

**EXAMPLE 6**

Consider the parallelepiped with a vertex at the origin and edges  $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ , and  $\mathbf{w} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ . Then

$$\mathbf{v} \times \mathbf{w} = 5\mathbf{i} - 5\mathbf{k}.$$

Hence  $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = -10$ . Thus the volume  $V$  is given by (8) as

$$V = |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |-10| = 10.$$

We can also compute the volume by Equation (9) as

$$V = \left| \det \begin{pmatrix} 1 & -2 & 3 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{pmatrix} \right| = |-10| = 10.$$

**Key Terms**

Cross product

Jacobi identity

**5.1 Exercises**

In Exercises 1 and 2, compute  $\mathbf{u} \times \mathbf{v}$ .

1. (a)  $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{v} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$   
 (b)  $\mathbf{u} = (1, 0, 1)$ ,  $\mathbf{v} = (2, 3, -1)$   
 (c)  $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$   
 (d)  $\mathbf{u} = (2, -1, 1)$ ,  $\mathbf{v} = -2\mathbf{u}$
2. (a)  $\mathbf{u} = (1, -1, 2)$ ,  $\mathbf{v} = (3, 1, 2)$   
 (b)  $\mathbf{u} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ ,  $\mathbf{v} = \mathbf{i} + 3\mathbf{k}$   
 (c)  $\mathbf{u} = 2\mathbf{j} + \mathbf{k}$ ,  $\mathbf{v} = 3\mathbf{u}$   
 (d)  $\mathbf{u} = (4, 0, -2)$ ,  $\mathbf{v} = (0, 2, -1)$
3. Let  $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ ,  $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ ,  $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ , and  $c = -3$ . Verify properties (a) through (d) of Theorem 5.1.
4. Let  $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ , and  $\mathbf{w} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ .
  - (a) Verify Equation (3).
  - (b) Verify Equation (4).
5. Let  $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ , and  $\mathbf{w} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ .
  - (a) Verify Equation (3).

(b) Verify Equation (4).

6. Verify that each of the cross products  $\mathbf{u} \times \mathbf{v}$  in Exercise 1 is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

7. Verify that each of the cross products  $\mathbf{u} \times \mathbf{v}$  in Exercise 2 is orthogonal to both  $\mathbf{u}$  and  $\mathbf{v}$ .

8. Verify Equation (7) for the pairs of vectors in Exercise 1.

9. Find the area of the triangle with vertices  $P_1(1, -2, 3)$ ,  $P_2(-3, 1, 4)$ ,  $P_3(0, 4, 3)$ .

10. Find the area of the triangle with vertices  $P_1$ ,  $P_2$ , and  $P_3$ , where  $\overrightarrow{P_1 P_2} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  and  $\overrightarrow{P_1 P_3} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ .

11. Find the area of the parallelogram with adjacent sides  $\mathbf{u} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$  and  $\mathbf{v} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$ .

12. Find the volume of the parallelepiped with a vertex at the origin and edges  $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ , and  $\mathbf{w} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ .

13. Repeat Exercise 12 for  $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ , and  $\mathbf{w} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$ .

**Theoretical Exercises**

T.1. Prove Theorem 5.1.

T.2. Show that  $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ .

T.3. Show that  $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$ ,  $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$ ,  $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$ .

T.4. Show that

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}.$$

T.5. Show that  $\mathbf{u}$  and  $\mathbf{v}$  are parallel if and only if  $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ .

T.6. Show that  $\|\mathbf{u} \times \mathbf{v}\|^2 + (\mathbf{u} \cdot \mathbf{v})^2 = \|\mathbf{u}\|^2 \|\mathbf{v}\|^2$ .

T.7. Prove the Jacobi identity:

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} + (\mathbf{v} \times \mathbf{w}) \times \mathbf{u} + (\mathbf{w} \times \mathbf{u}) \times \mathbf{v} = \mathbf{0}.$$

①

## EX 5.1

In Exercises 1 and 2 compute  $UV$ .

