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Reg No: 19PWCSE1795
Section: B
Paper: Linear Algebra:

Q1:

(a) Given:

let x_1 denote lower sulpher & x_2 denote higher sulpher for each ton.

Then

	L.S		H.S		
B.P	$5x_1$	+	$4x_2$	$= 3\times 60$	$= 180$
R.P	$4x_1$	+	$2x_2$	$= 2\times 60$	$= 120$

(or)

$$5x_1 + 4x_2 = 180 \quad — (1)$$

$$4x_1 + 2x_2 = 120 \quad — (2)$$

(2)

xing eq (ii) by (2) then
subtract from (i)

$$5x_1 + 4x_2 = 180$$
$$-8x_1 \pm 4x_2 = -240$$

$$-3x_1 = -60$$

$$x_1 = \frac{-60}{-3}$$

$\boxed{x_1 = 20}$ tons

put $x_1 = 20$ in eq (i)

$$5 \times 20 + 4x_2 = 180$$
$$4x_2 = 180 - 100$$

$$x_2 = \frac{80}{4}$$

$x_2 = 20$ tons

$\boxed{x_1 = x_2 = 20 \text{ tons}}$ Ans

<u>Unique Solution.</u>

P e T x O

Q1:

(b) Given

0 = OFF

1 = ON

$$A = \begin{bmatrix} ON & ON & OFF \\ OFF & ON & OFF \\ OFF & ON & ON \end{bmatrix}$$

According to given Conditions

B = ???

$$A + B = \begin{bmatrix} ON & ON & ON \\ ON & ON & ON \\ ON & ON & ON \end{bmatrix}$$

$$\begin{bmatrix} ON & ON & OFF \\ OFF & ON & OFF \\ OFF & ON & ON \end{bmatrix} + B = \begin{bmatrix} ON & ON & ON \\ ON & ON & ON \\ ON & ON & ON \end{bmatrix}$$

Replace ON by 1

& OF by 0 So

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$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} + B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1-1 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-0 \\ 1-0 & 1-1 & 1-1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ Ans

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P. T. O

Q2:

(a) Given:

$$S_1 = [18.95 \quad 14.75 \quad 8.98]$$
$$S_2 = [17.80 \quad 13.50 \quad 10.79]$$

(i)

In matrix form

$$\begin{array}{cccc} \text{item-1} & \text{item-2} & \text{item-3} & \\ & & & \end{array}$$
$$A = \begin{bmatrix} 18.95 & 14.75 & 8.98 \\ 17.80 & 13.50 & 10.79 \end{bmatrix} \begin{array}{l} \text{Store-A} \\ \text{Store-B} \end{array}$$

(ii) Price of each item reduce 20% so,

$$A = \begin{bmatrix} 18.95 \times 80\% & 14.75 \times 80\% & 8.98 \times 80\% \\ 17.80 \times 80\% & 13.50 \times 80\% & 10.79 \times 80\% \end{bmatrix}$$

$$A = \begin{bmatrix} 15.16 & 11.8 & 7.18 \\ 14.24 & 10.8 & 8.63 \end{bmatrix} \text{ Ans}$$

x X ——— x X ———

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Q2:

(b) Given:

$$P(x) = ax^2 + bx + c$$

Condition:

$$P(1) = f(1)$$
$$P'(1) = f'(1)$$
$$P''(1) = f''(1)$$

&

$$f(x) = xe^{x-1}$$

Soln

$$P(x) = ax^2 + bx + c \quad \text{—— (1)}$$
$$\& \; f(x) = xe^{x-1} \quad \text{—— (2)}$$

Put $x = 1$ in $P(x)$

$$\rightarrow P(1) = a(1)^2 + b(1) + c$$
$$P(1) = a + b + c$$

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Put $x=1$ in $f(x)$

$f(1) = 1.e^{1-1}$

$f(1) = 1$

But given,

$P(1) = f(1)$

So

$\boxed{a+b+c = 1}$ —— (i)

