Circuits and Systems 1 - Week 6

Chapter 4 - Methods of Analysis of Resistive Circuits

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In this week inshaAllah, we will study two more techniques for circuit analysis. **WHY?**

Chapter 4 - Methods of Analysis of Resistive Circuits

In simple maths, you need n equations for n unknowns. If the circuit elements increase, then the number of equations also increase.

Obtain the equations for current at node a in this circuit below.

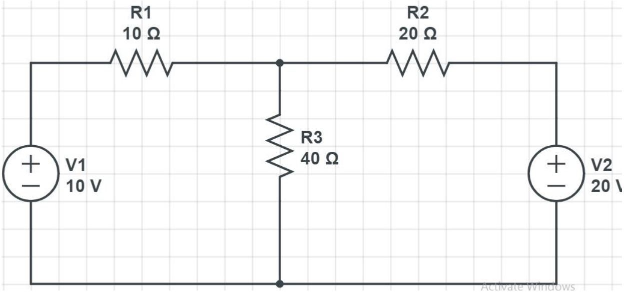

Figure: Example for motivation

Chapter 4 - Methods of Analysis of Resistive Circuits

When the number of circuit elements increases (or nodes or loops), the analysis using KCL and KVL become complex.

Two more/extra techniques which are used in combination with KCL and KVL.

1. Node voltage method or Nodal method
2. Mesh current method or Mesh method

Nodal Analysis

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So writing equations means applying KCL and writing equations for current.

Nodal Analysis Example 1

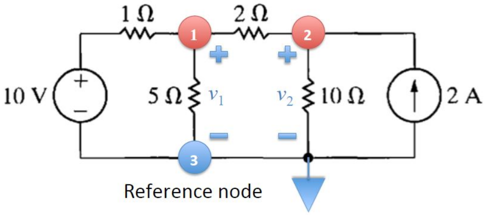

Figure: Example 1 using Nodal Analysis

At node 1:

$$\frac{10 - v_1}{1} = \frac{v_1 - v_2}{2} + \frac{v_1}{5} \tag{1}$$

At node 2:

$$\frac{v_1 - v_2}{2} + 2 = \frac{v_2}{10} \tag{2}$$

Nodal Analysis Example 2

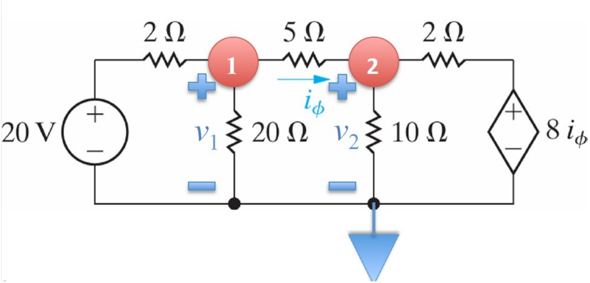

Figure: Example 2 using Nodal Analysis

Nodal Analysis Example 2

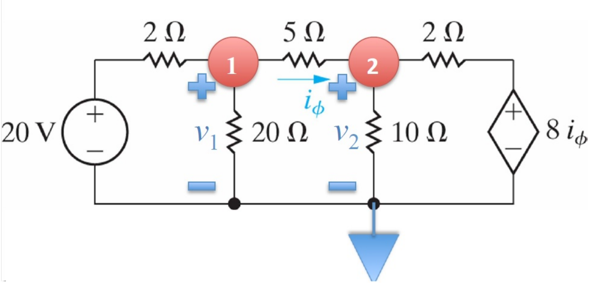

Figure: Example 2 using Nodal Analysis

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_1 - v_2}{5} = \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2}$$

$$i_\phi = \frac{v_1 - v_2}{5}$$

Nodal Analysis Tips

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Now lets proceed to the definition of super node.

Super Node

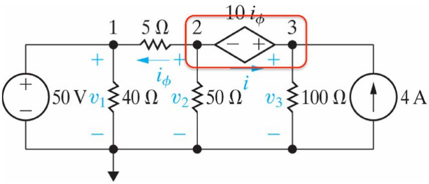

Figure: Example 3 - supernode concept

Obtain equations for node $\mathbf{2}$ and node $\mathbf{3}$

Super Node

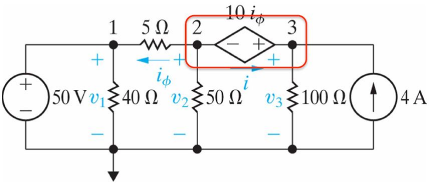

Figure: Example 3 - supernode concept

Obtain equations for node $\mathbf{2}$ and node $\mathbf{3}$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + i = 0$$

$$i + 4 = \frac{v_3}{100}$$

$$i_\phi = \frac{v_2 - v_1}{5}$$

$$v_1 = v_{1-v_{\text{ref}}} = 50$$

$$v_3 - v_2 = 10 i_\phi$$

Super Node

Now $v_1 = 50$ is known and i can be eliminated

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We can write the following:

