Chapter 7

Energy Storage Elements

Preview

In this chapter we describe the characteristics of two energy storage elements: the *capacitor* and the *inductor*.

Ideally, these elements do not dissipate energy.

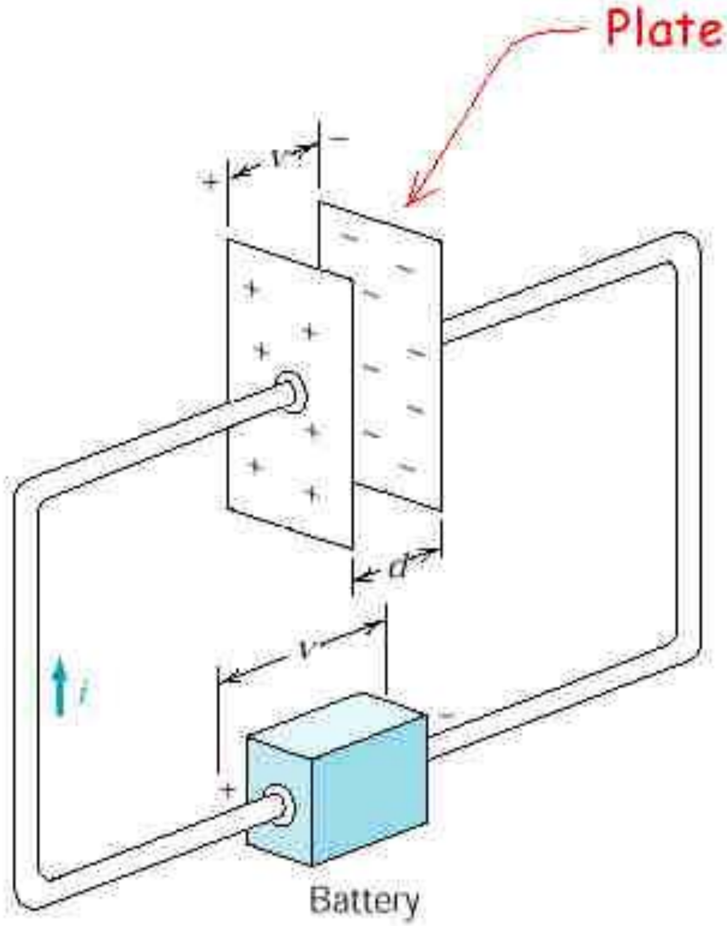
Capacitors and inductors are used in circuits to store energy and deliver it back to the rest of the circuit.

Capacitors

Capacitance

A capacitor is a two-terminal element that is a model of a device consisting of two conducting plates separated by a nonconducting material.

When a voltage is applied to the terminals of this device as part of an electric circuit, current will flow in the circuit and equal charge is deposited on the two plates of this device.



Capacitors

Capacitance

The capacitance is given by:

$$C = \frac{\varepsilon A}{d}$$

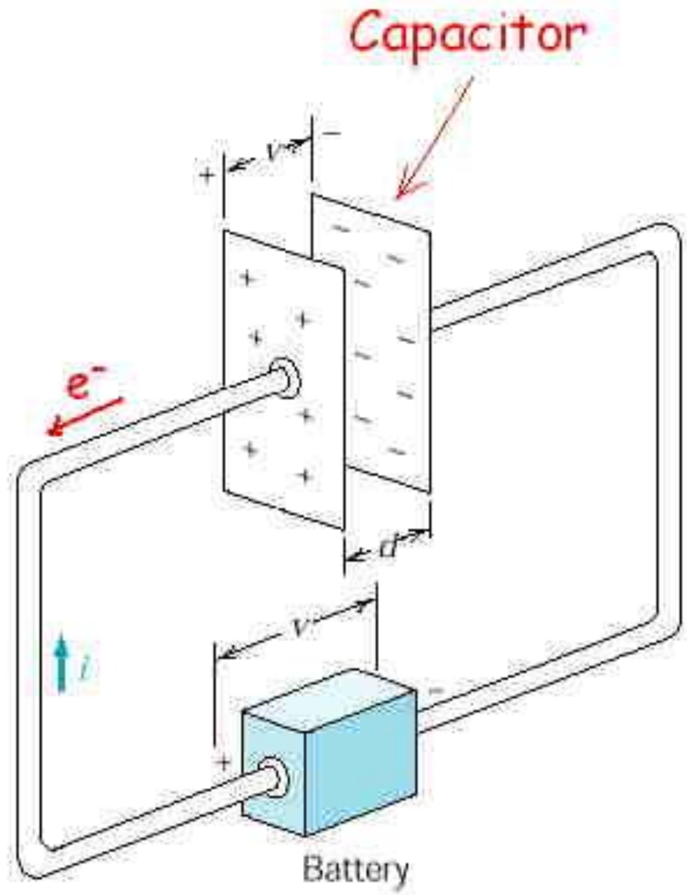
A: area of plates (in meter sq).

d: distance between plates (in meter).

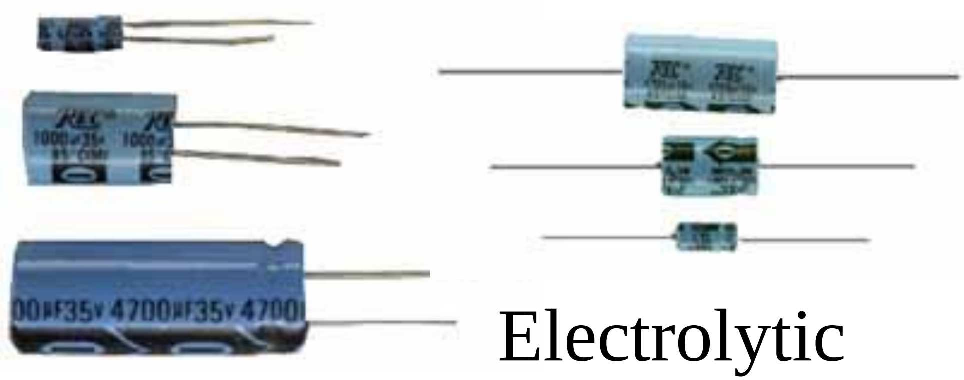
ε: dielectric constant.

Relative Dielectric Constant

Material	ε_r
Mica	7
Nylon	2
Bakelite	5
Paper	3.5



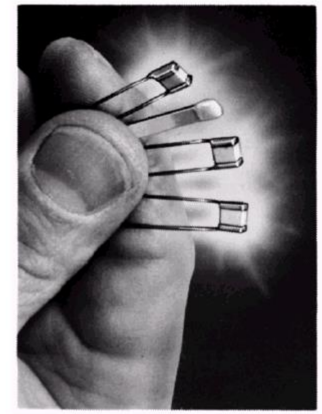
where $\boxed{\varepsilon = \varepsilon_r \varepsilon_o}$, and ε_o is permittivity of free space: $\varepsilon_o = 8.85\times10^{-12}$ Farad/Meter