Differentiate $p(x)$ & $f(x)$ w.r.t x

$p'(x) = 2ax + b$

$p'(1) = 2a(1) + b$

$p'(1) = 2a + b$

and

$f'(x) = 1e^{x-1} + x(e^{x-1})\frac{d}{dx}(x-1)$

$f'(x) = e^{x-1} + xe^{x-1}(1)$

$f'(x) = e^{x-1} + x e^{x-1}$

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Now

$$f'(1) = e^{1-1} + 1e^{1-1}$$
$$= e^{0} + e^{0}$$
$$f'(1) = 1+1$$
$$f'(1) = 2$$

But According to given Condition,

$$p'(1) = p'(1)$$
$$\boxed{2a+b = 2} \longrightarrow (ii)$$

Now again differentiate $p'(x)$ & $f'(x)$

$$p''(x) = 2a$$
$$p''(1) = 2a$$

&

$$f''(x) = e^{x-1} + 1e^{x-1} + xe^{x-1}\frac{d}{dx}(x-1)$$
$$= e^{x-1} + e^{x-1} + xe^{x-1}(1)$$
$$= e^{x-1}(1+1+x)$$

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$f''(x) = e^{x-1}(2+x)$

$f''(1) = e^{1-1}(2+1)$

$f''(1) = e^{0}(2+1)$

$f''(1) = 3$

But According to given Condition

$p''(1) = f''(1)$

$\boxed{2a = 3}$ ——— (iii)

from (iii)

$\boxed{a = 3/2}$

Put "a" in (ii)

$2(3/2) + b = 2$

$3 + b = 2$

$b = 2 - 3$

$\boxed{b = -1}$

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put — ~~a~~ — &

Put "a" & "b" in eq (i)

$\frac{3}{2} - 1 + c = 1$

$\frac{3}{2} + c = 1 + 1$

$c = 2 - \frac{3}{2}$

$\boxed{c = \frac{1}{2}}$

therefore the quadratic polynomial that satisfy given condition is,

$$\boxed{P(x) = \frac{3}{2}x^2 - x + \frac{1}{2}}$$

Ans

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Q3:

(a) Given

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

L-U Factorization = ???

Sol

$$A = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 1 & 0 & 0.25 & -1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 4 & 2.2 & 0.3 & -2.4 \end{bmatrix} \quad b = \begin{bmatrix} -3 \\ -1.5 \\ 5.6 \\ 2.2 \end{bmatrix}$$

~ Row operation on matrix "A" to Obtain Upper Triangular matrix "U"

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ -2 & -1.1 & 0.25 & 6.2 \\ 1 & 2.2 & 0.3 & -2.4 \end{bmatrix} \begin{matrix} \sim R_2 \\ R_2 - (0.5) R_1 \end{matrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & -0.1 & 0.25 & 2.2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \sim R_4 \quad R_4 - 2R_1$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0.2 & 0.3 & 5.6 \end{bmatrix} \sim R_3 \quad R_3 - (0.2)R_2$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0.4 & 6 \end{bmatrix} \sim R_4 \quad R_4 - (0.4R_2)$$

$$= \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix} \sim R_4 \quad R_4 - 2R_3$$

Therefore the upper tingular matrix is,

$$U = \begin{bmatrix} 2 & 1 & 0 & -4 \\ 0 & -0.5 & 0.25 & 1 \\ 0 & 0 & 0.2 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

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And lower trianguler matrix is,

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix}$$

As $Lz = b$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ -1 & 0.2 & 1 & 0 \\ 2 & -0.4 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1.8 \\ 5.6 \\ 2.2 \end{bmatrix}$$

$\Rightarrow$

$$1z_1 = -3 \quad \text{—— (1)}$$

$$0.5z_1 + z_2 = -1.5 \quad \text{—— (2)}$$

$$-1z_1 + 0.2z_2 + z_3 = 5.6 \quad \text{—— (3)}$$

$$2z_1 - 0.4z_2 + 2z_3 + z_4 = 2.2 \quad \text{—— (4)}$$

$$\boxed{z_1 = -3}$$

Put value of z_1 in eq (2)

$$0.5(-3) + z_2 = -1.5$$

$$-1.5 + z_2 = -1.5$$

$$z_2 = -1.5 + 1.5$$

$\boxed{Z_2 = 0}$

Put z_1 & z_2 in eq ③

$$-1(-3) + 0.2(0) + z_3 = 5.6$$

$$+3 + 0 + z_3 = 5.6$$

$$\boxed{z_3 = 2.6}$$

Put z_1, z_2, z_3 in eq ④

$$2(-3) - 0.4(0) + 2(2.6) + z_4 = 2.2$$

$$-6 - 0 + 5.2 + z_4 = 2.2$$

$$-0.8 + z_4 = 2.2$$

$$\boxed{z_4 = 3}$$

therefore

$$\boxed{\begin{aligned} z_1 &= -3 \\ z_2 &= 0 \\ z_3 &= 2.6 \\ z_4 &= 3 \end{aligned}}$$ Ans

