Q NO 1: (A) Linear transformation:

A linear transformation L of R^n into R^m is a function assigning a unique vector $L(u)$ in R^m to each u in R^n such that

a) $L(u+v) = L(u) + L(v)$, for every u and v in R^n. ~~such that~~

(b) $L(ku) = kL(u)$, for every u in R^n and every scalar k.

The vector $L(u)$ in R^m is called linear transformation. the image of u. the set of all image in R^m of the vectors in R^n is called the range of L.

We shall write the fact that L maps R^n into R^m, even if it is not a linear transformation as

$$L: R^n \to R^m$$

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(7)

if $n=m$, a linear transformation.

$L: R^n \to R^n$ is also called a linear operator on R^n.

Some well know linear transformations.

1) Decode
2) Encode
3) Rotations
4) Projections

Q No: (1) part (B):

Let $L = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$

$\phi = 30^\circ$

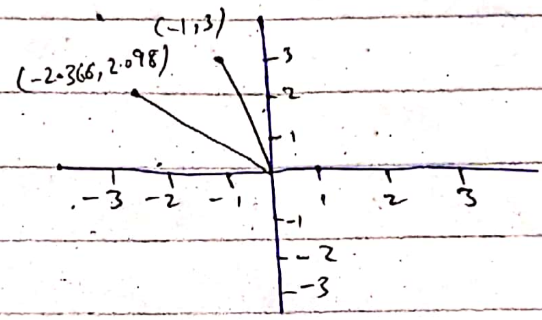
$p(-1,3) = 4$

$$L_4 \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

⑧ ③

$$= \begin{bmatrix} -\cos 30^\circ & -3\sin 30^\circ \\ -\sin 30^\circ & +3\cos 30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} -2.366 & , 2.098 \end{bmatrix}$$



Q2: (A)

Sol: For Encoding we will use the matrix A and for decoding we will use its inverse A^{-1},

S	E	N	D	H	I	M	M	O	N	E	Y
19	5	14	4	8	9	13	13	15	14	5	25

and

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix}$$

Now $L(x) = AX$

$$Ax = A\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix}$$

1
2
3
4

$$= \begin{bmatrix} 1\times19 + 2\times5 + 3\times14 \\ 1\times19 + 1\times5 + 2\times14 \\ 0\times19 + 1\times5 + 2\times14 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 19 + 10 + 42 \\ 19 + 5 + 28 \\ 0 + 5 + 28 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$$

2 B: Sol:

Decode:

67, 44, 41, 49, 39, 19, 113, 76
62, 104, 67, 55

Now will break it into Vector in R^3.

$\begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix}$

$3 \quad 2 \quad 2 \; \begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix}$

$+ 3\times14$
$+ 2\times14$
$+ 2\times14$

$= \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$

9, 19, 113, 76

it into

$L_1(x_1) \quad L(x_2) \quad L(x_3) \quad L(x_4)$

$$\begin{bmatrix} 67 \\ 49 \\ 41 \end{bmatrix} \quad \begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} \quad \begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix} \quad \begin{bmatrix} 104 \\ 67 \\ 55 \end{bmatrix}$$

As A^{-1} is given

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$L(x) = Ax = x = A^{-1}L(x)$$

$$x_1 = A^{-1}L(x_1) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}$$

$$X_2 = A^{-1}L(x_2) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} = \begin{bmatrix} 20 \\ 1 \\ 9 \end{bmatrix}$$

$$X_3 = A^{-1}L(x_3) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 113 \\ 76 \\ 12 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 25 \end{bmatrix}$$

(6)

(iv)

$$X_4 = A^{-1}L(x_4) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix} = \begin{bmatrix} 14 \\ 15 \\ 20 \end{bmatrix}$$

So the message is

3 5 18 20 1 9 14 12 25 14 15 20

C E R T A I N L Y N O T.