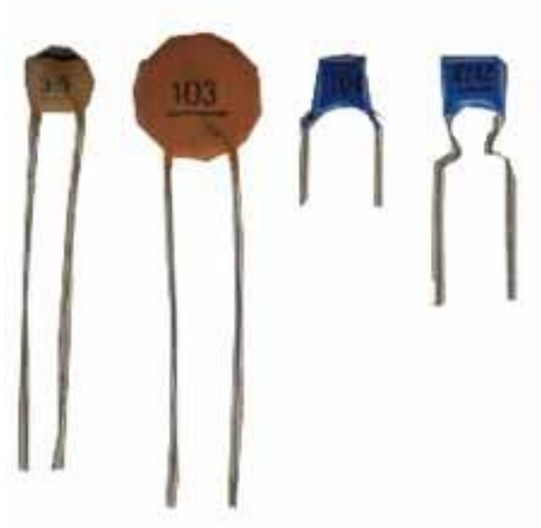
Electrolytic

Polyester

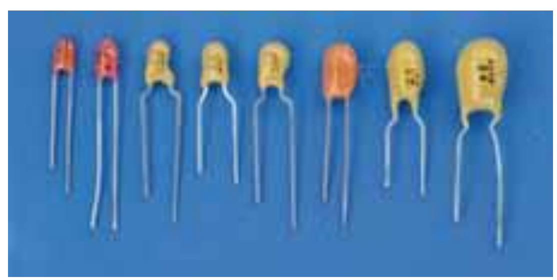
Trimmer



Polycarbonate



Ceramic



Tantalum

Power Capacitor

Capacitors

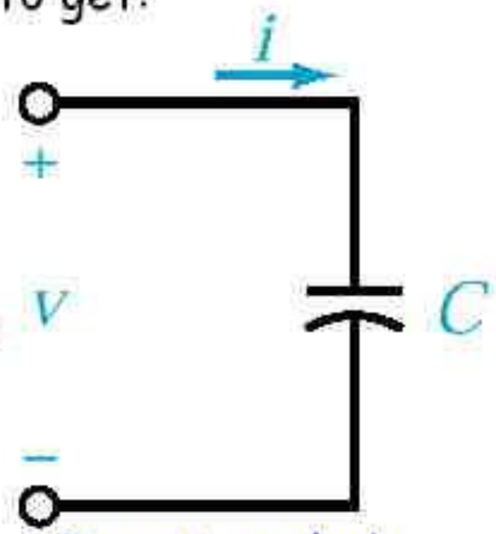
Current Voltage Relationship for a Capacitor

Recall that: $\boxed{q = C\,v}$

Current is the rate of change of charge. Therefore, the current in a capacitor is found by differentiating the above equation to get:

$$\boxed{i = \frac{dq}{dt} = C\frac{dv}{dt}}$$

with current direction and voltage polarity as shown here:



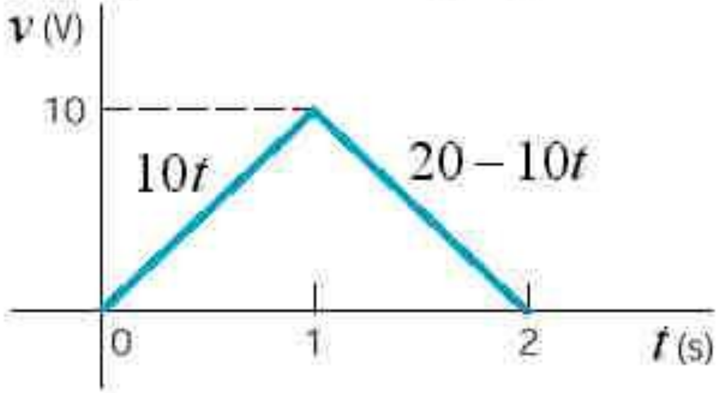
Circuit symbol for a capacitor

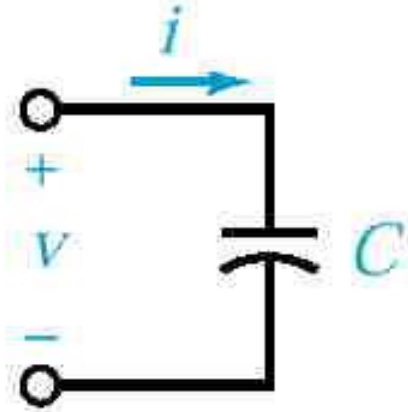
Note: The current-voltage relationship for a capacitor is linear (assuming C is a constant). Therefore, capacitors are linear elements.

Capacitors

Current Voltage Relationship for a Capacitor

Example: Find the current in a capacitor $C = 1$ mF when the voltage across the capacitor is represented by the following signal:

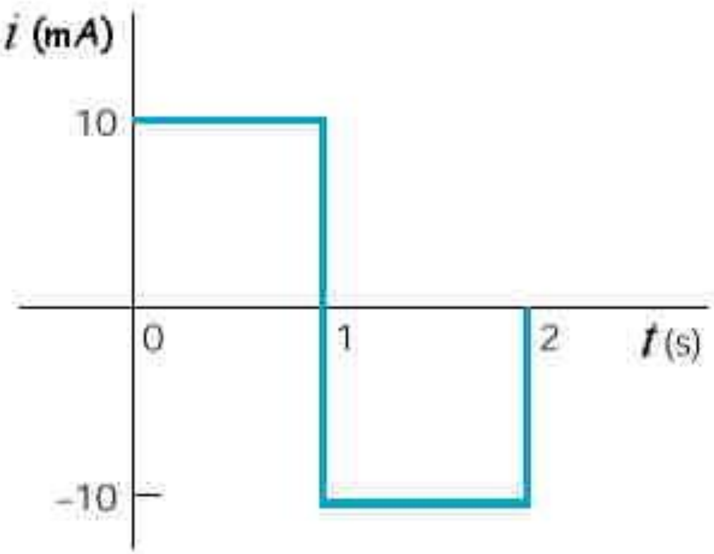




Solution: Since $i = C\dfrac{dv}{dt} = 0.001\dfrac{dv}{dt}$

we obtain

$$i = \begin{cases} 0 & t \le 0 \\ 10^{-2} & 0 < t \le 1 \\ -10^{-2} & 1 < t \le 2 \\ 0 & t > 2 \end{cases}$$

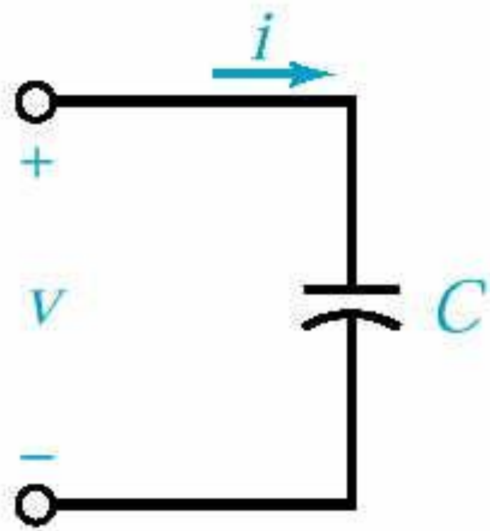


Capacitors

Current Voltage Relationship for a Capacitor

$$\boxed{i = C\frac{dv}{dt}}$$

Let us now find the voltage across a capacitor in terms of the current. By integration:



$$\int_{t_0}^{t} i\,d\tau = C\int_{v(t_0)}^{v(t)} \frac{dv}{d\tau}d\tau = C[(v(t) - v(t_0)]$$

or $$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)}$$

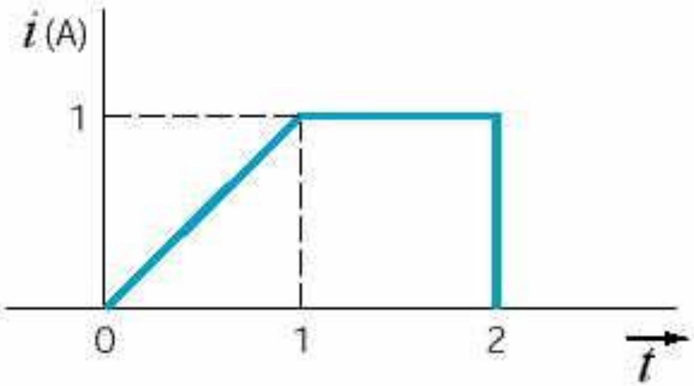
This equation says that the capacitor voltage $v(t)$ can be found by integrating the capacitor current from some convenient time t_0 until time t, provided that we also know the capacitor initial voltage $v(t_0)$. $v(t_0)$ is also known as the capacitor initial condition. Frequently, it is convenient to select $t_0 = 0$.