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Q3:

(b) Given,

$f : R^2 \longrightarrow R^2$

$f(v) = Av$

$A = \begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix}$

Sol

$A = \begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix}$

$K = 1/2$

"R" is a Unite Square

(0, 0) (0, 1) (1, 0) (1, 1)

$V = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Now

$f(v) = Av$

p ⊕ T ⊕ 0

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$$f(u) = \begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

for $K = 1/2$

$$f(v) = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} 0+0 & 0+0 & 1+0 & 1+0 \\ 0+0 & 0+1/2 & 0+0 & 0+1/2 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$$

Sketch:

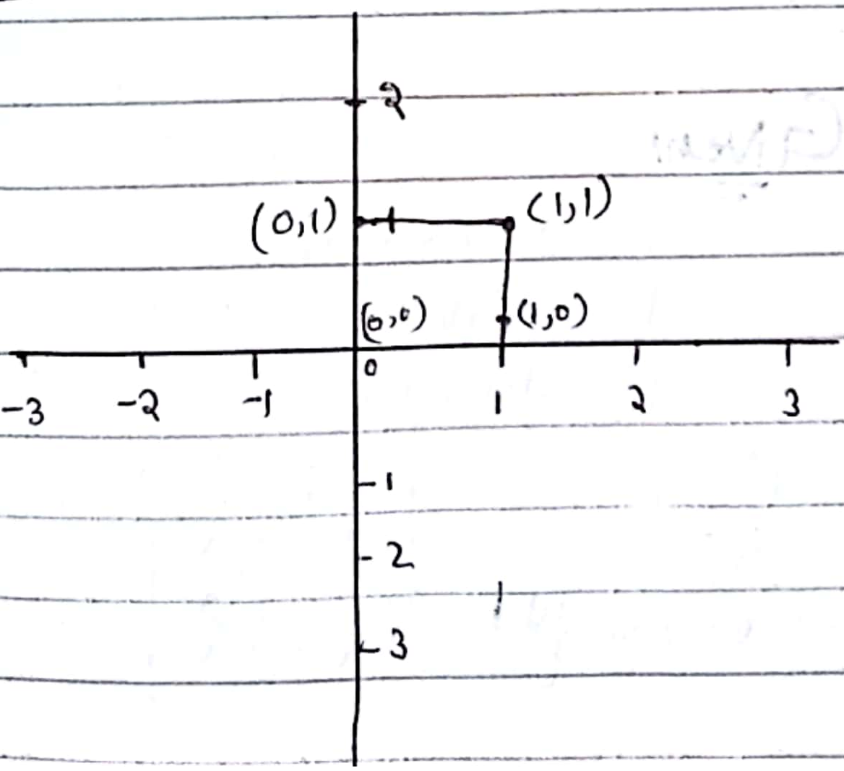
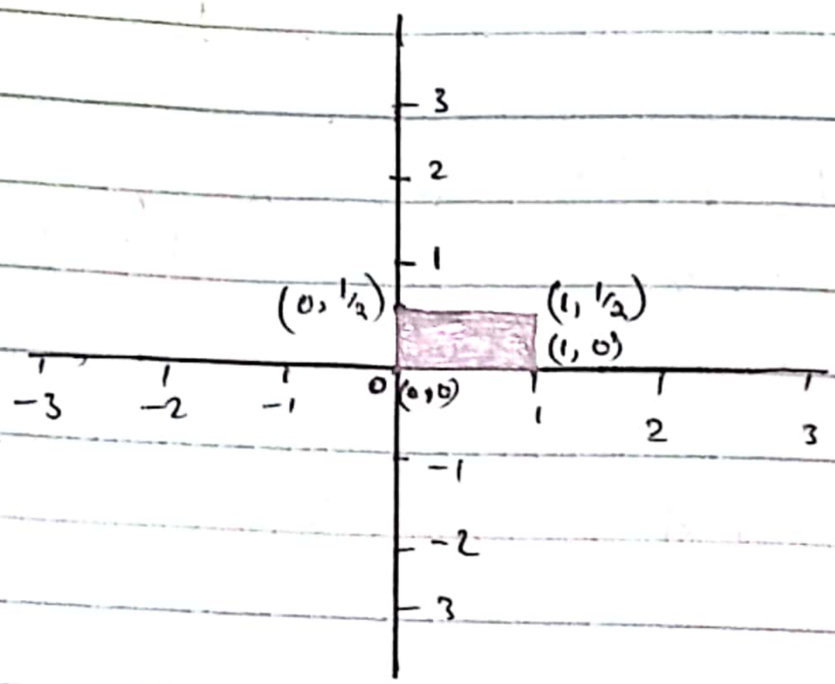