(Q1(a))  $U = 2i + 3j + 4k$ ,  $V = -i + 3j - k$

$$UV = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$i \begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} - j \begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} + k \begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix}$$

$$i(-3-12) - j(-2+4) + k(6+3)$$

$$i(-15) - j(2) + k(9)$$

$$-15i - 2j + 9k.$$

(b)  $U = (1, 0, 1)$ ,  $V = (2, 3, -1)$

$$\vec{U} = i + j + k \quad \vec{V} = 2i + 3j - k$$

$$UV = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 3 & -1 \end{vmatrix} = i(0-3) - j(-1-2) + k(3) \\ = -3i + 3j + 3k.$$

(c) Same as Part (a)

(d)  $U = (2, -1, 1)$ ,  $V = -2U = -2(2, -1, 1) = (-4, 2, -2)$

$$\vec{U} = 2i - j + k \quad \vec{V} = -4i + 2j - 2k$$

$$\vec{U} \vec{V} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ -4 & 2 & -2 \end{vmatrix} = i(2-2) - j(-4+4) + k(4-4) \\ = 0i + 0j + 0k.$$

$\theta_2$  is same as  $\theta_1$ .

Q3 Let  $U = i + 2j - 3k$ ,  $V = 2i + 3j + k$ ,  $W = 5i - j + 2k$ , &  $C = -3$

(i)  $UXV = -(VXU)$ .

$$UXV = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= 11i - 7j - k \quad \text{--- } \textcircled{1}$$

$$-(VXU) = - \begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= -(-11i + 7j + k) = 11i - 7j - k \quad \text{--- } \textcircled{2}$$

From eq  $\textcircled{1}$  & eq  $\textcircled{2}$  we have:

$$UXV = -(VXU) \text{ Proved}$$

Part b, c, d Same as Part (a).

②

## Ex 5.1

Q4

$$U = 2i - j + 3k$$

$$V = 3i + j - k$$

$$W = 3i + j + 2k$$

(a) Verify equation ③  $(U \times V) \cdot W = U \cdot (V \times W)$ .

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix}$$

$$= i - 2j + 11k$$

$$(U \times V) \cdot W = (-2i + 11j + 5k) \cdot (3i + j + 2k)$$

$$= -6 + 11 + 10 = 15 \quad \text{--- Q.}$$

Now

$$V \times W = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 3 & 1 & +2 \end{vmatrix} = 3i - 9j + 5k$$

$$U \cdot (V \times W) = (2i - j + 3k) \cdot (3i - 9j + 5k)$$

$$= 6 + 9 + 0 = 15 \quad \text{--- Q.}$$

From eq ① & eq ② we have:

$(U \times V) \cdot W = U \cdot (V \times W)$  Proved.

Part (b) is same as Part (a).

Q5 is same as Q4.

Orthogonal: If  $UXV$  is orthogonal to both  $U$  &  $V$  i.e

$$(UXV) \cdot U = 0.$$

$$\text{d} (UXV) \cdot V = 0.$$

Q6 Exercise a — to — (d)

(a)  $\vec{U} = 2\hat{i} + 3\hat{j} + 4\hat{k}$  &  $\vec{V} = -\hat{i} + 3\hat{j} - \hat{k}$

$$UXV = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$= -15\hat{i} - 2\hat{j} + 9\hat{k}$$

$$\text{Now } (UXV) \cdot U = (-15\hat{i} - 2\hat{j} + 9\hat{k}) \cdot (2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$= -30 - 6 + 36$$

$$= -36 + 36 = 0.$$

$$\text{i.e. } (UXV) \cdot U = 0.$$

$$\text{Also } (UXV) \cdot V = (-15\hat{i} - 2\hat{j} + 9\hat{k}) \cdot (-\hat{i} + 3\hat{j} - \hat{k})$$

$$= 15 - 6 - 9 \Rightarrow 15 - 15 = 0$$

$$(UXV) \cdot V = 0$$

which is orthogonal.

(3)

## Ex 5.1

Q7 is same as Q6.

$$\underline{Q9} \quad \text{Area of triangle} = \frac{1}{2} |\vec{U} \times \vec{V}| \rightarrow \textcircled{1}$$

$$\text{Let } \vec{U} = P_1P_2 = (-3-1, 1-(-2), 4-3) \\ = (-4, 3, 1)$$

$$\text{And } \vec{V} = P_1P_3 = (0-1, 4-(-2), 3-3) \\ = (-1, 6, 0)$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ -4 & 3 & 1 \\ -1 & 6 & 0 \end{vmatrix}$$

$$\vec{U} \times \vec{V} = -6i - j - 21k$$

$$\text{Now } |\vec{U} \times \vec{V}| = \sqrt{(-6)^2 + (-1)^2 + (-21)^2} = \sqrt{478}$$

$$\text{From Q9} \Rightarrow \text{Area of triangle} = \frac{1}{2} |\vec{U} \times \vec{V}| \\ = \frac{1}{2} \sqrt{478} \text{ (units)}^2.$$

Q10 is same as Q9.