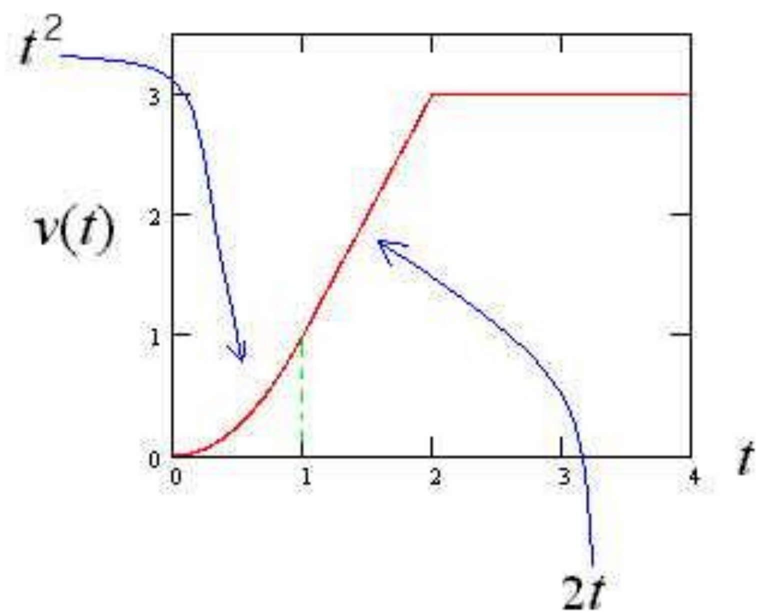
Capacitors

Current Voltage Relationship for a Capacitor

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)}$$

Example: Find the voltage across a capacitor C = 1/2 F when the current is as shown here.





Capacitors

The voltage across a capacitor cannot change instantaneously

The principle of conservation of charge states that the amount of electric charge can not change instantaneously. Thus, $q(t)$ must be continuous over time. Recall that $q(t) = Cv(t)$. Thus, the voltage across a capacitor cannot change instantaneously.

$$\boxed{v(t_0^+) = v(t_0^-) \quad \text{for all } t_0}$$

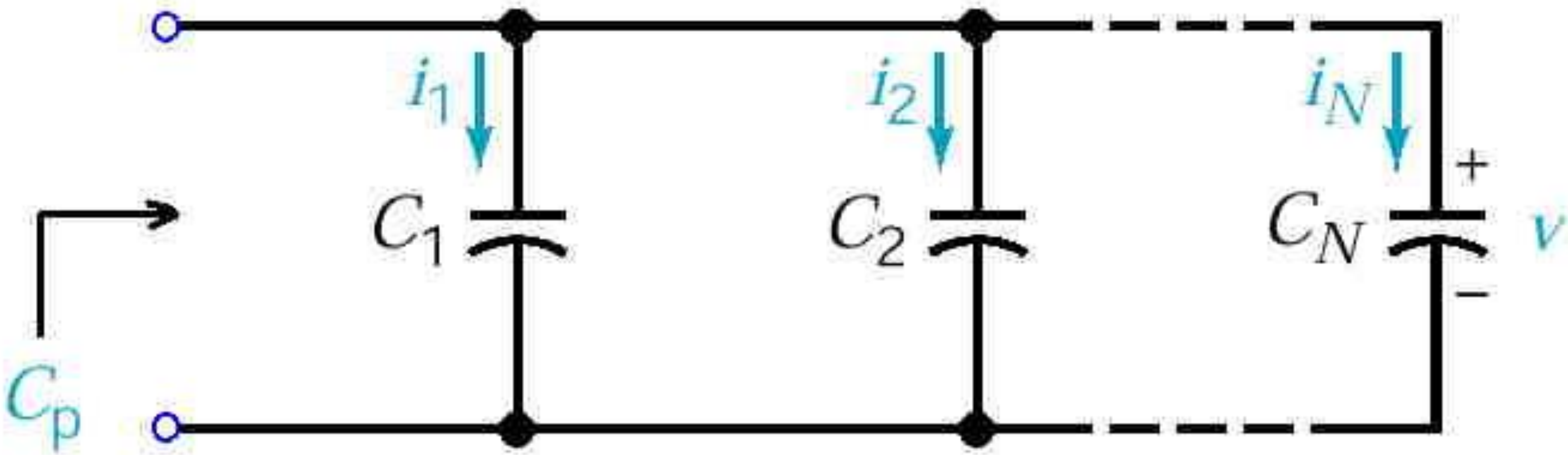
Note that the current in a capacitor can change instantaneously.

Capacitors

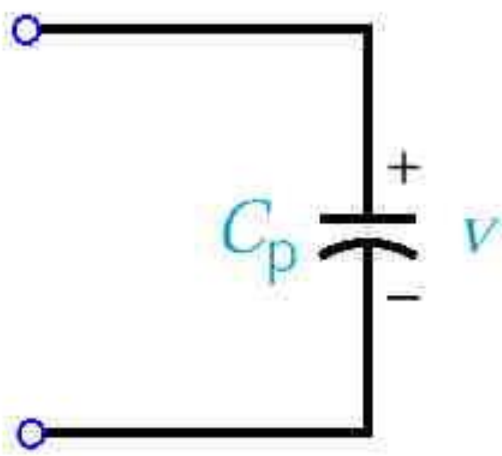
Actual capacitors have some resistance associated with them. In capacitors the dielectric material between the plates is not a perfect insulator and has a small conductivity. This can be represented by a very high resistance (hundreds of megaohms) in parallel with an ideal capacitor. This resistance is usually ignored.

Ordinary capacitors can hold a charge for hours. Therefore, one should be very carefull when handeling large capacitors, even when they are not connected in a circuit.