Q No 3: (a) Find a line that passes through the point $(-2,5,-3)$ and is perpendicular to the plane

$$2x-3y+4z+7=0$$

Sol (a) let $2x-3y+4z+7=0$

$2x-3y+4z+7$ and $c_1=(2,-3,4)$

The vector normal to the plane.

Now the equation of line passes through the point $(-2,5,-3)$ and parallel to the vector $u=(2,-3,4)$ is

$$\frac{x+2}{2}=\frac{y-5}{-3}=\frac{z+3}{4}$$

(b)

Let $p_1(0,1,2)$, $p_2(3,-2,5)$ $p_3(2,3,4)$

$$\overrightarrow{p_1p_2}=\vec{p_2}-\vec{p_1}=(3,-3,3)$$
$$\overrightarrow{p_1p_3}=\vec{p_3}-\vec{p_1}=(2,2,2)$$

Now $N=\overrightarrow{p_1p_2}\times\overrightarrow{p_1p_3}$ So

$$V = \begin{bmatrix} i & j & k \\ 3 & -3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

$$V = \begin{bmatrix} i & j & k \\ 3 & -3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$

$$V = i(-6-6) - j(6-6) + k(6+6)$$

$$V = -12\hat{i} + 0\hat{j} + 12\hat{k}$$

Now the equation of plane by taking point $(0,1,2)$ and

$$\vec{V} = -12i + 0j + 12k$$

$$\Rightarrow -12(x-0) + 0(y-1) + 12(z-2)$$

$$\Rightarrow -12x + 0y + 12z - 24 = 0$$

$$\Rightarrow -12x + 12z - 24 = 0$$

Equation of plane

So $\boxed{x - z + 2 = 0}$

(4) (a) Let $L: R^3 \to R^3$ be the Linear transformation defined by

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

is w is in Range L?

Solution: Here $w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, if w is in range L then ~~there~~ there exist

$\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$ Such that $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w$

So

$$\begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Now we will Find determin-nant From 1st row.

$$-1\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} -2\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 0$$

$-1(1+1) \quad -2(1-2)$

$-2+2=0$

So the determinant is 0.
But if we operate row operation an augmented matrix (A/b):

$$(A/b) = \left[\begin{array}{ccc|c} -1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & -1 & 1 & -1 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} -1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & 3 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 + R_1 \\ R_3 + 2R_1 \end{array}$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} -1 & 2 & 0 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right] \quad R_3 - R_2$$

So Rank $= 2$ of $(A) \neq$ Rank of augmented matrix $(A/b) = 3$, the System of equation is not consistent. So we can Conclude that ~~there~~ there exist no $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = w \,, \quad \text{So } w = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

is Not in Range L.

4(b) Find all Constant a such that the vectors (a, 2) and (a, −2) are orthogonal.

Sol: We know that vectors are orthogonal if

$$\cos\theta = 0$$

$$(a,2)\cdot(a,-2) = 0$$

$$a(a,-2) + 2(a,-2) = 0$$

$$a^2 - \cancel{2a} + \cancel{2a} - 4 = 0$$

$$a^2 + (-4) = 0$$

$$a^2 = 4$$

So $\boxed{a = \pm 2}$

(5) (a) :- (A) Sol:- Closure Law

$u \in R \oplus \; u \in R\oplus \Rightarrow u \oplus v = uv \in R\oplus$

closed under $\oplus$

$u \in R\oplus \Rightarrow \; c \cdot v = u^c \in R\oplus$

$c \in f$

$\Rightarrow$ closed under $\odot$

$u \oplus v = uv = uv = v \oplus u$

$u \oplus (v \oplus w) = u(vw) = (uv)w$

$(v \oplus v) \oplus w$

$u \oplus 1 = u1 = u \Rightarrow 1$

$u \oplus \frac{1}{u} = 1u1 = 0$

hence $\frac{1}{u}$ is additive

$\Rightarrow a(bc)u = (u^{bc})^a = u^{abc}$

$(u^{ab})^c = c(ab)u$

$(a+b)u = u^{a+b} = u^a u^b = a \cdot u \oplus b \cdot v$

$a(u+v) = auv = (u \cdot v)^a = u^a \cdot v^a = u^a + v^a$

$= av$

$1 \cdot u = u^1 = u \quad \forall u \in R\oplus$

from the above it is proof that it is a vector space.