Q11 Area of //gm =  $|\vec{U} \times \vec{V}|$ .  $\rightarrow$  ①

Now  $\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 3 & -1 & -1 \end{vmatrix}$

$$= -5i - 5j - 10k.$$

Now  $|\vec{U} \times \vec{V}| = \sqrt{(-5)^2 + (-5)^2 + (-10)^2} = \sqrt{150}$

$\Rightarrow$  Area of //gm =  $\sqrt{150}$  (unit)<sup>2</sup>.

Q12  $U = 2i - j$ ,  $V = i - 3j - 2k$  &  $W = 3i - j + k$ .

Volume of //Poid =  $\begin{vmatrix} U_1 & U_2 & U_3 \\ V_1 & V_2 & V_3 \\ W_1 & W_2 & W_3 \end{vmatrix}$

$$= \begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & -2 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= 2(-2 - 2) + 1(1 + 6) + 0$$

$$= 2(-4) + 7$$

$$= -8 + 7 = -1$$

$\nexists$  //P = 1 (unit)<sup>3</sup>

Q13 is same as Q12

Two planes are either parallel or they intersect in a straight line. They are parallel if their normals are parallel. In the following example we determine the line of intersection of two planes.

**EXAMPLE 10**

Find parametric equations of the line of intersection of the planes

$$\pi_1: 2x + 3y - 2z + 4 = 0 \quad \text{and} \quad \pi_2: x - y + 2z + 3 = 0.$$

**Solution**

Solving the linear system consisting of the equations of  $\pi_1$  and  $\pi_2$ , we obtain (verify)

$$\begin{aligned} x &= -\frac{13}{5} - \frac{4}{5}t \\ y &= \frac{2}{5} + \frac{6}{5}t \\ z &= -0 + t \end{aligned} \quad -\infty < t < \infty$$

as parametric equations of the line  $L$  of intersection of the planes (see Figure 5.9).

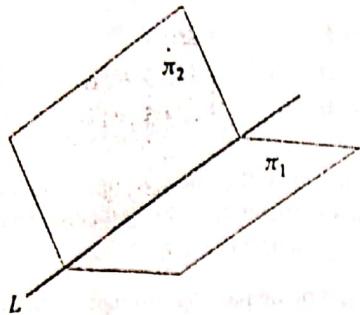


Figure 5.9 ▲

Three planes in  $R^3$  may intersect in a plane, in a line, in a unique point, or have no set of points in common. These possibilities can be detected by solving the linear system consisting of their equations.

**Key Terms**

Parametric equation(s) of a line

Normal to a plane

Symmetric form of a line

Skew lines

**5.2 Exercises**

1. In each of the following, find an equation of the line in  $R^2$  determined by the given points.

- (a)  $P_1(-2, -3), P_2(3, 4)$
- (b)  $P_1(2, -5), P_2(-3, 4)$
- (c)  $P_1(0, 0), P_2(-3, 5)$
- (d)  $P_1(-3, -5), P_2(0, 2)$

2. In each of the following, find the equation of the line in  $R^2$  determined by the given points.

- (a)  $P_1(1, 1), P_2(2, 2)$
- (b)  $P_1(1, 2), P_2(1, 3)$
- (c)  $P_1(2, -4), P_2(-3, -4)$
- (d)  $P_1(2, -3), P_2(3, -2)$

3. State which of the following points are on the line

$$\begin{aligned} x &= 3 + 2t \\ y &= -2 + 3t \quad -\infty < t < \infty \\ z &= 4 - 3t. \end{aligned}$$

- (a)  $(1, 1, 1)$
- (b)  $(1, -1, 0)$
- (c)  $(1, 0, -2)$
- (d)  $(4, -\frac{1}{2}, \frac{5}{2})$

4. State which of the following points are on the line

$$\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z-4}{-5}.$$

- (a)  $(0, 1, -6)$

- (b)  $(1, 2, 3)$

- (c)  $(4, -3, 4)$

- (d)  $(0, 1, -1)$

5. In each of the following, find the parametric equations of the line through the point  $P_0(x_0, y_0, z_0)$ , which is parallel to the vector  $\mathbf{u}$ .