Parallel Capacitors



Parallel connection of N capacitors



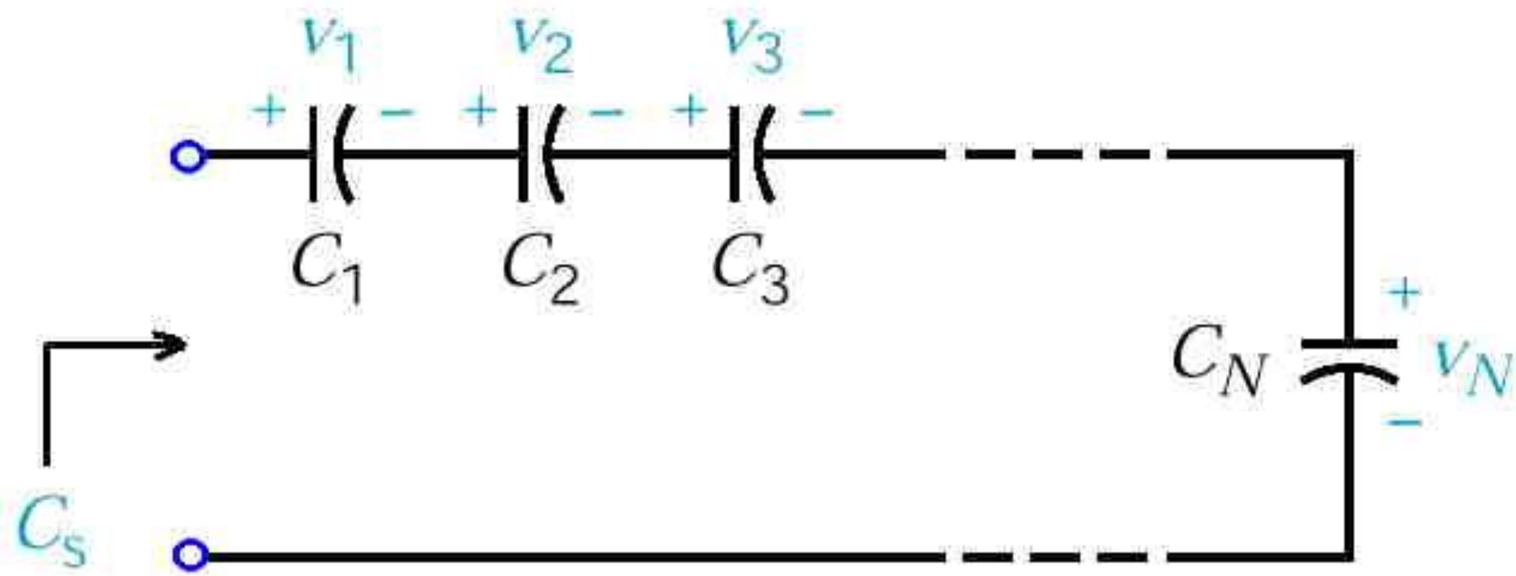
Equivalent circuit

$$\boxed{C_p = C_1 + C_2 + \cdots + C_N}$$

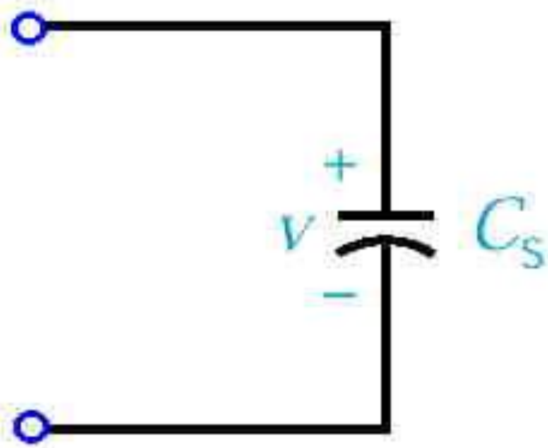
Note that:

Initial voltage on C_p is same as C_i.

Series Capacitors



Series connection of N capacitors

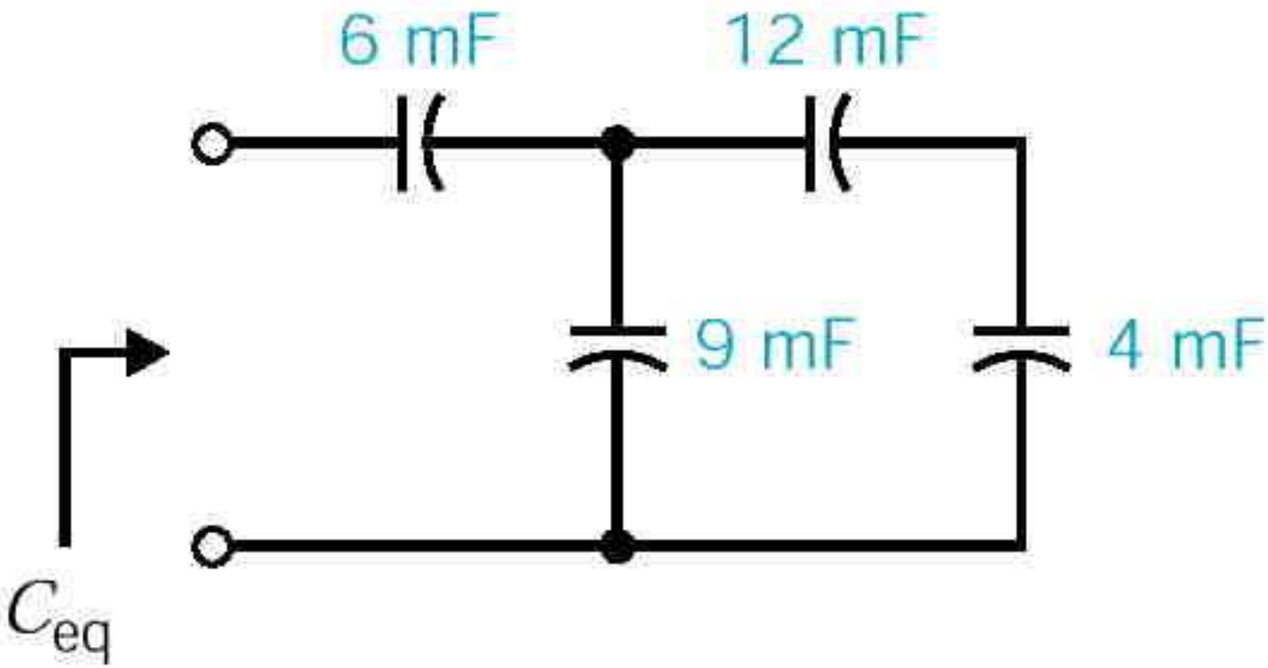


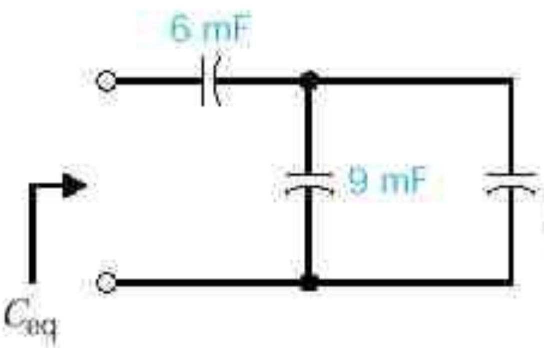
Equivalent circuit

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}}$$

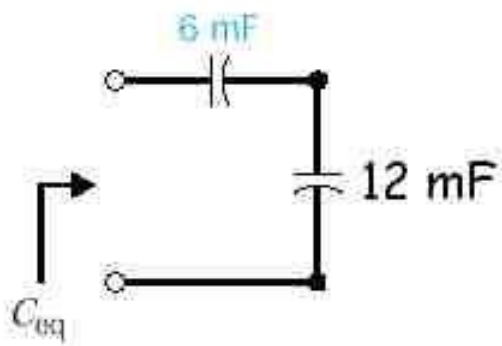
$$\boxed{v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)}$$

