since we must have $x_1 \geq 0$, $x_2 \geq 0$, and $x_3 \geq 0$. When $x_3 = 10$, we have

$$x_1 = 5, \quad x_2 = 10, \quad x_3 = 10$$

while

$$x_1 = \tfrac{13}{2}, \quad x_2 = 13, \quad x_3 = 7$$

when $x_3 = 7$. The reader should observe that one solution is just as good as the other. There is no best solution unless additional information or restrictions are given.

Key Terms

Linear equation
Unknowns
Solution to a linear equation
Linear system
Solution to a linear system
Method of elimination
Unique solution
No solution
Infinitely many solutions
Manipulations on a linear system

1.1 Exercises

In Exercises 1 through 14, solve the given linear system by the method of elimination.

1.
$$\begin{aligned} x + 2y &= 8 \\ 3x - 4y &= 4 \end{aligned}$$

2.
$$\begin{aligned} 2x - 3y + 4z &= -12 \\ x - 2y + z &= -5 \\ 3x + y + 2z &= 1 \end{aligned}$$

3.
$$\begin{aligned} 3x + 2y + z &= 2 \\ 4x + 2y + 2z &= 8 \\ x - y + z &= 4 \end{aligned}$$

4.
$$\begin{aligned} x + y &= 5 \\ 3x + 3y &= 10 \end{aligned}$$

5.
$$\begin{aligned} 2x + 4y + 6z &= -12 \\ 2x - 3y - 4z &= 15 \\ 3x + 4y + 5z &= -8 \end{aligned}$$

6.
$$\begin{aligned} x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \end{aligned}$$

7.
$$\begin{aligned} x + 4y - z &= 12 \\ 3x + 8y - 2z &= 4 \end{aligned}$$

8.
$$\begin{aligned} 3x + 4y - z &= 8 \\ 6x + 8y - 2z &= 3 \end{aligned}$$

9.
$$\begin{aligned} x + y + 3z &= 12 \\ 2x + 2y + 6z &= 6 \end{aligned}$$

10.
$$\begin{aligned} x + y &= 1 \\ 2x - y &= 5 \\ 3x + 4y &= 2 \end{aligned}$$

11.
$$\begin{aligned} 2x + 3y &= 13 \\ x - 2y &= 3 \\ 5x + 2y &= 27 \end{aligned}$$

12.
$$\begin{aligned} x - 5y &= 6 \\ 3x + 2y &= 1 \\ 5x + 2y &= 1 \end{aligned}$$

13.
$$\begin{aligned} x + 3y &= -4 \\ 2x + 5y &= -8 \\ x + 3y &= -5 \end{aligned}$$

14.
$$\begin{aligned} 2x + 3y - z &= 6 \\ 2x - y + 2z &= -8 \\ 3x - y + z &= -7 \end{aligned}$$

15. Given the linear system

$$\begin{aligned} 2x - y &= 5 \\ 4x - 2y &= t, \end{aligned}$$

(a) determine a value of t so that the system has a solution.

(b) determine a value of t so that the system has no solution.

(c) how many different values of t can be selected in part (b)?

H.W

16. Given the linear system

$$\begin{aligned} 2x + 3y - z &= 0 \\ x - 4y + 5z &= 0, \end{aligned}$$

(a) verify that $x_1 = 1$, $y_1 = -1$, $z_1 = -1$ is a solution.

(b) verify that $x_2 = -2$, $y_2 = 2$, $z_2 = 2$ is a solution.

(c) is $x = x_1 + x_2 = -1$, $y = y_1 + y_2 = 1$, and $z = z_1 + z_2 = 1$ a solution to the linear system?

(d) is $3x$, $3y$, $3z$, where x, y, and z are as in part (c), a solution to the linear system?

17. Without using the method of elimination, solve the linear system

$$\begin{aligned} 2x + y - 2z &= -5 \\ 3y + z &= 7 \\ z &= 4. \end{aligned}$$

H.W

18. Without using the method of elimination, solve the linear system

$$\begin{aligned} 4x \qquad\qquad &= 8 \\ -2x + 3y \qquad &= -1 \\ 3x + 5y - 2z &= 11. \end{aligned}$$

19. Is there a value of r so that $x = 1$, $y = 2$, $z = r$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 2x + 3y - z &= 11 \\ x - y + 2z &= -7 \\ 4x + y - 2z &= 12 \end{aligned}$$

H.W

20. Is there a value of r so that $x = r$, $y = 2$, $z = 1$ is a solution to the following linear system? If there is, find it.

$$\begin{aligned} 3x \quad\quad - 2z &= 4 \\ x - 4y + z &= -5 \\ -2x + 3y + 2z &= 9 \end{aligned}$$

21. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.2.

