Circuits and Systems 1 - Last Week Lecture

Problem 7.2-2 on page 294

If the capacitance of a capacitor is equal to $C = 0.125F$ and voltage across capacitor is $v(t) = 12\cos(2t + 30^o)V$, then compute the current across the capacitor?

Solution: The formula for computing current is as follows:

$$i = C\frac{dv}{dt}$$

Also, the following formula is stated:

$$\frac{d}{dt}A\cos(\omega t + \theta) = A\omega\cos(\omega t + (\theta + \frac{\pi}{2}))$$

Problem 7.2-2 on page 294

Let us first compute $\frac{dv}{dt}$ as follows:

$$\frac{d}{dt}A\cos(\omega t+\theta) = A\ \omega\ \cos\left(\omega t + (\theta + \frac{\pi}{2})\right)$$
$$\frac{d}{dt}12\cos(2t+30^o) = 12(2)\cos\left(2t + (30^o + \frac{\pi}{2})\right)$$
$$= 24\cos(2t+120^o)$$

When we plug-in the values, the following is obtained:

$$i = C\frac{dv}{dt}$$
$$= 0.125 \times 24\cos(2t+120^o)$$
$$= 3\cos(2t+120^o)$$

Problem 7.2-3 on page 295

Determine $v(t)$ for the circuit shown in below. Assume $v(0) = -1mV$.

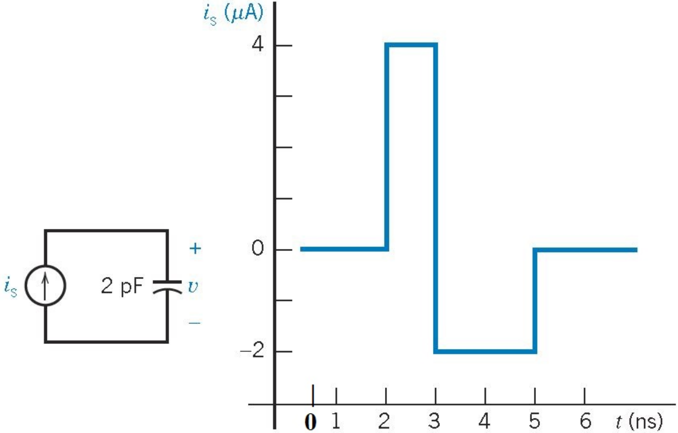

Figure: Problem 7.23 on page 295 of book

Problem 7.2-3 on page 295 - Solution

First, we would write equations of current from this graph (Note: the units of time are nano seconds and units of current are micro Amperes)

$$\boldsymbol{i_s(t) = \begin{cases} 0 & 0 < t \le 2 \\ 4 & 2 < t \le 3 \\ -2 & 3 < t \le 5 \\ 0 & t > 5 \end{cases}}$$

Problem 7.2-3 on page 295 - Solution

First, we would write equations of current from this graph (Note: the units of time are nano seconds and units of current are micro Amperes)

$$\boldsymbol{i_s(t) = \begin{cases} 0 & 0 < t \leq 2 \\ 4 & 2 < t \leq 3 \\ -2 & 3 < t \leq 5 \\ 0 & t > 5 \end{cases}}$$

The formula for voltage across capacitor is as follows:

$$\boldsymbol{v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + v(0)}$$

Problem 7.2-3 on page 295 - Solution

For $\mathbf{0 < t \leq 2}$ nano seconds, we obtain the following:

$$v(t) = \frac{1}{2 \times 10^{-12}} \int_0^t 0 d\tau + (-1mV) = -10^{-3}V$$

Problem 7.2-3 on page 295 - Solution

For $0 < t \leq 2$ nano seconds, we obtain the following:

$$v(t) = \frac{1}{2 \times 10^{-12}} \int_0^t 0 d\tau + (-1mV) = -10^{-3}V$$

For $2 < t \leq 3$ nano seconds, we obtain the following:

$$v(t) = \frac{1}{2 \times 10^{-12}} \int_{2\times10^{-9}}^t 4\times10^{-6} d\tau + (-10^{-3}) = -5\times10^{-3} + (2\times10^{6})t$$