Series and Parallel Capacitors: Example





$$\frac{(4)(12)}{(4+12)} = 3 \text{ mF}$$



$$C_{eq} = \frac{(6)(12)}{(6+12)} = 4 \text{ mF}$$

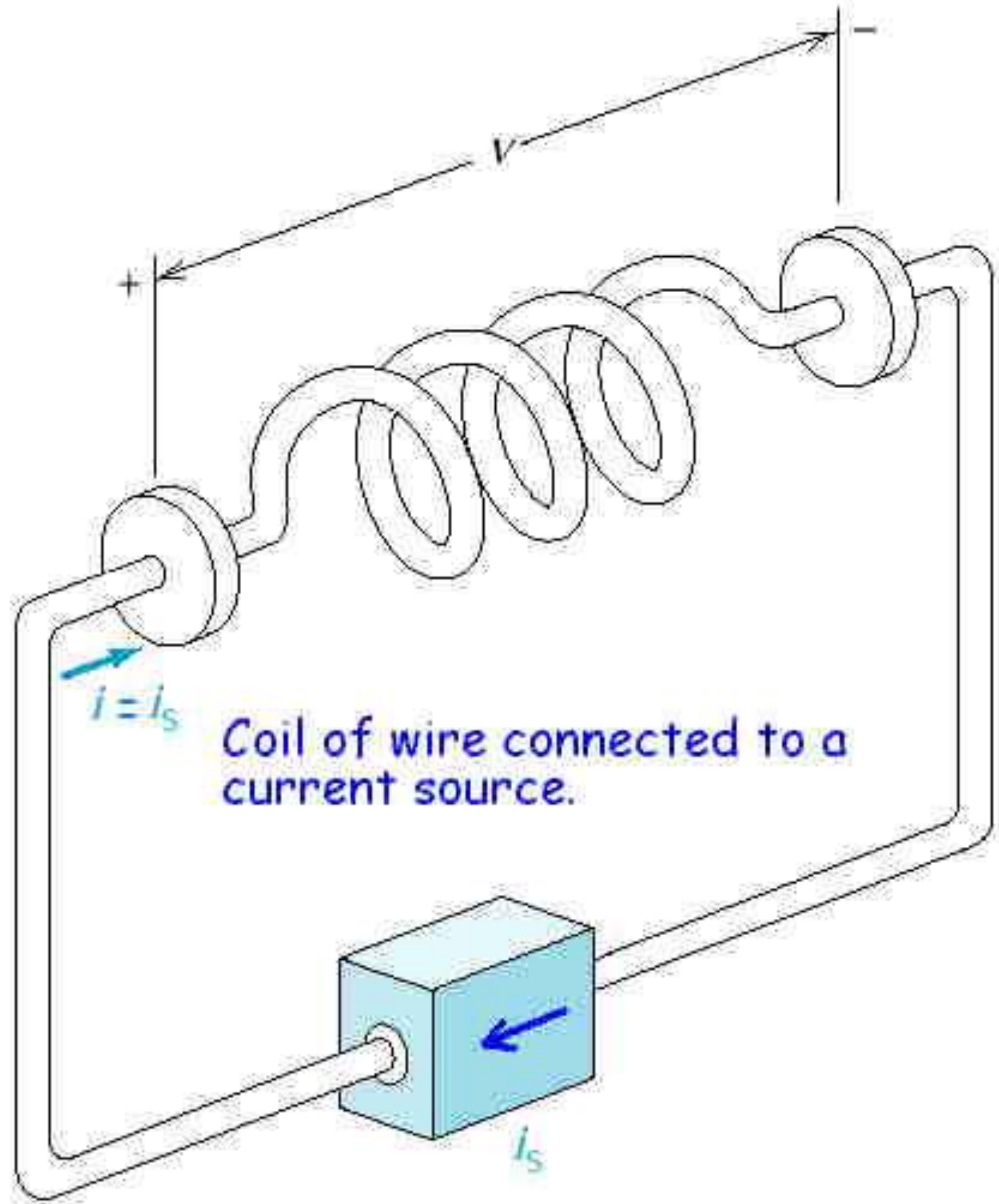
Inductors

An inductor is a two-terminal element consisting of winding of N turns for introducing inductance into an electric circuit.

Inductance is defined as the property of an electric device by which a time-varying current through the device produces a voltage across it:

$$\boxed{v = L\frac{di}{dt}}$$

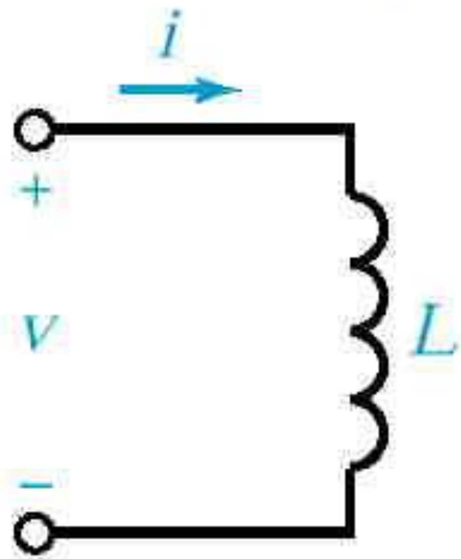
where L is the constant of proportionality called *inductance* and is measured in henrys (H).



Coil of wire connected to a current source.

Inductor Current Continuity Property

$$v = L\frac{di}{dt}$$

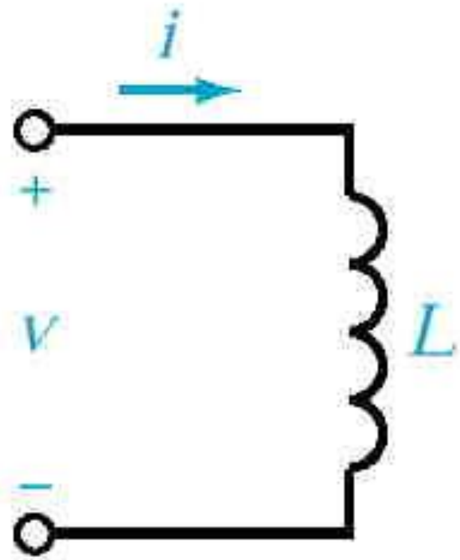


An abrupt (or instantaneous) change in current is impossible since an infinite voltage would be required. This means that the inductor current must be continuous:

$$\boxed{i(t_0^+) = i(t_0^-) \quad \text{for all } t_0}$$

Example:

$$\boxed{v = L\frac{di}{dt}}$$



Find the voltage across the inductor, $L = 0.1$ H, when the current in the inductor is:

$$i = 10t + 5 \quad \text{A}$$

$$v = L\frac{di}{dt} = (0.1)(10) = 1\ \cancel{\text{A}}\ \text{V}$$

Current-voltage Relationship for an inductor

The current in an inductor in terms of the voltage across it may be determined by integrating the relationship

$$v = L\frac{di}{dt}$$

from t_0 to t.

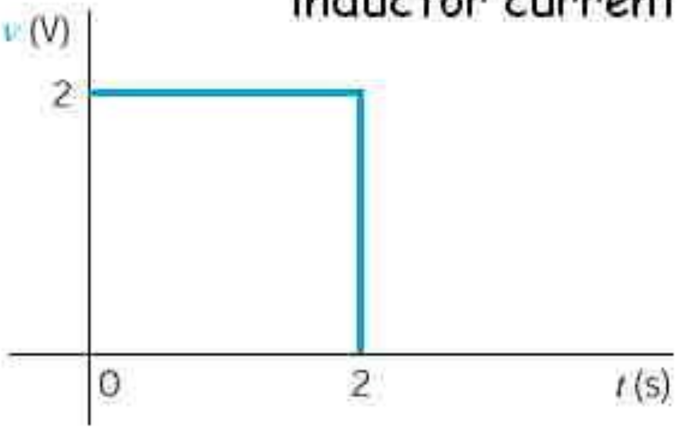
$$di = \frac{v}{L}dt$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)}$$

Current-voltage Relationship for an inductor

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)}$$

Example: Consider an inductor with L = 0.1 H and $i(0)$ = 2 A. Find the inductor current for the following inductor voltage.