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0 \tag{3}$$

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We can write the following:

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0 \tag{3}$$

Further we have the following 3 equations:

$$\frac{v_2 - 50}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0 \tag{4}$$

$$i_\phi = \frac{v_2 - 50}{5} \tag{5}$$

$$v_3 - v_2 = 10 i_\phi \tag{6}$$

Super Node Example 4.3.2 page 120

Determine v_a and v_b in this circuit shown below

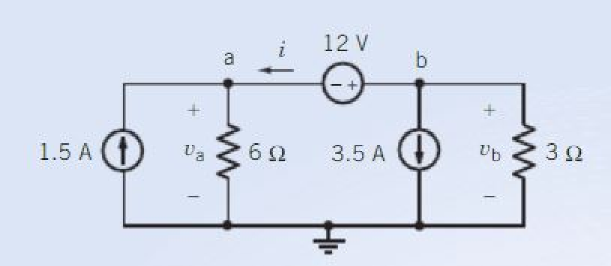

Figure: Example 4.3.2 on page 120 and 121

Super Node Example 4.3.2 page 120

KCL at node a:

$$\mathbf{1.5} + \boldsymbol{i} = \frac{\boldsymbol{v_a}}{\mathbf{6}}$$

KCL at node b:

$$\boldsymbol{i} + \mathbf{3.5} + \frac{\boldsymbol{v_b}}{\mathbf{3}} = \mathbf{0}$$

Super node:

$$\boldsymbol{v_b} - \boldsymbol{v_a} = \mathbf{12} \Longrightarrow \boldsymbol{v_b} = \boldsymbol{v_a} + \mathbf{12}$$

Super Node Example 4.3.2 page 120

KCL at node a:

$$1.5 + i = \frac{v_a}{6}$$

KCL at node b:

$$i + 3.5 + \frac{v_b}{3} = 0$$

Super node:

$$v_b - v_a = 12 \Longrightarrow v_b = v_a + 12$$

Substitute v_b is the above equation, we have the following 2 equations:

$$1.5 + i = \frac{v_a}{6}$$

$$i + 3.5 + \frac{v_a + 12}{3} = 0$$

Super Node Example 4.3.2 page 120

KCL at node a:

$$1.5 + i = \frac{v_a}{6}$$

KCL at node b:

$$i + 3.5 + \frac{v_b}{3} = 0$$

Super node:

$$v_b - v_a = 12 \implies v_b = v_a + 12$$

Substitute v_b is the above equation, we have the following 2 equations:

$$1.5 + i = \frac{v_a}{6}$$

$$i + 3.5 + \frac{v_a + 12}{3} = 0$$

Finally, we obtain $v_a = -12$ and $v_b = 0V$. Remember: Obtaining $0V$ is not an issue. If you can obtain negative voltages, then you can obtain $0V$ also - its zero voltage with reference to reference node.

Mesh Analysis Introduction

Mesh and loop are same terminologies.

Mesh Current Analysis involves obtaining equations in loops.

The important thing to consider is the direction of current flowing (and the direction of current opposing it).

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Which circuit laws we apply in loops or meshes? KCL or KVL?

Mesh Analysis Introduction

Problem 4.5.1 on page 155

Mesh Analysis Introduction

Mesh 1 Equation:

$$4i_1 + 18(i_1 - i_3) + 6(i_1 - i_2) = 0$$

Mesh 2 Equation:

$$6(i_2 - i_1) + 12(i_2 - i_3) + 30 = 0$$

Mesh 3 Equation:

$$18(i_3 - i_1) + 12(i_3 - i_2) - 42 = 0$$

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Mesh 1 Equation:

$$\mathbf{4}i_1 + \mathbf{18}(i_1 - i_3) + \mathbf{6}(i_1 - i_2) = \mathbf{0}$$

Mesh 2 Equation:

$$\mathbf{6}(i_2 - i_1) + \mathbf{12}(i_2 - i_3) + \mathbf{30} = \mathbf{0}$$

Mesh 3 Equation:

$$\mathbf{18}(i_3 - i_1) + \mathbf{12}(i_3 - i_2) - \mathbf{42} = \mathbf{0}$$

Now we have 3 equations and 3 unknowns. Further simplification gives us the following:

$$\begin{aligned} \mathbf{28}i_1 - \mathbf{6}i_2 - \mathbf{18}i_3 &= \mathbf{0} \\ -\mathbf{6}i_1 + \mathbf{18}i_2 - \mathbf{12}i_3 &= -\mathbf{30} \\ -\mathbf{18}i_1 - \mathbf{12}i_2 + \mathbf{30}i_3 &= \mathbf{42} \end{aligned} \tag{7}$$