Image:



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Q4:

(a) Given:

P = Professional

F = Father

L = Laborer

$$\begin{array}{cc} & \text{Father's occupation} \\ \text{Son's occupation} & \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \end{array}$$

Sol

(i) probability that the grandchild of a professional will also be a professional

$$T = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$Tx = x'$$

$$x' = T.x = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 0.8 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.8 \times 0.8 + 0.3 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.5 \times 0.1 + 0.2 \times 0.1 \\ 0.1 \times 0.8 + 0.2 \times 0.1 + 0.6 \times 0.1 \end{bmatrix}$$

$$x' = \begin{bmatrix} 0.64 + 0.03 + 0.02 \\ 0.08 + 0.05 + 0.02 \\ 0.08 + 0.02 + 0.06 \end{bmatrix}$$

p + T + P 0

$$x' = \begin{bmatrix} 0.69 \\ 0.18 \\ 0.66 \end{bmatrix}$$

this is the probability that grand child of a professional will also be a professional.

(iii) In the long run, what proportional of the population will formers?

Markov's chain

$Tx = x$ $\qquad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$(T - I)x = 0$

$$\left(\begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) x = 0$$

$$\begin{bmatrix} -0.2 & 0.3 & 0.2 \\ 0.1 & -0.5 & 0.2 \\ 0.1 & 0.2 & -0.4 \end{bmatrix} x = 0$$

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Homogeneous System of linear eq.

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0.1 & 0.2 & -0.4 & 0 \end{array}\right] R_3 - R_2$$

$$\sim \left[\begin{array}{ccc|c} -0.2 & 0.3 & 0.2 & 0 \\ 0.1 & -0.5 & 0.2 & 0 \\ 0 & 0.7 & -0.6 & 0 \end{array}\right] R_1 - R_2$$

$$\sim \begin{bmatrix} -0.3 & 0.8 & 0 \\ 0.1 & -0.5 & 0.2 \\ 0 & 0.7 & -0.6 \end{bmatrix}$$

1st Row

$\Rightarrow -0.3a + 0.8b = 0$

$0.3a = 0.8b$

$0.3a = 0.8b$

$a = \frac{0.8}{0.3} b$

$$\boxed{a = \frac{8}{3} b}$$

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$\Rightarrow$ 3rd row

$0.7b - 0.6c = 0$

$0.7b = 0.6c$

$b = 6/7 \, c$

$$\boxed{c = 7/6 \, b}$$

Probability of vector is,

$a + b + c = 1$

$\frac{8}{3} b + b + \frac{7}{6} b = 1$

$$\boxed{b = 6/29 = 0.207}$$

$\Rightarrow a = \frac{8}{3}\left(6/29\right)$

$$\boxed{a = \frac{16}{29} = 0.551}$$

P 1 T 4 0

and

$$c = \frac{7}{\cancel{16}}\left(\frac{\cancel{16}}{29}\right)$$

$$\boxed{c = \frac{7}{29} = 0.241}$$

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 16/29 \\ 6/29 \\ 7/29 \end{bmatrix} = \begin{bmatrix} 0.551 \\ 0.207 \\ 0.241 \end{bmatrix}$$

Ans

xx ——— xx ——— xx

Q4:

(b) Sol:

$\Rightarrow$ Tugboat along negative x-axis $= \bar{OA} = -400$

$\Rightarrow$ Tugboat along negative y-axis $= \bar{OB} = -300$

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23.