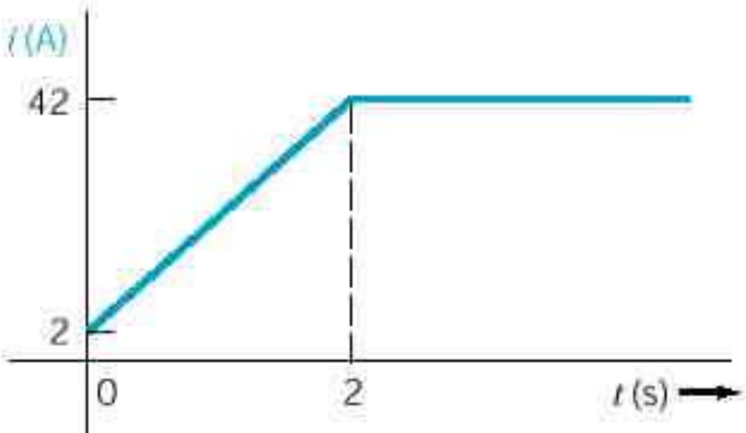


Solution: Current between t = 0 and t = 2

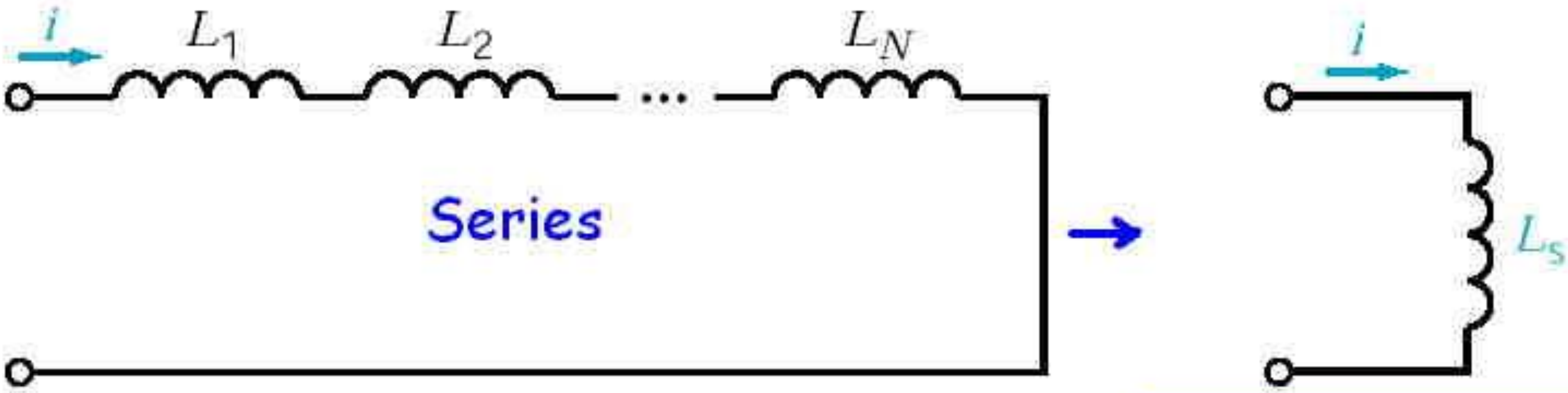
$$i(t) = \frac{1}{0.1}\int_{0}^{t} 2\,d\tau + 2 = 20t + 2 \text{ A}$$

Current for t > 2

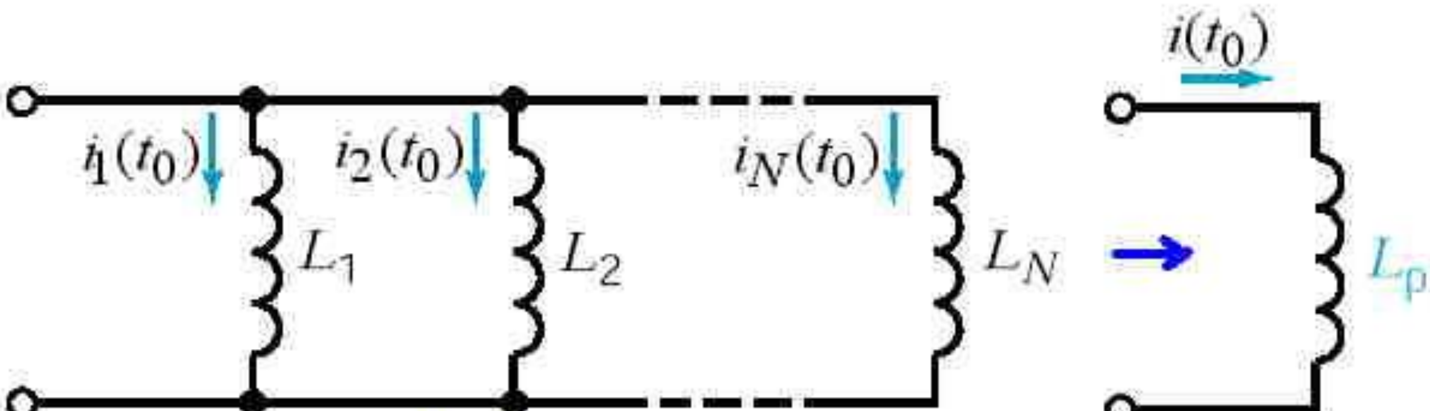
$$i(t) = \frac{1}{0.1}\int_{2}^{t} 0\,d\tau + i(2) = 42 \text{ A}.$$



Series and parallel Inductors



$$\boxed{L_s = L_1 + L_2 + \cdots + L_N}$$



parallel

$$\boxed{i(t_0) = i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)}$$

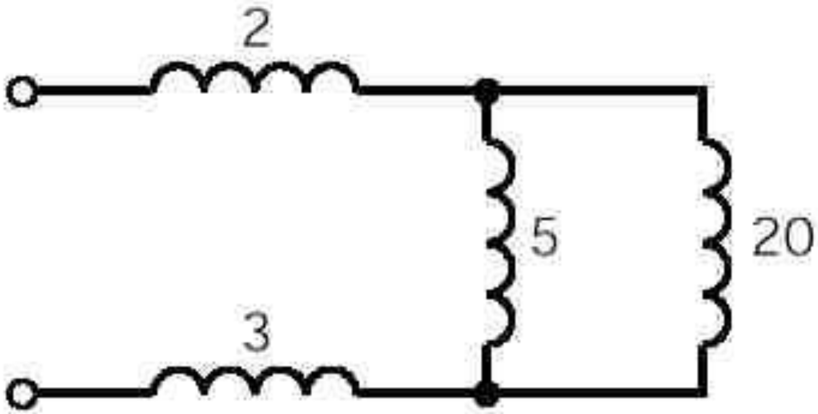
$$\boxed{\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}}$$

