Quiz No 2

Linear Algebra

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Section: B

Q1: (a)

Yes the Set V consisting of Single element 0 is a Vector Space, it will not be Considered as a Vector only in case if it does not Satisfies following Properties.

i) $0+0=0$

ii) $C.0=0$

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Q1:
(b)
Sol

let $W_3 = \{[a, b, c, d] \in R^4 : a > 0, b > 0\}$.
we have that $[1\ -1\ 0\ 0] \in W_3$
but $(-1)\ [1\ -1\ 0\ 0] \notin W_3$.

Thus W_3 is not closed under Scaler multiplication of vector and therefore it is not a Subspace of R^4.

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Q3:

(a) Given

$$\begin{aligned} x + y \quad &= 0 \\ y + z &= 0 \\ x \quad + z &= 0 \end{aligned}$$

Sol

Augmented matrix is,

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow \text{In Ecolon form}$$

Thus

$x = -z$
$y = -z$ where z is Either 1 or 0.

Hence the Solution Set Consist of all vectors in B^3 of the form $\begin{bmatrix} z \\ z \\ z \end{bmatrix}$ and So There are exactly two Solution $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

A basis for Solution Space is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

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Q3 (b)

Ans:

We need to check whether The rows of a 7×3 matrix are linearly Independent or linearly dependent whose Rank is 3.
Since the rank of matrix A is 3, then the no of nonzero rows in the reduced row Ecolon form of ~~three~~ Matrix A is 3.
thus we Conclude that only three rows of matrix A are linearly Independend and no of row of matrix A is 7.

Hence the rows of Matrix A are linearly dependent.

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Q4. (a)

Compute nullity of the matrix

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

Sol

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

the first step is to find reduced row Ecolon form.

$$\sim \begin{bmatrix} 1 & 4/3 \\ 1 & 2 \end{bmatrix} \frac{R_1}{3}$$

$$\sim \begin{bmatrix} 1 & 4/3 \\ 0 & 2/3 \end{bmatrix} R_2 - R_1$$

$$\sim \begin{bmatrix} 1 & 4/3 \\ 0 & 1 \end{bmatrix} \frac{3}{2} R_2$$

$$\sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} R_1 - \frac{4}{3} R_2$$

Now Solve matrix equation.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Since this equation has a unique Solution, then the null space contain only a zero vector.

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Q4: (b)

find all the Eigen-values and associated Eigen vector of the matrix.

$$A = \begin{bmatrix} i & 1 & 0 \\ 1 & i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol

$$A = \begin{bmatrix} i & 1 & 0 \\ 1 & i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Start from forming a new matrix by subtracting λ from the diagonal entries of the given matrix:

$$\begin{bmatrix} -\lambda+i & 1 & 0 \\ 1 & -\lambda+i & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$$\begin{vmatrix} -\lambda+i & 1 & 0 \\ 1 & -\lambda+i & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -\lambda+i & 1 \\ 1 & -\lambda+i \end{vmatrix}$$

$$= (1-\lambda)\left[(-\lambda+i)\cdot(-\lambda+i) - 1\right.$$

$$= -\lambda^3 + \lambda^2 + 2i\lambda^2 + 2\lambda + 2i\lambda - 2$$

This is a characteristic polynomial. Solve the equation

$$-\lambda^3 + \lambda^2 + 2i\lambda^2 + 2\lambda + 2i\lambda - 2 = 0$$

by solving this we get.

$\lambda_1 = 1+i$
$\lambda_2 = -1+i$
$\lambda_3 = 1$

These are Eigenvalues.

→ Next find the Eigen vectors.

(a) $\lambda = 1+i$

$$\begin{bmatrix} -\lambda+i & 1 & 0 \\ 1 & -\lambda+i & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -i \end{bmatrix}$$

Now perform row operation to obtain rref.

$$\sim \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -i \end{bmatrix} - R_1$$

$$\sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} - R_2 - R_1$$

Now Solve matrix equation.

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

if we take $V_2 = t$ then $V_1 = t$ $V_3 = 0$

Therefore $$V = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} t$$

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(b) $\lambda = -1+i$

$$\begin{bmatrix} -\lambda+i & 1 & 0 \\ 1 & -\lambda+i & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2-i \end{bmatrix}$$

perform row operation to obtain rref

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2-i \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2+R_1$$

by using calculator

Now Solve matrix for equation

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

if we take $v_2 = t$ then $v_1 = -t$, $v_2 = t$
$v_3 = 0$

therefore

$$v = \begin{bmatrix} -t \\ t \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} t$$

(c) $\lambda = 1$

$$\begin{bmatrix} -1+i & 1 & 0 \\ 1 & -1+i & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

by using calculator.

Now Solve matrix equation.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

if we take $v_3 = t$ Then $v_1 = 0$ $v_2 = 0$
$v_3 = t$

Therefore

$$v = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} t$$

Answers:

☆ Eigenvalue : $1+i = 1.0 + 1.0i$,

eigenvector : $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

☆ Eigenvalue: $-1+i = -1.0 + 1.0i$

eigenvector: $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

☆ Eigenvalue: 1

eigenvector : $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

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Q2 (a)

Sol

$$V_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \& \quad V_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

let $V = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ be any vector in B^3

where a, b, and c are any of the bits 0 or 1.

we must determine if there are scalers c_1, c_2 and c_3 (which are bits 0 or 1) such that

$$c_1 + c_2 = a$$
$$c_1 \quad + c_3 = b$$
$$c_2 + c_3 = c$$

we form the augmented matrix and obtaind its reduced row echelon form,

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & a+a+b \\ 0 & 1 & 1 & a+b \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

the system is inconsistent if the choice of the bits for a, b and c are such that $a+b+c=0$.

for example if $V = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ then the system is inconsistent.

hence V_1, V_2 & V_3 donot span V.

Q2 (b)

$S = \{\cos^2 t, \sin^2 t, \cos 2t\}$.

find basis for the sub-space $W = \text{Span}\, S$.

What is $\dim W = ?$

Sol

$S = \{\cos^2 t, \sin^2 t, \cos 2t\}$

$= \{\cos^2 t, 1-\cos^2 t, 2\cos^2 t - 1\}$

So

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$

So v_1 & v_2 form basis for W. Dim $W = 2$.

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The END