University of engineering & technology Peshawar

Linear algebra

Quizz#01

Fall 2020

Submitted by: **Ashfaq Ahmad**

Section: **B**

Reg No: **19PWCSE1795**

Semester: **3rd**

"On my honor, as a student of University of Engineering and Technology Peshawar, I have neither given nor received unauthorized assistance on this academic work"

Student signature: ____________________

Submitted to:
Prf: jamal nasir

Department Of Computer System Engineering

Quizz No 1 Linear Algebra.

Name: ASHFAQ AHMAD

Reg No: 19PW CSE 1795

Section: B

Semester: 3rd.

—— xx —— xx —— xx ——

Q 1: Given:

$$2x - y = 5$$
$$4x - 2y = t$$

Sol

(a) $[AB] = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 4 & -2 & t \end{array}\right]$

$\Rightarrow R_2 = R_2 - 2R_1$

So

$[AB] = \left[\begin{array}{cc|c} 2 & -1 & 5 \\ 0 & 0 & t-10 \end{array}\right]$

$\Rightarrow t - 10 = 0$

$t = 10$

Rank of A = Rank of $(AB) \leq n = 1$

$\therefore$ the System has Infinite many Solution.

(b) The System has no Solution only if

$p \neq T \neq 0$

$t - 10 \neq 0$

$t \neq 0$

Rank of A $\neq$ Rank of AB

System has no Solution.

(3) $t \neq 10$

$(-\infty, 10) \cup (10, \infty)$

— xx —— xx —— xx —— xx x

Q2: let R be Rectangle with vertexs $(1,1)(1,4)$. $(3,1)$ & $(3,4)$. let f be the shear in the x-direction with $k=3$. find & sketch Image of R

Sol

From given Condition we can say that

$f: R^2 \longrightarrow R^2$

$f(v) = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} v$

$\because k = 3$

$f(v) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} v$

Using Rectangle verteics

⇒ Image of $(1,1)$ is,

$$f(1,1) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

⇒ Image of vertex $(1,4)$ is,

$$f(1,4) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 13 \\ 4 \end{bmatrix}$$

⇒ Image of $(3,1)$ is,

$$f(3,1) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

⇒ Image of $(3,4)$ is,

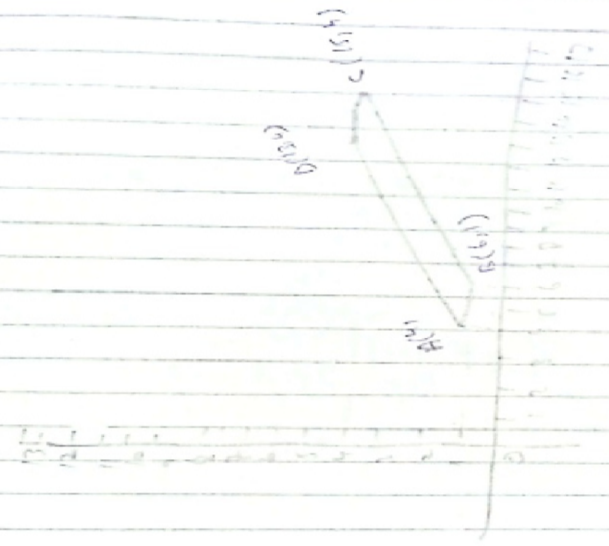
$$f(3,4) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \end{bmatrix}$$

Image of Vertex $(4,1)$, $(13,4)$, $(6,1)$, $(15,4)$

So we can draw the sketch (rectangle of graph)

P.T.O

page (4)



Q3: Given

~~$2x+3y(a^2-1)z = a+1$~~

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2-1)z = a+1$

(a)

Write an augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & \end{array}\right]$$

Now Reduce to echelon form by row operations

$R_1 = R_1 - R_3$, $R_3 = R_3 - R_2$

we get

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right]$$

Note that if in 3rd row of Augmented matrix a^2-3 then

$a = \pm\sqrt{3}$ and then

$0 = \pm\sqrt{3}$ which is not Possible

thus if $a = \pm\sqrt{3}$ then given System has no Solution.

(b) for unique Solution

let $a^2 - 3 \neq 0$ that is $a \neq \pm\sqrt{3}$ then we convert Matrix into row echelon form we apply $R_3 = \frac{1}{a^2-3} \times R_3$ formula.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{a-4}{a^2-3} \end{array}\right]$$

Now $R_1 = R_1 - R_3$ we get

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{a-4}{a^2-3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{a-4}{a^2-3} \end{array}\right]$$

P T O

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{a^2-a+1}{a^2-3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{a-4}{a^2-3} \end{array}\right]$$

So Solution to given System is,

$x = \frac{a^2-a+1}{a^2-3}$

$y = 1$

$z = \frac{a-4}{a^2-3}$

$\therefore$ System has unique Solution.