- (a)  $P_0 = (3, 4, -2), \mathbf{u} = (4, -5, 2)$
- (b)  $P_0 = (3, 2, 4), \mathbf{u} = (-2, 5, 1)$
- (c)  $P_0 = (0, 0, 0), \mathbf{u} = (2, 2, 2)$
- (d)  $P_0 = (-2, -3, 1), \mathbf{u} = (2, 3, 4)$

6. In each of the following, find the parametric equations of the line through the given points.

- (a)  $(2, -3, 1), (4, 2, 5)$
- (b)  $(-3, -2, -2), (5, 5, 4)$
- (c)  $(-2, 3, 4), (2, -3, 5)$
- (d)  $(0, 0, 0), (4, 5, 2)$

7. For each of the lines in Exercise 6, find the equations in symmetric form.

8. State which of the following points are on the plane  $3(x-2) + 2(y+3) - 4(z-4) = 0$ .

- (a)  $(0, -2, 3)$
- (b)  $(1, -2, 3)$
- (c)  $(1, -1, 3)$
- (d)  $(0, 0, 4)$

- 9.** In each of the following, find an equation of the plane passing through the given point and perpendicular to the given vector  $\mathbf{n}$ .
- (0, 2, -3),  $\mathbf{n} = (3, -2, 4)$
  - (-1, 3, 2),  $\mathbf{n} = (0, 1, -3)$
  - (-2, 3, 4),  $\mathbf{n} = (0, 0, -4)$
  - (5, 2, 3),  $\mathbf{n} = (-1, -2, 4)$
- 10.** In each of the following, find an equation of the plane passing through the given three points.
- (0, 1, 2), (3, -2, 5), (2, 3, 4)
  - (2, 3, 4), (-1, -2, 3), (-5, -4, 2)
  - (1, 2, 3), (0, 0, 0), (-2, 3, 4)
  - (1, 1, 1), (2, 3, 4), (-5, 3, 2)
- 11.** In each of the following, find parametric equations of the line of intersection of the given planes.
- $2x + 3y - 4z + 5 = 0$  and  $-3x + 2y + 5z + 6 = 0$
  - $3x - 2y - 5z + 4 = 0$  and  $2x + 3y + 4z + 8 = 0$
  - $-x + 2y + z = 0$  and  $2x - y + 2z + 8 = 0$
- 12.** In each of the following, find a pair of planes whose intersection is the given line.
- $x = 2 - 3t$   
 $y = 3 + t$   
 $z = 2 - 4t$
  - $\frac{x - 2}{-2} = \frac{y - 3}{4} = \frac{z + 4}{3}$
  - $x = -4t$   
 $y = 1 + 5t$   
 $z = 2 - t$
- 13.** Are the points (2, 3, -2), (4, -2, -3), and (0, 8, -1) on the same line?
- 14.** Are the points (-2, 4, 2), (3, 5, 1), and (4, 2, -1) on the same line?
- 15.** Find the point of intersection of the lines
- $$\begin{array}{ll} x = 2 - 3s & x = 5 + 2t \\ y = 3 + 2s & \text{and} \\ z = 4 + 2s & y = 1 - 3t \\ & z = 2 + t \end{array}$$
- 16.** Which of the following pairs of lines are perpendicular?
- $x = 2 + 2t$   
 $y = -3 - 3t$   
 $z = 4 + 4t$  and  $x = 2 + t$   
 $y = 4 - t$   
 $z = 5 - t$
  - $x = 3 - t$   
 $y = 4 + t$   
 $z = 2 + 2t$  and  $x = 2t$   
 $y = 3 - 2t$   
 $z = 4 + 2t$
- 17.** Show that the following parametric equations define the same line.
- $$\begin{array}{ll} x = 2 + 3t & x = -1 - 9t \\ y = 3 - 2t & \text{and} \\ z = -1 + 4t & y = 5 + 6t \\ & z = -5 - 12t \end{array}$$
- 18.** Find parametric equations of the line passing through the point (3, -1, -3) and perpendicular to the line passing through the points (3, -2, 4) and (0, 3, 5).
- 19.** Find an equation of the plane passing through the point (-2, 3, 4) and perpendicular to the line passing through the points (4, -2, 5) and (0, 2, 4).
- 20.** Find the point of intersection of the line
- $$\begin{array}{l} x = 2 - 3t \\ y = 4 + 2t \\ z = 3 - 5t \end{array}$$
- and the plane  $2x + 3y + 4z + 8 = 0$ .
- 21.** Find a plane containing the lines
- $$\begin{array}{ll} x = 3 + 2t & x = 1 - 2t \\ y = 4 - 3t & \text{and} \\ z = 5 + 4t & y = 7 + 4t \\ & z = 1 - 3t \end{array}$$
- 22.** Find a plane that passes through the point (2, 4, -3) and is parallel to the plane  $-2x + 4y - 5z + 6 = 0$ .
- 23.** Find a line that passes through the point (-2, 5, -3) and is perpendicular to the plane  $2x - 3y + 4z + 7 = 0$ .

