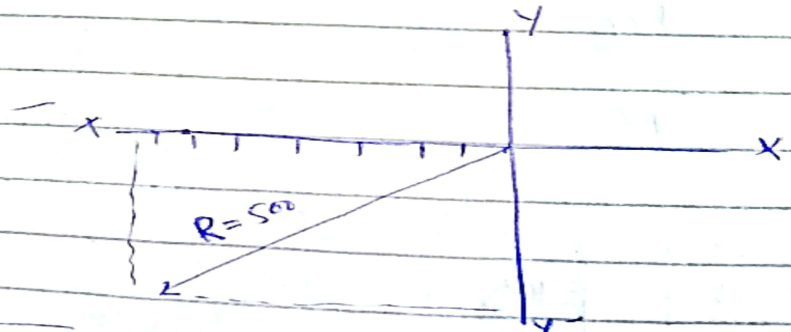
$$\bar{OB} = -300$$

Now by pethagours Theorem

$$(\bar{OC})^2 = (\bar{OA})^2 + (\bar{OB})^2$$

$$\sqrt{(\bar{OC})^2} = \sqrt{(-400)^2 + (-300)^2}$$

$$\boxed{OC = 500}$$



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The END

THE END