University of engineering & technology Peshawar

Differential equation

Final-term exam

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Section: B

Reg No: 19PWCSE1795

Semester: 2nd

"On my honor, as a student of University of Engineering and Technology Peshawar, I have neither given nor received unauthorized assistance on this academic work"

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Paper : Calculus

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Part ⇒ 1

MCQs:

~~Answer:~~

1. → d
2. → d
3. → d
4. → a
5. → c
6. → d
7. → d
8. → b
9. → d
10. → d
11. → b

Part ⇒ 2

Q1: $y'' + y + x^2 y = 0$

Sol

let

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 \text{———} \quad (1)$$

xing "x^2" on both side

$$x^2 y = a_0 x^2 + a_1 x^3 + a_2 x^4 + a_3 x^5 + a_4 x^6 + a_5 x^7 + \cdots$$

Differentiate eq (1)

$$y' = 0 + a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + 6a_6 x^5 + 7a_7 x^6$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_3 x^3 + 30a_4 x^4 + 42a_7 x^5 + 56a_6 x^6 + 72a_7 x^7 + \cdots$$

equating the coffient to zero.

$$2a_2 + a_0 = 0 \Rightarrow \boxed{a_2 = \frac{-a_0}{2}} \quad (a)$$

$$6a_3 + a_1 = 0 \Rightarrow \boxed{a_3 = \frac{-a_1}{6}} \quad (b)$$

$$12a_4 + a_0 + a_2 = 0$$

$$\Rightarrow a_4 = -\frac{a_0}{12} - \frac{a_2}{12} = -\frac{a_0}{12} - \frac{1}{12}\left(-\frac{a_0}{2}\right)$$

from eq (a)

$$20a_5 + a_1 + a_3 = 0$$

$$\Rightarrow a_5 = -\frac{a_1}{20} - \frac{a_3}{20} = -\frac{a_1}{20} - \frac{1}{20}\left(-\frac{a_1}{6}\right)$$

from eq (b)

$$30a_6 + a_2 + a_4 = 0 \Rightarrow a_6 = -\frac{a_2}{30} - \frac{a_4}{30}$$

$$a_6 = -\frac{1}{30}\left(-\frac{a_0}{2}\right) - \frac{1}{30}\left(\frac{a_0}{24}\right)$$

$$42a_7 + a_3 + a_5 = 0$$

$$a_7 = -\frac{a_3}{42} - \frac{a_5}{42} = -\frac{1}{42}\left(-\frac{a_1}{6}\right) - \frac{1}{42}\left(-\frac{a_1}{24}\right)$$

$$\therefore y = a_0 + a_1 x + \left(-\frac{a_0}{2}\right)x^2 + \left(-\frac{a_1}{6}\right)x^3 + \left(-\frac{a_0}{24}\right)x^4 + \left(-\frac{a_1}{24}\right)x^5 + \frac{12a_0}{720}x^6 + \cdots$$

$$\cancel{y = a_0\left(1 - \frac{x^2}{2} - \frac{x^4}{24} + \frac{12x^6}{720}\right)}$$

$$y = a\left(1 - \frac{x^2}{2} - \frac{x^4}{24} + \frac{13x^6}{720} + \cdots\right)$$

$$+ a_1\left(x - \frac{x^3}{6} - \frac{x^5}{24} + \frac{5x^7}{1008} + \cdots\right)$$

Ans

Q2: Using laplace transffer. to solve given D.E

$$y'' + 3y' + 2.25y = 9t^3 + 64$$

$$y(0) = 0 \quad y'(0) = 31.5$$

Sol $y'' + 3y' + 2.25y = 9t^3 + 64$ — (1)

As we know that

$$L(f') = s\,L(f) - f(0)$$

and

$$L(f'') = s^2 L(f) - s f(0)$$

So eq (1) is

$$(s^2 Y + s - 1) + 2(sY + 1) + y = \frac{1}{s+1}$$

$$(s^2 y + s - 1) + 2(sy + 1) + y = \frac{1}{s+1}$$

Compare y - term

$$(s^2 + 2s + 1)y = (s+1)^2 y$$

$$= -\frac{1}{s+1} + \frac{1}{(s-1)^3}$$

$$\boxed{y = \left(\left(\frac{1}{2}t^2 - 1\right)e^{-t}\right)}$$

Ans

Q 3: $(y - x)\frac{dy}{dx} = (x+y)\frac{dy}{dx}$

Sol

$(y-x)y' = (x+y)y'$

$yy' - xy' = y'x + yy'$

$yy' - yy' = y'x + y'x$

$0 = y'x + y'x$

$2y'x = 0$

$y' = 0$

$\boxed{y = c}$ Ans

Q4: find orthogonal Trajectory

$$y = x - 1 + ce^{-x}$$

Sol

$$y = x - 1 + ce^{-x}$$

$$y' = 1 - ce^{-x}$$

Derivative of Normal curve

$$y' = -\frac{1}{1 - ce^{-x}} = \frac{1}{ce^{-x} - 1}$$

$$\int dy = \int \frac{1}{ce^{-x} - 1} dx$$

$$\int dy = \int \frac{e^{x}}{c - e^{x}} dx$$

$$y = -\ln(c - e^{x}) + \ln k$$

$$y = \ln k - \ln(c - e^{x})$$

$$\boxed{y = \ln\left(\frac{k}{c - e^{x}}\right)} \text{ Ans}$$