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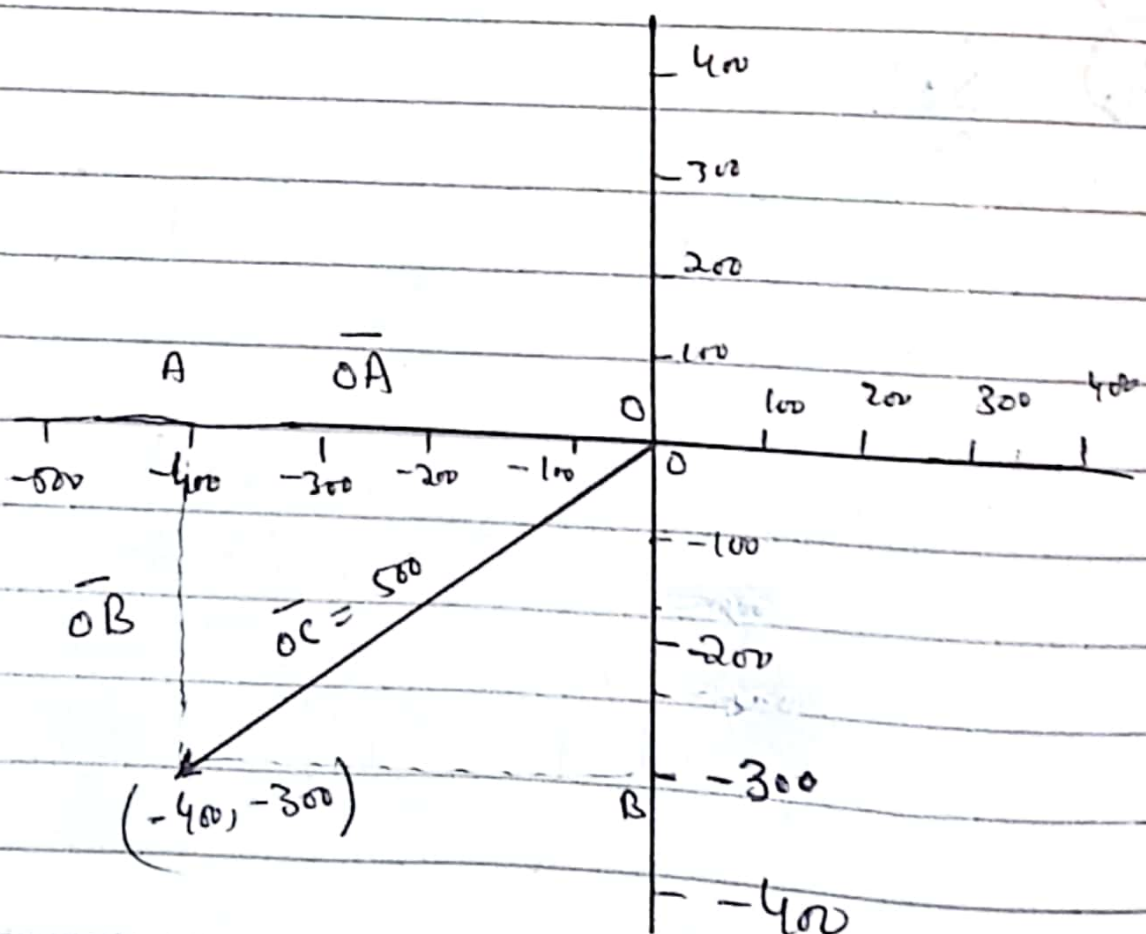
Using pethagoras Theorim.

$$(\overline{OC})^2 = (\overline{OA})^2 + (OB)^2$$

$$\overline{OC} = \sqrt{\overline{OA}^2 + \overline{OB}^2}$$

$$\overline{OC} = \sqrt{(-400)^2 + (-300)^2}$$

$$\boxed{\overline{OC} = 500}$$



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