## Theoretical Exercises

- T.1.** Show that the graph of the equation  $ax + by + cz + d = 0$ , where  $a, b, c$ , and  $d$  are constants with  $a, b$ , and  $c$  not all zero, is a plane with normal  $\mathbf{n} = (a, b, c)$ .
- T.2.** Let the lines  $L_1$  and  $L_2$  be given parametrically by  $L_1: \mathbf{x} = \mathbf{w}_0 + s\mathbf{u}$  and  $L_2: \mathbf{x} = \mathbf{w}_1 + t\mathbf{v}$ . Show that
- $L_1$  and  $L_2$  are parallel if and only if  $\mathbf{u} = k\mathbf{v}$  for some scalar  $k$ .
- T.3.** The lines  $L_1$  and  $L_2$  in  $R^3$  are said to be skew if they are not parallel and do not intersect. Give an example of skew lines  $L_1$  and  $L_2$ .
- $L_1$  and  $L_2$  are identical if and only if  $\mathbf{w}_1 - \mathbf{w}_0$  and  $\mathbf{u}$  are both parallel to  $\mathbf{v}$ .
  - $L_1$  and  $L_2$  are perpendicular if and only if  $\mathbf{u} \cdot \mathbf{v} = 0$ .
  - $L_1$  and  $L_2$  intersect if and only if  $\mathbf{w}_1 - \mathbf{w}_0$  is a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ .

①  
Ex 5.2

Q1 (a) Let  $P_1(x_1, y_1) = (-2, -3)$  &  $P_2(x_2, y_2) = (3, 4)$ .

Now  $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x & y & 1 \\ -2 & -3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

$$x(-3-4) - y(-2-3) + 1(-8+9) = 0$$

$$-7x + 5y + 1 = 0$$

This is the required equation of line for the given points.

Part (b), (c), (d) same as Part (a).

Q2 is same as Q1.

Q3  $x = 3+2t \Rightarrow t = \frac{x-3}{2}$

$$y = -2+3t \Rightarrow t = \frac{y+2}{3}$$

$$z = 4-3t \Rightarrow t = \frac{z-4}{-3}$$

$$\text{Now } \frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{-3} \quad -\textcircled{1}$$

(a)  $(1, 1, 1)$

$$\text{eq } \textcircled{1} \Rightarrow \frac{1-3}{2} = \frac{1+2}{3} = \frac{1-4}{-3}$$

$$\frac{-2}{2} = \frac{3}{3} = \frac{-3}{-3}$$

$$-1 \neq 1 \neq 1$$

Hence all the value of  $t$  are not same, Therefore  
pts are not lies on the lines.

Part (b) & (c) same as part (a)

(d)  $(4, -y_2, 5/2)$

$$\text{eq } \textcircled{1} \Rightarrow \frac{4-3}{2} = \frac{-y_2+2}{3} = \frac{5/2-4}{-3}$$

$$1 = y_2 = 1/2$$

Hence all the value of  $t$  are same so its lie on line.

$\Omega_4$  is same as  $\Omega_3$

②

EX S.2

Q5 (a)  $P_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = (3, 4, -2)$ ,  $U = \begin{pmatrix} a & b & c \\ 4 & -5 & 2 \end{pmatrix}$ ,  $-\infty < t < \infty$

$$x = x_0 + at \Rightarrow x = 3 + 4t$$

$$y = y_0 + bt \Rightarrow y = 4 - 5t$$

$$z = z_0 + ct \Rightarrow z = -2 + 2t$$

Part b, c, d same as Part (a)

