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Final-term Examination

NAME: ASHFAQ AHMAD

Reg No: 19PWCSE1795

Section: B

Paper: PME

Total Pages: 13 (excluding title page)

Submitted to:

Dr. Safdar Merwat.

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Castelli

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Question NO : 1

Given:

$$S_v = \{-a, b, c, d\}$$

$$\text{Interval} = [1, 9]$$

$$z = v^3$$

Required

$$E[z] = ?$$

Sol

~~let~~ Suppose I Select

$a = 2$

$b = 3$

$c = 5$

$d = 8$

then

$$S_v = \{-2, 3, 5, 8\}$$

As $v = -2$ $\quad v^3 = -8$

$v = 3$ $\quad v^3 = 27$

$v = 5$ $\quad v^3 = 125$

$v = 8$ $\quad v^3 = 512$

So

$$S_z = \{-8, 27, 125, 512\}$$

As we know that

$$E[z] = \sum_{\text{All } j} z_j P_z(z_j) \qquad j = 1, 2, 3, 4$$

As

$$P_z(-8) = 1/4$$

$$P_z(27) = 1/4$$

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$P_z(125) = 1/4$

$P_z(512) = 1/4$

Now

$E[Z] = \cancel{-8} \times \frac{1}{4} + 27 \times \frac{1}{4} + 125 \times \frac{1}{4} + 512 \times \frac{1}{4}$

$E[Z] = -2 + 6.75 + 31.25 + 128$

$\boxed{E[Z] = 164}$ Ans

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Question No: 2:

Given:

a, b, c denote any three distinct integers in interval [1,9]

$S_4 = \{-a, b, c\}$

$P = V^2/R$

$R = 1/2$

Required:

(a) $E[P] = ?$

(b) $STD[P] = ?$

$p \quad +7+0$

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Sol

let Suppose I choose

$a = 2$
$b = 4$
$c = 6$

Then

$S_V = \{-2, 4, 6\}$

Part (a)

As $\Rightarrow V = -2$ Then $P = \frac{V^2}{R} = \frac{(-2)^2}{1/2}$

$P = 4 \times 2$

$\boxed{P = 8}$

$\Rightarrow V = 4$ Then $P = \frac{(4)^2}{1/2}$

$P = 16 \times 2$

$\boxed{P = 32}$

$\Rightarrow V = 6$ Then $P = \frac{(6)^2}{1/2}$

$P = 36 \times 2$

$\boxed{P = 72}$

Now

$S_P = \{8, 32, 72\}$

As we know that

$E[P] = \sum_{\text{all } j} P_j \cdot P_P(P_j)$ ——— (1)

$j = 1, 2, 3$

As $P_P(8) = 1/3$

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$$P_p(32) = 1/3$$

$$P_p(72) = 1/3$$

Now eq (1) became

$$E[P] = 8 \times 1/3 + 32 \times 1/3 + 72 \times 1/3$$

$$E[P] = 2.66 + 10.66 + 24$$

$$\boxed{E[P] = 37.32}$$ Ans

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Part → (b)

$$STD[P] = ?$$

For STD[P] first we find variance of P

$$\sigma_p^2 = ?$$

As we know that

$$\sigma_p^2 = E[P - E[P]]^2$$

(or)

$$\sigma_p^2 = \sum_{all\ j} [P_j - E[P]]^2 \cdot P_p(P_j) \quad \text{—} (1)$$

As E[P] we already finded in Part (a) i.e

$$E[P] = 37.32$$

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As

$$S_p = \{8, 32, 72\}$$

So

$$P_p(8) = 1/3$$

$$P_p(32) = 1/3$$

$$P_p(72) = 1/3$$

eq (1) became

$$\sigma^2 p = (8 - 37.32)^2 \times 1/3 + (32 - 37.32)^2 \times 1/3 + (72 - 37.32)^2 \times 1/3$$

$$\sigma^2 p = (-29.32)^2 \times 1/3 + (-5.32)^2 \times 1/3 + (34.68)^2 \times 1/3$$

$$\sigma^2 p = 859.66 \times 1/3 + 28.30 \times 1/3 + 1202.70 \times 1/3$$

$$\sigma^2 p = 186.55 + 9.43 + 400.9$$

$$\sigma^2 p = 596.88$$

Now Take Square root on both Side to get STD[P]

$$STD[P] = \sqrt{\sigma^2 p} = \sqrt{596.88}$$

$$\boxed{STD[P] = 24.43}$$ Ans

Required STD[P].

