

# Probability Methods in Engineering

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Lecture 21





## Important Continuous RVs

#### > Uniform Random Variable

- $\triangleright S_X = [a, b]$
- $f_X(x) = 1/(b a) \ a \le x \le b$
- $\triangleright$   $E[X] = (a + b)/2, VAR[X] = (b a)^2/12$

#### Exponential Random Variable

- $\triangleright S_X = [0, \infty)$
- $ightharpoonup f_X(x) = \lambda \ e^{-\lambda t}, \ x \ge 0, \ \lambda > 0$
- $\triangleright$   $E[X] = 1/\lambda$ ,  $VAR[X] = 1/\lambda^2$





### Transform Methods

- > Multiplication of huge numbers without calculators
  - □ Tedious and error-prone
- > Instead, use log tables
  - $\square$  x multiply by y can be computationally complex
  - $\square$  Find antilog(log(x)+log(y)) instead
- > Similarly, convolution of two functions (signals) is complex
  - ☐ Use "some" transform for convenience
- > Two transform in our course
  - ☐ The characteristic function
  - ☐ The probability generating function





### The Characteristic Function

> The characteristic function of a continuous RV is

$$\Phi_X(\omega) = E[e^{j\omega X}] = \int_{-\infty}^{\infty} f_X(x)e^{j\omega x}dx$$

> The inverse is taken as

$$f_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_X(\omega) e^{-j\omega x} d\omega$$





# The Characteristic Function (cont.)

> The characteristic function of a discrete RV is

$$\Phi_X(\omega) = \sum_k p_X(x_k) e^{j\omega x_k}$$

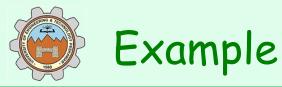
> The characteristic function of an integer-valued RV is

$$\Phi_X(\omega) = \sum_{k=-\infty}^{\infty} p_X(k) e^{j\omega k}$$

> The inverse for integer-valued is taken as

$$p_X(k) = \frac{1}{2\pi} \int_{0}^{2\pi} \Phi_X(\omega) e^{-j\omega k} d\omega$$





> Determine the characteristic function for an exponentially distributed random variable





# Examples (cont.)

Find the characteristic function for a geometric random variable