Q6 (a)  $P_0 = (2, -3, 1)$  &  $P_1 = (4, 2, 5)$

$$\vec{U} = \overrightarrow{P_0 P_1} = (4-2, 2+3, 5-1) = (2, 5, 4)$$

Now  $P_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = (2, -3, 1)$  &  $U = \begin{pmatrix} a & b & c \\ 2 & 5 & 4 \end{pmatrix}$

$$x = x_0 + at = 2 + 2t$$

$$y = y_0 + bt \Rightarrow y = -3 + 5t$$

$$z = z_0 + ct \Rightarrow z = 1 + 4t$$

Part (b), (c) & (d) same as Part (a).

$$Q7(a) P_0 = (x_0, y_0, z_0) \text{ & } U = \begin{pmatrix} a & b & c \\ 2 & 5 & 4 \end{pmatrix}$$

For Symmetric form

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-4}{4}$$

(b), (c), + (d) Part as same as Part (a)

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Q8

$$3(x-2) + 2(y+3) - 4(z-4) = 0 \quad \textcircled{1}$$

(a)  $(0, -2, 3)$

$$\text{eq } \textcircled{1} \Rightarrow 3(0-2) + 2(-2+3) - 4(3-4) = 0$$

$$0 = 0$$

So it's lie on plane.

(b), (c), + (d) Same as Part (a).

③

Ex 5.2

Q9 (d)  $P_0 = (x_0, y_0, z_0) = (5, 2, 3)$

$\vec{n} = (a, b, c) = (-1, -2, 4)$

Equation of plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$-1(x - 5) - 2(y - 2) + 4(z - 3) = 0$$

$$-x - 2y + 4z - 3 = 0$$

Part (a), (b) & (c) Same as Part (d)

Q10 (b)  $P_1(2, 3, 4), P_2(-1, -2, 3), P_3(-5, -4, 2)$

Let  $\vec{U} = \vec{P_1 P_2} = (-3, -5, -1)$

$\vec{V} = \vec{P_1 P_3} = (-7, -7, -2)$

Now  $\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ -3 & -5 & -1 \\ -7 & -7 & -2 \end{vmatrix}$

$$\Rightarrow x(10 - 7) - y(6 - 7) + z(21 - 35) = 0$$

$$\Rightarrow 3x + y - 14z = 0$$

which is the required equation of Plane.

Others parts are same as

$$\underline{\text{Q11}} \text{ (h)} \quad \begin{aligned} 3x - 2y - 5z + 4 &= 0 \quad \text{--- (1)} \\ 2x + 3y + 4z + 8 &= 0 \quad \text{--- (2)} \end{aligned}$$

Xing eq (1) by "3" + eq (2) by "5"

$$9x - 6y - 15z + 12 = 0$$

$$\underline{4x + 6y + 8z + 16 = 0}$$

$$\underline{13x - 7z + 28 = 0}$$

$$\frac{13x + 28}{7} = z \quad \text{--- (3)}$$

Xing eq (1) by "2" + eq (2) by "3" + Subtracting

$$6x + 4y - 10z + 8 = 0$$

$$\underline{6x + 9y + 12z + 24 = 0}$$

$$\underline{-13y - 22z - 16 = 0}$$

$$\frac{-13y - 16}{22} = z \quad \text{--- (4)}$$

Comparing eq (3) + eq (4)

$$\frac{13x + 28}{7} = \frac{-13y - 16}{22} = z = t \quad (\text{say})$$

(1)

Ex 5:-

$$\text{Now } \frac{13x+28}{7} = t$$

$$\Rightarrow x = \frac{7t - 28}{13} \rightarrow (a)$$

$$\text{And } \frac{-13y-16}{22} = t$$

$$\Rightarrow y = \frac{22t + 16}{-13} \rightarrow (b)$$

$$\text{Also } z = t \rightarrow (c)$$

Equation (a), (b) & (c) are the required Parametric equations of the line of intersection of the given planes.

$$\text{Q12 (b)} \quad \frac{x-2}{-2} = \frac{y-3}{4} = \frac{z+4}{3}$$

$$\text{Now } \frac{x-2}{-2} = \frac{y-3}{4}$$

$$\Rightarrow 4x + 2y - 14 = 0 \rightarrow (1) \text{ which is } 1^{\text{st}} \text{ equation of plane}$$

$$\text{And } \frac{x-2}{-2} = \frac{z+4}{3}$$

$$3x + 2z + 2 = 0 \text{ which is } 2^{\text{nd}} \text{ equation of plane}$$

Plane.