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Q NO: (4)

Given:

$$S_x = \{1, 2, 3, \dots\}$$

it is geometric R.V.

Required:

$$E[X^2] = ? \quad \text{2nd-moment.}$$

Sol

As we know that

$$E[X] = \sum_{\text{all } x} x \cdot P_X(x) \quad \text{—— (1)}$$

here $X = X^2$ & Sample Space is Infinite so limit will be from 1 to ∞.

eq (1) become

$$E[X^2] = \sum_{K=1}^{\infty} K^2 \cdot P_X(K)$$

According to geometric P. law

$$E[X^2] = \sum_{K=1}^{\infty} K^2 q^{K-1} \cdot p$$

$$= p \sum_{K=1}^{\infty} (K^2 + K - K) \cdot q^{K-1}$$

For simplicity we add & sub K.

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$$E[x^2] = P \sum_{K=1}^{\infty} \left[(K^2+K).q^{K-1} - Kq^{K-1}\right]$$

$$= P \sum_{K=1}^{\infty} (K^2+K) q^{K-1} - P \sum_{K=1}^{\infty} K.q^{K-1}$$

As $P \sum_{K=1}^{\infty} K.q^{K-1} = E[X]$ so

$$E[x^2] = P \sum_{K=1}^{\infty} (K^2+K) q^{K-1} - E[x]$$

For simplicity let

$$a = P \sum_{K=1}^{\infty} (K^2+K) q^{K-1}$$

Integrate w.r.t q

$$\int a\,dq = \int P \sum_{K=1}^{\infty} K(K+1) q^{K-1}$$

write constant term outside of $\int$

$$\int a\,dq = P \sum_{K=1}^{\infty} K(K+1) \int q^{K-1} dq$$

$$= P \sum_{K=1}^{\infty} K(K+1) \cdot \frac{q^K}{K}$$

$$\int a\,dq = P \sum_{K=1}^{\infty} (K+1) q^K$$

Now expand it

$$\int a\,dq = P(2q + 3q^2 + 4q^3 + \dots)$$

Now Integrate w.r.t q again.

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$$\iint a\,dq\,dq = \int p(2q + 3q^2 + 4q^3 + \cdots)\,dq$$

$$\iint a\,dq\,dq = p(q^2 + q^3 + q^4 + \cdots)$$

$$= pq^2(1 + q + q^2 + \cdots)$$

it is geometric series so

$$\iint a\,dq\,dq = pq^2\left(\frac{1}{1-q}\right)$$

Now for removing integration take derivative on both side w.r.t q

$$\frac{d}{dq}\iint a\,dq\,dq = \frac{d}{dq}\left(pq^2\left(\frac{1}{1-q}\right)\right)$$

$$\int a\,dq = \frac{p(2q)(1-q) - q^2(-1)}{(1-q)^2}$$

$$\int a\,dq = \frac{p(2q - 2q^2 + q^2)}{(1-q)^2}$$

$$\int a\,dq = \frac{p(2q - q^2)}{(1-q)^2}$$

Take derivative again w.r.t q

$$\frac{d}{dq}\int a\,dq = \frac{d}{dq}\left(\frac{p(2q - q^2)}{(1-q)^2}\right)$$

$$a = p\left[\frac{(2-2q)(1-2q+q^2) - (2q-q^2)(-2+2q)}{(1-q)^4}\right]$$

p.T.O

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$$a = \frac{p\left[2 - \cancel{4q} + \cancel{2q^2} - 2q + \cancel{4q^2} - \cancel{2q^3} + \cancel{4q} - \cancel{4q^2} - \cancel{2q^2} + \cancel{2q^3}\right]}{(1-q)^4}$$

$$a = \frac{p(2-2q)}{(1-q)} = \frac{2p(1-q)}{(1-q)^4}$$

$$a = \frac{2p}{(1-q)^3} \qquad 1-q = p$$

$$a = \frac{2\cancel{p}}{\cancel{p^3}}$$

$$\boxed{a = 2/p^2}$$

So

$$E[x^2] = a - E[x]$$

$$E[x^2] = \frac{2}{p^2} - E[x] \text{ ——— } (1)$$

Now we find $E[x]$

$$E[x] = \sum_{\text{all } x} x \cdot P_x(x)$$ it is geometric R.V so we will use g.p. law.

