Probability Methods in Engineering

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Lecture 22

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Moment Theorem

- The moment theorem is

$$E\left[X^n\right] = \frac{1}{j^n}\frac{d^n}{d\omega^n}\Phi_X(\omega)\Big|_{\omega=0}$$

Example

- Determine the mean and variance of an exponentially distributed random variable using moment theorem

Probability Generating Function

- The probability generating function for discrete RVs with
  - Non-negative values
  - Integer values

$$G_N(z) = E\left[z^N\right] = \sum_{k=0}^{\infty} p_N(k) z^k$$

- Inverse of generating function is

$$p_N(k) = \frac{1}{k!} \frac{d^k}{dz^k} G_N(z)\Big|_{z=0}$$

Probability Generating Function (cont.)

- Mean using generating function

$$E[N] = \frac{d}{dz} G_N(z)\Big|_{z=1}$$

- 2nd moment using generating function

$$E[N^2] = \frac{d^2}{dz^2} G_N(z)\Big|_{z=1} + E[N]$$

- Variance using generating function

$$VAR[N] = \frac{d^2}{dz^2} G_N(z)\Big|_{z=1} + E[N] - E^2[N]$$

Examples (cont.)

- Find the mean and variance of Poisson RV using probability generating function