Let us find the voltage at $t = 3$ nano seconds from the above equation:

$$v(3 \times 10^{-9}) = -5 \times 10^{-3} + (2 \times 10^{6})(3 \times 10^{-9}) = 10^{-3}$$

Problem 7.2-3 on page 295 - Solution

For $3 < t \leq 5$ nano seconds, we obtain the following:

$$v(t) = \frac{1}{2 \times 10^{-12}} \int_{3\times 10^{-9}}^{t} -2 \times 10^{-6} d\tau + (10^{-3}) = 4 \times 10^{-3} + (-10^{6})t$$

Let us find the voltage at $t = 5$ nano seconds from the above equation:

$$v(5 \times 10^{-9}) = 4 \times 10^{-3} + (-10^{6})(5 \times 10^{-9}) = -10^{-3}$$

For $t > 5$ nano seconds, we obtain the following:

$$v(t) = \frac{1}{2 \times 10^{-12}} \int_{5\times 10^{-9}}^{t} 0 d\tau + (-10^{-3}) = -10^{-3}t$$

Problem 7.2-3 on page 295 - Solution

Finally, we summarize the result here as follows:

$$\boldsymbol{v(t) = \begin{cases} -10^{-3} & 0 < t \leq 2 \\ -5 \times 10^{-3} + (2 \times 10^{6})t & 2 < t \leq 3 \\ 4 \times 10^{-3} - (10^{6})t & 3 < t \leq 5 \\ -10^{-3} & t > 5 \end{cases}}$$

Problem 7.2-3 on page 295 - Plot

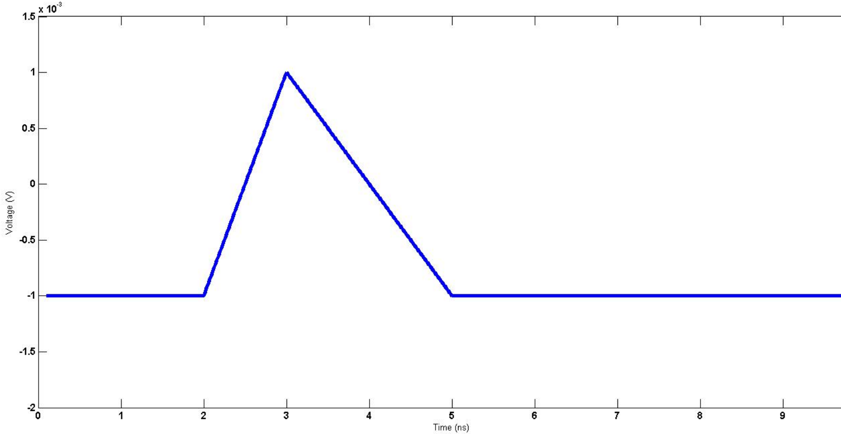

Figure: Plot of voltage

Problem 7.5-2 on page 300

A current $\boldsymbol{i(t) = 4te^{-t}A}$ flows through series combination of $\mathbf{20\Omega}$ resistor and $\boldsymbol{0.2H}$ inductor. Find the voltage across the combination (resistor plus inductor).

Problem 7.5-2 on page 300 - Solution

The formula is as follows:

$$\begin{aligned} v &= L\frac{di}{dt} + iR \\ &= 0.2 \times \frac{d}{dt}(4te^{-t}) + (4te^{-t}) \times 20 \\ &= 0.2 \times (4e^{-t} - 4te^{-t}) + 80te^{-t} \\ &= 0.8e^{-t} - 0.8te^{-t} + 80te^{-t} \\ &= 0.8e^{-t} + 79.2te^{-t}V \end{aligned}$$