$$E[x] = 1 \times q^{1-1} p + 2 \times q^{2-1} p + 3 q^{3-1} p + 4 q^{4-1} p + \cdots$$

$$E[x] = p + 2qp + 3q^2 p + 4q^3 p + \cdots$$

Take Integration w.r.t q

$$\int E[x]\, dq = \int \left[p + 2qp + 3q^2 p + 4q^3 p + \cdots\right] dq$$

$$= \left[pq + q^2 p + q^3 p + q^4 p + \cdots\right]$$

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$\int E[x]\,dq = pq(1+q+q^2+q^3+\cdots)$

it is geometric series so

$\int E[x]\,dq = pq\left(\frac{1}{1-q}\right)$

Take derivative w.r.t q

$$\frac{d}{dq}\int E[x]\,dq = \frac{d}{dq}\left(\frac{pq}{1-q}\right)$$

$$E[x] = \frac{(1-q)\frac{d}{dq}pq - pq\frac{d}{dq}(1-q)}{(1-q)^2}$$

$$E[x] = \frac{p(1-q)+pq}{(1-q)^2} \qquad p=1-q$$

$$E[x] = \frac{p-\cancel{pq}+\cancel{pq}}{p^2}$$

$$E[x] = \cancel{p}/p^{\cancel{2}}$$

$$E[x] = 1/p$$

Now eq (1) become

$$E[x^2] = 2/p^2 - 1/p$$

$$\boxed{E[x^2] = \frac{2-p}{p^2}}$$

Required 2nd moment of geometric random Variable X.

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Question No: 3

Given:

No of Customers in 1 second = N

$\alpha = \lambda t$

$\lambda = \dfrac{\text{Customers}}{\text{second}}$ λ = arrival Rate

here arrival Rate = 2 Customers/per minute.

Required:

Probability = ?

i) more than one Customers in 30 seconds

ii) less than or equal to 1 Customer in 2 minute.

Sol

mean rate $= \dfrac{2\,\text{Customer}}{60s} = \dfrac{2}{60s}$

mean Rate $= \frac{1}{30}s$

Part (i)

$t = 30$ sec

$\alpha = \dfrac{1}{\cancel{30}} \times \cancel{30}$

$\alpha = 1$

$P[N>1] = 1 - P[N \le 1]$ — (1)

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$$P[N \leq 1] = \sum_{k=0}^{1} \frac{\alpha^{t}}{k!} e^{-\alpha}$$

$$P[N \leq 1] = \sum_{k=0}^{1} \frac{(1)^k}{k!} \times e^{-1}$$

$$= \frac{(1)^0}{0!} e^{-1} + \frac{(1)}{1!} e^{-1}$$

$$= \frac{1}{1} e^{-1} + \frac{1}{1} e^{-1}$$

$$= e^{-1} + e^{-1}$$

$$= \frac{1}{e} + \frac{1}{e}$$

$$P[N \leq 1] = \frac{1}{2.71} + \frac{1}{2.71}$$

$$\boxed{P[N \leq 1] = 0.728}$$

$$P[N > 1] = 1 - 0.738$$

$$= 0.262$$

Part ii, less or equal to 1 Customers in 2 minute.

$t = 2 \times 60 = 120 sec$

$\alpha = \frac{1}{\cancel{30}} \times \cancel{120}$

$\cancel{\alpha = 4}$

$\alpha = 4$

$$P[N \leq 1] = \sum_{k=0}^{1} \frac{\alpha^K}{K!} \times e^{-\alpha}$$

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$$P[N \le 1] = \frac{4\bar{e}^{4}}{0!} + \frac{4e^{-4}}{1!}$$

$$= \frac{(4)^0 e^{-4}}{1} + \frac{4e^{-4}}{1}$$

$$= (4)^0 e^{-4} + 4e^{-4}$$

$$= e^{-4} + 4e^{-4}$$

$$= 5e^{-4}$$

$$= 5/e^4$$

$$P[N \le 1] = \frac{5}{(2.731)^4}$$

$$= \frac{5}{53.14}$$

$$P[N < 1] = 0.09$$

$$\boxed{P[N \le 1] = 0.09}$$ Ans

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END OF FINAL TERM PAPER