22. Describe the number of points that simultaneously lie in each of the three planes shown in each part of Figure 1.3.

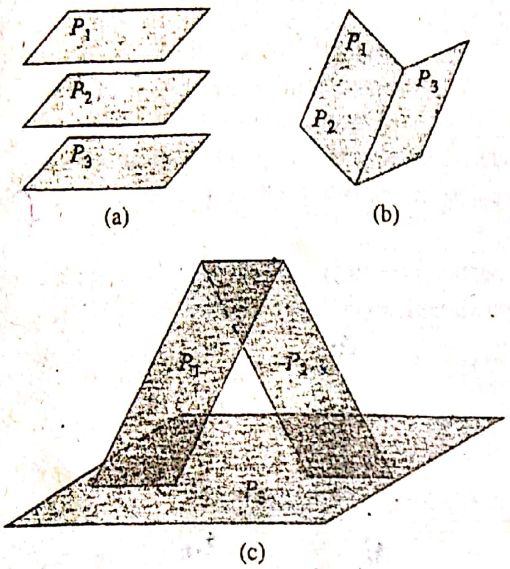

Figure 1.3 ▲

23. An oil refinery produces low-sulfur and high-sulfur fuel. Each ton of low-sulfur fuel requires 5 minutes in the blending plant and 4 minutes in the refining plant; each ton of high-sulfur fuel requires 4 minutes in the blending plant and 2 minutes in the refining plant. If the blending plant is available for 3 hours and the refining plant is available for 2 hours, how many tons of each type of fuel should be manufactured so that the plants are fully utilized?

24. A plastics manufacturer makes two types of plastic: regular and special. Each ton of regular plastic requires 2 hours in plant A and 5 hours in plant B; each ton of special plastic requires 2 hours in plant A and 3 hours in plant B. If plant A is available 8 hours per day and plant B is available 15 hours per day, how many tons of each type of plastic can be made daily so that the plants are fully utilized?

25. A dietician is preparing a meal consisting of foods A, B, and C. Each ounce of food A contains 2 units of protein, 3 units of fat, and 4 units of carbohydrate. Each ounce of food B contains 3 units of protein, 2 units of fat, and 1 → 3 unit of carbohydrate. Each ounce of food C contains 3 units of protein, 3 units of fat, and 2 units of carbohydrate. If the meal must provide exactly 25 units of protein, 24 units of fat, and 21 units of carbohydrate, how many ounces of each type of food should be used?

HW

26. A manufacturer makes 2-minute, 6-minute, and 9-minute film developers. Each ton of 2-minute developer requires 6 minutes in plant A and 24 minutes in plant B. Each ton of 6-minute developer requires 12 minutes in plant A and 12 minutes in plant B. Each ton of 9-minute developer requires 12 minutes in plant A and 12 minutes in plant B. If plant A is available 10 hours per day and plant B is available 16 hours per day, how many tons of each type of developer can be produced so that the plants are fully utilized?

Q1(b)

27. Suppose that the three points $(1, -5)$, $(-1, 1)$, and $(2, 7)$ lie on the parabola $p(x) = ax^2 + bx + c$.

(a) Determine a linear system of three equations in three unknowns that must be solved to find a, b, and c.

(b) Solve the linear system obtained in part (a) for a, b, and c.

H.W

28. An inheritance of \$24,000 is to be divided among three trusts, with the second trust receiving twice as much as the first trust. The three trusts pay interest at the rates of 9%, 10%, and 6% annually, respectively, and return a total in interest of \$2210 at the end of the first year. How much was invested in each trust?

Theoretical Exercises

T.1. Show that the linear system obtained by interchanging two equations in (2) has exactly the same solutions as (2).

T.2. Show that the linear system obtained by replacing an equation in (2) by a nonzero constant multiple of the equation has exactly the same solutions as (2).

T.3. Show that the linear system obtained by replacing an equation in (2) by itself plus a multiple of another equation in (2) has exactly the same solutions as (2).

T.4. Does the linear system

$$\begin{aligned} ax + by &= 0 \\ cx + dy &= 0 \end{aligned}$$

always have a solution for any values of a, b, c, and d?

