Probability Methods in Engineering

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Lecture 23

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Entropy

- Measure of uncertainty in random experiment
- Uncertainty of value k

$$I(X=k)=\log_2\frac{1}{P[X=k]}=-\log_2 P[X=k]$$

- Entropy (in bits) of RV defined as

$$H_X=E[I(X)]=\sum_{k=1}^{K}P[X=k]\log_2\frac{1}{P[X=k]}$$

$$=-\sum_{k=1}^{K}P[X=k]\log_2 P[X=k]$$

Entropy (cont.)

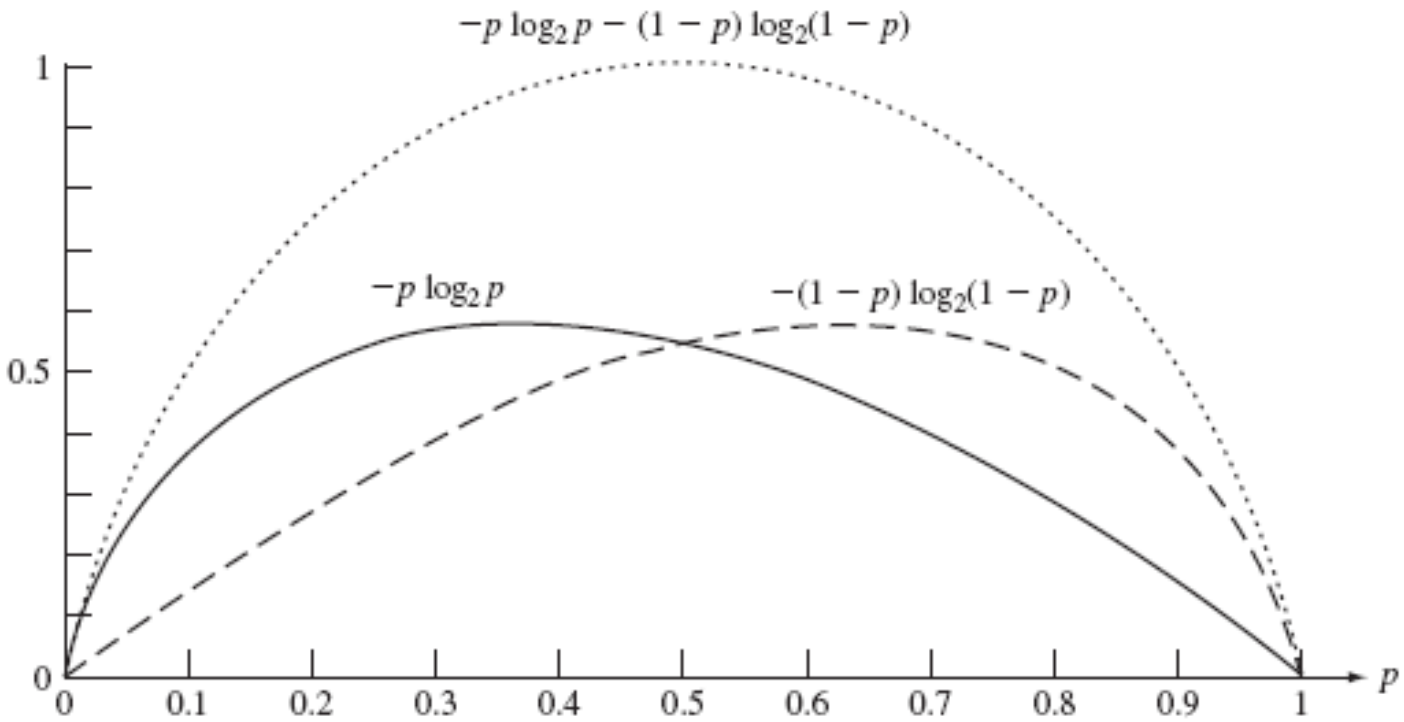
- Entropy is measure of expected information
- Example: Order a cup of tea, waiter needs some "bits" of information
  - With or without milk
  - With or without sugar
  - With or without cardamom
- 2^3 possible orders
  - Without milk without sugar without cardamom
  - Without milk without sugar with cardamom
  - Without milk with sugar without cardamom
  - Without milk with sugar with cardamom
  - With milk without sugar without cardamom
  - With milk without sugar with cardamom
  - With milk with sugar without cardamom
  - With milk with sugar with cardamom

Entropy (cont.)

- Getting more information reduces entropy
- Play the game of 20 questions
  - After each answer, entropy is reduced

Examples

- Determine the entropy of a Bernoulli distributed RV



Examples (cont)

- Flip a fair coin three times. The RV X is given as $S_X = \{000, 001, 010, \ldots, 111\}$. Find the entropy of X. Assume that an event A that the first outcome is 1 has occurred. Find the reduction in entropy.

Examples (cont.)

- Draw a ball from an un containing eight balls. Four balls are numbered as 1, two as 2 and remaining as 3 and 4. X is the RV denoting the number of the ball. Find the entropy of X.