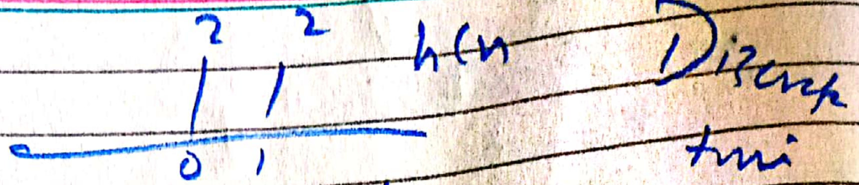
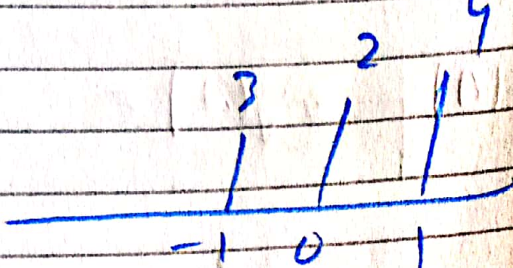
Best example of Convolution Sum



یہ بہت آسان طریقہ ہے کہ h(n) اور x(n) کو unite impulse میں توڑنا ہے اور impulse کو پورے impulse response کے ساتھ xing کرنا۔

$2h(n) + 2h(n-1)$

Xing

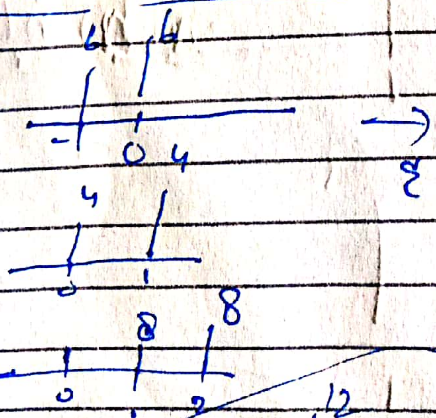
$3\delta(n+1) + 2\delta(n) + 4(n-1)$

Ans $= 6h(n+1) + 6h(n) + 4h(n) + 4h(n-1$
$+ 8h(n-1) + 8h(n-2)$

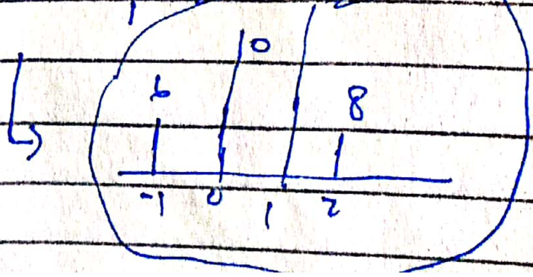
$= 6h(n+1) + 10h(n) + 12h(n-1) + 8h(n-$

Ans

given is Partition method



یہ سروالا طریقہ ہے

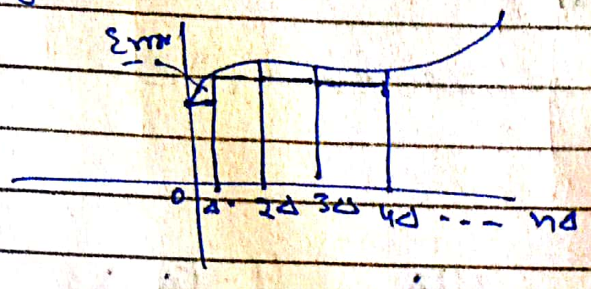


# Continuous time LTI Systems

→ we use Convolution Integral.

~~there~~ we divide continuous graph to small interval $\Delta$

i.e

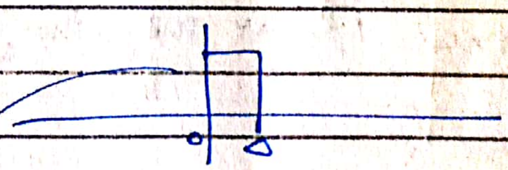



→ 0 to $\Delta$ and so on are real ~~tot~~ Impulse.

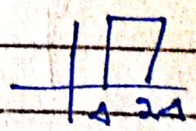
→ If we take $\lim \Delta \to 0$ then it became ideal Impulse.

→ If we decrease $\Delta$ then Error will reduce.

Now




$x(0) = x(0) \cdot \delta_\Delta(t)$



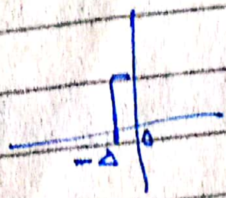

$x(\Delta) = x(\Delta) \cdot \delta_\Delta(t - \Delta)$

Similar

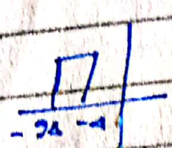
$x(2\Delta) = x(2\Delta) \cdot \delta_\Delta(t - 2\Delta)$

⋮ ⋮ ⋮

For -ive




$$x(-\Delta) = x(-\Delta)\,\delta_\Delta(t+\Delta)$$




$$x(-2\Delta) = x(-2\Delta)\,\delta_\Delta(t+2\Delta)$$

Generally

$$x(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\cdot\delta_\Delta(t-k\Delta)\,\Delta$$

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\,\delta_\Delta(t-k\Delta)\,\Delta}$$