$1 - 2 + 2x = -7$

$4 + 2 - 2y = 12$

EX: 1.1

Q23 Let x_1 denotes lower-sulfur

& x_2 denotes high-sulfur for each ton.

L.S H.S

B.P $5x_1+4x_2=3\times 60=180$

R.P $4x_1+2x_2=2\times 60=120$

or

$5x_1+4x_2=180$ —— (i)

$4x_1+2x_2=120$ —— (ii)

Xing eq (ii) by 2 then subtr: from (i)

$$\begin{array}{r} 5x_1+\cancel{4x_2}=180 \\ \underline{-8x_1\pm\cancel{4x_2}=-240} \\ -3x_1=-60 \end{array} \Rightarrow x_1=\frac{\cancel{-60}^{20}}{\cancel{-3}}=20$$

$\boxed{x_1=20}$

Put $x_1=20$ in eq (i)

$5\times 20+4x_2=180$

$4x_2=180-100=80$

$4x_2=80$

$x_2=80/4=20$

$\boxed{x_1=x_2=20 \text{ tons}}$

unique solution

(24) Let x_1 denotes regular plastic ton

& x_2 denotes special plastic ton.

$$\begin{array}{lcccl} & R.P & & S.P & \\ \text{Now } P.A & 2x_1 & + & 2x_2 = 8 & \text{---- (i)} \\ P.B & 5x_1 & + & 3x_2 = 15 & \text{---- (ii)} \end{array}$$

Solving (i) & (ii)

$x_1 = 1.5$ tons

& $x_2 = 2.5$ tons

—————X—————

Q25 Let x_1 denotes of ounce A

x_2 denotes of ounce B

& x_3 denotes of ounce C.

$$\begin{array}{lcccl} & A & B & C & \\ \text{Now } P & 2x_1 & +3x_2 & +3x_3 = 25 & \text{---- (i)} \\ F & 3x_1 & +2x_2 & +.3x_3 = 24 & \text{---- (ii)} \\ C & 4x_1 & +x_2 & +2x_3 = 21 & \text{---- (iii)} \end{array}$$

xing eq (iii) by 2 then subtr: from (ii)

$$3x_1 + \cancel{2x_2} + 3x_3 = 24$$

$$\underline{8x_1 + \cancel{2x_2} + 4x_3 = 42}$$

$$-5x_1 - 3x_3 = -18$$

$$\Rightarrow 5x_1 + 3x_3 = 18 \text{ ———— (iv).}$$

Ex: 1.1

Xing eq (iii) by 3 then subtr. from (i)

$$2x_1 + 3x_2 + 3x_3 = 25$$
$$\underline{12x_1 + 3x_2 + 6x_3 = 63}$$
$$-10x_1 - 3x_3 = -38$$

$$\Rightarrow 10x_1 + 3x_3 = 38 \quad \text{——} (v)$$

Xing eq (iv) by 2 then subtr. from (v)

$$10x_1 + 2x_3 = 36$$
$$\underline{10x_1 + 3x_3 = 38}$$
$$-x_3 = -2 \quad \Rightarrow \boxed{x_3 = 2}$$

Xing eq (iv) by 3 then subtr. from (v)

$$15x_1 + 3x_3 = 54$$
$$\underline{10x_1 + 3x_3 = 38}$$
$$+5x_1 = 16 \quad \Rightarrow x_1 = 16/5 = 3.2$$

$$\boxed{x_1 = 3.2}$$

Put $x_1 = 3.2$ & $x_3 = 2$ in eq (i)

$$\text{eq (i)} \Rightarrow 2(3.2) + 3x_2 + 3(2) = 25$$
$$3x_2 = 25 - 12.5 = 12.6$$
$$3x_2 = 12.6 \Rightarrow x_2 = 12.6/3 = 4.2.$$

$$\boxed{x_2 = 4.2}$$

Q26 Let x_1 denotes tons of 2 minute
x_2 denotes tons of 6-minute
& x_3 denotes tons of 9-minute.