$$(1) \quad x = 4t \rightarrow (i)$$

$$y = 11st \rightarrow (ii)$$

$$z = 2-t \rightarrow (iii)$$

$$\text{eq}(i) \Rightarrow \frac{x}{4} = t \rightarrow (i)$$

$$\text{eq}(ii) \Rightarrow \frac{y-1}{5} = t \rightarrow (ii)$$

$$\text{eq}(iii) \Rightarrow 2-z = t \rightarrow (iii)$$

From eq(i), (ii) & eq(iii)

$$\frac{x}{4} = \frac{y-1}{5} = 2-z = t$$

$$\text{Now } \frac{x}{4} = \frac{y-1}{5}$$

$$\Rightarrow 5x - 4y + 4 = 0 \quad \text{--- (1) which is 1st eq of Plane.}$$

$$\text{And } \frac{y-1}{5} = 2-z$$

$$y + 5z - 11 = 0 \quad \text{--- (2) which is 2nd eq of Plane.}$$

Plane:

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⑤

Ex 5.2

Q14

$$\text{let } P_1(x_1, y_1, z_1) = (-2, 4, 2)$$

$$P_2(x_2, y_2, z_2) = (3, 5, 1)$$

$$P_3(x_3, y_3, z_3) = (4, 2, -1)$$

Now  $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$

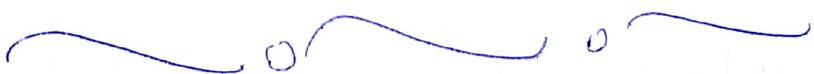
$$\begin{vmatrix} -2 & 4 & 2 \\ 3 & 5 & 1 \\ 4 & 2 & -1 \end{vmatrix} = 0$$

$$-2(-5-2) - 4(6-2) + 2(6-2) = 0$$

$$14 \neq 0$$

These points are not on the same plane.

Q13 is same as Q14.



Q5

$$x = 2 - 3s \quad x = s + 2t$$

$$y = 3 + 2s \quad \text{and} \quad y = 1 - 3t$$

$$z = 4 + 2s \quad z = 2 + t$$

$$2 - 3s = s + 2t \quad \text{--- (1)}$$

$$3 + 2s = 1 - 3t \quad \text{--- (2)}$$

$$4 + 2s = 2 + t \quad \text{--- (3)}$$

Subtracting (3) from eq (2), we get

$$\begin{array}{r} 3 + 2s = 1 - 3t \\ 4 + 2s = 2 + t \\ \hline -1 = -1 - 4t \end{array}$$

$$-4t = 0$$

$$\boxed{t = 0}$$

$$\text{eq (1)} \Rightarrow 2 - 3s = s + 2(0) \Rightarrow \boxed{s = -1}$$

$$\text{eq (2)} \Rightarrow 3 + 2s = 1 - 3(0)$$

$$3 + 2s = 1 \Rightarrow 2s = -2 \Rightarrow \boxed{s = -1}$$

$$\therefore t = 0, s = -1$$

Now

$$x = 2 - 3s$$

$$x = 2 - 3(-1)$$

$$\boxed{x = 5}$$

$$x = s + 2t$$

$$x = s + 2(0)$$

$$\boxed{x = s}$$

(6)  
Ex 5.9

$$y = 3 + 2s$$

$$y = 3 + 2(-1)$$

$$y = 3 - 2 = 1$$

$$\boxed{y = 1}$$

$$z = 4 + 2s$$

$$z = 4 + 2(-1)$$

$$z = 4 - 2$$

$$\boxed{z = 2}$$

$$y = 1 - 3t$$

$$y = 1 - 3(0)$$

$$\boxed{y = 1}$$

$$z = 2 + t$$

$$z = 2 + 0$$

$$\boxed{z = 2}$$

Therefore the point of intersection of the given line

is  $P(u, y, z) = P(5, 1, 2)$ .

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Q16 (a)  $x = 2 + 2t$        $u = 2 + t$   
 $y = -3 - 3t$  and  $v = 4 - t$   
 $z = 4 + 4t$        $w = 5 - t$

Let  $\vec{U} = (2, -3, 4)$

&  $\vec{V} = (1, -1, -1)$

$$\text{Now } \vec{U} \cdot \vec{V} = (2, -3, 4) \cdot (1, -1, -1).$$

$$= 2 + 3 - 4 = 1$$

$$\vec{U} \cdot \vec{V} \neq 0$$

The given lines are not  $\perp$ .

Note  $\vec{U} \cdot \vec{V} = 0$  then  $\perp$