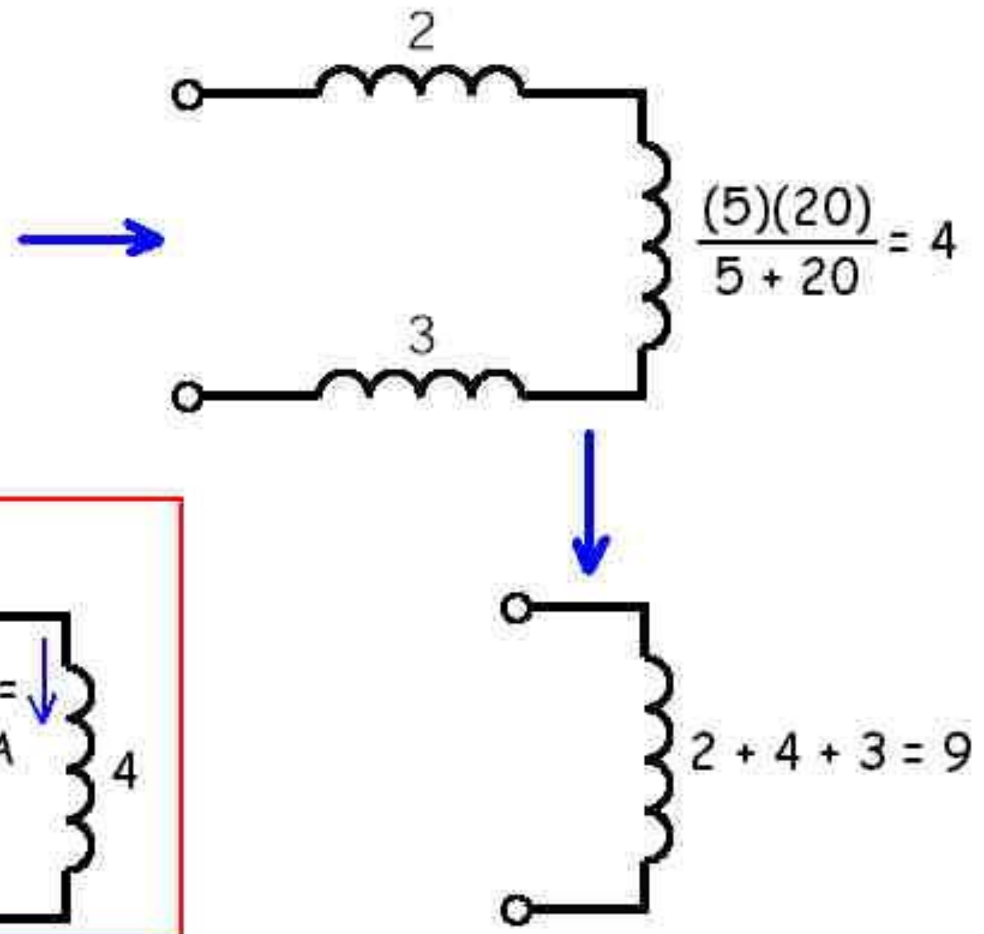
Series and parallel Inductors

Example: Find the equivalent inductance for the following circuit. Assume all initial inductor currents are zero (why?).
All inductances are in H.

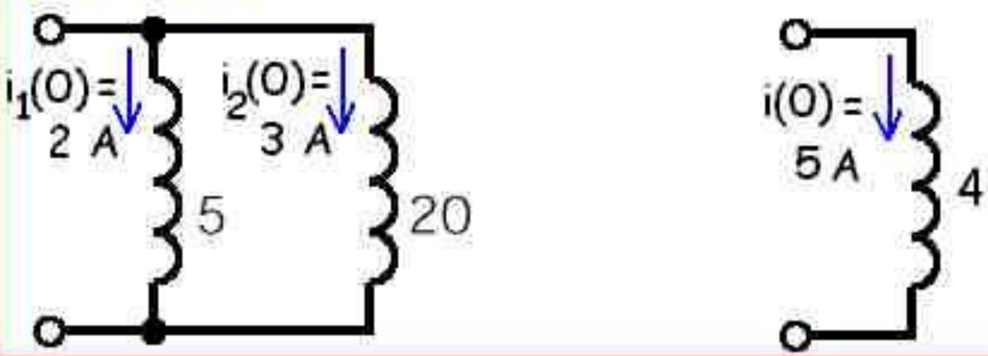
$$\boxed{L_s = L_1 + L_2 + \cdots + L_N}$$

$$\boxed{\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}}$$





Example 2:



Inductors: Summary of Main Results

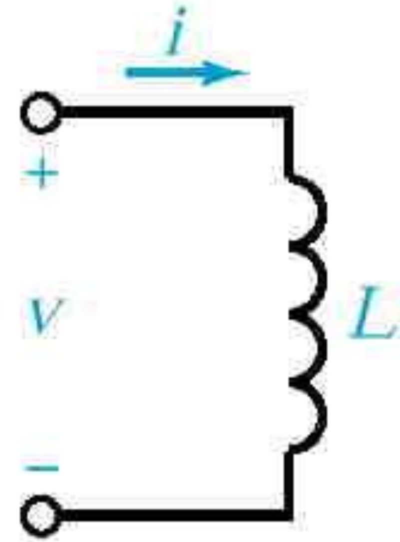
$$L = \frac{\mu N^2 A}{l + 0.45d} \text{ H}$$

$$v = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

$$p = vi = \left(L\frac{di}{dt}\right)i$$

$$w = \frac{1}{2}Li^2$$



Circuit Symbol for an inductor

The current throught an inductor cannot change instantaneously:

$$i(t_0^+) = i(t_0^-) \text{ for all } t_0$$

$$L_s = L_1 + L_2 + \cdots + L_N$$ series inductors

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$$ parallel inductors

Note that the voltage across an inductor can change instantaneously.

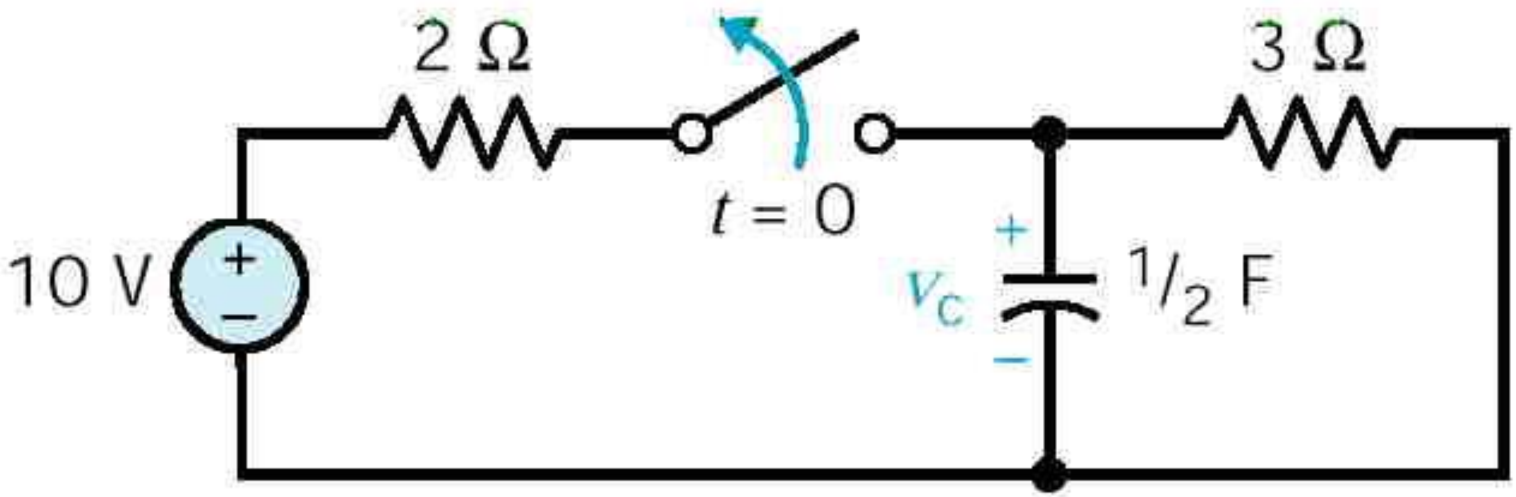
The inductor behaves a s a short circuit under pure dc excitation and assuming steady-state operation.

DC Steady State for RC Circuits

In dc steady-state operation, all voltages and currents in an RC circuit are constants. Since a capacitor current is proportional to the derivative of the capacitor voltage, the current is zero. The capacitor appears as an open circuit.