(c) the value of a^2-3 can either be zero or non-zero therefore when $a^2-3=0$. System has no Solution. when $a^2-3 \neq 0$ System has unique Solution.

Hence

No, Infinity many Solution exist.

~~~~~ X x x ~~~~~
~~~~~

Q3 (b):

$$T = \begin{bmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 0 & 0.5 & 0.7 \end{bmatrix}$$

Sol

We have need to find Same value of T.

$$\begin{bmatrix} 0.06 & 0+0.018+0.03 & 0+0.018+0.042 \\ 0.3 & 0.06+0.072+0.15 & 0+0.072+0.21 \\ 0.64 & 0.14+0.21+0.32 & 0+0.21+0.48 \end{bmatrix}$$

$$\begin{bmatrix} 0.06 & 0.048 & 0.06 \\ 0.3 & 0.282 & 0.282 \\ 0.64 & 0.67 & 0.658 \end{bmatrix}$$

Since all the entries of T^5 are Possible. Thus we set T is regular.

$$TU = U$$

$$(1-T)U = 0$$

In this case we have to Solve the homogeneous System

P - P T P 0

$$\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] - \begin{pmatrix} 0 & 0.2 & 0.0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{pmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -0.2 & 0.0 \\ 0 & 0.7 & -0.3 \\ -1 & 0.5 & 0.3 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = 0$$

$M_1 = 0.2U_2 = 0$

$0.7M_2 - 0.3U_3 = 0$

$1M_1 - 0.5M_2 + 0.3M_3 = 0$

Therefore we set

$M_1 = 0.2U_2$

$= \frac{2}{10} U_2$

thus

we get

$M_1 = \frac{1}{5} U_2$

and $0.3U_2 = 0.7U_2$

$M_1 = \frac{0.7}{0.3} U_3$

$M_1 = \frac{7}{3} M_3$

Since $M_1 + M_2 + M_3 = 1$ therefore we get

$$\frac{1}{5} M_2 + M_3 + \frac{7}{3} = 1$$

$$\cancel{3M_2 + M_3 + 7}$$

$$\frac{3M_2 + 15M_2 + 35M_2}{15} = 1$$

$$\frac{53M_2}{15} = 1$$

$$M_2 = \frac{15}{53}$$

Now

$$M_1 = \frac{1}{5} M_2$$

$$M_1 = \frac{1}{\cancel{5}} \times \frac{\cancel{15}}{53}$$

$$M_1 = \frac{3}{53}$$

Now

$$M_1 = \frac{7}{3} M_3$$

$$M_3 = \frac{3}{7} M_1$$

$$M_3 = \frac{3}{7}\left(\frac{3}{53}\right) = \frac{9}{371}$$

Hence

$$U = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 3/53 \\ 15/53 \\ 9/371 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.057 \\ 0.283 \\ 0.024 \end{bmatrix}$$ Ans

Q4. (g) ~~step~~ evaluate

$$\det\left(\begin{bmatrix} \lambda-1 & -1 & -2 \\ 0 & \lambda-2 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}\right)$$

Sol

$$0\cdot(-1)^{3+1}\begin{vmatrix} -1 & -2 \\ \lambda-2 & 2 \end{vmatrix} + 0\cdot(-1)^{3+2}\begin{vmatrix} \lambda-1 & -2 \\ 0 & \lambda-3 \end{vmatrix}$$

$$+(\lambda-3)\cdot(-1)^{3+3}\begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{vmatrix}$$

$$= (\lambda-3)\begin{vmatrix} \lambda & -1 \\ 0 & \lambda-2 \end{vmatrix}$$

the determinant of 2x2 matrix is

$$\begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2) - (-1)\cdot(0)$$

$$= (\lambda-2)(\lambda-1)$$

Now

$$(\lambda-3)((\lambda-2)(\lambda-1))$$

$$= (\lambda-3)(\lambda-2)(\lambda-1)$$

Hence

$$\boxed{(\lambda-3)(\lambda-2)(\lambda-1)}$$ Ans

— xx — xx x

Q4 (b)

Solution

Re writing in matrix form

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{array}\right]$$

P T O

$R_2 - 2R_1 \rightarrow R_2$ &

$R_3 - 3R_1 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -5 & -5 & -10 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

Now $R_2/-5 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 1 & 2 \\ 0 & -6 & -10 & -24 \end{array}\right]$$

$R_1 - 2R_2 \rightarrow R_1$ &

$R_3 + 6R_2 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & -12 \end{array}\right]$$

$R_3/-4 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Page (14)

$R_1 - 1R_3 \longrightarrow R_1$ &

$R_2 - 1R_3 \longrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= -1 \\ x_3 &= 3 \end{aligned}}$$ Solution Set

—— xx —— xx ✓

END UP

The end