5 (b) = Sol: ~~Order~~

let $W_2 = \{ a_2t^2 + a_1t + a_0 \in P_2 : a_0 = 2 \}$

we have that $p(t) = t^2 + 2 \in W_2$

but $2p(t) = 2t^2 + 4 \notin W_2$ thus

W_2 is not Closed under scalar multiplication of vector and therefore it is not a Subspace of P_2.

5 (c) :- (a) $(a, b, 2)$

Let $\vec{u} = (a, b, 2)$ and $\vec{v} = (a', b', 2)$ are two vector in W.

then,

$$\vec{u} \oplus \vec{v} = (a, b, 2) \oplus (a', b', 2)$$
$$= (a+a', b+b', 4) \notin W$$

which is not in W, hence W is not a Subspace

(b) Let $\vec{u} = (a, b, c)$ and $\vec{v} (a', b', c')$ are two vector in W then

$a+b=c$, $a'+b'=c'$

(a) $\vec{u} \oplus \vec{v} = (a, b, c) \oplus (a', b', c')$
$= (a+a', b+b', c+c') \in W.$

(c) $k \odot v = k \odot (a, b, c)$

$= k \odot (a, b, a+b)$

$= (k \odot a, k \odot b, k \odot (a+b)) \in W$

$= (k \odot a, k \cdot b, k \odot c) \in W$

hence W is the Subspace of R^3.

(c) (a, b, c), where $c > 0$

for $S = [(a, b, c) \in R^3 : c > 0\}$

we note that $(1, 1, 1) \in S$ but

$(-1)(1, 1, 1) = (-1, -1, -1) \notin S$

because $-1 < 0$, hence S is not a Subspace of R_3.

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(6) Solution:

For the given space $V = \{\vec{x} = \{(x,y) \in R^2\}$ with the usual operation. if $\vec{x} = (x,y)$, $\vec{x}' = (x'.y')$

$\vec{x}'' = (x', y'$

(a) $\vec{x} + \vec{x}' = (x,y) + (x',y') = (x+x', y+y')$
$= (x'+x, y'+y) = \vec{x}' + \vec{x}$

(b) $\vec{x} + (\vec{x}' + \vec{x}'') = (x,y) + [(x',y') + (x'',y'')]$
$= (x,y) + (x'+x''+y'+y'')$

$((x+x')+x'')(y+y')+y'') + (x+x'-y+y') + (x'',y'')]$
$= (x,y) + (x'+x''+y'+y'')$
$= ((x+x')+x'')(y+y')+y'') = (x+x', y+y') + (x'' y'')$

$(\vec{x}' + \vec{x}) + \vec{x}'' =$

(c) we note that for ~~any~~ any x belong to V $x \in V$, $\vec{x} + \vec{0} = \vec{0} + \vec{x} = \vec{0}$ and $\vec{0} = (0,0)$ is in V.

(d) Let $(-\vec{x}) \in R^2$ Such that $\vec{x} + (-\vec{x})$
$= \vec{a}' \to (x,y) + (a,b) = (0,0)$

$a = -x$ and $b = -y$ but $x \in V$

$x \leq 0 \Rightarrow 0 = -(x) \geq 0$

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$\Rightarrow -\vec{x} \notin V$, So this property fail.