2min 6min 9min

Now P.A $6x_1 + 12x_2 + 12x_3 = 10 \times 60 = 600$
P.B $24x_1 + 12x_2 + 12x_3 = 16 \times 60 = 960$

or

$6x_1 + 12x_2 + 12x_3 = 600$
$24x_1 + 12x_2 + 12x_3 = 960$

or

$x_1 + 2x_2 + 2x_3 = 100$ —— (i)
$2x_1 + x_2 + x_3 = 80$ —— (ii)

Xing eq (ii) by 2 then subtr. from (i)

$$x_1 + \cancel{2x_2} + \cancel{2x_3} = 100$$
$$4x_1 + 2x_2 + 2x_3 = 160$$

$-3x_1 = -60 \Rightarrow x_1 = \frac{-60}{-3} = 20$

$$\boxed{x_1 = 20}$$

Xing eq (i) by 2 then ~~subtr~~ subtr. from (ii)

$$\cancel{2x_1} + 4x_2 + 4x_3 = 200$$
$$2x_1 + x_2 + x_3 = 80$$

$3x_2 + 3x_3 = 120$

$\Rightarrow x_2 + x_3 = 40$ —— (iii)

let $x_3 = r$ any real no: & +ve.

then eq (iii) $\Rightarrow$ $x_2 + r = 40$ $\Rightarrow$ $\boxed{x_2 = 40 - r}$

Hence $\boxed{x_1 = 20}$, $\boxed{x_2 = 40 - r}$, $\boxed{x_3 = r}$ Ans.

Q27 (a) Given that $P_1(x) = \cancel{(x,y)}\ (1,-5)$

$P_2(x) = (-1, 1)$

$P_3(x) = (2, 7)$

& Parabola $P(x) = ax^2 + bx + c$.

(a) $P_1(x) = y = ax^2 + bx + c$

$-5 = a(1) + b(1) + c$

$-5 = a + b + c$ ——— (i)

$P_2(x) = y = ax^2 + bx + c$

$1 = a(-1)^2 + b(-1) + c$

$1 = a - b + c$ ——— (ii)

&

$P_3(x) = y = a(2)^2 + b(2) + c$

$7 = 4a + 2b + c$ ——— (iii)

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(b) $a=?,\ b=?,\ c=?$

Now $a+b+c=-5$ — (i)

$a-b+c=1$ — (ii)

$4a+2b+c=7$ — (iii)

(i) — (ii)

$$\begin{array}{r} \cancel{a}+b+\cancel{c}=-5 \\ \underline{\cancel{a}-b+\cancel{c}=1} \\ 2b=-6 \end{array} \Rightarrow \boxed{b=-3}$$

Xing eq (i) by 4 then subtr. from (iii)

$$\begin{array}{r} \cancel{4a}+4b+4c=-20 \\ \underline{\cancel{4a}+2b+c=7} \\ 2b+3c=-27 \end{array}$$ — (iv)

Put $b=-3$ in eq (iv)

$2(-3)+3c=-27$

$3c=-27+6=-21 \Rightarrow c=-21/3=-7$

$\boxed{c=-7}$

eq (i) $\Rightarrow a-3-7=-5 \Rightarrow -5+10=5$

$\boxed{a=5}$

Hence $\boxed{a=5,\ b=-3,\ c=-7}$ Ans.

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Ex: 1.1

Q28 Let x_1, y_1 & z_1 be three trusts.
Also given 2nd trust receiving twice as much as the 1st trust i.e $y_1 = 2x_1$

Now $x_1 + y_1 + z_1 = 24000\$$

$x_1 + 2x_1 + z_1 = 24000\$$

$3x_1 + z_1 = 24000\$$ — (i)

& $9\% x_1 + 10\% y_1 + 6\% z_1 = 2210\$$

or

$\frac{9}{100}x_1 + \frac{10}{100}y_1 + \frac{6}{100}z_1 = 2210\$$

$\Rightarrow 9x_1 + 10(2x_1) + 6z_1 = 221000\$$

$\Rightarrow 9x_1 + 20x_1 + 6z_1 = 221000\$$

$\Rightarrow 29x_1 + 6z_1 = 221000\$$ — (ii)

Xing eq (i) by 6 then subtr: from (ii)

$$18x_1 + \cancel{6z_1} = 144000$$
$$\underline{\underset{-}{29x_1} + \underset{-}{\cancel{6z_1}} = \underset{-}{221000}}$$
$$-11x_1 = -77000$$

$\Rightarrow \boxed{x_1 = 7000\$}$

& $y_1 = 2x_1 = 2(7000) = 14000\$$

$\boxed{y_1 = 14000\$}$

Q(B) $\Rightarrow$ $Z_1 = 24000 - 3x_1$

$Z_1 = 24000 - 3(7000)$

$Z_1 = 24000 - 21000$

$\boxed{Z_1 = 3000\$}$

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