Example:

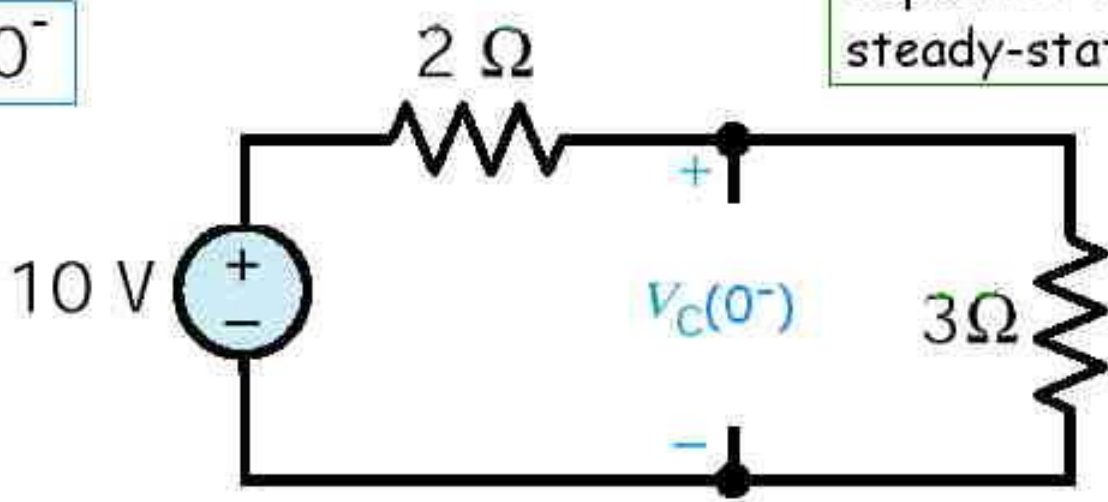
Find $v_c(0^-)$ and $v_c(\infty)$. Assume the switch has been closed for a long time.



Example: Steady-State Operation

Find $v_c(0-)$ and $v_c(\infty)$. Assume the switch has been closed for a long time.

$t = 0^-$

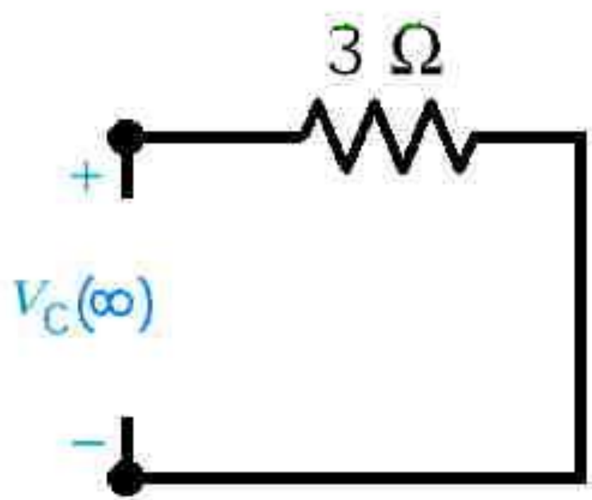


Capacitor is open circuit for steady-state operation

By voltage division:

$V_C(0^-) = (3)(10)/(2+3)$
$= 6\ V$

$t = \infty$

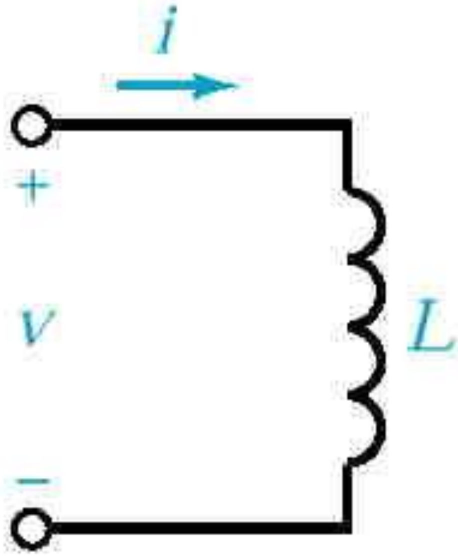


Current in mesh is zero because of open circuit.

$V_C(\infty) = 3i = (3)(0) = 0\ V$

DC Steady-State Property of Inductors

$$\boxed{v = L\frac{di}{dt}}$$

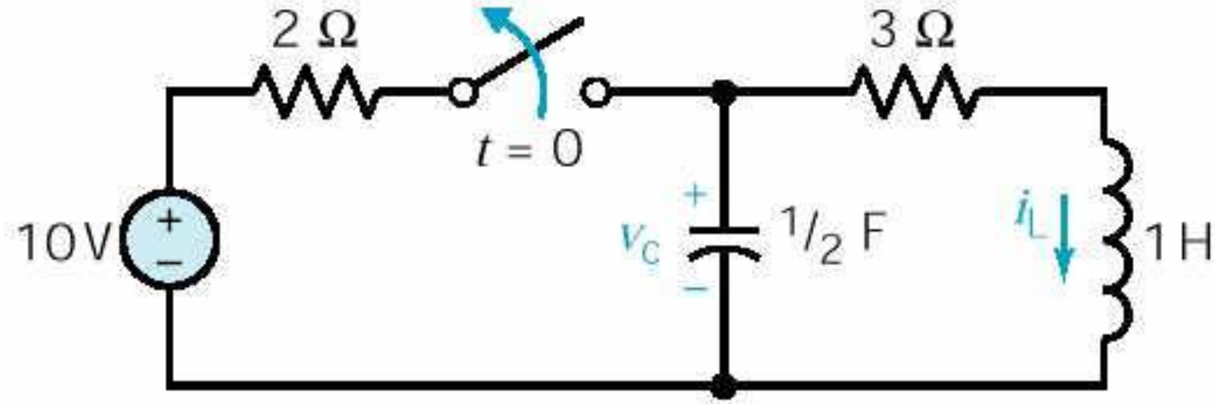


Consider a circuit with inductors (and other elements) and only dc sources. If the circuit stays for a very long time without any switching taking place (i.e, circuit is in steady-state mode), then every inductor behaves as a short circuit. This is because all currents in the circuit would be constants and their derivatives are zero.

DC Steady-State Property of Inductors

Example:

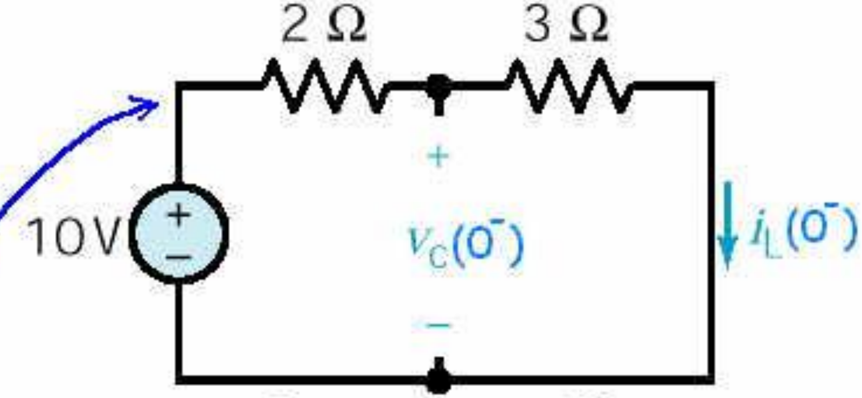
Find the inductor current at $t = 0^-$ and 0^+. Assume that the switch has been closed for a very long time.



Solution:

Circuit operates under dc steady-state mode for negative time (close to zero), since source is dc and switch has been closed for a long time.

Therefore, the capacitor behaves as open circuit and the inductor behaves as a short circuit.



Circuit at $t = 0^-$

By ohm's law:

$$i_L(t = 0^-) = 10/(2+3) = 2\ A$$

By inductor current continuity:

$$i_L(t = 0^+) = i_L(t = 0^-)$$

$\boxed{i_L(t = \infty) = ?}$