Probability Methods in Engineering

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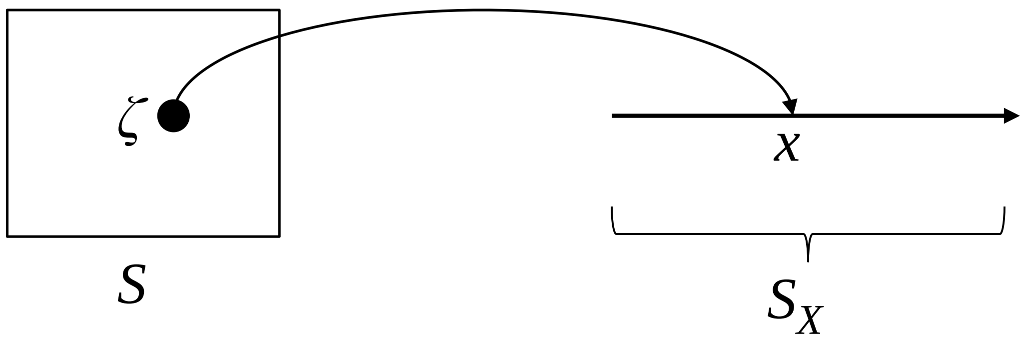
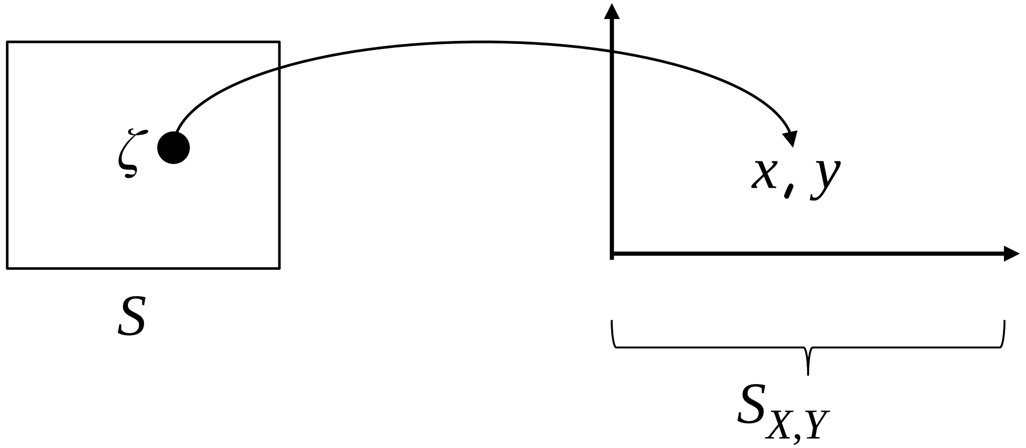
Lecture 24

Pairs of RVs

- RV X assigns number $X(\zeta) = x$, to each outcome ζ in the sample space of a random experiment



- RV X, Y assign numbers $X(\zeta) = x$ and $Y(\zeta) = y$, to each outcome ζ in the sample space of a random experiment

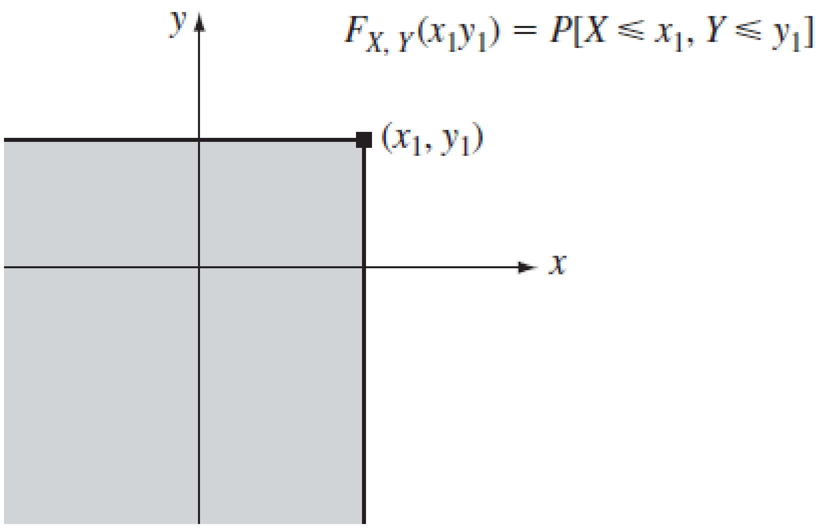


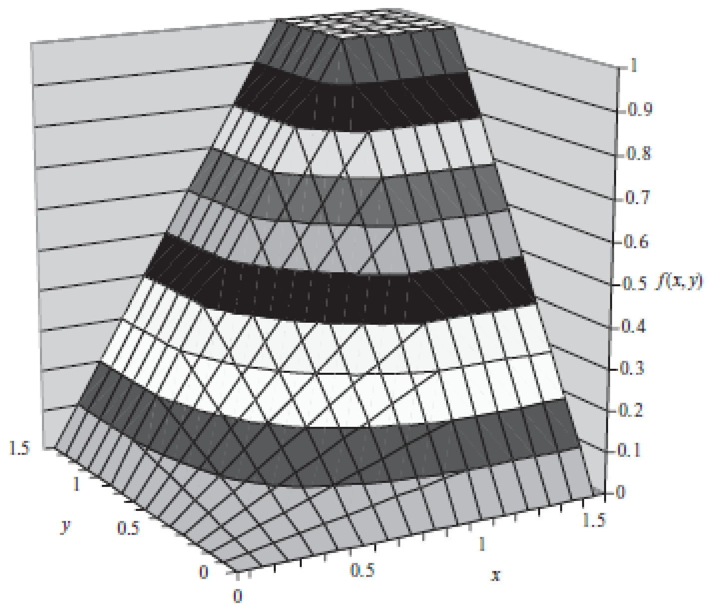
Pairs of RVs (cont.)

- Joint pmf and cdf

$$p_{X,Y}(x,y) = P[X = x, Y = y]$$

$$F_{X,Y}(x,y) = P[X \le x, Y \le y]$$





Example

- A pair of fair, four-sided dice are rolled in which outcome of the red die is denoted by RV X and the that of black by Y. Find the joint pmf and cdf of X and Y.

Marginal pmf

- Let X and Y be discrete random variables
- X and Y have joint pmf $p(x, y)$
- pmf X alone, called the marginal pmf of X, is defined as

$$p_X(x_j) = P[X = x_j]$$
$$= P[X = x_j, Y = \text{anything}]$$
$$= P[X = x_j, Y = y_1] + P[X = x_j, Y = y_2] + \ldots$$
$$= \sum_{k=1}^{\infty} p_{X,Y}(x_j, y_k)$$

- Marginal pmf of Y is defined as

$$p_Y(y_k) = P[Y = y_k]$$
$$= \sum_{j=1}^{\infty} p_{X,Y}(x_j, y_k)$$

CCDF

- Complementary Cumulative Distributive Function
- Sometimes useful to study opposite question
  - How often the random variable is above a particular level?
- Known as ccdf or tail distribution or exceedance

$$\bar{F}_X(x) = 1 - F_X(x)$$

$$\bar{F}_X(x) = P[X > x]$$

- A right-continuous function

Examples

- Find the ccdf of RV X where X is the number of dots facing up when a fair die is rolled.

Examples (cont.)

- Find the ccdf of exponential random variable.