General form of Continuous signal if we take $\lim_{\Delta\to 0}$

(or)

$$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$

For 0, it is removed

→ For Input Signal

→ General Form for Convolutional Integral

If

$x(t) = x(0)\delta_\Delta(t) + x(\Delta)\delta_\Delta(t-\Delta$
$+ x(2\Delta)\delta_\Delta(t-2\Delta) + \cdots$

for -ive

$x(t) = x(-\Delta)\cdot\delta_\Delta(t+\Delta) + x(-2\Delta)\delta_\Delta(t-2\Delta$
$+ \text{———}$

Then

$y(t) = \cdots x(-2\Delta)\cdot h(t+2\Delta) + x(+\Delta)\cdot h(t+\Delta)$
$x(0)\cdot h(t) + x(\Delta)\cdot h(t+\Delta)$
$x(2\Delta)\cdot h(t+2\Delta) + \cdots$

$$\boxed{y(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\cdot h(t-k\Delta)\Delta}$$

If we apply limit for less error for

$$y(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty} x(k\Delta)\cdot h(t-k\Delta)\Delta$$

$\boxed{k\Delta = \tau}$

$$\boxed{y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau}$$

→ for output signal

Convolution Property:

For discrete & Continuous

① Commutative Property.

$$y(n) = x(n) * h(n) = h(n) * x(n)$$

$$= \sum_{k=-\infty}^{+\infty} x(k)\, h(n-k)$$

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

③ Distributive

$$x[n] * (h_1(n) + h_2(n)) =$$

$$x[n] * h_1(n) + x[n] * h_2(n)$$

$$x(t) * (h_1(t) + h_2(t)) =$$

$$x(t) * h_1(t) + x(t) * h_2(t)$$

# Coefficient of Fourier Series.

general form

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$ xing by $e^{-jn\omega_0 t}$

$x(t) e^{-jn\omega_0}$

$$x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \cdot e^{-j\omega n t}$$

$$x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{j(k-n)\omega_0 t}$$

Integrate $\int_0^T$

$$\int_0^T x(t) e^{-jn\omega_0 t} dt = \int_0^T \sum_{k=-\infty}^{+\infty} a_k e^{j(k-n)\omega_0 t} dt$$

$$\int_0^T x(t) e^{-jn\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k \int_0^T e^{j(k-n)\omega_0 t} dt \quad \text{--- (1)}$$

$$\int_0^T e^{j(k-n)\omega_0 t} dt = ?$$

Euler theorem

$$= \int_0^T \cos((k-n)\omega_0 t) + j \sin((k-n)\omega_0 t) dt$$

$$\int_0^T e^{j(k-n)\omega_0 t} dt = \begin{cases} T & k = n \\ 0 & k \neq n \end{cases}$$

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For only $k = n$
whole $\int = a_n T$
other wise "0"

So

$$\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt = a_n T$$

$$\boxed{a_n = a_k = \frac{1}{T}\int_0^T x(t)\, e^{-j\omega_0 n t}\, dt}$$

$\omega_0 = \frac{2\pi}{T}$

(or)

while

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k\, e^{jk\omega_0 t}$$

Any periodic signal can be expressed as a linear combination

## Properties of Continuous time F.S

### ① Linearity

if

$$x(t) \xrightarrow{F.S} F_x \xrightarrow{\text{coefficient}} a_k$$

$$\& \quad y(t) \xrightarrow{F.S} F_y \rightarrow b_k$$

$$\boxed{Aa_k + Bb_k}$$

then linearity

$$\boxed{a\,x(t) + b\,y(t) \xrightarrow{F.S} aF_x + bF_y}$$

### ③ Time Shifting

if

$$x(t) \xrightarrow{F.S} a_k$$

then T. Shifting

$$x(t-t_0) \xrightarrow{F.S} e^{-jw_0 n t_0}\, a_k$$

### ③ Frequency Shifting Property:

if $x(t) \xrightarrow{F.S} a_k$

then

$$e^{jn_0 w_0 t} \cdot x(t) \xrightarrow{F.S} a(k-n_0)$$

④ Multiplication & Convolution

If

$x(t) \xrightarrow{F.S} a_k$

$y(t) \xrightarrow{F.S} b_k$

Multiplication

$x(t) \cdot y(t) \xrightarrow{F.S} T_0 (a_k * b_k)$

Convolution

$x(t) * y(t) \xrightarrow{F.S} T_0 (a_k \cdot b_k)$

⑤ Differentiation-

if

$x(t) \xrightarrow{F.S} a_k$

then

$\frac{d}{dt} x(t) \xrightarrow{F.S} jk\omega_0 a_k = jk\left(\frac{2\pi}{T}\right) a_k$

⑥ Time Scaling:

If

$x(t) \xrightarrow{F.S} a_k$

then $x(at) \xrightarrow{F.S} a_k$

Time Scaling change only frequency component. $\frac{1}{T_0}$ — $t_0$ — $\frac{a}{T_0}$

8) Time reversal

If
$$x(t) \xrightarrow{FS} a_k$$
then
$$x(-t) \xrightarrow{FS} a_{-k}$$

9) Conjugate property

If
$$x(t) \xrightarrow{F.S} a_k$$
then
$$x^*(t) \xrightarrow{FS} a^*_{-k}$$

10) Parseval's Relation

$$x(t) \rightarrow a_k$$
$$[x(t)]^2 \rightarrow a_k^2$$

12) Integration property:

$$x(t) \rightarrow a_k$$
$$\int x(t)\,dt = \frac{1}{jk\omega_0} a_k$$

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