(e) $c(\vec{x}+\vec{x}') = c(x+x', y+y') =$

$(cx+cx', cy+cy') = (cx, cy)+(cx', cy')$

$= c\vec{x} + c\vec{x}'$

(f) $(c+d)\vec{x} = ((c+d)x, (c+d)y)$

$= (cx+dx, cy+dy)$

$= ((cx, cy)+(dx, dy)$

$= c\vec{x} + d\vec{x}$

(g) $c(d\vec{x}) = c(dx, dy)$

$= (c(dx), c(dy)) = ((cd)x, (cd)y)$

$= (cd)(x, y) = (cd)\vec{x}$

(h) $1\vec{x} = (1x, 1y)$

$= (x, y) = \vec{x}$

So this is not a vector space and the property (d) fails to hold.

$$x + y = 0$$

Q10 part A : $$y + z = 0$$

$$x + z = 0$$

Now form the Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

Now find reduce echeolon form of this

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_3 \times R_1 \\ R_3 \times R_2 \end{array}$$

So we have

$$x = -z$$

& $$y = -z$$

Where z is either 0 or 1 (Recall that negative of a bit is itself).

hence the set of solutions consists of all vectors in B^3 of the form

$$\begin{bmatrix} z \\ z \\ z \end{bmatrix}$$

and so there are exactly two solutions.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

A basis for the solution space is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Q10.

Part B:-

Since the rank of matrix A is 3, then the number of non-zero rows in the reduced row echelon form of matrix A is 3.

thus we conclude that only three row of matrix A are linearly independent and number of row of matrix A is 7.

Hence, the rows of the matrix A, are linearly independent

let A be 7×3 matrix, hence A has 7 rows vectors in R_3. because Rank = 3 just 3 rows vectors of A are linearly independent

thus the rows of A are linearly dependent.

(a) (11)

Finding Row echelon form of matrix

$$\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4/3 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4/3 \\ 0 & 2/3 \end{bmatrix} \quad R_2 - R_1$$

$$\begin{bmatrix} 1 & 4/3 \\ 0 & 1 \end{bmatrix} \quad \tfrac{3}{2} R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R_1 - \tfrac{4}{3} R_2$$

Now solve the matrix equation.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

equation has a unique solution.

i.e: null Space contains only a zero Vector.

(ii) Let $A = \begin{bmatrix} i & 1 & 0 \\ 1 & i & 0 \\ 0 & 0 & 1 \end{bmatrix}$

P(b)

then $(xI_3 - A) = \begin{bmatrix} x-i & -1 & 0 \\ -1 & x-i & 0 \\ 0 & 0 & x-1 \end{bmatrix}$

Let characteristic polynomial of A be $p(x)$, then

$p(x) = \det(xI_3 - A)$

$= (x-i)[(x-i)(x-1)-0]+1[-x+1-0]+0$

$= (x-i)(x-i)(x-1)-(x-1)$

$= (x-1)[(x-i)^2-1]$

$= (x-1)(x^2-2ix-1-1)$

$= (x-1)(x^2-2ix-2)$

$= (x-1)[x-(1+i)][x-(-1+i)]$

thus the required characteristics polynomial is $(x-1)[x-(1+i)][x-(-1+i)]$.

This implies that the roots of $p(x)$ are $1, (1+i), (-1+i)$.

hence the eigen values of A are $1, (1+i), (-1+i)$.

Part (a):

the Set V consisting of Single element 0 is a vector Space. it will not be Considered as a vector only in case if it doesn't satisfied following properties.

1) $0+0=0$
2) $c \cdot 0 = 0$

part (b):

Subgroup of groups:

A proper Subgroup of a group G is a Subgroup of H is a proper Subset of G i-e $H \neq G$
This is usually represented by $H < G$. if H is a Subgroup of G, then G is Sometimes called over group

Solution: Part (B)

$$S = \{\cos^2 t, \sin^2 t, \cos 2t\}$$

$$= \{\cos^2 t, -\cos^2 t, 2\cos^2 t - 1\}$$

$$= \{\cos^2 t, -\cos^2 t + 1, 2\cos^2 t - 1\}$$

So

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$$

$$c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \quad R_1 + R_2$$

So v_1 & v_2 form for lo.

So Dim $W = 2$.